Measurement and Geometry

2. parallelogram

Solutions Key

ARE YOU READY?

3. radius; diameter

Nearest ten: 1,540

Nearest hundred: 1,500

1. trapezoid

4. 1,535

5. 294

	Nearest ten: 290 Nearest hundred: 300		
6.	30,758 Nearest ten: 30,760 Nearest hundred: 30,80	0	
7.	497 Nearest ten: 500 Nearest hundred: 500		
8.	6.18 Nearest whole number: Nearest tenth: 6.2	6	
9.	10.50 Nearest whole number: Nearest tenth: 10.5	11	
10.	513.93 Nearest whole number: Nearest tenth: 513.9	514	L
11.	29.06 Nearest whole number: Nearest tenth: 29.1	29	
12.	$5.63 \cdot 8 = 45.04$	13.	9.67 · 4.3 = 41.581
14.	8.34 · 16 = 133.44	15.	$6.08 \cdot 0.56 = 3.4048$
16.	0.82 · 21 = 17.22	17.	$2.74 \cdot 6.6 = 18.084$
18.	40 · 9.54 = 381.60	19.	$0.33 \cdot 0.08 = 0.0264$
20.	$2 \cdot 9 + 2 \cdot 9$ $18 + 12$ 30	21.	2(15 + 8) 2(23) 46
22.	$4 \cdot 6.8 + 7 \cdot 9.3$ 27.2 + 65.1 92.3	23.	14(25.9 + 13.6) 14(39.5) 553
24.	$\begin{array}{c} (27.3 + 0.7) \div 2^2 \\ 28 \div 4 \\ 7 \end{array}$	25.	$5 \cdot 3^3 - 8.02$ $5 \cdot 27 - 8.02$ 135 - 8.02 126.98
26.	$(63 \div 7) \cdot 4^2$ 9 \cdot 16 144	27.	$\begin{array}{r} 1.1 + 3 \cdot 4.3 \\ 1.1 + 12.9 \\ 14 \end{array}$
28.	$66 \cdot [5 + (3 + 3)^2] \\ 66 \cdot [5 + 36] \\ 66 \cdot [41] \\ 2,706$	29.	trapezoid
30.	triangle	31.	parallelogram

LESSON 1

Think and Discuss

- 1. Possible answer: Add the lengths of the four sides. Or multiply twice the length and twice the width, and then find the sum of the products.
- **2.** Possible answer: Multiply the radius by 2 to find the diameter, and then substitute the value for the diameter into the formula.

Exercises

1. P = 6 + 4 + 8P = 18The perimeter of the triangle is 18 m. **2.** P = 7 + 5 + 7 + 5P = 24The perimeter of the parallelogram is 24 in. **3.** P = 8 + 8 + 8 + 8P = 32The perimeter of the square is 32 ft. **4.** $P = 2\ell + 2w$ $P = (2 \cdot 12) + (2 \cdot 6)$ P = 24 + 12P = 36 The perimeter of the rectangle is 36 in. **5.** $P = 2\ell + 2w$ $P = (2 \cdot 8) + (2 \cdot 2)$ P = 16 + 4P = 20The perimeter of the rectangle is 20 m. **6.** $P = 2\ell + 2w$ $P = \left(2 \cdot 4\frac{1}{2}\right) + \left(2 \cdot 1\frac{1}{2}\right)$ P = 9 + 3P = 12The perimeter of the rectangle is 12 ft. **7.** $C = \pi d$ $C \approx 3.14 \cdot 12$ $C \approx 37.68$ The circumference of the circle is about 37.7 m. **8.** $C = \pi d$ $C \approx 3.14 \cdot 3$ $C \approx 9.42$ The circumference of the circle is about 9.4 ft. **9.** $C = 2\pi r$ $C \approx 2 \cdot \frac{22}{7} \cdot 21$ $C \approx 132$ The circumference of the circle is about 132 in. 10. $C = \pi d$ 440 \approx 3.14 \cdot d $440 \div 3.14 \approx 3.14d \div 3.14$ $140.127 \approx d$ The diameter of the Ferris wheel is about 140 ft. **11.** P = 12 + 12 + 12 + 12P = 48The perimeter of the rhombus is 48 cm. **12.** P = 7 + 13 + 10P = 30The perimeter of the triangle is 30 ft. **13.** P = 8 + 10 + 10 + 16P = 44The perimeter of the trapezoid is 44 m. **14.** $P = 2\ell + 2w$ $P = (2 \cdot 8) + (2 \cdot 5)$ P = 16 + 10P = 26The perimeter of the rectangle is 26 in. **15.** $P = 2\ell + 2w$ $P = (2 \cdot 3) + (2 \cdot 1)$ P = 6 + 2*P* = 8 The perimeter of the rectangle is 8 ft. **16.** $P = 2\ell + 2w$ $P = (2 \cdot 10.2) + (2 \cdot 8)$ P = 20.4 + 16P = 36.4The perimeter of the rectangle is 36.4 cm. **17.** $C = \pi d$ $C \approx \frac{22}{7} \cdot 35$ $C \approx \frac{770}{7}$ $C \approx 110$ The circumference of the circle is about 110 cm. **18.** $C = 2\pi r$ $C\approx 2\cdot 3.14\cdot 3$ $C \approx 18.84$ The circumference of the circle is about 18.8 m. **19.** $C = 2\pi r$ $C \approx 2 \cdot 3.14 \cdot 5.1$ $C \approx 32.028$ The circumference of the circle is about 32.0 in. 20. $C = \pi d$ $91 \approx 3.14 \cdot d$ $91 \div 3.14 \approx 3.14 d \cdot 3.14$ $28.98 \approx d$ The diameter is about 29 in. 21. $C = \pi d$ $17.8 \approx 3.14 \cdot d$ $17.8 \div 3.14 \approx 17.8d \cdot 3.14$ $5.67 \approx d$ $r = d \div 2$ $= 5.67 \div 2$ = 2.835The diameter is about 5.7 m, and the radius is about 2.8 m. **22.** $C = 2\pi r$ $C\approx 2\cdot 3.14\cdot 6.7$ $C \approx 42.076$ $d = r \cdot 2$ $= 6.7 \cdot 2$ = 13.4The circumference is about 42.1 yd, and the diameter is about 13.4 yd.

23. $C = \pi d$ $C \approx 3.14 \cdot 10.6$ $C \approx 33.284$ $r = d \div 2$ $= 10.6 \div 2$ = 5.3 The circumference is about 33.3 in., and the radius is about 5.3 in. **24.** $C = \pi d$ $\pi \approx \pi \cdot d$ $\pi \div \pi \approx \pi d \div \pi$ $1 \approx d$ $r = d \div 2$ $= 1 \div 2$ $=\frac{1}{2}$ The diameter is about 1 unit, and the radius is about a $\frac{1}{2}$ unit. $C = \pi d$ 25. $C \approx \pi \cdot 24.2$ $C \approx 75.988$ ft $\frac{12 \text{ in.}}{1 \text{ ft}} = \frac{x}{76 \text{ ft}}$ 1 ft $1 \cdot x = 12 \cdot 76$ x = 912 in. $C \approx 912$ in. 912 in. \div 57-in. strands = 16 strands 16 strands of lights are needed to surround the patio edge. **26.** P = 34 + 45 + 62 = 141He travels 141 mi roundtrip. $C = \pi d$ 27. $301.5 \approx 3.14 \cdot d$ $301.5 \div 3.14 \approx 3.14d \div 3.14$ $96 \approx d$ The diameter of the rotunda is about 96 ft. 28. Possible answer: Lay the string all the way around the outside of the object and cut it. Then take the string and measure it with a ruler. 29. Possible answer: How many feet of ribbon would be needed to trim the edge of the bulletin board? 30. Subtract twice the length from the perimeter and divide the result by 2. **31.** $25\frac{1}{2} \div 9 = \frac{51}{2} \cdot \frac{1}{9} = \frac{17}{2} \cdot \frac{1}{3} = \frac{17}{6} = 2\frac{5}{6}$ Each side of the nonagon is $2\frac{5}{6}$ in. 32. D; 47.1 inches $C = \pi d$ $C \approx 3.14 \cdot 15$ $C \approx 47.1$ The circumference of the circle is about 47.1 in. 33. 28 $P = 2\ell + 2w$ $P=2\cdot 6+2\cdot 8$ P = 12 + 16P = 28The perimeter is 28 ft.

Think and Discuss

- 1. Possible answer: The formula for the area of a circle uses the radius of a circle. If the diameter is given, you must first divide the diameter by 2.
- 2. Possible answer: a clock; estimate the radius of the clock, square that number, and then multiply by 3.14.
 (6 in.)² · 3.14 = 113.04 in²

Exercises

1. $A = \pi r^2$ $A \approx 3.14 \cdot 5^2$ $A \approx 3.14 \cdot 25$ $A \approx 78.5$ The area of the circle is about 78.5 in². **2.** $A = \pi r^2$ $A \approx 3.14 \cdot 8^2$ $A \approx 3.14 \cdot 64$ $A \approx 200.96$ The area of the circle is about 201.0 cm². **3.** $A = \pi r^2$ $A \approx 3.14 \cdot 10^2$ $A \approx 3.14 \cdot 100$ $A \approx 314$ The area of the circle is about 314 yd². **4.** $A = \pi r^2$ $A \approx 3.14 \cdot 1.1^2$ $A \approx 3.14 \cdot 1.21$ $A \approx 3.7994$ The area of the circle is about 3.8 m^2 . **5.** $A = \pi r^2$ $A \approx \frac{22}{7} \cdot 7^2$ $A \approx \frac{22}{7} \cdot 49$ $A \approx \frac{22}{1} \cdot 7$ $A \approx 154$ The area of the pizza is about 154 in². 6. The diameter of the circle is 1.2 cm. $A = \pi r^2$ $A \approx 3.14 \cdot 0.6^2$ *A* ≈ 1.1304 Half of the circle is shaded, so $1.1304 \div 2 = 0.5652$. The area of the shaded part of the circle is about 0.6 cm². **7.** $A = \pi r^2$ $A \approx 3.14 \cdot 3^2$ $A \approx 3.14 \cdot 9$ $A \approx 28.26$ The area of the circle is about 28.3 in². **8.** $A = \pi r^2$ $A \approx 3.14 \cdot 16^2$ $A \approx 3.14 \cdot 256$ $A \approx 803.84$ The area of the circle is about 803.8 ft².

9. $A = \pi r^2$ $A \approx 3.14 \cdot 3.2^2$ $A \approx 3.14 \cdot 10.24$ *A* ≈ 32.1536 The area of the circle is about 32.2 yd². **10.** $A = \pi r^2$ $A \approx 3.14 \cdot 7.5^2$ $A \approx 3.14 \cdot 56.25$ $A \approx 176.625$ The area of the circle is about 176.6 cm^2 . **11.** $A = \pi r^2$ $A \approx \frac{22}{7} \cdot 14^2$ $A \approx \frac{22}{7} \cdot 196$ $A \approx \frac{22}{1} \cdot 28$ $A \approx 616$ The area of the pizza is about 616 cm². 12. The diameter of the circle measures 3 cm. $A = \pi r^2$ $A \approx 3.14 \cdot 1.5^2$ $A \approx 7.065$ A quarter of the circle is shaded, so $7.065 \div 4 = 1.76625$. The area of the shaded part of the circle is about 1.8 cm². **13.** $A = \pi r^2$ $A \approx 3.14 \cdot 75^2$ $A \approx 3.14 \cdot 5,625$ $A \approx 17,662.5$ The area of the circle is about 17,662.5 mi². **14.** $A = \pi r^2$ $A \approx 3.14 \cdot 4^2$ $A \approx 3.14 \cdot 16$ $A \approx 50.24$ The area of Kay's flower bed is about 50.2 ft². **15.** $A = \pi r^2$ $A \approx 3.14 \cdot 3^2$ $A \approx 3.14 \cdot 9$ $A \approx 28.26$ The area of one lid is about 28.3 cm². **16.** $C = 2\pi r$ $C \approx 2 \cdot 3.14 \cdot 7$ $C \approx 43.96$ The circumference is about 44.0 m. $A = \pi r^2$ $A \approx 3.14 \cdot 7^2$ $A \approx 3.14 \cdot 49$ $A \approx 153.86$ The area is about 153.9 m². **17.** $C = \pi d$ $C \approx 3.14 \cdot 18$ $C \approx 56.52$ The circumference is about 56.5 in. $A = \pi r^2$ $A \approx 3.14 \cdot 9^2$ $A \approx 3.14 \cdot 81$ $A \approx 254.34$ The area is about 254.3 in².

18. $C = \pi d$ $C \approx 3.14 \cdot 24$ $C \approx 75.36$ The circumference is about 75.4 ft. $A = \pi r^2$ $A \approx 3.14 \cdot 12^2$ $A \approx 3.14 \cdot 144$ $A \approx 452.16$ The area is about 452.2 ft². **19.** $C = 2\pi r$ $C \approx 2 \cdot 3.14 \cdot 6.4$ $C \approx 40.192$ The circumference is about 40.2 cm. $A = \pi r^2$ $A \approx 3.14 \cdot 6.4^2$ $A \approx 3.14 \cdot 40.96$ *A* ≈ 128.6144 The area is about 128.6 cm². $A = \pi r^2$ 20. **21.** $A = \pi r^2$ $113.04 \approx 3.14r^2$ $3.14 \approx 3.14r^2$ $1 \approx r^2$ $36 \approx r^2$ $6 \approx r$ $1 \approx r$ The radius is 6 cm. $A = \pi r^2$ 22. $28.26 \approx 3.14r^2$ $9 \approx r^2$ $3 \approx r$ The radius is 3 in.

- The radius is 1 ft.
- 23. a. The hiker walked 6 miles in any direction. This is equivalent to a circle with a radius of 6.
 - $A = \pi r^2$
 - $A \approx 3.14 \cdot 6^2$ $A \approx 3.14 \cdot 36$
 - $A \approx 113.04$
 - They would have to search about 113 mi². b. The hiker would have walked 9 miles in any
 - direction, which is a circle with a radius of 9. $A = \pi r^2$
 - $A \approx 3.14 \cdot 9^2$ $A \approx 3.14 \cdot 81$
 - $A \approx 254.34$

They would have to search about 254 mi². 254 - 113 = 141This is an additional 141 mi².

- 24. The diameter of the turbine is 187 ft.
 - $A = \pi r^2$
 - $A \approx 3.14 \cdot 93.5^{2}$
 - $A \approx 3.14 \cdot 8,742.25$
 - $A \approx 27,450.665$

The area covered by the turbine when it is rotating is about 27,450.7 ft².

- 25. No: the combined area of the two circles of the same radius is $2\pi r^2$, while the area of a circle with twice the radius is $\pi(2r)^2 = 4\pi r^2$.
- 26. Possible answer: What is the area that Chang painted, using 3.14 for π ?
- 27. Possible answer: First use the formula for the circumference, $C = 2\pi r$, to solve for the radius, *r*. Then plug the radius into the formula for the area of a circle, $A = \pi r^2$.

- **28.** The area is multiplied by a factor of n^2 . For instance, a circle with a radius of 5 has an area of about 78.5 units². If the radius is multiplied by 5, then the area is about 1,962.5 units², which is 5² times, or 25 times, as large.
- 29. A, 3.7 square feet $A = \pi r^2$

$$30 \approx 3.14r^{2}$$

$$9.55 \approx r^{2}$$

$$3.09 \approx r$$

$$3.09 - 2 = 1.09$$

$$A = \pi r^{2}$$

$$A \approx 3.14 \cdot 1.09^{2}$$

$$A \approx 3.73$$
The area of the assessed size

The area of the second circle is 3.7 square feet.

- 30. No, the 24-in. pizza is actually 4 times the size of the 12-in. pizza. When the diameter is doubled, the area becomes 2^2 , or 4, times as large. 12-in. diameter pizza: $A = \pi r^2$
 - $A \approx 3.14 \cdot 6^2$ $A \approx 113.04 \text{ in}^2$ 24-in. diameter pizza: $A = \pi r^2$ $A \approx 3.14 \cdot 12^2$ $A \approx 452.16 \text{ in}^2$

LESSON 3

Think and Discuss

- 1. Possible answer: Divide the figure horizontally into two rectangles (with areas 12 in² and 16 in²), or divide the figure vertically into two rectangles (with areas 24 in² and 4 in²).
- 2. Possible answer: The area will stay the same, 28 in², but the perimeter will change. The perimeter of the irregular figure is 24 in., whereas the perimeter of the two rectangles is 16 in. + 20 in. = 36 in.

Exercises

- **1.** 27 full squares + 4 partially-filled squares is about 28 ft².
- **2.** 8 full squares + 6 half-filled squares + 4 partially-filled squares is about 14 ft².
- 3. Top rectangle:

$$A = \ell w$$

$$A = 8 \cdot 10$$

$$A = 80$$

Bottom rectangle:

 $A = \ell w$ $A = 18 \cdot 8$ *A* = 144

80 + 144 = 224The area of the figure is 224 ft².

4. Rectangle: $A = \ell w$ $A = 12 \cdot 26$ A = 312Half circle: $A = \frac{1}{2}\pi r^2$ $A\approx\frac{1}{2}\cdot3.14\cdot5^2$ $A \approx \frac{1}{2} \cdot 78.5$ $A \approx 39.25$ 312 + 39.25 = 351.25 The area of the figure is 351.25 m². 5. Rectangle: $A = \ell w$ $A = 3 \cdot 2$ A = 6Triangle: $A = \frac{1}{2}bh$ $A = \frac{1}{2} \cdot 8 \cdot 8$ $A = \frac{1}{2} \cdot 64$ A = 326 + 32 = 38The area of the figure is 38 ft². 6. Rectangle: $A = \ell w$ $A = 4.5 \cdot 2$ A = 9Two half circles: $A = 2 \cdot \left(\frac{1}{2}\pi r^2\right)$ $A \approx 2 \cdot \left(\frac{1}{2} \cdot 3.14 \cdot 1^2\right)$ $A \approx 2 \cdot (1.57)$ $A \approx 3.14$ 9 + 3.14 = 12.14The area of the figure is 12.14 ft². 7. 21 full squares + 4 half-filled squares + 3 partially-filled squares is about 25 ft². 8. 22 full squares + 6 partially-filled squares is about 26 ft². 9. Rectangle: $A = \ell w$ $A = 12 \cdot 6$ A = 72 Two half circles: $A = 2 \cdot \left(\frac{1}{2}\pi r^2\right)$ $A \approx 2 \cdot \left(\frac{1}{2} \cdot 3.14 \cdot 2^2\right)$ $A \approx 2 \cdot (6.28)$ $A \approx 12.56$ 72 + 12.56 = 84.56The area of the figure is 84.56 m^2 .

10. Rectangle 1: $A = \ell w$ $A = 8 \cdot 10$ A = 80Rectangle 2: $A = \ell w$ $A = 4 \cdot 7$ A = 28Rectangle 3: $A = \ell w$ $A = 3 \cdot 3$ A = 9 80 + 28 + 9 = 117 The area of the figure is 117 ft². **11.** Rectangle: $A = \ell w$ $A = 9 \cdot 4$ A = 36 Triangle 1: $A = \frac{1}{2}bh$ $A = \frac{1}{2} \cdot 4 \cdot 2$ $A = \frac{1}{2} \cdot 8$ A = 4Triangle 2: $A = \frac{1}{2}bh$ $A = \frac{1}{2} \cdot 4 \cdot 3$ $A=\frac{1}{2}\cdot 12$ A = 636 + 4 + 6 = 46The area of the figure is 46 cm^2 . 12. Trapezoid: A = bh $A = 20 \cdot 6$ A = 120Half circle: $A = \frac{1}{2}\pi r^2$ $A \approx \frac{1}{2} \cdot 3.14 \cdot 10^2$ $A \approx 157$ 120 + 157 = 277 The area of the figure is 277 m². 13. Top rectangle: $A = \ell w$ $A = 3 \cdot 4$ A = 12 Bottom rectangle: $A = \ell w$ $A = 9 \cdot 2$ A = 18 12 + 18 = 30The area of the figure is 30 ft^2 . P = 2 + 3 + 4 + 3 + 4 + 3 + 2 + 9 = 30The perimeter of the figure is 30 ft.

14. Square: $A = \ell w$ $A = 5 \cdot 5$ A = 25Triangle: $A = \frac{1}{2}bh$ $A = \frac{1}{2} \cdot 3 \cdot 4$ $A = \frac{1}{2} \cdot 12$ A = 625 + 6 = 31The area of the figure is 31 m^2 . P = 5 + 5 + 4 + 5 + 2 + 5 = 26The perimeter of the figure is 26 m. 15. Rectangle: $A = \ell w$ $A = 18 \cdot 12$ A = 216Half circle: $A = \frac{1}{2}\pi r^2$ $A \approx \frac{1}{2} \cdot 3.14 \cdot 5^2$ $A \approx \frac{1}{2} \cdot 78.5$ $A \approx 39.25$ 216 + 39.25 = 255.25The area of the figure is 255.25 m^2 . $C = \pi d$ $C \approx 3.14 \cdot 10$ $C \approx 31.4$ Half circle: $C\approx 31.4\,\div\,2=15.7$ P = 12 + 18 + 12 + 15.7 + 8 = 65.7The perimeter of the figure is 65.7 m. 16. No, rearranging the composite figure does not change its perimeter or area. P = s + s + s + s + s17. 44 = x + x + x + 7 + 744 = 3x + 1444 - 14 = 3x + 14 - 1430 = 3x10 ft = x18. 8,5) (D(3,2) H(-8, -4)G (6, Rectangle 1: $A = \ell w$ $A = 4 \cdot 9$ A = 36Rectangle 2: $A = \ell w$

 $A = 7 \cdot 6$ A = 42Rectangle 3: $A = \ell w$ $A = 3 \cdot 2$ A = 636+42+6=84The area of the figure is 84 units². P = 9 + 4 + 3 + 7 + 4 + 3 + 2 + 14 = 46The perimeter of the figure is 46 units. 19. A; 5 m Draw a Picture and Guess and Check Area of total figure = 32.5; area of triangle = 7.532.5 - 7.5 = 25, area of square $A = s^2$ $25 = s^2$ 5 m = s20. Draw a vertical line to divide the figure into a rectangle (area 30 in²) and triangle (area 27 in²). The total area is 30 + 27 = 57 in². 21. Find the area of the entire rectangle. Then subtract the area of the two half circles. Rectangle: $A = \ell w$ $A = 10 \cdot 8$ A = 80Two half circles: $A = 2 \cdot \left(\frac{1}{2}\pi r^2\right)$ $A \approx 2 \cdot \left(\frac{1}{2} \cdot 3.14 \cdot 4^2\right)$ $A \approx 2 \cdot (25.12)$ $A \approx 50.24$ 80 - 50.24 = 29.76The area of the figure is 29.76 cm². $C = \pi d$ $C \approx 3.14 \cdot 8$ $C \approx 25.12$ Half circle: $C\approx 31.4\div 2=12.56$ P = 10 + 12.56 + 10 + 12.56 = 45.12The perimeter of the figure is 45.12 cm. 22. B; 24 in. If the area of one triangle is 6 in², use the formula for the area of a triangle to find the possible base and height lengths. $A = \frac{1}{2}bh$ $6 = \frac{1}{2}bh$ 12 = bhPossible bh combinations that equal 12: 1×12 2×6 3×4 Try these combinations as perimeters. 1×12 combination: P = 1 + 12 + 1 + 12 = 26 in. 2×6 combination: P = 2 + 6 + 2 + 6 = 16 in. 3×4 combination: P = 3 + 4 + 3 + 4 = 14 in. 24 inches is the only choice that cannot be the perimeter of this rectangle.

READY TO GO ON?

1. P = 10.2 + 3.0 + 10.2 + 3.0 = 26.4The perimeter is 26.4 m. 2. $C = \pi d$ $94 \approx 3.14 \cdot d$ $29.936 \approx d$ The diameter is about 30 cm. **3.** P = 6.5 + 22 + 6.5 + 22 = 57The perimeter is 57 cm. **4.** P = 12 + 5 + 9 + 6 = 32The perimeter is 32 in. **5.** P = 50 + 30 + 40 = 120The perimeter is 120 ft. **6.** $A = \pi r^2$ $A \approx 3.14 \cdot 14^2$ $A \approx 615.44 \approx 615.4$ OR $A \approx \frac{22}{7} \cdot 14^2$ $A \approx \frac{22}{7} \cdot \frac{196}{1}$ $A \approx \frac{22}{1} \cdot \frac{28}{1} \approx 616$ The area of the circle is 615.4 ft² or 616 ft². **7.** $A = \pi r^2$ $A \approx \frac{22}{7} \cdot \left(\frac{35}{4}\right)^2$ $A\approx\frac{22}{7}\cdot\frac{1,225}{16}$ $A \approx \frac{11}{1} \cdot \frac{175}{8}$ $A \approx \frac{1,925}{8} \approx 240\frac{5}{8}$ The area of the clock face is 240 in² (if using 3.14 for π) or 241 in² (if using $\frac{22}{7}$ for π). **8.** $A = \pi r^2$ $\begin{array}{l} 28.26\approx 3.14r^2\\ 9\approx r^2 \end{array}$ $3 \approx r$ The radius is 3 mm. **9.** $A = \pi r^2$ $153.86\approx 3.14r^2$ $49\approx r^2$ $7 \approx r$ The radius is 7 yd. **10.** $A = \pi r^2$ $50.24 \approx 3.14r^2$ $16 \approx r^2$ $4 \approx r$ The radius is 4 in.

 $A = \ell w$ $A = 21 \cdot 15$ A = 315 Rectangle 2: $A = \ell w$ $A = 15 \cdot 6$ A = 90315 + 90 = 405The area of the figure is 405 cm². 12. Rectangle: $A = \ell w$ $A = 13 \cdot 7$ A = 91Triangle 1: $A = \frac{1}{2}bh$ $A = \frac{1}{2} \cdot 4 \cdot 3$ A = 6Triangle 2: $A = \frac{1}{2}bh$ $A = \frac{1}{2} \cdot 7 \cdot 3$ A = 11.5 91 + 6 + 10.5 = 107.5The area of the figure is 107.5 ft². 13. Rectangle: $A = \ell w$ $A = 9 \cdot 11$ A = 99Two half circles: $A = 2 \cdot \left(\frac{1}{2}\pi r^2\right)$ $A \approx 2 \cdot \left(\frac{1}{2} \cdot 3.14 \cdot 5.5^2\right)$ $A \approx 2 \cdot (47.4925)$ $A \approx 94.985$ 99 + 94.985 = 193.985

11. Rectangle 1:

LESSON 4

Think and Discuss

The area of the figure is 194 yd^2 .

- Possible answer: Prisms and pyramids are polyhedrons, so all of their faces are polygons. A prism has two parallel, congruent bases, and a pyramid has one base.
- 2. Possible answer: Both have two parallel, congruent bases. The bases of cylinders are circles, and the bases of prisms are polygons. The bases of cylinders are connected by a curved surface, and the bases of prisms are connected by polygonal faces.

Exercises

 There is one base, and it is a pentagon. The other faces are triangles. The figure is a pentagonal pyramid.

- **2.** There are two bases, and they are octagons. The other faces are rectangles. The figure is a octagonal prism.
- There are two bases, and they are triangles. The other faces are rectangles. The figure is a triangular prism.
- **4.** The figure is not a polyhedron. The figure is a cone.
- **5.** The figure is a polyhedron. The figure is a hexagonal pyramid.
- **6.** The figure is not a polyhedron. The figure is a sphere.
- There is one base, and it is a triangle. The other faces are triangles. The figure is a triangular pyramid.
- There are two bases, and they are rectangles. The other faces are rectangles. The figure is a rectangular prism.
- **9.** There is one base, and it is a hexagon. The other faces are triangles. The figure is a hexagonal pyramid.
- **10.** The figure is a polyhedron. The figure is an octagonal prism.
- **11.** The figure is not a polyhedron. The figure is a cylinder.
- **12.** The figure is a polyhedron. The figure is a triangular pyramid.
- **13.** The figure is made up of two parallel, congruent square bases and four other polygonal faces. Square prism
- The figure is made up of two parallel, congruent circular bases and one curved surface. Cylinder
- **15.** The figure is made up of one triangular base and three other triangular faces. Triangular pyramid
- **16.** By definition, all points or a surface that are equidistance from a given point form a sphere.
- 17. The figure is made up of two parallel, congruent bases.Possible answer: cylinder, rectangular prism
- **18.** The figure is made up of one base. Possible answer: cone, rectangular pyramid
- 19. Rectangular pyramid
- 20. Possible answer: cylinders, rectangular prism
- 21. Cylinder
- **22.** Possible answer: The Unisphere is not a true sphere because it has gaps in its surface.
- 23. A; Rectangular prism 24. G; Pyramid

LESSON 5

Think and Discuss

- **1.** Possible answer: A cubic unit is the unit used when finding volume; cubic yards, or yd³.
- 2. Possible answer: Both are measured in cubic units, and both are found by multiplying the area of the base times the height. The areas of their bases are found differently: One is the area of a circle and the other is the area of a polygon.

Exercises

1. V = Bh $= (8 \cdot 6)(5)$ $= 240 \text{ in}^3$ The volume is 240 in³. **3.** V = Bh **2.** V = Bh $= (\frac{1}{2} \cdot 10 \cdot 5)(20)$ $= 500 \text{ mm}^3$ The volume is 500 mm³.

$$= (2.25 \cdot 0.5)(3.5) \\= 3.9375 \text{ in}^3$$

The volume is 3.9375 in³.

4.
$$V \approx \pi r^2 h$$

 $\approx \pi(2)^2(6)$ $\approx \pi(4)(6)$

$$\approx$$
 75.4 cm³

The volume of the can is about 75.4 cm³.

5.
$$V = Bh + \pi r^2 h$$

$$\approx (6)(3)(7) + (3.14)(2)^{2}(5)$$

≈ 126 + 62.8 ≈ 188.8

The volume of the composite figure is about 188.8 m^3 .

6.
$$V = Bh + Bh$$

= (8)(5)(4)+ $\frac{1}{2}$ (3)(8)(5)
= 160 + 60

= 220

The volume of the composite figure is 220 ft³.

7.
$$V = Bh$$

 $= (\frac{1}{2} \cdot 4 \cdot 8)(12)$
 $= 192 \text{ ft}^3$
The volume is 192 ft³.
8. $V = Bh$
 $= (20 \cdot 15)(9)$
 $= 2,700 \text{ cm}^3$
The volume is 2,700 cm³.

)

9.
$$V = Bh$$

= $(6 \cdot 0.4)(5.6)$
= 13.44 in.³
The volume is 13.44 in³.
10. $V \approx \pi r^2 h$
 $\approx \pi (2)^2 (28)$
 $\approx \pi (4) (28)$

The volume of the paper towel roll is about 351.7 cm³.

11.
$$V = Bh + \pi r^2 h$$

= (5)(5)(5) + $\frac{1}{2}$ (6)(5)(5)
= 125 + 75
= 200

 \approx 351.7 cm³

The volume of the composite figure is 200 in³.

12. $V = Bh + \pi r^2 h$ $\approx (8)(6)(3) + (3.14)(2)^{2}(5)$ $\approx 144 + 62.8$ ≈ 206.8 The volume of the composite figure is about 206.8 cm³. **13.** First find the missing leg of the triangle. $a^2 + b^2 = c^2$ $6^2 + b^2 = 10^2$ $36 + b^2 = 100$ -36 $\overline{b^2} = \frac{-36}{64}$ b = 8The length of the missing leg is 8 m. The area of each triangular base is $\frac{1}{2} \cdot 6 \cdot 8 = 24$. Find the volume of the prism. V = Bh $= \left(\frac{1}{2} \cdot 6 \cdot 8\right) (12)$ $= 288 \text{ m}^3$ **14.** $V \approx \pi r^2 h$ $= (3.14)(1.5)^{2}(4)$ = 28.26 The candle is 28.26 in³. (26.28)(16.38) = 462.90The candle is 462.90 cm³. **15.** *V* = *Bh* $=\left(\frac{1}{2}\cdot 4.5\cdot 3.5\right)(6)$ $= 47.25 \text{ ft}^3$ There are 47.25 ft³ of space in the tent. 16. Possible answer: The volumes are equal because a cylinder with a 3-inch diameter has a 1.5-inch radius. 17. Possible answer: For both, the volume is the area of the base times the height; the formulas for the base areas are different. 18. Find the volume of the outer cylinder. $V \approx \pi r^2 h$ $\approx \pi (4.2)^2 (15)$ $\approx 3(17.64)(15)$ \approx 830.8 cm³ Find the volume of the inner cylinder. $V \approx \pi r^2 h$

 $\approx \pi (3)^{2} (15)$ $\approx 3.14(9)(15)$ $\approx 423.9 \text{ cm}^{3}$ Subtract the two volumes. $830.8 - 423.9 \approx 406.9 \text{ cm}^{3}$ **19.** B; V = Bh $= (\frac{1}{2} \cdot 7 \cdot 4)(10)$ $= 140 \text{ in}^{3}$ The volume is 140 ft³. $\approx 72 \text{ in.}^{3}$ II: $V = (\frac{1}{2} \cdot 3 \cdot 3)(16)$ $\approx 72 \text{ in.}^{3}$ III: $V \approx \pi r^{2}h$

LESSON 6

Think and Discuss

1. Possible answer: Find the sum of the areas of the five faces, including only one base.

Exercises

1. $S = 2\ell w + 2\ell h + 2wh$ $S = (2 \cdot 9 \cdot 7) + (2 \cdot 9 \cdot 5) + (2 \cdot 7 \cdot 5)$ S = 126 + 90 + 70S = 286 The surface area of the prism is 286 ft^2 . **2.** $S = 2(\frac{1}{2})bh + bh + bh + b$ $= 2\left(\frac{1}{2}\right)(6)(10) + (10)(2.5) + (8)(2.5)$ + (8)(2.5)= 60 + 25 + 20 + 20 = 125 The surface area of the prism is 125 cm². **3.** $S = 2\pi r^2 + 2\pi rh$ $S \approx (2 \cdot 3.14 \cdot 3^2) + (2 \cdot 3.14 \cdot 3 \cdot 10)$ $S \approx 56.52 + 188.4$ $S \approx 244.92$ $S \approx 244.9$ The surface area of the cylinder is 244.9 m². **4.** $S = 2\pi r^2 + 2\pi rh$ $S \approx (2 \cdot 3.14 \cdot 15^2) + (2 \cdot 3.14 \cdot 15 \cdot 5)$ $S \approx 1,413 + 471$ S ≈ 1,884.0 The surface area of the cylinder is 1,884.0 in². 5. $S = bh + 2bh + 2bh + \frac{1}{4}\pi r^2 + \frac{1}{4}2\pi rw$ $\approx (8+8)(16) + 2(8)(8) + 2(16)(8)$ $+ 2 \cdot \frac{1}{4} (3.14)(8)^2 + \frac{1}{4} 2(3.14)(8)(16)$ $\approx 256 + 128 + 256 + 100.48 + 200.96$ ≈ 941 The surface area of the breadbox is about 941 in². To calculate the reasonableness of the answer, calculate the surface area of a prism breadbox. S = 2bh + 2bh + 2bh= 2(16)(16) + 2(8)(16) + 2(8)(16)= 512 + 256 + 256 = 1.024The surface area of a prism breadbox would be 1,024 cm². Therefore the answer is reasonable. **6.** $S = 2\ell w + 2\ell h + 2wh$ $S = (2 \cdot 20 \cdot 16) + (2 \cdot 20 \cdot 4) + (2 \cdot 16 \cdot 4)$ S = 640 + 160 + 128S = 928 The surface area of the prism is 928 in.² **7.** $S = 2 \cdot \frac{1}{2}bh + 2bh + bh$ $= 2 \cdot \frac{1}{2}(8)(3) + 2(5)(6) + (8)(6)$ = 24 + 60 + 48= 132 The surface area of the prism is 132 yd^2 .

 $\approx \pi (2)^2 (7)$ $\approx \pi (4) (7)$ $\approx 87.9 \text{ in.}^3$

8. $S = 2\pi r^2 + 2\pi rh$ $S \approx (2 \cdot 3.14 \cdot 6^2) + (2 \cdot 3.14 \cdot 6 \cdot 15)$ $S \approx 226.08 + 565.2$ $S \approx 791.28$ $S \approx 791.3$ The surface area of the cylinder is 791.3 in². **9.** $S = 2\pi r^2 + 2\pi rh$ $S \approx (2 \cdot 3.14 \cdot 1.5^2) + (2 \cdot 3.14 \cdot 1.5 \cdot 18.5)$ $S \approx 14.13 + 174.27$ $S \approx 188.4$ The surface area of the cylinder is 188.4 cm^2 . **10.** $S = 4bh + bh + 2bh + 2 \cdot \frac{1}{2}bh$ $= 4(4)(12) + (4)(4) + \overline{2(2\sqrt{2})}(4)$ $+2\cdot\frac{1}{2}(4)(2)$ $\approx 192 + 16 + 23 + 8$ ≈ 239

The surface area of the milk carton is about 239 in². To check the surface area for reasonableness, find the area of a rectangular prism.

S = 4bh + 2bh= 4(12)(4) + 2(4)(4) = 192 + 32 = 224

The area of a similar rectangular prism is 224 in². Therefore the answer is reasonable.



About 158.0 cm² of metal are used to make each can.

c. The label fits the area of the lateral surface, $2\pi rh$. From the steps in part b, the lateral surface area is about 85.408. So, about 85.4 cm² of paper are needed for each label.

12. a.Box 1: *V* = *ℓwh*

= (3)(8)(9)= 216 in³ Box 2: $V = \ell wh$ = (4)(6)(9) = 216 in³ Box 3: $V = \ell wh$ = (6)(6)(6) = 216 in³ All the boxes have a volume of 216 in³. **b.** Box 1: S = 2bh + 2bh + 2bh= 2(3)(8) + 2(3)(9) + 2(8)(9) = 48 + 54 + 144 = 246 in² Box 2: S = 2bh + 2bh + 2bh= 2(4)(6) + 2(4)(9) + 2(6)(9) = 48 + 72 + 108 = 228 in² Box 3: S = 2bh + 2bh + 2bh= 6(6)(6) = 216 in²

Box 3 requires the least amount of material to wrap. **c**.The edge lengths must be equal to give the least surface area.

13. $C = 2\pi r$ $20\pi = 2\pi r$ 10 = 2rThe radius is 10 cm. $S = 2\pi r^2 + C(\frac{1}{2}r)$ $= 2\pi (10)^2 + 20\pi (\frac{1}{2}(10))$ $= 200\pi + 100\pi$ $= 300\pi$

The surface area of the cylinder is 300π cm².

- **14.** Possible answer: Because a cube has 6 congruent square faces, divide 512 by 6 to find the surface area of each face. Take the square root of the area of one face to get the length of each side.
- **15.** Find the surface area of the large rectangular prism. $S = 2\ell w + 2\ell h + 2wh$ $S = (2 \cdot 25 \cdot 20) + (2 \cdot 25 \cdot 12) + (2 \cdot 20 \cdot 12)$ S = 1,000 + 600 + 480 S = 2,080Subtract off the 2 square openings. $2,080 - (2 \cdot 8 \cdot 8) = 1,952$ Find the surface area of the hole. $S = 2\ell w + 2wh$ $S = (2 \cdot 8 \cdot 20) + (2 \cdot 8 \cdot 20)$ S = 320 + 320 S = 640Find the sum of the two surface areas. 1,952 + 640 = 2,592The total surface area is 2,592 cm².
- **16.** C; 708 in² **17.** $S = 2\pi r^2 + 2\pi rh$ $S \approx (2 \cdot 3.14 \cdot 5^2) + (2 \cdot 3.14 \cdot 5 \cdot 15)$ $S \approx 157 + 471$ $S \approx 628$ The surface area of the cylinder is 628 cm².

READY TO GO ON?

1. $S = 2\ell w + 2\ell h + 2wh$ $S = (2 \cdot 15 \cdot 3) + (2 \cdot 15 \cdot 8) + (2 \cdot 3 \cdot 8)$ S = 90 + 240 + 48 S = 378The surface area of the prism is 378 in². 2. $S = 2\ell w + 2\ell h + 2wh$ $S = (2 \cdot 2 \cdot 5) + (2 \cdot 2 \cdot 2) + (2 \cdot 5 \cdot 2)$

$$S = (2 \cdot 2 \cdot 5) + (2 \cdot 2 \cdot 2) + (2 \cdot 5 \cdot 2)$$

$$S = 20 + 8 + 20$$

$$S = 48$$

The surface area of the prism is 48 cm².

3. $S = 2\pi r^2 + 2\pi rh$ $S \approx 2 \cdot 3.14 \cdot 7^2 + 2 \cdot 3.14 \cdot 7 \cdot 15$ $S \approx 307.72 + 659.4$ S ≈ 967.1 The surface area of the cylinder is 967.1 in². 4. $S = 2bh + 2bh + bh + 2 \cdot \frac{1}{2}\pi r^2 + \frac{1}{2} \cdot 2\pi r\ell$ = 2(0.5)(6) + 2(0.5)(9) + (6)(9) $+2 \cdot \frac{1}{2}(3.14)(3)^2 + \frac{1}{2} \cdot 2(3.14)(3)(9)$ = 6 + 9 + 54 + 28.26 + 84.78= 182.04The surface area of the composite figure is about 182.0 m². 5. The figure is not a polyhedron. The figure is a cone. 6. The figure is a polyhedron. The figure is a hexagonal pyramid. 7. The figure is a polyhedron. The figure is a triangular pyramid. 8. $V = \ell w h$ $V = 6 \cdot 2 \cdot 3$ V = 36The volume is 36 ft³. **9.** $V = \pi r^2 h$ The radius of the cylinder is 2.6 m, and the height is 2.3 m. $V \approx 3.14 \cdot 2.6^2 \cdot 2.3$ $V \approx 48.8$ The volume is about 48.8 cm³. **10.** $V = Bh + \pi r^2 h$ $\approx (12)(5)(3) + (3.14)(3)^{2}(8)$ $\approx 180 + 226.08$ ≈ 406.1 The volume of the composite figure is about 406.1 in³. **11.** V = Bh + Bh $= (2)(4)(3) + \frac{1}{2}(4)(3)(2)$ = 24 + 12= 36The volume of the composite figure is 36 cm³. **12.** $V \approx 3,785 \cdot 4^3$ $V \approx 3,785 \cdot 64$ $V \approx 242,240$ The volume of the larger container is 242,240 in³. $3,785 \div 231 = 16.4$ The small container can hold 16.4 gallons. 242,240 ÷ 231 = 1,048.7 The larger container can hold 1,048.7 gallons. 1,048.7 - 16.4 = 1,032.3The larger container can hold about 1,032 more gallons than the smaller container. **STUDY GUIDE: REVIEW**

1. cylinder

2. surface area

3. polyhedron **4.** cone

5.
$$P = 12 + 24 + 15 + 32$$

P = 83The perimeter of the trapezoid is 83 m.

6. $P = 2\ell + 2w$ $P = (2 \cdot 24.9) + (2 \cdot 15.8)$ P = 49.8 + 31.6*P* = 81.4 The perimeter of the rectangle is 81.4 cm. **7.** $C = \pi d$ $C \approx 3.14 \cdot 13$ $C \approx 40.82$ The circumference of the circle is about 40.8 ft. **8.** $C = 2\pi r$ $C \approx 2 \cdot 3.14 \cdot 7.8$ $C \approx 48.984$ The circumference of the circle is about 49.0 in. **9.** $A = \pi r^2$ $A \approx 3.14 \cdot 3.4^2$ $A \approx 3.14 \cdot 11.56$ $A \approx 36.3$ The area of the circle is 36.3 m^2 . **10.** $A = \pi r^2$ $A \approx 3.14 \cdot 8.5^2$ $A \approx 3.14 \cdot 72.25$ $A \approx 226.865$ The area of the circle is 226.9 ft². **11.** $A = \pi r^2$ $A \approx 3.14 \cdot 92$ $A \approx 3.14 \cdot 81$ $A \approx 254.34$ The area of the circle is 254.34 in^2 . **12.** Area of rectangle: $A = \ell w$ $A = 7 \cdot 3.5$ A = 24.5Area of half circle: $A = \frac{1}{2}\pi r^2$ $A \approx \frac{1}{2} \cdot 3.14 \cdot 2.5^2$ $A \approx 9.8125$ 24.5 + 9.8125 = 34.3125The area of the figure is 34.31 ft². 13. Area of rectangle 1: $A = \ell w$ $A = 2 \cdot 6$ A = 12Area of rectangle 2: $A = \ell w$ $A = 3 \cdot 1$ A = 3Area of rectangle 3: $A = \ell w$ $A = 2 \cdot 3$ A = 612 + 3 + 6 = 21The area of the figure is 21 m^2 . 14. cylinder 15. rectangular pyramid **16.** triangular prism 17. cone

18. *V* = *Bh* $V = \left(\frac{1}{2} \cdot 8 \cdot 7\right) \cdot 13$ $V = 28 \cdot 13$ V = 364The volume of the triangular prism is 364 cm³. **19.** $V = \pi r^2 h$ $V \approx 3.14 \cdot 1.8^2 \cdot 11$ $V \approx 111.9 \, {\rm ft}^3$ **20.** $S = 2\ell w + 2\ell h + 2wh$ $S = (2 \cdot 5 \cdot 10) + (2 \cdot 5 \cdot 5) + (2 \cdot 10 \cdot 5)$ S = 100 + 50 + 100S = 250The surface area of the prism is 250 m^2 . **21.** $S = 2\ell w + 2\ell h + 2wh$ $S = (2 \cdot 8 \cdot 1) + (2 \cdot 8 \cdot 1) + (2 \cdot 1 \cdot 1)$ S = 16 + 16 + 2S = 34 The surface area of the prism is 34 cm^2 . **22.** $S = 2\pi r^2 + 2\pi rh$ $S \approx (2 \cdot 3.14 \cdot 2.4^2) + (2 \cdot 3.14 \cdot 2.4 \cdot 15)$ $S \approx 36.1728 + 226.08$ $S \approx 262.2528$ $S \approx 262.3$ The surface area of the cylinder is about 262.3 cm². **23.** $S = 2\pi r^2 + 2\pi rh$ $\approx (2 \cdot 3.14 \cdot 8^2) + (2 \cdot 3.14 \cdot 8 \cdot 8)$ $\approx 401.92 + 401.92$ $\approx 803.8 \text{ ft}^2$

CHAPTER TEST

 The figure is not a polyhedron. The figure is a cone. 2. The figure is a polyhedron. The figure is a triangular prism. 3. The figure is a polyhedron. The figure is a hexagonal pyramid. 4. $V = \ell w h$ $V = 24 \cdot 15 \cdot 13$ V = 4,680.0The volume is 4,680.0 in³. **5.** $V = \pi r^2 h$ $V \approx 3.14 \cdot 72 \cdot 8.4$ $V \approx 1.292.4$ The surface area of the cylinder is about 1,292.4 m³. **6.** V = Bh $B = \frac{1}{2} \cdot 4.2 \cdot 3.9 = 8.19 \text{ mm}^2$ $V = 8.19 \cdot 6.7$ V = 54.9The volume is 54.9 mm³. **7.** P = 10.5 + 6.3 + 17.2 + 9.1*P* = 43.1 The perimeter of the trapezoid is 43.1 in.

8. *A* = *bh* $A = 13.6 \cdot 8.7$ A = 118.32The area of the parallelogram is 118.32 ft². 9. Area of rectangle 1: $A = \ell w$ $A = 18 \cdot 7$ A = 126 Area of rectangle 2: $A = \ell w$ $A = 6 \cdot 5$ A = 30126 + 30 = 156The area of the figure is 156 m^2 . **10.** $A = \frac{1}{2}h(b_1 + b_2)$ $A = \frac{1}{2} \cdot 4\frac{1}{4} \left(2\frac{1}{2} + 6 \right)$ $A = \frac{1}{2} \cdot 4 \frac{1}{4} \left(8 \frac{1}{2} \right)$ $A = 18\frac{1}{16}$ The area of the trapezoid is $18\frac{1}{16}$ mi² or 18.0625 mi². **11.** $C = 2\pi r$ $C \approx 2 \cdot 3.14 \cdot 5\frac{1}{2}$ $C \approx 34.54$ The circumference of the circle is about 34.5 ft. **12.** $A = \pi r^2$ $A \approx 3.14 \cdot \left(5\frac{1}{2}\right)^2$ $A \approx 3.14 \cdot 30\frac{1}{4}$ $A \approx 94.985$ The area of the circle is about 95.0 ft². **13.** $S = 2\ell w + 2\ell h + 2wh$ $S = (2 \cdot 19 \cdot 13) + (2 \cdot 19 \cdot 8) + (2 \cdot 13 \cdot 8)$ S = 494 + 304 + 208S = 1,006The surface area of the prism is 1,006 in². **14.** $S = 2\pi r^2 + 2\pi rh$ $S = (2 \cdot 3.14 \cdot 5.5^2) + (2 \cdot 3.14 \cdot 5.5 \cdot 6.8)$ S = 189.97 + 234.872S = 424.842The surface area of the cylinder is about 424.8 cm². **15.** $S = 2\pi rh + 2bh + 2bh + 2bh$ $\approx (2 \cdot 3.14 \cdot 1 \cdot 1) + (2 \cdot 2 \cdot 3)$ $+ (2 \cdot 2 \cdot 7) + (2 \cdot 3 \cdot 7)$ $\approx 6.28 + 12 + 28 + 42$ \approx 88.3 ft²