Probability

Solutions Key

ARE YOU READY?

1. ratio
2. even number
3. percent
4. composite number
5. odd number
6. $\frac{6}{9} = \frac{6 \div 3}{9 \div 3} = \frac{2}{3}$
7. $\frac{12}{15} = \frac{12 \div 3}{15 \div 3} = \frac{4}{5}$
8. $\frac{8}{10} = \frac{8 \div 2}{10 \div 2} = \frac{4}{5}$
9. $\frac{20}{24} = \frac{20 \div 4}{24 \div 4} = \frac{5}{6}$
10. $\frac{2}{4} = \frac{2 \div 2}{4 \div 2} = \frac{1}{2}$
11. $\frac{7}{35} = \frac{7 \div 7}{35 \div 7} = \frac{1}{5}$
12. $\frac{12}{22} = \frac{12 \div 2}{22 \div 2} = \frac{6}{11}$
13. $\frac{72}{81} = \frac{72 \div 9}{81 \div 9} = \frac{8}{9}$
14. $\frac{3}{5} = 3 \div 5 = 0.6$
15. $\frac{9}{20} = 9 \div 20 = 0.45$
16. $\frac{57}{100} = 57 \div 100 = 0.57$
17. $\frac{12}{25} = 12 \div 25 = 0.48$
18. $\frac{3}{25} = 3 \div 25 = 0.12$
19. $\frac{1}{2} = 1 \div 2 = 0.5$
20. $\frac{7}{10} = 7 \div 10 = 0.7$
21. $\frac{9}{5} = 9 \div 5 = 1.8$
22. 0.14 = 14%
23. 0.08 = 8%
24. 0.75 = 75%
25. 0.38 = 38%
26. 0.27 = 27%
27. 1.89 = 189%
28. 0.234 = 23.4%
29. 0.0025 = 0.25%
30. $\frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$
31. $\frac{2}{3} \cdot \frac{3}{5} = \frac{2}{1} \cdot \frac{1}{5} = \frac{2}{5}$
32. $\frac{3}{10} \cdot \frac{1}{2} = \frac{3}{20}$
33. $\frac{5}{6} \cdot \frac{3}{4} = \frac{5}{2} \cdot \frac{1}{4} = \frac{5}{8}$
34. $\frac{5}{14} \cdot \frac{7}{17} = \frac{5}{2} \cdot \frac{1}{17} = \frac{5}{34}$
35. $-\frac{1}{8} \cdot \frac{3}{4} = -\frac{3}{64}$
36. $-\frac{2}{15} \cdot \left(\frac{-2}{3}\right) = \frac{4}{45}$
37. $\frac{1}{4} \cdot \left(\frac{-1}{6}\right) = -\frac{1}{24}$

LESSON 1

Think and Discuss

1. Possible answer: 0% probability—pulling a red marble from a bag containing green and blue marbles; 100% probability—pulling a white marble from a bag containing only white marbles.
2. Possible answer: Spinning a spinner and rolling a number cube
3. Possible answer: getting homework or not getting homework

Exercises

1. There is only one number greater than 5 on a number cube, so it is unlikely.
2. There are no blue marbles in the bag; therefore it is impossible to pull out a blue marble.
3. $$\begin{aligned} P(\text{event}) + P(\text{complement}) &= 1 \\ P(\text{pink}) + P(\text{not pink}) &= 1 \\ \frac{1}{6} + P(\text{not pink}) &= 1 \\ -\frac{1}{6} \qquad\qquad & -\frac{1}{6} \\ P(\text{not pink}) &= \frac{5}{6} \end{aligned}$$
4. Since it is Saturday and she does not have to work, it is likely that she will sleep in.
5. Since all the cards are red or pink, it is certain that you will pick a red or pink card.
6. Since there are only two outcomes, heads and tails, it is as likely as not that you will flip a tail.
7. Since there is only one way to roll a 6 on each roll, it is unlikely to roll a 6 five times in a roll.
8. $$\begin{aligned} P(\text{event}) + P(\text{complement}) &= 1 \\ P(5 \text{ or } 6) + P(\text{not 5 or 6}) &= 1 \\ \frac{1}{3} + P(\text{not 5 or 6}) &= 1 \\ -\frac{1}{3} \qquad\qquad & -\frac{1}{3} \\ P(\text{not 5 or 6}) &= \frac{2}{3} \end{aligned}$$
9. $$\begin{aligned} P(\text{event}) + P(\text{complement}) &= 1 \\ P(\text{green}) + P(\text{red or blue}) &= 1 \\ \frac{3}{5} + P(\text{red or blue}) &= 1 \\ -\frac{3}{5} \qquad\qquad & -\frac{3}{5} \\ P(\text{red or blue}) &= \frac{2}{5} \end{aligned}$$
10. Since Tim rarely watches more than 30 minutes of TV, it is not likely that he is watching TV at 5:00 P.M.
11. Since there is the same number of red and black checkers, it is as likely as not that you will pull a red checker.
12. There are no white checkers in the bag, so it is impossible to draw a white checker.
13. Since there are only red and black checkers in the bag, it is certain you will draw a red or black checker.
14. Since there is the same number of red and black checkers, it is as likely as not that you will pull a black checker.
15. Since it is cold and rainy, it is not likely Luka will go for a jog.
16. The plant would be more likely to have purple flowers because there are 500 more plants with purple flowers than plants with white flowers.
17. **a.** It is very likely, since most fish are bony fish.
    b. It is impossible, since a shark is not a bony fish.
18. It is not likely, since carbon dioxide levels increased steadily between 1970 and 2000.
19. Possible answer: Rolling an odd number on a number cube is as likely as not.

20. Possible answer: There are just as many ways that the event can happen, as there are ways that the event cannot happen.

21. Since there are now 8 red and 8 blue marbles, the events are equally likely.

22. D; There are 5 outcomes that are even numbers: 2, 4, 6, 8, and 10.

$$P(\text{even number}) = \frac{\text{number of ways event can occur}}{\text{total possible outcomes}} = \frac{5}{5} = 1\text{, or } 100\%$$

23.

$$P(\text{event}) + P(\text{complement}) = 1$$
$$P(1, 2, \text{ or } 3) + P(\text{not } 1, 2, \text{ or } 3) = 1$$
$$\frac{1}{2} + P(\text{not } 1, 2, \text{ or } 3) = 1$$
$$\frac{1}{2} - \frac{1}{2} + P(\text{not } 1, 2, \text{ or } 3) = 1 - \frac{1}{2}$$
$$P(\text{not } 1, 2, \text{ or } 3) = \frac{1}{2}$$

24. Possible answer: It is likely taht a test will be given over this chapter.

LESSON 2

Think and Discuss

1. Possible answer: Estimating the probability that people will eat lunch between 1:00 P.M. and 2:00 P.M. can be based on past experiences, which is experimental probability.

2. Possible answer: You can use experimental probability to forecast the weather by observing past weather patterns and their results.

Exercises

1.

$$P \approx \frac{\text{number of times an event occurs}}{\text{total number of trials}}$$
$$P(\text{hit}) \approx \frac{\text{number of hits made}}{\text{number of hits attempted}} \approx \frac{14}{20} \approx \frac{7}{10} = 0.7 = 70\%$$

The experimental probability that Teri will hit the target on her next try is approximately 70%.

2. a. $P(\text{vote for}) \approx \frac{\text{number to vote for bill}}{\text{total number surveyed}} \approx \frac{65}{75} \approx \frac{13}{15}$

The experimental probability that the next person surveyed would say they would vote for the bill is approximately $\frac{13}{15}$.

b. $P(\text{vote against}) \approx \frac{\text{number to vote against bill}}{\text{total number surveyed}} \approx \frac{10}{75} \approx \frac{2}{15}$

The experimental probability that the next person surveyed would say they would vote against the bill is approximately $\frac{2}{15}$.

3.

$$P \approx \frac{\text{number of times an event occurs}}{\text{total number surveyed}}$$
$$P(\text{hit}) \approx \frac{\text{number of hits made}}{\text{number of hits attempted}} \approx \frac{13}{30} = 0.4\overline{3} \approx 43\%$$

The experimental probability that Jack will hit the baseball on his next try is approximately $\frac{13}{30}$.

4.

$$P \approx \frac{\text{number of times an event occurs}}{\text{total number surveyed}}$$
$$P(\text{hit}) \approx \frac{\text{number of hits made}}{\text{number of hits attempted}} \approx \frac{8}{15}$$

5. a. $P(\text{50 or more}) \approx \frac{\text{days of 50 or more people}}{\text{total number of days}} \approx \frac{9}{14}$

The experimental probability that there will be 50 or more people at the park on the fifteenth day is approximately $\frac{9}{14}$.

b. $P(\text{fewer than 50}) \approx \frac{\text{days of fewer than 50 people}}{\text{total number of days}} \approx \frac{5}{14}$

The experimental probability that there will be fewer than 50 people at the park on the fifteenth day is approximately $\frac{5}{14}$.

6. $P(\text{strike}) \approx \frac{\text{number of strikes made}}{\text{number of frames}} \approx \frac{4}{10} \approx \frac{2}{5}$

The experimental probability that Alexis will roll a strike on the first frame of the next game is approximately $\frac{2}{5}$.

7. $P(\text{wearing jacket}) \approx \frac{\text{number wearing jacket}}{\text{total number of people}} \approx \frac{16}{25}$

The experimental probability that the next person to enter the store will be wearing a jacket is approximately $\frac{16}{25}$.

8. $P(\text{blue jay}) \approx \frac{\text{days of seeing a blue jay}}{\text{total number of days}} \approx \frac{12}{30} \approx \frac{2}{5}$

The experimental probability of seeing a blue jay on the first day of July is $\frac{2}{5}$.

9.

$$P(\text{waking up}) + P(\text{not waking up}) = 1$$
$$\frac{8}{11} + P(\text{not waking up}) = 1$$
$$-\frac{8}{11} \qquad\qquad -\frac{8}{11}$$
$$P(\text{not waking up}) = \frac{3}{11} \approx 27\%$$

The experimental probability that Claudia's cat does not wake her up is $\frac{3}{11}$.

10. a. $(9.1 + 9.1) \div 2 = 9.1$
The median depth of snow from the 10-day period is 9.1 inches.

b. $P(\text{less than 6 in,}) \approx \frac{\text{days with less than 6 in.}}{\text{total number of days}}$
$\approx \frac{0}{10}$
≈ 0
The experimental probability that the snow will be less than 6 inches deep on the eleventh day is approximately 0.

c. $P(\text{more than 10 in.}) \approx \frac{\text{days with more than 10 in.}}{\text{total number of days}}$
$\approx \frac{2}{10}$
$\approx \frac{1}{5}$
The experimental probability that the snow will be more than 10 inches deep on the eleventh day is approximately $\frac{1}{5}$.

11. a. $P(\text{below 90 °F}) \approx \frac{\text{days with high below 90 °F}}{\text{total number of days}}$
$\approx \frac{3}{8}$
The experimental probability that the high temperature on July 4, 2002 is below 90 °F is approximately $\frac{3}{8}$.

b. $P(\text{above 100 °F}) \approx \frac{\text{days with high above 100 °F}}{\text{total number of days}}$
$\approx \frac{0}{8}$
≈ 0
The experimental probability that the high temperature on July 4, 2002 is above 100 °F is approximately 0.

12. $\frac{3}{50} \cdot 1{,}800 = 108$
There are likely to be about 108 defective balls.

13. D; $\frac{26}{32} = 0.8125 \approx 80\%$

14. F; $24 - 18 = 6$
$\frac{6}{24} = 0.25$, or 25%

LESSON 3

Think and Discuss

1. Possible answer: A tree diagram shows all the possible outcomes of an experiment. The Fundamental Counting Principle gives the total number of possibilities.

2. There are 32 possible outcomes in the sample space.

Exercises

1. Let H = heads and T = tails.

Coin	**Spinner**
H	1
H	2
T	1
T	2

The possible outcomes are H1, H2, T1, and T2. There are 4 possible outcomes in the sample space.

2. cake-vanilla, cake-chocolate, cake-strawberry, cake-pistachio, cake-coffee, waffle-vanilla, waffle-chocolate, waffle-strawberry, waffle-pistachio, waffle-coffee, cup-vanilla, cup-chocolate, cup-strawberry, cup-pistachio, cup-coffee
15 outcomes

3. Use the Fundamental Counting Principle.
Number of ways the number cube can land: 6
Number of ways the spinner can land: 4
$6 \cdot 4 = 24$
There are 24 possible outcomes.

4. Let F = football, B = basketball, D = documentary, M = movie, and C = concert.

Noon	**3:00**
F	F
F	M
F	C
B	F
B	M
B	C
D	F
D	M
D	F
D	C

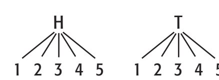
The possible outcomes are FF, FM, FC, BF, BM, BC, DF, DM, DC. There are 9 possible outcomes in the sample space.

5. Let H = heads and T = tails

Spinner	**Nickel**
1	H
1	T
2	H
2	T
3	H
3	T
4	H
4	T

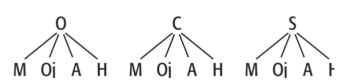
The possible outcomes are 1H, 1T, 2H, 2T, 3H, 3T, 4H, 4T. There are 8 possible outcomes in the sample space.

6. Let H = heads and T = tails.



H1, H2, H3, H4, H5, T1, T2, T3, T4, T5
There are 10 possible outcomes in the sample space.

7. Let O = oatmeal, C = corn flakes, S = scrambled eggs, M = milk, Oj = orange juice, A = apple juice, and H = hot chocolate.



OM, OOj, OA, OH, CM, COj, CA, CH, SM, SOj, SA, SH
There are 12 possible outcomes in the sample space.

8. Use the Fundamental Counting Principle.
Number of crusts: 3
Number of toppings: 9
$3 \cdot 9 = 27$
27 different one-topping pizzas can be ordered.

9. Number of sweaters: 3
Number of shirts: 2
$3 \cdot 2 = 6$
She can wear a sweater and a shirt together in 6 different ways.

10. blue-blue, blue-red, blue-green, red-red, red-green, green-green

11. a. $3 \cdot 3 = 9$
There are 9 possible outcomes.
b. $3 \cdot 2 = 6$
There are 6 possible outcomes.
c. $3 \cdot 4 = 12$
There are 12 possible outcomes.

12. a. $4 \cdot 3 \cdot 2 \cdot 1 = 24$
There are 24 ways to arrange 4 classes.
b. $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$
There are 120 ways to arrange 5 classes.

13. $3 \cdot 4 = 12$
There are 12 possible routes.

14. How many outcomes are possible if he draws one card from each pile?

15. Possible answer: Use the Fundamental Counting Principle. Each number cube can land 6 ways:
$6 \cdot 6 \cdot 6 = 216$

16. Make an organized list. The eight possible outcomes are: HHH, HHT, HTH, HTT, THH, THT, TTH, TTT.

17. D; $6 \cdot 6 = 36$
36 possible outcomes

18. $3 \cdot 2 \cdot 4 = 24$
There are 24 possible sandwich choices.
white–American–beef; white–American–turkey; white–American–ham; white–American–pork; white–Swiss–beef; white–Swiss–turkey; white–Swiss–ham; white–Swiss–pork; rye–American–beef; rye–American–turkey; rye–American–ham; rye–American–pork; rye–Swiss–beef; rye–Swiss–turkey; rye–Swiss–ham; rye–Swiss–pork; garlic–American–beef; garlic–American–turkey; garlic–American–ham; garlic–American–pork; garlic–Swiss–beef; garlic–Swiss–turkey; garlic–Swiss–ham; garlic–Swiss–pork

LESSON 4

Think and Discuss

1. Possible answer: If the Bingo balls had varied densities, then the denser balls would sink and be less likely to be selected.

2. Possible answer: If Mr. Ashley does not choose randomly, then each outcome is not equally likely and theoretical probability is not applicable.

Exercises

1. $P = \dfrac{\text{number of ways the event can occur}}{\text{total number of equally likely outcomes}}$

$P(\text{red}) = \dfrac{\text{total number of red marbles}}{\text{total number of marbles}}$
$= \dfrac{15}{90}$
$= \dfrac{1}{6}$
$\approx 0.17 \approx 17\%$

The theoretical probability of choosing a red marble is $\frac{1}{6}$, about 0.17, or about 17%.

2. $P = \dfrac{\text{number of ways the event can occur}}{\text{total number of equally likely outcomes}}$

$P(\text{H, H}) = \dfrac{\text{number of outcomes with two heads}}{\text{number of possible outcomes}}$
$= \dfrac{1}{4}$
$= 0.25 = 25\%$

The theoretical probability of both pennies landing heads up is $\frac{1}{4}$, 0.25, or 25%.

3. $P(\text{yellow}) = \dfrac{\text{number of yellow cards}}{\text{number of all cards}}$
$= \dfrac{15}{35} = \dfrac{3}{7}$

4. $P(\text{green}) = \dfrac{\text{number of green cards}}{\text{number of all cards}}$
$= \dfrac{10}{35} = \dfrac{2}{7}$

5. $P(\text{not yellow or green}) = \dfrac{\text{cards not yellow or green}}{\text{number of all cards}}$
$= \dfrac{10}{35} = \dfrac{2}{7}$

6. $P = \dfrac{\text{number of ways the event can occur}}{\text{total number of equally likely outcomes}}$

$P(\text{heart or club}) = \dfrac{\text{number of hearts and clubs}}{\text{total number of cards}}$
$= \dfrac{26}{52}$
$\dfrac{1}{2} = 0.5 = 50\%$

The theoretical probability of drawing a heart or club is $\frac{1}{2}$, 0.5, or 50%.

7. $P = \dfrac{\text{number of ways the event can occur}}{\text{total number of equally likely outcomes}}$

$P(\text{purple}) = \dfrac{\text{number of purple disks}}{\text{total number of disks}}$
$= \dfrac{13}{52}$
$= \dfrac{1}{4} = 0.25 = 25\%$

The theoretical probability of drawing a purple disk is $\frac{1}{4}$, 0.25, or 25%.

8. $P = \dfrac{\text{number of ways the event can occur}}{\text{total number of equally likely outcomes}}$

$P(\text{G or O}) = \dfrac{\text{number of Gs or Os}}{\text{total number of Bingo balls}}$
$= \dfrac{30}{75}$
$= \dfrac{2}{5}$ or 0.4 or 40%

9. $P(\text{girl}) = \frac{\text{number of girls}}{\text{number of students}}$
$= \frac{6}{14} = \frac{3}{7}$

10. $P(\text{boy}) = \frac{\text{number of boys}}{\text{number of students}}$
$= \frac{8}{14} = \frac{4}{7}$

11. Use the fundamental Counting Principle to find total number of outcomes.
$6 \cdot 6 = 36$
There are two outcomes that total 3: 1, 2; 2, 1.
$P = \frac{\text{number of ways the event can occur}}{\text{total number of equally likely outcomes}}$
$P(\text{total of } 3) = \frac{2}{36}$
$= \frac{1}{18}$

12. Use the fundamental Counting Principle to find total number of outcomes.
$6 \cdot 6 = 36$
There are six outcomes that total 7: 1, 6; 2, 5; 3, 4; 4, 3; 5, 2; 6, 1.
$P = \frac{\text{number of ways the event can occur}}{\text{total number of equally likely outcomes}}$
$P(\text{total of } 7) = \frac{6}{36}$
$= \frac{1}{6}$

13. Use the fundamental Counting Principle to find total number of outcomes.
$6 \cdot 6 = 36$
There are 3 outcomes that total 4: 1, 3; 2, 2; 3, 1.
$P = \frac{\text{number of ways the event can occur}}{\text{total number of equally likely outcomes}}$
$P(\text{total of } 4) = \frac{3}{36}$
$= \frac{1}{12}$

14. Use the fundamental Counting Principle to find total number of outcomes.
$6 \cdot 6 = 36$
There is one outcome that totals 2: 1, 1.
$P = \frac{\text{number of ways the event can occur}}{\text{total number of equally likely outcomes}}$
$P(\text{total of } 2) = \frac{1}{36}$

15. Use the fundamental Counting Principle to find total number of outcomes.
$6 \cdot 6 = 36$
There are 4 outcomes that total 9: 3, 6; 4, 5; 5, 4; and 6, 3.
$P = \frac{\text{number of ways the event can occur}}{\text{total number of equally likely outcomes}}$
$P(\text{total of } 9) = \frac{4}{36}$
$= \frac{1}{9}$

16. Use the fundamental Counting Principle to find total number of outcomes.
$6 \cdot 6 = 36$
There are no outcomes that total 13.
$P = \frac{\text{number of ways the event can occur}}{\text{total number of equally likely outcomes}}$
$P(\text{total of } 13) = \frac{0}{36}$
$= 0$

17. $P = \frac{\text{number of ways the event can occur}}{\text{total number of equally likely outcomes}}$
$P(\text{less than } 3) = \frac{4}{10}$
$= \frac{2}{5}$

18. $P = \frac{\text{number of ways the event can occur}}{\text{total number of equally likely outcomes}}$
$P(5) = \frac{2}{10}$
$= \frac{1}{5}$

19. $P = \frac{\text{number of ways the event can occur}}{\text{total number of equally likely outcomes}}$
$P(8) = \frac{0}{10}$
$= 0$

20. $P = \frac{\text{number of ways the event can occur}}{\text{total number of equally likely outcomes}}$
$P(\text{less than } 6) = \frac{10}{10}$
$= 1$

21. $P = \frac{\text{number of ways the event can occur}}{\text{total number of equally likely outcomes}}$
$P(\text{greater than or equal to } 4) = \frac{4}{10}$
$= \frac{2}{5}$

22. $P = \frac{\text{number of ways the event can occur}}{\text{total number of equally likely outcomes}}$
$P(3) = \frac{2}{10}$
$= \frac{1}{5}$

23. **a**. There is only one sector with a 2, two for each of 1 and 4, and three for 3. The outcomes for landing on a specific number are not equally likely. So, the experiment is unfair for landing on 2.

b. There are two sectors of each of four colors. The outcomes for landing on a specific colour are equally likely. So, the experiment is fair for landing on blue.

24. **a.** $P(\text{Disney World}) = \frac{15{,}640{,}000}{42{,}120{,}000} \approx 0.37 \approx 37\%$
The probability that the person visited Disney World is about 37%.

b. $P(\text{California}) = \frac{13{,}680{,}000 + 3{,}700{,}000}{42{,}120{,}000} \approx 0.41$
$\approx 41\%$
The probability that the person visited one of the parks in California is about 41%.

25. $P(\text{red lettuce}) = \frac{50}{200} = \frac{1}{4} = 25\%$
The probability that the seed will be a red lettuce seed is 25%.

26. Use logical reasoning.
Make a table and work the clues one at a time.

	Tan	Orange	Purple	Aqua
Francis	✗	✓	✗	✗
Amanda	✗	✗	✗	✓
Raymond	✓	✗	✗	✗
Albert	✗	✗	✓	✗

Francis wore orange. Raymond wore tan. Albert wore purple. Amanda wore aqua.

27. The numerator means there are 3 favorable outcomes, and the denominator means there are 8 possible outcomes.

28. $P(\text{red or green}) = \frac{1}{2} + \frac{1}{6}$
$= \frac{3}{6} + \frac{1}{6}$
$= \frac{4}{6} = \frac{2}{3}$

The probability of the spinner landing on red or green is $\frac{2}{3}$.

29. D; $P(\text{blue}) = \frac{\text{number of blue marbles}}{\text{total number of marbles}}$
$= \frac{2}{8}$
$= \frac{1}{4}$

30. $P = \frac{\text{number of ways the event can occur}}{\text{total number of equally likely outcomes}}$

$P(\text{not yellow}) = \frac{5 + 7}{3 + 5 + 7}$
$= \frac{12}{15}$
$= \frac{4}{5}$ or 0.8

LESSON 5

Think and Discuss

1. Possible answer: A prediction based on experimental probability is based on a ratio of the number of times an event can occur to the number of total trial. A prediction based on theoretical probability is based on a ratio of the number of ways an event can occur to the total number of equally likely outcomes.

2. Possible answer: No, it is will not always match the results because a prediction is what you expect to happen but this may not always match the actual results.

Exercises

1. Set up an equation and solve for x.
25% of 730 $= x$
$\frac{1}{4} \cdot 730 = x$
$182.5 = x$
$183 \approx x$
Floridians can expect to hear thunder on about 183 out of 730 days.

2. Set up an equation and solve for x.
40% of 50 $= x$
$\frac{4}{10} \cdot 50 = x$
$20 = x$
The player can be expected to reach first base 20 out of 50 times at bat.

3. P(heads) $= \frac{1}{2}$
$\frac{1}{2} = \frac{x}{18}$
$18 \cdot \frac{1}{2} = 18 \cdot \frac{x}{18}$
$9 = x$
You can expect heads to appear 9 out of 18 times.

4. $P(\text{white}) = \frac{\text{number of white marbles}}{\text{total number of marbles}}$
$= \frac{6}{10}$
$= \frac{3}{5}$
$\frac{3}{5} = \frac{x}{35}$
$\frac{3}{5} \cdot 35 = \frac{x}{35} \cdot 35$
$21 = x$
You can expect to remove a white marble 21 out of 35 times.

5. Understand the problem
The answer will be whether or not the Escobars should go to Boulder.

List the important information:
Boulder had 23 days when it did not snow last December (in 31 days).
The Escobars would like it to snow at least 3 days of their 7-day vacation.

Make a Plan
If it did not snow on 23 of 31 days, then it did snow on 8 of 31 days. Find the number of days it is expected to snow in 7 days, and compare to the Escobars desire for at least 3 days.

Solve
$\frac{8}{31} = \frac{x}{7}$
$\frac{8}{31} \cdot 7 = \frac{x}{7} \cdot 7$
$1.806 \approx x$
$2 \approx x$

Look Back
The Estobars are not likely to get three or more days of snow based on last year. So, they should not go to Boulder.
$\frac{8}{31} \approx \frac{1}{4} = 25\%$ $\quad \frac{2}{7} \approx \frac{1}{4} = 25\%$
Since both ratios are about 25%, the answer is reasonable.

6. Set up an equation and solve for x.
16% of 30 $= x$
$0.16 \cdot 30 = x$
$4.8 = x$
$5 \approx x$
Mobile can expect to have about 5 rainy days out of 30.

7. Set up an equation and solve for x.
72% of 450 $= x$
$0.72 \cdot 450 = x$
$324 = x$
This student can expect to make 324 free throws out of 450 attempts.

8. 2, 4, 6, and 8 are evenly divisible by 2. There are 8 sectors in all.
$P(\text{evenly divisible by 2}) = \frac{1}{2}$
$\frac{1}{2} = \frac{x}{20}$
$\frac{1}{2} \cdot 20 = \frac{x}{20} \cdot 20$
$10 = x$
You can expect to get a number evenly divisible by 2 in 10 out of 20 spins.

9. 1, 2, 3, 4, and 5 are less than 6. There are 8 sectors in all.
$P(\text{less than } 6) = \frac{5}{8}$
$$\frac{5}{8} = \frac{x}{150}$$
$$\frac{5}{8} \cdot 150 = \frac{x}{150} \cdot 150$$
$$93.75 = x$$
$$94 \approx x$$
You can expect to get a number less than 6 in 94 out of 150 spins.

10. 8 + 3 = 11 marbles white or red. There are 15 marbles in all.
$P(\text{white or red}) = \frac{11}{15}$
$$\frac{11}{15} = \frac{x}{45}$$
$$\frac{11}{15} \cdot 45 = \frac{x}{45} \cdot 45$$
$$33 = x$$
You can expect to get a white or red marble 33 out of 45 times.

11. **Understand the problem**
The answer will be whether or not the Alexia should take the light rail train to work.

List the important information:
The light rail train is on time 24 out of 25 times. Alexia does not want to be late more than 5% of the time.

Make a Plan
If the train is on time 24 out of 25 times, then it is late 1 out of 25 times. Find the number of times it is expected to be late as a percent, and compare to the Alexia's desire for no more than 5% of the time.

Solve
$$\frac{1}{25} = \frac{x}{100}$$
$$\frac{1}{25} \cdot 100 = \frac{x}{100} \cdot 100$$
$$4 = x$$

Look Back
Alexia is not likely to be late any more than 5% of the time, only 4%. So, she should take the light rail train.

12. Set up a proportion and solve for x.
$$\frac{12}{15} = \frac{x}{1{,}000{,}000}$$
$$\frac{12}{15} \cdot 1{,}000{,}000 = \frac{x}{1{,}000{,}000} \cdot 1{,}000{,}000$$
$$800{,}000 = x$$
You can expect 800,000 out of 1,000,000 people to recycle aluminum cans.

13. $16, $32, and $64 are spins of $16 or over. There are 7 sectors in all.
$P(\$16 \text{ or over}) = \frac{3}{7}$
$$\frac{3}{7} = \frac{x}{84}$$
$$\frac{3}{7} \cdot 84 = \frac{x}{84} \cdot 84$$
$$36 = x$$
You can expect spins of $16 or over in 36 out of 84 spins.

14. $P(\text{hot chocolate}) = \frac{72}{150}$
$$\frac{72}{150} = \frac{x}{500}$$
$$\frac{72}{150} \cdot 500 = \frac{x}{500} \cdot 500$$
$$240 = x$$
You can expect to 240 out of 500 people to name hot chocolate as their favorite hot drink.

15. 5 is the only odd number greater than 3 on a fair number cube. There are 6 numbers in all.
$P(\text{odd number greater than } 3) = \frac{1}{6}$
$$\frac{1}{6} = \frac{x}{18}$$
$$\frac{1}{6} \cdot 18 = \frac{x}{18} \cdot 18$$
$$3 = x$$
You can expect to get an odd number greater than 3 when rolling a fair number cube 3 out of 18 times.

16. Set up a proportion and solve for x.
$$\frac{7}{10} = \frac{x}{500}$$
$$\frac{7}{10} \cdot 500 = \frac{x}{500} \cdot 500$$
$$350 = x$$
Based on this information, you can expect 350 people to own a laptop computer. This is not a good prediction because the people surveyed were exiting an electronic store and this sample is not representative of the general population.

17. Ella set up the right side of the proportion incorrectly; it should be $\frac{x}{80}$.

18. Check students' work. For most places, one week is not representative of a year.

19. Let x be the month's box office receipts in millions of dollars.
$$25\% \text{ of } x = 80$$
$$0.25 \cdot x = 80$$
$$\frac{0.25x}{0.25} = \frac{80}{0.25}$$
$$x = 320$$
$$40\% \text{ of } 320 = 128$$
For 3 months, $3 \cdot 128 = 384$
Movie A would be expected to earn $384 million in 3 months if its rate of success continues.

20. Set up a proportion and solve for x.
$$\frac{4}{23} = \frac{x}{414}$$
$$\frac{4}{23} \cdot 414 = \frac{x}{414} \cdot 414$$
$$72 = x$$
You can expect 72 out of 414 toys to be defective.

21. Set up an equation and solve for x.
$$89\% \text{ of } 5{,}000 = x$$
$$0.89 \cdot 5{,}000 = x$$
$$4{,}450 = x$$
You can expect 4,450 of 5,000 flights in a year to arrive on time.

22. black to white = 5 to 7 = 5:7 = $\frac{5}{7}$

READY TO GO ON

1. There is only one outcome that sums to 2, so it is unlikely.
2. There are only two choices, so it is as likely as not.
3. $P(\text{winning}) + P(\text{not winning}) = 1$

$$\frac{7}{10} + P(\text{not winning}) = 1$$
$$-\frac{7}{10} \qquad\qquad \frac{7}{10}$$
$$P(\text{not winning}) = \frac{3}{10}$$

4. $P(\text{no dog or cat}) \approx \dfrac{\text{people with neither dog nor cat}}{\text{total number of people}}$

$$\approx \frac{7}{31}$$

The experimental probability that the next person will not have a dog or a cat is approximately $\frac{7}{31}$.

5. The possible outcomes are H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6.
There are 12 possible outcomes in the sample space.
6. Number of flavors: 4
Number of toppings: 3
$4 \cdot 3 = 12$
You can have 12 different desserts.
7. There is 1 favorable outcome.
$P(5) = \frac{1}{10} = 0.1 = 10\%$
8. There are 4 prime numbers: 2, 3, 5, and 7.
$P(\text{prime number}) = \frac{4}{10} = \frac{2}{5} = 0.4 = 40\%$.
9. There is not a sector labeled 20.
$P(20) = \frac{0}{10} = 0 = 0\%$
10. $P(\text{CD}) = \frac{8}{13}$
$P(\text{DVD}) = \frac{5}{13}$
11. Set up an equation and solve for x.

$$15\% \text{ of } 20 = x$$
$$0.15 \cdot 20 = x$$
$$3 = x$$

Sally can expect to put a ball in a hole 3 out of 15 times.

12. 4, 5, and 6 are the numbers greater than 3 on a number cube. There are 6 numbers on a number cube.

$$P(\text{greater than 3}) = \frac{1}{2}$$
$$\frac{1}{2} = \frac{x}{25}$$
$$\frac{1}{2} \cdot 25 = \frac{x}{25} \cdot 25$$
$$12.5 = x$$

You can expect to get a number greater than 3 in 12 or 13 out of 25 rolls.

LESSON 6

Think and Discuss

1. Possible answer: Both are products of individual probabilities, but for dependent events, you have to take into account the first event when calculating the probability of the second event.
2. Possible answer: The probability of two events is less than or equal to the probability of the individual events. If you find the product of two numbers between 0 and 1, the product will be less than either factor, and any number times 1 is equal to itself.

Exercises

1. Because the outcome of flipping one coin does not affect the outcomes of flipping the second coin, the events are independent.
2. Because, on the second time, the student cannot choose the same marble as on the first, and since there are fewer marbles to choose from, the events are dependent.
3. $P(\text{heads and 5 or 6}) = P(\text{heads}) \cdot P(\text{5 or 6})$

$$= \frac{1}{2} \cdot \frac{2}{6}$$
$$= \frac{1}{2} \cdot \frac{1}{3}$$
$$= \frac{1}{6}$$

4. $P(\text{5 and 2}) = P(5) \cdot P(2)$

$$= \frac{1}{10} \cdot \frac{1}{6}$$
$$= \frac{1}{60}$$

5. The first selection changes the number of students left, and may change the number of girls left, so the events are dependent.
$P(\text{first girl}) = \frac{10}{25} = \frac{2}{5}$
$P(\text{second girl}) = \frac{9}{24}$
$P(\text{first girl, then second girl}) = P(\text{A}) \cdot P(\text{B after A})$

$$= \frac{2}{5} \cdot \frac{9}{24}$$
$$= \frac{1}{5} \cdot \frac{3}{4}$$
$$= \frac{3}{20}$$

The probability that Mr. Samms chooses 2 girls is $\frac{3}{20}$.

6. Because, on the second time, the student cannot choose the same fiction book as on the first, and since there are fewer fiction books to choose from, the events are dependent.
7. Because the outcome of choosing a lily from one bunch does not affect the outcome of choosing a tulip from the second bunch, the events are independent.

8. $P(\text{red then blue}) = P(\text{red}) \cdot P(\text{blue})$
$$= \frac{6}{10} \cdot \frac{4}{10}$$
$$= \frac{3}{5} \cdot \frac{2}{5}$$
$$= \frac{6}{25}$$

9. $P(\text{even then odd}) = P(\text{even}) \cdot P(\text{odd})$
$$= \frac{3}{6} \cdot \frac{3}{6}$$
$$= \frac{1}{2} \cdot \frac{1}{2}$$
$$= \frac{1}{4}$$

10. The first removal changes the number of quarters left, and may change the number of Delaware quarters left, so the events are dependent.
$P(\text{first Delaware}) = \frac{3}{7}$
$P(\text{second Delaware}) = \frac{2}{6} = \frac{1}{3}$
$P(\text{first Delaware}) = P(\text{A}) \cdot P(\text{B after A})$
$$= \frac{3}{7} \cdot \frac{1}{3}$$
$$= \frac{1}{7} \cdot \frac{1}{1}$$
$$= \frac{1}{7}$$
The probability that Francisco removes two Delaware quarters is $\frac{1}{7}$.

11. Because the outcome of the first pick affects the outcome of the second, the events are dependent.
$P(\text{first even}) = \frac{4}{8} = \frac{1}{2}$
$P(\text{second even}) = \frac{3}{7}$
$P(\text{first even, then second even}) = \frac{1}{2} \cdot \frac{3}{7}$
$$= \frac{3}{14}$$
The probability of both events occurring is $\frac{3}{14}$.

12. Because the answer on the first question has no affect on the answer of the second, the events are independent.
$P(\text{first wrong}) = \frac{4}{5}$
$P(\text{second wrong}) = \frac{4}{5}$
$P(\text{wrong and wrong}) = \frac{4}{5} \cdot \frac{4}{5}$
$$= \frac{16}{25}$$
The probability that the student gets both answers wrong is $\frac{16}{25}$.

13. **a.** Follow the branch of the diagram and multiply the probabilities.
$P(\text{green, then red}) = \frac{1}{3} \cdot 1 = \frac{1}{3}$

b. Follow the branch of the diagram and multiply the probabilities.
$P(\text{red, then red}) = \frac{2}{3} \cdot \frac{1}{2} = \frac{1}{3}$

c. Follow the branch of the diagram and multiply the probabilities.
$P(\text{green, then green}) = \frac{1}{3} \cdot 0 = 0$

14. Possible answer: Dependent. There are 15 boys and 17 girls in MsTran's class. She randomly chooses 2 students from the class to be on the Student Council. What is the probability that both the students she chooses will be boys?

15. Dependent; with dependent events, the outcome of the first event influences the outcome of the second event.

16. The events are independent because if it rains one day, that does not affect if it will rain on the next day.
$P(\text{rain on first day}) = \frac{4}{5}$
$P(\text{rain on second day}) = \frac{4}{5}$
$P(\text{rain on both days}) = \frac{4}{5} \cdot \frac{4}{5}$
$$= \frac{16}{25}$$
The probability that they will accurately predict rain two days is a row is $\frac{16}{25}$.

17. B; $P(\text{first red}) = \frac{5}{10} = \frac{1}{2}$
$P(\text{second purple}) = \frac{5}{9}$
$P(\text{first red, then purple}) = P(\text{A}) \cdot P(\text{B after A})$
$$= \frac{1}{2} \cdot \frac{5}{9}$$
$$= \frac{5}{18}$$

18. The events are dependent because the removal of one sock will affect the number of socks left,
$P(\text{first black}) = \frac{6}{14} = \frac{3}{7}$
$P(\text{second black}) = \frac{5}{13}$
$P(\text{first black, then second black}) = \frac{3}{7} \cdot \frac{5}{13}$
$$= \frac{15}{91}$$

LESSON 7

Think and Discuss

1. Possible answer: Begin with book 1, and make a branch for each of the other 4 books. Repeat for book 2, book 3, book 4, and book 5. Then cross out all the repeated pairs leaving each pair listed only once.
2. Possible answer: You can use combinations to find the denominator "total number of equally likely outcomes."

Exercises

1. Begin by listing all of the possible groupings of fruit taken two at a time. Because order does not matter, you can eliminate repeated pairs. For example, apple, pear is already listed, so pear, apple can be eliminated.

apple, pear	~~pear, apple~~	~~orange, apple~~	~~plum, apple~~
apple, orange	pear, orange	~~orange, pear~~	~~plum, pear~~
apple, plum	pear, plum	orange, plum	~~plum, orange~~

There are 6 different combinations of two fruits.

2. Begin by listing all of the possible groupings of five letters taken three at a time. Because order does not matter, you can eliminate duplications. For example, ARI is already listed, so EAI and IAE can be eliminated.

AEI ~~EAI~~ ~~IAE~~ ~~OAE~~ ~~UAE~~
AEO ~~EAO~~ ~~IAO~~ ~~OAI~~ ~~UAI~~
AEU ~~EAU~~ ~~IAU~~ ~~OAU~~ ~~UAO~~
AIO EIO ~~IEO~~ ~~OEI~~ ~~UEI~~
AIU EIU ~~IEU~~ ~~OEU~~ ~~UEO~~
AOU EOU IOU ~~OUI~~ ~~UIO~~

There are 10 different combinations of three letters.

3. Let B = blueberry, A = apricot, G = grape, P = peach, and O = orange.
You can make a tree diagram using the following combinations.
B: AG, AP, AO, GO, PO
A: BG, BP, BO, GP, GO, PO
G: BA, BP, BO, AP, AO, PO
P: BA, BG, BO, AG, AO, GO
P: BA, BG, BP, AG, AP, GP
Each combination is listed three times. So Robin can make 30 ÷ 3 = 10 different combinations.

4. Let R = red, B = blue, G = green, Y = yellow, O = orange, and W = white.
You can make a tree diagram using the following combinations.
R: B, G, Y, O, W
B: R, G, Y, O, W
G: R, B, Y, O, W
Y: R, B, G, O, W
O: R, B, G, Y, W
W: R, B, G, Y, O
Each combination is listed twice. So Eduardo can choose 30 ÷ 2 = 15 combinations of colors.

5. Let B = bacon, O = onions, M = mushrooms, S = Swiss, and C = cheddar.

BO ~~OB~~ ~~MB~~ ~~SB~~ ~~CB~~
BM OM ~~MO~~ ~~SO~~ ~~CO~~
BS OS MS ~~SM~~ ~~CM~~
BC OC MC SC ~~CS~~

You can build 10 burgers with different toppings.

6. Let P = Paris, N = New York, M = Moscow, and L = London.

PNM ~~NMP~~ ~~MPN~~ ~~LPN~~
PNL ~~NPL~~ ~~MPL~~ ~~LPM~~
PML NML ~~MNL~~ ~~LMN~~

There are 4 different combinations of cities possible.

7. Let C = carnations, R = roses, L = lilies, D = daisies, I = irises, and T = tulips.
You can make a tree diagram using the following combinations.
CR, CL, CD, CI, CT
RL, RD, RI, RT
LD, LI, LT
DI, DT
IT
There are 15 different combinations.

8. Let each student be represented by a number from 1–7.
You can make a tree diagram using the following combinations.
1: 2, 3, 4, 5, 6, 7
2: 1, 3, 4, 5, 6, 7
3: 1, 2, 4, 5, 6, 7
4: 1, 2, 3, 5, 6, 7
5: 1, 2, 3, 4, 6, 7
6: 1, 2, 3, 4, 5, 7
7: 1, 2, 3, 4, 5, 6
Each combination is listed twice, so 42 ÷ 2 = 21 tennis teams can be made.

9. Let H = hiking, M = mosaics, T = tennis, P = painting, V = volleyballs, R = rafting, Y = pottery, and S = swimming.
You can use a tree diagram using the following combinations.
HM, HT, HP, HV, HR, HY, HS
MH, MT, MP, MV, MR, MY, MS
TH, TM, TP, TV, TR, TY, TS
PH, PM, PT, PV, PR, PY, PS
VH, VM, VT, VP, VR, VY, VS
RH, HM, RT, RP, RV, RY, RS
YH, YM, YT, YP, YV, YR, YS
SH, SM, ST, SP, SV, SR, SY
Each combination is listed two times, so the camper can choose from 56 ÷ 2 = 28 different combinations.

10. Let R = Rob, C = Caryn, and S = Sari
RC, RS, CS
They can pair up 3 different ways.

11. Let C = Churchill, K = King, and M = Mandela
There are 3 different combinations of biographies.

12. Let 1–5 represent each photo in the series.
Make a tree diagram using the following combinations.
1: 23, 24, 25, 34, 35, 45
2: 13, 14, 15, 34, 35, 45
3: 12, 14, 15, 24, 25, 45
4: 12, 13, 15, 23, 25, 35
5: 12, 13, 14, 23, 24, 34
Each combination is listed three times, so there are 30 ÷ 3 = 10 combinations of photos possible.

13. Let R = Renoir, M = Monet, A = Manet, D = Degas, P = Pissarro, and C = Cassatt. Make a tree diagram using the following combinations.
R: M, A, D, P, C
M: R, A, D, P, C
A: R, M, D, P, C
D: R, M, A, P, C
P: R, M, A, D, C
C: R, M, A, D, P
Each combination is listed twice, so there are $30 \div 2 = 15$ different pairs of artists.

14. Let 15 represent each painting.
Here is the first branch of a tree diagram.
1: 234, 235, 245, 345
Each of the five branches will have 4 combinations. $5 \cdot 4 = 20$. Each combination will be listed four times, so there will be $20 \div 4 = 5$ different combinations of paintings.

15. Let the digits 1−9 represent the art works. To pick 7 at a time, it is easiest to eliminate 2 at a time from the list of 9 in an organized fashion.
12,34,568 12,34,569 12,34,589 12,34,789 12,36,789
12,56,789 14,56,789 34,56,789
12,34,568 12,34,579 12,34,689 12,35,789 12,46,789
13,56,789 24,56,789
12,34,578 12,34,679 12,35,689 12,45,789 13,46,789
23,56,789
12,34,678 12,35,679 12,45,689 13,45,789 23,46,789
12,35,678 12,45,679 13,45,689 23,45,789
12,45,678 13,45,679 23,45,689
13,45,678 23,45,679
23,45,678
There are 36 distinct combinations of the artist's works.

16. A; $2 \cdot 5 = 10$

17. AB, AC, AD, AE, AF
BA, BC, BD, BE, BF
CA, CB, CD, CE, CF
DA, DB, DC, DE, DF
EA, EB, EC, ED, EF
FA, FB, FC, FD, FE
Each of the combinations are listed twice, so $30 \div 2 = 15$ different combinations.

18. $\sqrt{76} \approx \sqrt{81} = 9$

19. $\sqrt{31} \approx \sqrt{36} = 6$

20. $\sqrt{126} \approx \sqrt{121} = 11$

21. $\sqrt{55} \approx \sqrt{49} = 7$

LESSON 8

Think and Discuss

1. Possible answer: I am less likely to leave out possible arrangements if I follow a pattern when listing permutations.

2. Possible answer: There are 8 choices for the first position, 7 choices for the second, 6 for the third and so on and their product is 8!.

Exercises

1.

1,234	2,134	3,124	4,123
1,243	2,143	3,142	4,132
1,324	2,314	3,214	4,213
1,342	2,341	3,241	4,231
1,423	2,413	3,412	4,312
1,432	2,431	3,421	4,321

There are 24 ways to arrange the numbers to make a 4-digit number.

2. $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$
There are 120 permutations of the letters in *quiet*.

3. Number of permutations $= 6!$
$= 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$
$= 720$
There are 720 different orders in which Sam can make the calls.

4. Number of permutations $= 7!$
$= 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$
$= 5{,}040$
There are 5,040 different orders in which the auditions can be done.

5.

Eric, Meera, Roger	Meera, Eric, Roger	Roger, Eric, Meera
Eric, Roger, Meera	Meera, Roger, Eric	Roger, Meera, Eric

There are 6 ways in which they can stand in line.

6. $3 \cdot 2 \cdot 1 = 6$
There are 6 permutations of the letters in *art*.

7. Number of permutations $= 10!$
$= 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$
$= 3{,}628{,}800$
There are 3,628,800 permutations of the letters *A* through *J*.

8. Number of permutations $= 8!$
$= 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$
$= 40{,}320$
There are 40,320 ways to match up the riders and horses.

9. Combinations; The order in which the books are chosen does not matter.

10. Permutations; The order in which they sit is important.

11. Permutations; The order of the digits is important.

12. Number of permutations $= 10!$
$= 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$
$= 3{,}628{,}800$
There are 3,628,800 different lineups the golf coach can make.

13. $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$
There are 24 ways the students can present their reports. Each student goes first in six of the ways.
$P(\text{Melba first}) = \frac{6}{24} = \frac{1}{4}$

14. Number of permutations $= 7!$
$= 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$
$= 5{,}040$
The county can assign 5,040 different numbers.

15. $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$
There are 120 different 5-digit numbers that can be made.

16. $12! = 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$
$= 479{,}001{,}600$
There are 479,001,600 different ways to play 12 songs on a CD.

17. first item = 5
second item = 4
third item = 3
$5 \cdot 4 \cdot 3 = 60$
There are 60 ways to put the items on the shelf.

18. 4% of 200
$0.04 \cdot 200 = 8$
8 people felt their memory was the same.
$8! = 40{,}320$
The interviews could be conducted in 40,320 ways.

19. There are 13 choices for the first book, 12 for the second, 11 for the third, and so on. Merina can read them in 13! ways.

20. **a.** $4! = 24$
b. 3; DEAR, READ, DARE

21. $P(\text{sunflower, then a bluebonnet}) = \frac{3}{7} \cdot \frac{2}{6}$
$= \frac{1}{7} \cdot \frac{2}{1}$
$= \frac{2}{7}$

22. The student added when he or she should have multiplied;
$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120.$

23. Possible answer: In combinations order is not important, such as when choosing 2 out of 5 students to serve on a committee. In permutations, order does matter, such as when choosing 2 students to serve as president and secretary of the committee.

24. $\frac{11!}{3!(11-3)!}$
$= \frac{11!}{3!(8!)}$
$= \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$
$= \frac{11 \cdot 10 \cdot 9}{3 \cdot 2 \cdot 1}$
$= 11 \cdot 5 \cdot 3$
$= 165$

25. D; $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

26. $7! = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$
$= 5{,}040$

LESSON 9

Think and Discuss

1. Possible answer: Organized lists, tree diagrams, and tables can be used to find all the possible outcomes of a compound event, which is the sample space, without missing any possible outcomes.

Exercises

1. Kai = K
Paula = P
Member3 = 3
Member4 = 4
Member5 = 5

KP	PK	3K	4K	5K
K3	P3	3P	4P	5P
K4	P4	34	43	53
K5	P5	35	45	54

$P(KP \text{ or } PK) = \frac{2}{20} = \frac{1}{10}$

2.

1-2	2-1	3-1	4-1	5-1	6-1	7-1
1-3	2-3	3-2	4-2	5-2	6-2	7-2
1-4	2-4	3-4	4-3	5-3	6-3	7-3
1-5	2-5	3-5	4-5	5-4	6-4	7-4
1-6	2-6	3-6	4-6	5-6	6-5	7-5
1-7	2-7	3-7	4-7	5-7	6-7	7-6

$P(6 \text{ \& } 7 \text{ or } 7 \text{ \& } 6) = \frac{2}{42} = \frac{1}{21}$

3.
A — B — T → ABT
A — T — B → ATB
B — A — T → BAT
B — T — A → BTA
T — A — B → TAB
T — B — A → TBA

$P(\text{word}) = \frac{2}{6} = \frac{1}{3}$

4. C = Chicago
M = Miami
D = Denver

C — M — D → CMD
C — D — M → CDM
M — C — D → MCD
M — D — C → MDC
D — C — M → DCM
D — M — C → DMC

$P(\text{C first}) = \frac{2}{6} = \frac{1}{3}$

5.

	1	2	3	4	5	6
1	1-1	1-2	1-3	1-4	1-5	1-6
2	2-1	2-2	2-3	2-4	2-5	2-6
3	3-1	3-2	3-3	3-4	3-5	3-6
4	4-1	4-2	4-3	4-4	4-5	4-6
5	5-1	5-2	5-3	5-4	5-5	5-6
6	6-1	6-2	6-3	6-4	6-5	6-6

$P(\text{sum} = 7) = \frac{6}{36} = \frac{1}{6}$

6. carrots = C
peas = P
sweet potatoes = S
broccoli = B

CP	PC	SC	BC
CS	PS	SP	BP
CB	PB	SB	BS

$P(PB \text{ or } BP) = \frac{2}{12} = \frac{1}{6}$

7. Jared = J
Matt = M
Player3 = 3
Player4 = 4
Player5 = 5
Player6 = 6

JM	MJ	3J	4J	5J	6J
J3	M3	3M	4M	5M	6M
J4	M4	34	43	53	63
J5	M5	35	45	54	64
J6	M6	36	46	56	65

$P(JM \text{ or } MJ) = \frac{2}{30} = \frac{1}{15}$

8.
A — R — T — S → ARTS
A — R — S — T → ARST
A — T — R — S → ATRS
A — T — S — R → ATSR
A — S — R — T → ASRT
A — S — T — R → ASTR
R — A — T — S → RATS
R — A — S — T → RAST
R — T — A — S → RTAS
R — T — S — A → RTSA
R — S — A — T → RSAT
R — S — T — A → RSTA
T — A — R — S → TARS
T — A — S — R → TASR
T — R — A — S → TRAS
T — R — S — A → TRSA
T — S — A — R → TSAR
T — S — R — A → TSRA
S — A — R — T → SART
S — A — T — R → SATR
S — R — A — T → SRAT
S — R — T — A → SRTA
S — T — A — R → STAR
S — T — R — A → STRA

$P(RATS \text{ or } TARS \text{ or } TSAR \text{ or } STAR) = \frac{4}{24} = \frac{1}{6}$

9. Ace of Hearts = H
Ace of Clubs = C
Ace of Diamonds = D
Ace of Spades = S

H — C → HC
H — D → HD
H — S → HS
D — H → DH
D — C → DC
D — S → DS
C — H → CH
C — D → CD
C — S → CS
S — H → SH
S — C → SC
S — D → SD

$P(H \text{ second}) = \frac{3}{12} = \frac{1}{4}$

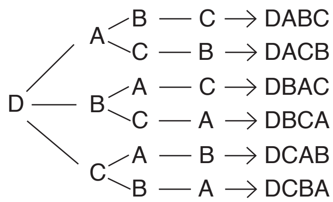
10. Sarah = S
David = D
Ali = A
student4 = 4
student5 = 5
student6 = 6
student7 = 7
student8 = 8
student9 = 9
student10 = 10
student11 = 11
student12 = 12

	S	A	D	4	5	6	…	11	12
S	~~SS~~	SA	SD	S4	S5	S6	…	S11	S12
A	**AS**	~~AA~~	AD	A4	A5	A6	…	A11	A12
D	**DS**	**DA**	~~DD~~	D4	D5	D6	…	D11	D12
4	**4S**	**4A**	**4D**	~~44~~	45	46	…	411	412
5	**5S**	**5A**	**5D**	**54**	~~55~~	56	…	511	512
6	**6S**	**6A**	**6D**	**64**	**65**	~~66~~	…	611	612
⋮	⋮	⋮	⋮	⋮	⋮	⋮		⋮	⋮
11	**11S**	**11A**	**11D**	**114**	**115**	**116**	---	~~1111~~	1112
12	**12S**	**12A**	**12D**	**124**	**125**	**126**	---	**1211**	~~1212~~

The sample space begins with 12 × 12, or 144 possible outcomes. First strike out the doubles because the same student cannot be chosen twice. Order is not important, so shade out all the duplicate outcomes, such as SA and AS. The size of the remaining sample space is: 1 + 2 + 3 + ... + 10 + 11 = 66. Circle the outcomes with 2 friends together.

$P(2 \text{ friends}) = \frac{3}{66} = \frac{1}{22}$

11.
A — B — C — D → ABCD
A — B — D — C → ABDC
A — C — B — D → ACBD
A — C — D — B → ACDB
A — D — B — C → ADBC
A — D — C — B → ADCB
B — A — C — D → BACD
B — A — D — C → BADC
B — C — A — D → BCAD
B — C — D — A → BCDA
B — D — A — C → BDAC
B — D — C — A → BDCA
C — A — B — D → CABD
C — A — D — B → CADB
C — B — A — D → CBAD
C — B — D — A → CBDA
C — D — A — B → CDAB
C — D — B — A → CDBA



There are 24 permutations.

$P(\text{any one}) = \frac{1}{24}$

12.

AB	~~BA~~	~~CA~~	~~DA~~	~~EA~~
AC	BC	~~CB~~	~~DB~~	~~EB~~
AD	BD	CD	~~DC~~	~~EC~~
AE	BE	CE	DE	~~ED~~

There are 10 combinations.

$P(\text{any one}) = \frac{1}{10}$

13. Drawing cards from the deck are dependent events because the cards are not replaced.

$P(\text{first card ace}) = \frac{4}{52}$

$P(\text{second card ace}) = \frac{3}{51}$

$P(\text{third card ace}) = \frac{2}{50}$

$P(\text{fourth card ace}) = \frac{1}{49}$

Recall that the Fundamental Counting Principle states that you can find the total number of outcomes for two or more experiments by multiplying the number of outcomes for each separate experiment.

$P(\text{first four cards aces})$

$= \frac{4}{52} \cdot \frac{3}{51} \cdot \frac{2}{50} \cdot \frac{1}{49}$

$= \frac{1}{13} \cdot \frac{1}{17} \cdot \frac{1}{25} \cdot \frac{1}{49}$

$= \frac{1}{13 \cdot 17 \cdot 25 \cdot 49}$

$= \frac{1}{270{,}725}$

14. **a.** The tree diagram shows the sample space has 8 possible outcomes. One of those consists of 2 blue plates.

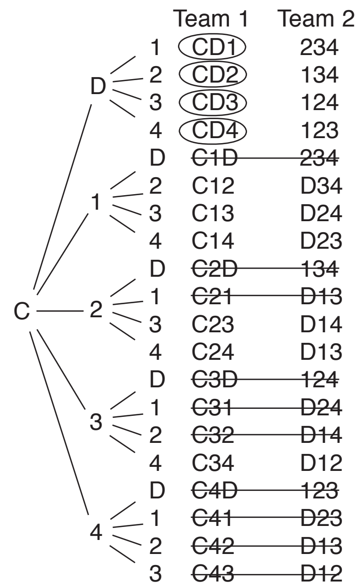
$P(\text{2 blue}) = \frac{1}{8}$

b. Five of the 8 possible outcomes are yellow and blue in any order.

$P(\text{mixed color}) = \frac{5}{8}$

15. Possible answer: Create tables of all the possible equally likely outcomes of an experiment. If order does not matter, do not duplicate repeated pairs. Use the tables to find the sample space and desired outcomes.

16. Cho = C Darla = D
other1 = 1 other2 = 2
other3 = 3 other4 = 4



$P(\text{CD teammates}) = \frac{4}{10} = \frac{2}{5}$

17. B; ${}_6P_1 = 6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$

$P(\text{1 of 6-letter permutations}) = \frac{1}{6!} = \frac{1}{720}$

The correct answer choice is B.

18. 120; the events are dependent, so there are $10 \cdot 9 \cdot 8 = 720$ permutations. But since order does not matter, there are 6 different ways to choose 3 president, which will cause duplicates. The sample space will be narrowed down to $720 \div 6 = 120$.

READY TO GO ON

1. Independent; the results of one number cube does not affect the results of the other.

2. Dependent; without replacement, the deck contains fewer cards from which to choose.

3. $P(\text{blue then yellow}) = P(\text{blue}) \cdot P(\text{yellow after blue})$

$= \frac{8}{15} \cdot \frac{7}{14}$

$= \frac{4}{15}$

4. $P(\text{blue and blue}) = P(\text{blue}) \cdot P(\text{blue})$

$= \frac{8}{15} \cdot \frac{8}{15}$

$= \frac{64}{225}$

5. $8 \cdot 7 = 56$
Each combination is listed twice, so there are $56 \div 2 = 28$ way to choose 2 juices.

6. $12 \cdot 11 = 132$
Each combination is listed twice, so there are $132 \div 2 = 66$ ways two students can volunteer.

7. $9 \cdot 8 = 72$
Each combination is listed twice, so there are $72 \div 2 = 36$ combinations of two sides.

8. $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$
There are 24 possible relays.

9. $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$
There are 120 different passcodes.

10. $P(\text{first vowel}) = \frac{9}{21}$

$P(\text{second vowel}) = \frac{8}{20}$

$P(\text{first vowel}) \cdot P(\text{second vowel}) = \frac{9}{24} \cdot \frac{8}{20}$

$= \frac{72}{420} = \frac{6}{35}$

The probability that Tina chooses two vowels is $\frac{6}{35}$.

11. This question is above the content of this lesson. The answer is incorrect. The answer given did not account for all 12 players being available for a starting position in the second week.

STUDY GUIDE: REVIEW

1. independent events **2.** combination

3. Unlikely, only one outcome has a sum of 12.

4. Impossible, there is no outcome that has a sum of 24.

5. $P(\text{above } 82) \approx \frac{\text{number of grades above 82}}{\text{number of grades}}$

$\approx \frac{10}{15}$

$\approx \frac{2}{3}$

The experimental probability that Sami's next grade is above 82 is approximately $\frac{2}{3}$.

6. $P(\text{82 or below}) \approx \frac{\text{number of grades 82 or below}}{\text{number of grades}}$

$\approx \frac{5}{15}$

$\approx \frac{1}{3}$

The experimental probability that Sami's next grade is 82 or below is approximately $\frac{1}{3}$.

7. The possible outcomes are R1, R2, R3, R4, W1, W2, W3, W4, B1, B2, B3 and B4.

8. There are 12 possible outcomes.

9. $P(\text{girl}) = \frac{9}{21} = \frac{3}{7} \approx 0.43 \approx 43\%$

10. There are 8 possible outcomes. Tails on each is one outcome.
$\frac{1}{8}$, 0.125, 12.5%

11. Set up an equation and solve for x.
40% of 50 $= x$

$\frac{4}{10} \cdot 50 = x$

$20 = x$

Tim can expect to make 20 goals out of 50 attempts.

12. 1, 3, and 5 are the odd numbers on a number cube. There are 6 numbers in all

$P(\text{odd}) = \frac{1}{2}$

$\frac{1}{2} = \frac{x}{12}$

$\frac{1}{2} \cdot 12 = \frac{x}{12} \cdot 12$

$6 = x$

You can expect an odd number to appear 6 out of 12 times.

13. The first selection changes the number of tags left, and may change the number of multiples of 9 left, so the events are dependent.

$P(\text{multiple of } 5) = \frac{8}{40} = \frac{1}{5}$

$P(\text{multiple of } 9) = \frac{4}{39}$

$P(\text{multiple of 5, then multiple of 9})$
$= P(A) \cdot P(B \text{ after } A)$
$= \frac{1}{5} \cdot \frac{4}{39}$
$= \frac{4}{195}$

The probability that Glenn picks a multiple of 5 and then a multiple of 9 is $\frac{4}{195}$.

14. Because the outcome of the first drawing does not affect the outcome of the second, the events are independent.

$P(\text{i and i}) = P(\text{i}) \cdot P(\text{i})$

$= \frac{2}{11} \cdot \frac{2}{11}$

$= \frac{4}{121}$

15. Let the digits 1–5 represent each piece of fruit. Here is the first branch of a tree diagram.
1: 2, 3, 4, 5
Each of the 5 branches will have 4 combinations. $5 \cdot 4 = 20$. However, each combination will be listed three times, so there are $20 \div 2 = 10$ ways to select 2 pieces of fruit.

16. $7 \cdot 6 = 42$
Each of the combinations will be listed twice, so there are $42 \div 2 = 21$ different groups.

17. Let digits 1–9 represent the balloons.
1: 2, 3, 4, 5, 6, 7, 8, 9
2: 3, 4, 5, 6, 7, 8, 9
3: 4, 5, 6, 7, 8, 9
4: 5, 6, 7, 8, 9
5: 6, 7, 8, 9
6: 7, 8, 9
7: 8, 9
8: 9
There are 36 combinations.

18. 10! = 3,628, 800
There are 3,628, 800 different batting orders.

19. 6! = 720
There are 720 different ways to arrange the letters.

20. 5! = 120
They can line up 120 different ways.

21. There are $4 \cdot 3 \cdot 2 \cdot 1 = 24$ ways the four friends can sit at the table.
There are 8 ways that Luis and Pam will sit across from each other.

P(Luis and Pam will sit across from each other) $= \frac{8}{24} = \frac{1}{3}$

P(Luis and Pam will sit across from each other two days in a row) $= \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9}$

The probability that Luis and Pam will sit across from each other two days in a row is $\frac{1}{9}$.

22. Let C = Chase, T = Ty, L = Lincoln, and H = Hector.
C: CT, CL, CH
T: TC, TL, TH
L: LC, LT, LH
H: HC, HT, HL
Each combination is listed twice. So the coach can choose two players $12 \div 2 = 6$ ways.

$P(\text{Chase and Lincoln start}) = \frac{1}{6}$

$P(\text{Chase and Lincoln start two games in a row})$
$= \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$
The probability that Chase and Lincoln start two games in a row is $\frac{1}{36}$.

CHAPTER TEST

1. as likely as not; There is the same number of orange cubes as black cubes.

2. unlikely; There are less white cubes than any other colored cube.

3. impossible; There are not any purple cubes.

4. $\frac{7}{20} = 0.35 = \frac{35}{100}$
He can expect the coin to land heads up in 35 out of the next 100 tosses.

5. The number 10 does not appear on the spinner.
$P(10) = \frac{0}{8} = 0$

6. Number of sizes: 8
Number of colors: 3
$8 \cdot 3 = 24$
24 combinations are possible.

7. There are 12 different combinations he can use to plan vacations: train-skiing, train-skating, train-snowboarding, train-hiking, bus-skiing, bus-skating, bus-snowboarding, bus-hiking, plane-skiing, plane-skating, plane snowboarding, plane-hiking.

8. $P(\text{odd number}) = \frac{5}{10} = \frac{1}{2} = 0.5 = 50\%$

9. $P(\text{composite number}) = \frac{5}{10} = \frac{1}{2} = 0.5 = 50\%$

10. $P(\text{number greater than } 10) = \frac{0}{10} = 0.0 = 0\%$

11. $P(\text{red and tails}) = \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8}$

12. The first selection changes the number of cards left, and may change the number of cards labeled vanilla left, so the events are dependent.
$P(\text{vanilla}) = \frac{1}{4}$

$P(\text{chocolate}) = \frac{1}{3}$

$$P(\text{vanilla, then chocolate}) = P(\text{A}) \cdot P(\text{B after A}) = \frac{1}{4} \cdot \frac{1}{3} = \frac{1}{12}$$

The probability of choosing vanilla then chocolate is $\frac{1}{12}$.

13. 2, 3, 4, 5, and 6 are the numbers greater than 1 on a number cube. There are 6 numbers in all
$P(\text{greater than } 1) = \frac{5}{6}$

$$\frac{5}{6} = \frac{x}{12}$$
$$\frac{5}{6} \cdot 12 = \frac{x}{12} \cdot 12$$
$$10 = x$$

You can expect a number greater than 1 to appear 10 out of 12 times.

14. $10 \cdot 9 = 90$
Each of the combinations are listed twice, so there are $90 \div 2 = 45$ combinations.

15. $P(\text{Jen wore her favorite pair of shoes on Monday})$
$= \frac{1}{6}$
$P(\text{Jen wore her favorite pair of shoes on Tuesday})$
$= \frac{1}{6}$
$P(\text{Jen wore her favorite pair of shoes on Monday or Tuesday}) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$
The probability that Jen wore her favorite pair of shoes on Monday or Tuesday is $\frac{1}{3}$.