Chapter 8 Quadrilaterals

Prerequisite Skills for the chapter "Quadrilaterals"

1. $\angle 1$ and $\angle 4$ are vertical angles.
2. $\angle 3$ and $\angle 5$ are consecutive interior angles.
3. $\angle 7$ and $\angle 3$ are corresponding angles.
4. $\angle 5$ and $\angle 4$ are alternate interior angles.
5. $m\angle A + m\angle B + m\angle C = 180°$

   $x° + 3x° + (4x - 12)° = 180°$

   $8x - 12 = 180$

   $8x = 192$

   $x = 24$

   $m\angle A = x° = 24°$

   $m\angle B = 3x° = 3(24°) = 72°$

   $m\angle C = (4x - 12)° = (4(24) - 12)° = 84°$
6. $\angle 3$ and $\angle 1$ are corresponding angles, so $m\angle 1 = m\angle 3 = 105°$

   $\angle 1$ and $\angle 2$ are alternate interior angles, so $m\angle 2 = m\angle 1 = 105°$
7. Because $\angle 1$ and $\angle 3$ are corresponding angles, $m\angle 3 = m\angle 1 = 98°$
8. $\angle 4$ is congruent to the supplement of $\angle 3$ because they are corresponding angles. The supplement of $\angle 3$ is congruent to the supplement of $\angle 1$ because they are corresponding angles. So, $m\angle 4 + m\angle 1 = 180°$.

   $m\angle 4 + m\angle 1 = 180°$

   $82° + m\angle 1 = 180°$

   $m\angle 1 = 98°$
9. $\angle 2$ is congruent to the supplement of $\angle 4$ because they are alternate interior angles. So, $m\angle 4 + m\angle 2 = 180$.

   $m\angle 4 + m\angle 2 = 180°$

   $m\angle 4 + 102 = 180°$

   $m\angle 4 = 78°$

Lesson 8.1 Find Angle Measures in Polygons

Investigating Geometry Activity for the lesson "Find Angle Measures in Polygons"

STEP 3

Polygon	Number of sides	Number of triangles	Sum of measures of interior angles
Triangle	3	1	$1 \cdot 180° = 180°$
Quadrilateral	4	2	$2 \cdot 180° = 360°$
Pentagon	5	3	$3 \cdot 180° = 540°$
Hexagon	6	4	$4 \cdot 180° = 720°$

1. The sum of the measures of the interior angles of a convex heptagon is $5 \cdot 180° = 900°$. The sum of the measures of the interior angles of a convex octagon is $6 \cdot 180° = 1080°$. As the number of sides is increased by 1, so is the number that is multiplied by 180° to get the sum of the measures of the interior angles.
2. The sum of the measures of the interior angles of a convex n-gon is $(n - 2) \cdot 180°$.
3. The lengths of the sides does not affect the sum of the interior angle measures of a hexagon. Only the number of sides affects the sum.

Guided Practice for the lesson "Find Angle Measures in Polygons"

1. Use the Polygon Interior Angles Theorem. Substitute 11 for n.

   $(n - 2) \cdot 180° = (11 - 2) \cdot 180° = 9 \cdot 180° = 1620°$
2. Because the sum of the measures of the interior angles is 1440°, set $(n - 2) \cdot 180°$ equal to 1440° and solve for n.

   $(n - 2) \cdot 180° = 1440°$

   $n - 2 = 8$

   $n = 10$

   The polygon has 10 sides. It is a decagon.
3. $m\angle T = m\angle S$

   $m\angle P + m\angle Q + m\angle R + m\angle S + m\angle T = (n - 2) \cdot 180°$

   $93° + 156° + 85° + m\angle T + m\angle T = (5 - 2) \cdot 180°$

   $334° + 2m\angle T = 540°$

   $2m\angle T = 206°$

   $m\angle T = 103° = m\angle S$
4. Let $x°$ equal the measure of the fourth angle.

   $x° + 89° + 110° + 46° = 360°$

   $x + 245 = 360$

   $x = 115$
5. Use the Polygon Exterior Angles Theorem.

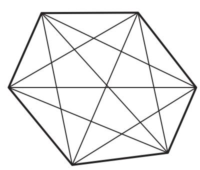
   $x° + 34° + 49° + 58° + 67° + 75° = 360°$

   $x + 283 = 360$

   $x = 77$
6. If the angles form a linear pair, they are supplementary so their sum is 180°. The measure of the interior angle could have been subtracted from 180° to find the measure of the exterior angle. $180° - 150° = 30°$

Exercises for the lesson "Find Angle Measures in Polygons"

Skill Practice

1. 

2. There are 2 • n exterior angles in an n-gon. However, only 1 angle at each vertex, or n-angles, is considered when using the Polygon Exterior Angles Theorem.

3. A nonagon has 9 sides.

$(n - 2) \cdot 180° = (9 - 2) \cdot 180° = 7 \cdot 180° = 1260°$

4. $(n - 2) \cdot 180° = (14 - 2) \cdot 180° = 12 \cdot 180° = 2160°$

5. $(n - 2) \cdot 180° = (16 - 2) \cdot 180° = 14 \cdot 180° = 2520°$

6. $(n - 2) \cdot 180° = (20 - 2) \cdot 180° = 18 \cdot 180° = 3240°$

7. $(n - 2) \cdot 180° = 360°$

$n - 2 = 2$

$n = 4$

The polygon has 4 sides. It is a quadrilateral.

8. $(n - 2) \cdot 180° = 720°$

$n - 2 = 4$

$n = 6$

The polygon has 6 sides. It is a hexagon.

9. $(n - 2) \cdot 180° = 1980°$

$n - 2 = 11$

$n = 13$

The polygon has 13 sides. It is a 13-gon.

10. $(n - 2) \cdot 180° = 2340°$

$n - 2 = 13$

$n = 15$

The polygon has 15 sides. It is a 15-gon.

11. $x° + 86° + 140° + 138° + 59° = (n - 2) \cdot 180°$

$x + 86 + 140 + 138 + 59 = (5 - 2) \cdot 180$

$x + 423 = 540$

$x = 117$

12. $x° + 121° + 96° + 101° + 162° + 90° = (n - 2) \cdot 180°$

$x + 121 + 96 + 101 + 162 + 90 = (6 - 2) \cdot 180$

$x + 570 = 720$

$x = 150$

13. $x° + 143° + 2x° + 152° + 116° + 125° + 140° + 139° = (n - 2) \cdot 180°$

$x + 143 + 2x + 152 + 116 + 125 + 140 + 139 = (8 - 2) \cdot 180$

$3x + 815 = 1080$

$3x = 265$

$x = 88.\overline{3}$

14. $x° + 78° + 106° + 65° = 360°$

$x + 249 = 360$

$x = 111$

15. $x° + 77° + 2x° + 45° + 40° = 360°$

$3x + 162 = 360$

$3x = 198$

$x = 66$

16. $x° + x° + 58° + 39° + 50° + 48° + 59° = 360°$

$2x + 254 = 360$

$2x = 106$

$x = 53$

17. The student's error was thinking the sum of the measures of the exterior angles of different polygons are different when in fact this sum is always 360°.

The student should have claimed that the sum of the measures of the interior angles of an octagon is greater than the sum of the measures of the interior angles of a hexagon because an octagon has more sides.

18. B; $x° + 2x° + 3x° + 4x° = (n - 2) \cdot 180°$

$x + 2x + 3x + 4x = (4 - 2) \cdot 180$

$10x = 360$

$x = 36$

Because $x = 36°$, then $4x° = 144°$

19. $(n - 2) \cdot 180° = (5 - 2) \cdot 180° = 540°$

The measure of each interior angle is $540 \div 5 = 108°$.
The measure of each exterior angle is $360 \div 5 = 72°$.

20. $(n - 2) \cdot 180° = (18 - 2) \cdot 180° = 2880°$

The measure of each interior angle is $2880° \div 18 = 160°$.
The measure of each exterior angle is $360° \div 18 = 20°$.

21. $(n - 2) \cdot 180° = (90 - 2) \cdot 180 = 15{,}840°$

The measure of each interior angle is $15{,}840° \div 90 = 176°$.
The measure of each exterior angle is $360° \div 90 = 4°$.

22. $\frac{ST}{RU} = \frac{KL}{JM}$

$\frac{6}{12} = \frac{10}{JM}$

$6 \cdot JM = 120$

$JM = 20$

The length of $\overline{JM}$ is 20.

23. The sides of each polygon are congruent, so the ratio of corresponding sides will always be the same. The measures of the angles in any regular pentagon are the same because the measures do not depend on the side length.

24. $(n - 2) \cdot 180° = 156° \cdot n$

$180n - 360 = 156n$

$180n = 360 + 156n$

$24n = 360$

$n = 15$

25. $(9°)n = 360°$

$n = 40$

26. The number of sides n, of a polygon can be calculated with the Polygon Interior Angles Theorem. n must be a positive, whole number.

a.

$(165°)n = (n - 2) \cdot 180°$
$165n = 180n - 360$
$-15n = -360$
$n = 24$; possible

b.

$(171°)n = (n - 2) \cdot 180°$
$171n = 180n - 360$
$-9n = -360$
$n = 40$; possible

c.

$(75°)n = (n - 2) \cdot 180°$
$75n = 180n - 360$
$-105n = -360$
$n = 3.43$; not possible

d.

$(40°)n = (n - 2) \cdot 180°$
$40n = 180n - 360$
$-140n = -360$
$n = 2.57$; not possible

27. An increase of one in the number of sides of a polygon results in an increase of 180° in the sum of the measures of the interior angles. If the sum is increased by 540°, the increase in the number of sides is $540° \div 180° = 3$.

Problem Solving

28. $(n - 2) \cdot 180° = (5 - 2) \cdot 180° = 3 \cdot 180° = 540°$

The sum of the interior angle measures of the playing field is 540°.

29. $(n - 2) \cdot 180° = (6 - 2) \cdot 180° = 4 \cdot 180° = 720°$

The sum of the interior angle measures of the playing field is 720°.

30. $(n - 2) \cdot 180° = (6 - 2) \cdot 180° = 720°$

Measure of one angle $= 720° \div 6 = 120°$

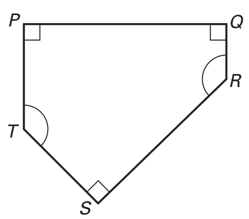
The measure of each interior angle of the hexagon is 120°.

31. Sum of interior angles

$(n - 2) \cdot 180° = (10 - 2) \cdot 180° = 1440°$

The measure of each interior angle is $1440° \div 10 = 144°$. The measure of each exterior angle is $360° \div 10 = 36°$.

32. a.



b. $(n - 2) \cdot 180° = (5 - 2) \cdot 180° = 540°$

c. $m\angle P + m\angle Q + m\angle R + m\angle S + m\angle T = 540°$

$90° + 90° + m\angle R + 90° + m\angle R = 540°$

$2(m\angle R) + 270° = 540°$

$2m\angle R = 270°$

$m\angle R = 135° = m\angle T$

33. Draw all of the diagonals of $ABCDE$ that have A as an endpoint. The diagonals formed, $\overline{AD}$ and $\overline{AC}$, divide $ABCDE$ into three triangles. By the Angle Addition Postulate, $m\angle CDE = m\angle CDA + m\angle ADE$. Similarly $m\angle EAB = m\angle EAD + m\angle DAC + m\angle CAB$ and $m\angle BCD = m\angle BCA + m\angle ACD$. The sum of the measures of the interior angles of $ABCDE$ is equal to the sum of the measures of the angles of triangles $\triangle ADE$, $\triangle ACD$, and $\triangle ABC$. By the Triangle Sum Theorem, the sum of the measures of the interior angles of each triangle is 180°, so the sum of the measures of the interior angles of $ABCDE$ is $(5 - 2) \cdot 180° = 3 \cdot 180° = 540°$.

34. By the Polygon Interior Angles Theorem, the sum of the measures of the interior angles of a regular polygon is $(n - 2) \cdot 180°$. A quadrilateral has 4 sides, so the sum of the measures of the interior angles is $(4 - 2) \cdot 180° = 2 \cdot 180° = 360°$.

35. Let A be a convex n-gon. At each vertex, each interior angle and one of the exterior angles form a linear pair, so the sum of their measures is 180°. Then the sum of the measures of the interior angles and one exterior angle at each vertex is $n \cdot 180°$. By the Polygon Interior Angles Theorem, the sum of the measures of the interior angles of A is $(n - 2) \cdot 180°$. So the sum of the measures of the exterior angles of A, one at each vertex, is

$$n \cdot 180° - [(n - 2) \cdot 180°] = n \cdot 180° - n \cdot 180° + 360° = 360°.$$

36. a. $h(n) = (n - 2) \cdot 180° \div n = \frac{1}{n}(n - 2) \cdot 180°$

b. $h(n) = \frac{1}{n}(n - 2) \cdot 180°$

$h(9) = \frac{1}{9}(9 - 2) \cdot 180$

$= \frac{1}{9}(1260)$

$= 140$

$h(n) = \frac{1}{n}(n - 2) \cdot 180°$

$150° = \frac{1}{n}(n - 2) \cdot 180°$

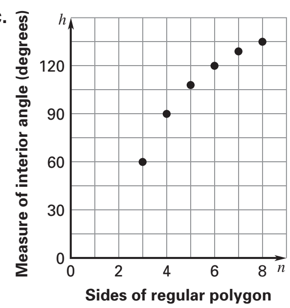
$150n = (n - 2) \cdot 180$

$150n = 180n - 360$

$-30n = -360$

$n = 12$

c.



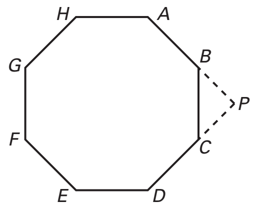
The value of $h(n)$ increases as the value of n increases. The graph shows that when n increases, $h(n)$ also increases.

37. a.

Polygon	Number of sides	Number of triangles	Sum of measures of interior angles
Quadrilateral	4	2	$2 \cdot 180° = 360°$
Pentagon	5	3	$3 \cdot 180° = 540°$
Hexagon	6	4	$4 \cdot 180° = 720°$
Heptagon	7	5	$5 \cdot 180° = 900°$

b. Using the results from the table in part (a), you can see that the sum of the measures of the interior angles of a concave polygon is given by $s(n) = (n - 2) \cdot 180°$ where n is the number of sides.

38.



The measure of one exterior angle of a regular octagon at each vertex is $360° \div 8 = 45°$. This means $\angle PBC$ and $\angle PCB$ each have a measure of 45°. Because the sum of the measures of the interior angles of a triangle is 180°, the measure of $\angle BPC$ is $180° - 45° - 45° = 90°$.

Lesson 8.2 Use Properties of Parallelograms

Investigating Geometry Activity for the lesson "Use Properties of Parallelograms"

STEP 3

The sides $\overline{AB}$ and $\overline{DC}$ remain parallel and their lengths remain equal to each other. Similarly, $\overline{AD}$ and $\overline{BC}$ remain parallel and their lengths remain equal to each other.

STEP 4

If point A is dragged out, the angle measures of $\angle A$ and $\angle C$ decrease, and the angle measures of $\angle B$ and $\angle D$ increase. Similarly, when point B is dragged away from the figure, the angle measures of $\angle B$ and $\angle D$ decrease while the angle measures of $\angle A$ and $\angle C$ increase. Whether point A is dragged or point B is dragged, $\angle A$ and $\angle C$ always have the same measure and $\angle B$ and $\angle D$ always have the same measure.

1. Both sets of opposite sides in the polygon are parallel.
2. Opposite side lengths and opposite angle measures in a parallelogram are always equal.
3. Answers will vary.

Guided Practice for the lesson "Use Properties of Parallelograms"

1. $FG = HE$ $\quad$ $m\angle G = m\angle E$

$FG = 8$ $\quad$ $m\angle G = 60°$

2. $JK = LM$ $\quad$ $m\angle J = m\angle L$

$18 = y + 3$ $\quad$ $2x° = 50°$

$15 = y$ $\quad$ $x = 25$

3. $NM = KN$

$NM = 2$

4. $KM = 2 \cdot KN = 2 \cdot 2 = 4$

5. $m\angle JML = 180° - m\angle KJM = 180 - 110 = 70$

6. $m\angle KML = m\angle JML - m\angle JMK = 70 - 30 = 40$

Exercises for the lesson "Use Properties of Parallelograms"

Skill Practice

1. That both pairs of opposite sides of a parallelogram are parallel is a property included in its definition. Other properties of parallelograms are their opposite sides and angles are congruent, consecutive angles are supplementary, and the diagonals bisect each other.

2. $m\angle C = 65°$ because $\angle C$ is opposite $\angle A$ in the parallelogram. Consecutive angles in a parallelogram are supplementary, so $\angle B$ and $\angle D$ each have a measure of $180° - 65° = 115°$.

3. $x = 9$

$y = 15$

4. $n = 12$ $\quad$ $m + 1 = 6$

$m = 5$

5. $a° = 55°$

$a = 55$

6. $2p° = 120°$

$p = 60$

7. $20 = z - 8$ $\quad$ $105° = (d - 21)°$

$28 = z$ $\quad$ $126 = d$

8. $16 - h = 7$ $\quad$ $(g + 4)° = 65°$

$-h = -9$ $\quad$ $g = 61$

$h = 9$

9. $m\angle A + m\angle B = 180°$

$51° + m\angle B = 180°$

$m\angle B = 129°$

10. $m\angle L + m\angle M = 180°$

$m\angle L + 95° = 180°$

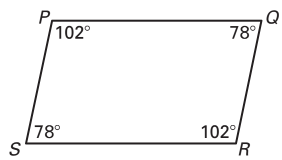
$m\angle L = 85°$

11. $m\angle X + m\angle Y = 180°$

$119° + m\angle Y = 180°$

$m\angle Y = 61°$

12.



$\angle R \cong \angle P$ and $\angle S \cong \angle Q$

$m\angle R + m\angle S = 180°$ $\quad$ $m\angle R = m\angle S + 24°$

$(m\angle S + 24°) + m\angle S = 180°$ $\quad$ $= 78° + 24°$

$24° + 2m\angle S = 180°$ $\quad$ $= 102°$

$2m\angle S = 156°$

$m\angle S = 78°$

13. $b - 1 = 9$ $\quad$ $5a = 15$

$b = 10$ $\quad$ $a = 3$

14. $4m = 16$ $\quad$ $2n = 9 - n$

$m = 4$ $\quad$ $3n = 9$

$n = 3$

15. $3x = 12$ $\quad$ $5y = 4y + 4$

$x = 4$ $\quad$ $y = 4$

16. A; Coordinates of midpoint M of $\overline{QO} = \left(\frac{2 + 0}{2}, \frac{5 + 0}{2}\right)$

$= \left(1, \frac{5}{2}\right)$

17. $\overline{AD} \cong \overline{BC}$

$\overline{AD}$ and $\overline{BC}$ are opposite sides of a parallelogram.

18. $\angle ADC \cong \angle ABC$

$\angle ADC$ and $\angle ABC$ are opposite angles of a parallelogram.

19. $\angle CBD \cong \angle ADB$

$\angle CBD$ and $\angle ADB$ are alternate interior angles.

20. $m\angle BCD = 92°$

$m\angle BCD = m\angle BAD$ because they are opposite angles of a parallelogram.

21. $m\angle BDC = 92°$

$m\angle BAD = m\angle BCD$ because they are opposite angles of a parallelogram. Because $\angle BCD \cong \angle BDC$, $m\angle BCD = m\angle BDC$.

22. $m\angle ADB = 48°$

$m\angle ADB = m\angle CBD$ because they are alternate interior angles.

23. $m\angle EJF = 180° - 60° = 120°$

$\angle EJF$ and $\angle FJG$ form a linear pair.

24. $m\angle EGF = m\angle HEG = 85°$

$\angle EGF$ and $\angle HEG$ are alternate interior angles.

25. $m\angle EGF = 85°$ because $\angle HEG$ and $\angle EGF$ are alternate interior angles. By the Triangle Sum Theorem, $m\angle J + m\angle G + m\angle F = 180°$ for $\triangle JGF$. $m\angle F = 180° - 85° - 60° = 35°$. So, $m\angle HFG = 35°$.

26. $m\angle GEF = 45°$

$\angle GEF$ and $\angle EGH$ are alternate interior angles.

27. $m\angle EGF = 85°$ because $\angle HEG$ and $\angle EGF$ are alternate interior angles. By the Angle Addition Postulate, $m\angle HGF = m\angle HGE + m\angle EGF$. So, $m\angle HGF = 45° + 85° = 130°$.

28. From Exercise 27, you know that $m\angle HGF = 130°$. Because consecutive angles of a parallelogram are supplementary, $m\angle EHG + m\angle HGF = 180°$. So, $m\angle EHG = 180° - 130° = 50°$.

29. C; Let p = perimeter of $\square ABCD$.

$p = AB + BC + CD + AD = 14 + 20 + 14 + 20 = 68$

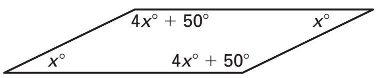
30.

$0.25x° + x° = 180°$ $\quad$ $0.25x° = 0.25(144) = 36$

$1.25x = 180$

$x = 144$

31.



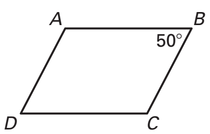
$4x° + 50° + x° = 180°$ $\quad$ $4x° + 50° = 4(26) + 50$

$5x + 50 = 180$ $\quad$ $= 154$

$5x = 130$

$x = 26$

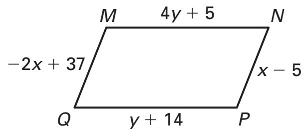
32.

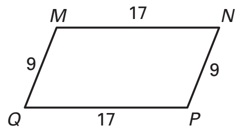


The student is incorrect because $\angle A$ and $\angle B$ are consecutive angles. Consecutive angles of a parallelogram are supplementary.

33. Because $\overline{ST} \cong \overline{QR} \cong \overline{UV}$, x, the length of $\overline{UV}$ is 20. Because $\angle UTS$ and $\angle TSV$ are supplementary, $m\angle TSV = 180° - 40° = 140°$. By the Angle Addition Postulate, $m\angle TSV = m\angle TSU + m\angle USV$. $140° = y° + 80°$. So, $y = 60°$.

34.





$4y + 5 = y + 14$ $\quad$ $x - 5 = -2x + 37$

$3y + 5 = 14$ $\quad$ $3x - 5 = 37$

$3y = 9$ $\quad$ $3x = 42$

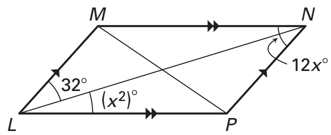
$y = 3$ $\quad$ $x = 14$

Let p represent the perimeter of $\square MNPQ$.

$p = 17 + 9 + 17 + 9 = 52$

35. *Sample answer:* Because $m\angle B = 124°$ and $m\angle A = 66°$, $m\angle B = m\angle A = 190°$. Consecutive angles are not supplementary, so $ABCD$ is not a parallelogram.

36.



$32° + (x^2)° = 12x°$

$x^2 - 12x + 32 = 0$

$(x - 8)(x - 4) = 0$

$x - 8 = 0$ or $x - 4 = 0$

$x = 8$ or $x = 4$

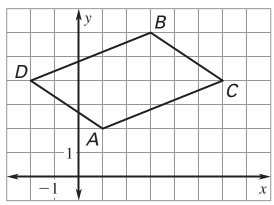
If $x = 8$: $\quad$ If $x = 4$:

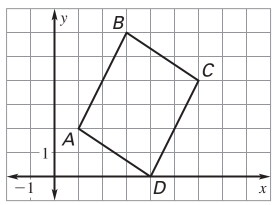
$m\angle MNP = 12 \cdot 8° = 96°$ $\quad$ $m\angle MNP = 12 \cdot 4° = 48°$

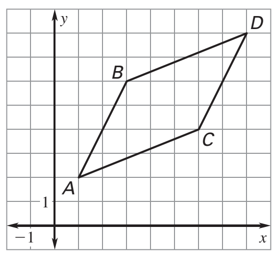
$96°$ is not an acute angle. $\quad$ $48°$ is an acute angle.

Because $x = 4$, $x^2 = (4)^2 = 16$. So $m\angle NLP = 16°$.

37.







In each quadrilateral, each pair of opposite sides is parallel.

Problem Solving

38. $m\angle D + m\angle C = 180°$

$m\angle D + 40° = 180°$

$m\angle D = 140°$

$\angle D$ and $\angle C$ are consecutive angles. So, $\angle D$ and $\angle C$ are supplementary.

39. a. $PQ = RS = 3$

The length of $\overline{RS}$ is 3 inches.

b. $m\angle Q = m\angle S = 70°$

c. $\angle P$ and $\angle Q$ are supplementary. When $m\angle Q$ increases, $m\angle P$ decreases. When $m\angle Q$ decreases, $m\angle P$, $m\angle R$ and the length of $\overline{QS}$ increase.

40. $\frac{LM}{MN} = \frac{4}{3}$

$4MN = 3LM$

$MN = \frac{3}{4}LM$

Let p = perimeter of $LMNO$.

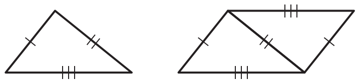
$p = 2 \cdot LM + 2 \cdot MN$

$28 = 2LM + 2\left(\frac{3}{4}LM\right)$

$28 = \frac{7}{2}LM$

$8 = LM$

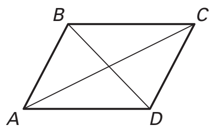
41.



The quadrilateral is a parallelogram because both pairs of opposite sides are congruent.

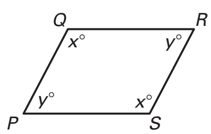
You can arrange eight such congruent triangles to make a parallelogram that is similar to the one shown above, but with all side lengths twice as long.

42. Given: $ABCD$ is a ▱.
Prove: $\angle A \cong \angle C$, $\angle B \cong \angle D$



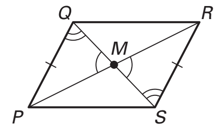
Statements	Reasons
1. $ABCD$ is a ▱.	1. Given
2. $\overline{BC} \parallel \overline{AD}$, $\overline{AB} \parallel \overline{CD}$	2. Definition of a parallelogram
3. $\angle CBD \cong \angle ADB$ $\angle CDB \cong \angle ABD$	3. Alternate Interior Angles Congruence Theorem
4. $\overline{BD} \cong \overline{BD}$	4. Reflexive Property of Congruence
5. $\triangle ABD \cong \triangle CDB$	5. ASA
6. $\angle A \cong \angle C$	6. Corr. parts of ≅ ▵s are ≅.
7. $m\angle CBD = m\angle ADB$, $m\angle CDB = m\angle ABD$	7. Definition of congruent angles
8. $m\angle B = m\angle ABD + m\angle CBD$ $m\angle D = m\angle ADB + m\angle CDB$	8. Angle Addition Postulate
9. $m\angle B = m\angle D$	9. Transitive Property of Congruence
10. $\angle B \cong \angle D$	10. Definition of congruent angles

43. Given: $PQRS$ is a parallelogram.
Prove: $x° + y° = 180°$



Statements	Reasons
1. $PQRS$ is a parallelogram.	1. Given
2. $m\angle Q = x°$ and $m\angle P = y°$	2. Given
3. $\overline{PS} \parallel \overline{QR}$	3. Definition of a parallelogram
4. $\angle P$ and $\angle Q$ are supplementary.	4. Consecutive Interior Angles Theorem
5. $m\angle Q + m\angle P = 180°$	5. Definition of supplementary angles
6. $x° + y° = 180°$	6. Substitution

44. Given: $PQRS$ is a parallelogram.
Prove: The diagonals bisect each other.



Statements	Reasons
1. $PQRS$ is a parallelogram.	1. Given
2. $\overline{PQ} \cong \overline{RS}$, $\overline{QR} \cong \overline{SP}$	2. If a quadrilateral is a parallelogram, then its opposite sides are congruent.
3. $\overline{QR} \parallel \overline{PS}$, $\overline{PQ} \parallel \overline{RS}$	3. Definition of a parallelogram
4. $\angle QPR \cong \angle SRP$, $\angle PQS \cong \angle RSQ$, $\angle RPS \cong \angle QRP$, $\angle PSQ \cong \angle RQS$	4. Alternate Interior Angles Congruence Theorem
5. $\triangle PMQ \cong \triangle RMS$, $\triangle QMR \cong \triangle SMP$	5. ASA
6. $\overline{QM} \cong \overline{SM}$, $\overline{PM} \cong \overline{RM}$	6. Corr. parts of ≅ ▵s are ≅.
7. $\overline{PR}$ bisects $\overline{QS}$ and $\overline{QS}$ bisects $\overline{PR}$.	7. Definition of segment bisector

45. *Sample answer:* $\triangle DCG \sim \triangle ACF$ and $\triangle DAE \sim \triangle ACF$ using the AA Similarity Postulate. $\frac{DG}{AF} = \frac{DC}{AC}$ and $\frac{DE}{AF} = \frac{DA}{AC}$ since the ratio of corresponding sides of similar triangles are equal. Adding, you get $\frac{DE}{AF} + \frac{DG}{AF} = \frac{DA}{AC} + \frac{DC}{AC}$, which implies $\frac{DE + DG}{AF} = \frac{DA + DC}{AC}$, which implies $\frac{DE + DG}{AF} = \frac{AC}{AC}$, which implies $\frac{DE + DG}{AF} = 1$, which implies $DE + DG = AF$.

Quiz for the lessons "Find Angle Measures in Polygons" and "Use Properties of Parallelograms"

1. $x° + 89° + 125° + 100° + 105° = (n - 2) \cdot 180°$

$x + 89 + 125 + 100 + 105 = (5 - 2) \cdot 180$

$x + 419 = 540$

$x = 121$

2. $x° + 115° + 84° + 139° + 150° + 90° = (n - 2) \cdot 180°$

$x + 115 + 84 + 139 + 150 + 90 = (6 - 2) \cdot 180$

$x + 578 = 720$

$x = 142$

3. $x° + 78° + 80° + 90° = 360°$

$x + 248 = 360$

$x = 112$

4. $6x - 3 = 21$ $\quad$ $7y - 6 = 15$

$6x = 24$ $\quad$ $7y = 21$

$x = 4$ $\quad$ $y = 3$

5. $2y - 1 = 9$ $\quad$ $x + 3 = 12$

$2y = 10$ $\quad$ $x = 9$

$y = 5$

6. $a° + (a - 10)° = 180°$ $\quad$ $b° = (a - 10)°$

$2a - 10 = 180$ $\quad$ $b = (95 - 10)$

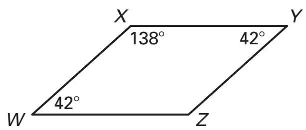
$2a = 190$ $\quad$ $b = 85$

$a = 95$

Lesson 8.3 Show that a Quadrilateral is a Parallelogram

Guided Practice for the lesson "Show that a Quadrilateral is a Parallelogram"

1.



$m\angle W + m\angle X + m\angle Y + m\angle Z = (n - 2) \cdot 180°$

$42 + 138 + 42 + m\angle Z = (4 - 2) \cdot 180$

$222 + m\angle Z = 360$

$m\angle Z = 138$

WXYZ is a parallelogram because both pairs of opposite angles are congruent.

2. If one pair of opposites of a quadrilateral are congruent and parallel, then the quadrilateral is a parallelogram.

3. If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

4. If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

5. $2x = 10 - 3x$

$5x = 10$

$x = 2$

For the quadrilateral to be a parallelogram, the diagonals must bisect each other. This only occurs when $x = 2$.

6. One way is to use the definition of a parallelogram. Find the slopes of all the sides of the quadrilateral. Because both pairs of opposite sides have the same slope, each pair of opposite sides are parallel. Another way is to use Theorem 8.7. Find the length of each side of the quadrilateral. Because both pairs of opposite sides have the same length, both pairs of opposite sides are congruent. Another way is to use Theorem 8.10. Draw and find the midpoint of the diagonals of the quadrilateral. Because the midpoint of each diagonal is the same point, the diagonals bisect each other.

Exercises for the lesson "Show that a Quadrilateral is a Parallelogram"

Skill Practice

1. By definition, if both pairs of opposite sides in a quadrilateral are parallel, the quadrilateral is a parallelogram. Knowing that $\overline{AB} \parallel \overline{CD}$ and $\overline{AD} \parallel \overline{BC}$ proves the quadrilateral is a parallelogram.

2. By Theorem 8.7, if both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

3. By Theorem 8.7, if both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram. The quadrilateral shown has two pairs of *adjacent* sides that are congruent.

4. Both pairs of opposite angles are congruent, so you can use Theorem 8.8 to show the quadrilateral is a parallelogram.

5. Both pairs of opposite sides are congruent, so you can use Theorem 8.7 to show the quadrilateral is a parallelogram.

6. The diagonals of the quadrilateral bisect each other, so you can use Theorem 8.10 to show the quadrilateral is a parallelogram.

7. Because both pairs of opposite sides are congruent, the quadrilateral *JKLM* is a parallelogram. This means both pairs of opposite sides are parallel, so $\overline{JK} \parallel \overline{ML}$.

8. $2x + 3 = x + 7$

$x + 3 = 7$

$x = 4$

9. $5x - 6 = 4x + 2$

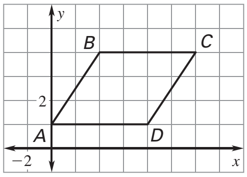
$x - 6 = 2$

$x = 8$

10. $6x = 3x + 2$

$3x = 2$

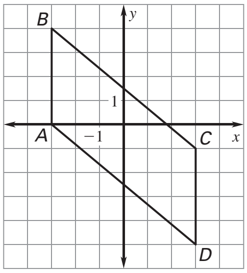
$x = \frac{2}{3}$

11. 

Midpoint of $\overline{BD} = \left(\frac{4+8}{2}, \frac{4+1}{2}\right) = \left(6, \frac{5}{2}\right)$

Midpoint of $\overline{AC} = \left(\frac{0+12}{2}, \frac{1+4}{2}\right) = \left(6, \frac{5}{2}\right)$

The diagonals of $ABCD$ bisect each other, so $ABCD$ is a parallelogram.

12. 

$AB = \sqrt{(-3-(-3))^2 + (4-0)^2} = \sqrt{0^2 + 4^2} = 4$

$BC = \sqrt{(3-(3))^2 + (-1-4)^2}$

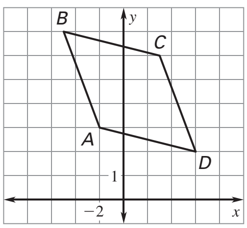
$= \sqrt{6^2 + (-5)^2} = \sqrt{61}$

$CD = \sqrt{(3-3)^2 + (-5-(-1))^2} = \sqrt{0^2 + (-4)^2} = 4$

$DA = \sqrt{(-3-3)^2 + (0-(-5))^2}$

$= \sqrt{(-6)^2 + 5^2} = \sqrt{61}$

Both pairs of opposite sides are congruent, so $ABCD$ is a parallelogram.

13. 

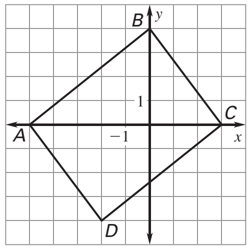
Slope of $\overline{AB} = \frac{7-3}{-5-(-2)} = -\frac{4}{3}$

Slope of $\overline{BC} = \frac{6-7}{3-(-5)} = -\frac{1}{8}$

Slope of $\overline{CD} = \frac{2-6}{6-3} = -\frac{4}{3}$

Slope of $\overline{DA} = \frac{2-3}{6-(-2)} = -\frac{1}{8}$

Both pairs of opposite sides are parallel, so $ABCD$ is a parallelogram.

14. 

$\overline{AB} = \sqrt{(0-(-5))^2 + (4-0)^2} = \sqrt{5^2 + 4^2} = \sqrt{41}$

$\overline{CD} = \sqrt{(-2-3)^2 + (-4-0)^2} = \sqrt{(-5)^2 + (-4)^2}$

$= \sqrt{41}$

Slope of $\overline{AB} = \frac{4-0}{0-(-5)} = \frac{4}{5}$

Slope of $\overline{CD} = \frac{-4-0}{-2-3} = \frac{4}{5}$

Because $\overline{AB} \cong \overline{CD}$ and $\overline{AB} \parallel \overline{CD}$, $ABCD$ is a parallelogram.

15. Use the SAS Congruence Postulate to prove $\triangle ADB \cong \triangle CBD$. Corresponding parts of congruent triangles are congruent, so $\overline{AD} \cong \overline{CB}$ and $\overline{AB} \cong \overline{CD}$. Because both pairs of opposite sides are congruent, $ABCD$ is a parallelogram.

16. Because $\angle ADB \cong \angle CBD$, $\angle ABD \cong \angle CDB$, and these angle pairs are alternate interior angles, $\overline{AB} \parallel \overline{DC}$ and $\overline{AD} \parallel \overline{BC}$. Because both pairs of opposite sides are congruent, $ABCD$ is a parallelogram.

17. Because $\angle B$ and $\angle C$ are congruent alternate interior angles, $\overline{AB} \parallel \overline{DC}$ by the Alternate Interior Angle Converse. Because $\angle C$ and $\angle D$ are congruent corresponding angles, $\overline{AD} \parallel \overline{BC}$ by the Corresponding Angles Converse. Both pairs of opposite sides are parallel so $ABCD$ is a parallelogram.

18. A; $\angle Y$ and $\angle W$ are not consecutive angles, so they are not necessarily supplementary.

19. $x° + 66° = 180°$

$x = 114$

20. $x° + 3x° = 180°$

$4x = 180$

$x = 45$

21. $(x + 10)° + (2x + 20)° = 180°$

$3x + 30 = 180$

$3x = 150$

$x = 50$

22. A quadrilateral is a parallelogram if and only if both pairs of opposite sides are congruent.

23. A quadrilateral is a parallelogram if and only if both pairs of opposite angles are congruent.

24. If the diagonals of a quadrilateral bisect each other, the quadrilateral is a parallelogram. Theorem 8.10

25. Midpoint of $\overline{AC} = \left(\frac{-2+3}{2}, \frac{-3+2}{2}\right) = \left(\frac{1}{2}, -\frac{1}{2}\right)$

Midpoint of $\overline{BD} = \left(\frac{4+x}{2}, \frac{-3+y}{2}\right)$

Because $ABCD$ is a parallelogram, the midpoint of $\overline{BD}$ is $\left(\frac{1}{2}, -\frac{1}{2}\right)$.

So, $\frac{4 + x}{2} = \frac{1}{2}$ and $\frac{-3 + y}{2} = -\frac{1}{2}$

$4 + x = 1$ $\qquad -3 + y = -1$

$x = -3$ $\qquad y = 2$

The coordinates of point D are $(-3, 2)$.

26. Midpoint of $\overline{AC} = \left(\frac{-4 + 6}{2}, \frac{1 + 5}{2}\right) = (1, 3)$

Midpoint of $\overline{BD} = \left(\frac{-1 + x}{2}, \frac{5 + y}{2}\right)$

Because $ABCD$ is a parallelogram, the midpoint of $\overline{BD}$ is $(1, 3)$.

So, $\frac{-1 + x}{2} = 1$ and $\frac{5 + y}{2} = 3$

$-1 + x = 2$ $\qquad 5 + y = 6$

$x = 3$ $\qquad y = 1$

The coordinates of point D are $(3, 1)$.

27. Midpoint of $\overline{AC} = \left(\frac{-4 + 3}{2}, \frac{4 + (-1)}{2}\right) = \left(-\frac{1}{2}, \frac{3}{2}\right)$

Midpoint of $\overline{BD} = \left(\frac{4 + x}{2}, \frac{6 + y}{2}\right)$

Because $ABCD$ is a parallelogram, the midpoint of $\overline{BD}$ is $\left(-\frac{1}{2}, \frac{3}{2}\right)$.

So, $\frac{4 + x}{2} = -\frac{1}{2}$ and $\frac{6 + y}{2} = \frac{3}{2}$

$4 + x = -1$ $\qquad 6 + y = 3$

$x = -5$ $\qquad y = -3$

The coordinates of point D are $(-5, -3)$.

28. Midpoint of $\overline{AC} = \left(\frac{-1 + 8}{2}, \frac{0 + (-6)}{2}\right) = \left(\frac{7}{2}, -3\right)$

Midpoint of $\overline{BD} = \left(\frac{0 + x}{2}, \frac{-4 + y}{2}\right)$

Because $ABCD$ is a parallelogram, the midpoint of $\overline{BD}$ is $\left(\frac{7}{2}, -3\right)$.

So, $\frac{x}{2} = \frac{7}{2}$ and $\frac{-4 + y}{2} = -3$

$x = 7$ $\qquad -4 + y = -6$

$y = -2$

The coordinates of point D are $(7, -2)$.

29. *Sample answer:* Use Theorem 8.7 to construct a parallelogram with two pairs of congruent sides. Use a straightedge to draw $\overline{AB}$ and $\overline{BC}$ intersecting at point B. At point A, use a compass to draw an arc with radius $\overline{BC}$. At point C, use a compass to draw an arc with radius $\overline{AB}$. The arcs intersect at point D. Draw $\overline{AD}$ and $\overline{CD}$.

30. $\overline{AD} \cong \overline{BC}$ because $ABCD$ is a parallelogram. $\angle ADB \cong \angle CBD$ because they are alternate interior angles. $\overline{BF} \cong \overline{DE}$. By the SAS Congruence Postulate, $\triangle AED \cong \triangle CFB$. $\overline{AE} \cong \overline{CF}$ because corresponding parts of congruent triangles are congruent. So, the length of AE is 8.

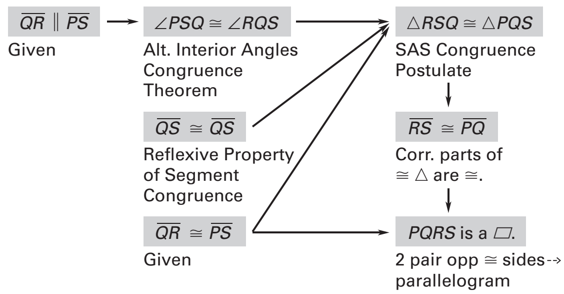
Problem Solving

31. a. $EFJK$; Both pairs of opposite sides are congruent. $EGHK$; Both pairs of opposite sides are congruent. $FGHJ$; Both pairs of opposite sides are congruent.

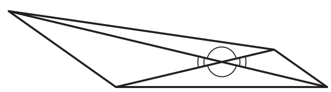
b. The lengths of $\overline{EG}$, $\overline{GH}$, $\overline{KH}$, and $\overline{EK}$ do not change as the lift moves. Because both pairs of opposite sides are always congruent, $EGHK$ is always a parallelogram, so $\overline{EG}$ and $\overline{HK}$ are always parallel.

32. $AEFD$ and $EBCF$ are parallelograms, so $\overline{AD} \cong \overline{EF}$, $\overline{AE} \cong \overline{DF}$, $\overline{BC} \cong \overline{EF}$, and $\overline{EB} \cong \overline{FC}$. Because both pairs of opposite sides are always congruent, $AEFD$ and $EBCF$ are always parallelograms. So, $\overline{AB}$ and $\overline{BC}$ remain parallel to $\overline{EF}$.

33.

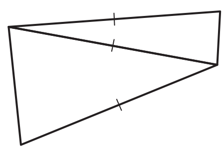


34.



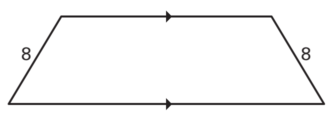
The point of intersection of the diagonals is not necessarily their midpoint.

35.



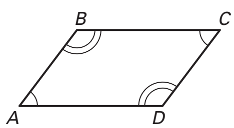
The opposite sides that are not marked in the given diagram are not necessarily the same length.

36.



The sides of length 8 are not necessarily parallel.

37. Converse of Theorem 8.5: In a quadrilateral, if consecutive angles are supplementary, then the quadrilateral is a parallelogram.



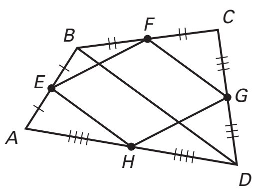
In $ABCD$, you are given $\angle A$ and $\angle B$ are supplementary, and $\angle C$ and $\angle B$ are supplementary, which gives you $m\angle A = m\angle C$. Also $\angle B$ and $\angle C$ are supplementary, and $\angle C$ and $\angle D$ are supplementary which gives you $m\angle B = m\angle D$. So, $ABCD$ is a parallelogram by Theorem 8.8.

38. The sum of the measures of the interior angles of a quadrilateral is 360°. $m\angle A + m\angle B + m\angle C + m\angle D = 360°$. It is given that $\angle A \cong \angle C$ and $\angle B \cong \angle D$, so $m\angle A = m\angle C$ and $m\angle B = m\angle D$. Let $x° = m\angle A = m\angle C$ and $y° = m\angle B = m\angle D$. By the Substitution Property of Equality, $x° + y° + x° + y° = 360°$. $2(x° + y°) = 360°$. $x° + y° = 180$. Using the definition of supplementary angles, $\angle A$ and $\angle B$, $\angle B$ and $\angle C$, $\angle C$ and $\angle D$, and $\angle D$ and $\angle A$ are supplementary. Using Theorem 8.5, *ABCD* is a parallelogram.

39. It is given that $\overline{KP} \cong \overline{MP}$ and $\overline{JP} \cong \overline{LP}$. $\angle KPJ \cong \angle LPM$ and $\angle KPL \cong \angle JPM$ by the Vertical Angles Congruence Theorem. $\triangle KPJ \cong \triangle MPL$ and $\triangle KPL \cong \triangle MPJ$ by the SAS Congruence Postulate. Because corresponding parts of congruent triangles are congruent, $\overline{KJ} \cong \overline{ML}$ and $\overline{KL} \cong \overline{MJ}$. Using Theorem 8.7, *JKLM* is a parallelogram.

40. It is given that *DEBF* is a parallelogram and $AE = CF$. Because *DEBF* is a parallelogram, you know that $FD = EB$, $\angle BFD \cong \angle DEB$, and $ED = FB$. $AE + EB = CF + FD$ which implies that $AB = CD$, which implies that $\overline{AB} \cong \overline{CD}$. $\angle BFC$ and $\angle BFD$, and $\angle DEB$ and $\angle DEA$ form linear pairs, thus making them supplementary. Using the Congruent Supplements Theorem, $\angle BFC \cong \angle DEA$ making $\triangle AED \cong \triangle CFB$ using the SAS Congruence Theorem. Because corresponding parts of congruent triangles are congruent, $\overline{AD} \cong \overline{CB}$. *ABCD* is a parallelogram by Theorem 8.7.

41.



$\overline{FG}$ is the midsegment of $\triangle CBD$ and therefore is parallel to $\overline{BD}$ and half its length. $\overline{EH}$ is the midsegment of $\triangle ABD$ and therefore is parallel to $\overline{BD}$ and half its length. This makes $\overline{EH}$ and $\overline{FG}$ both parallel and congruent. Using Theorem 8.9, *EFGH* is a parallelogram.

42. $\overline{FJ}$ is the midsegment of $\triangle AED$ and therefore is parallel to $\overline{AD}$ and half it length. $\overline{GH}$ is the midsegment of $\triangle BEC$ and therefore is parallel to $\overline{BC}$ and half its length. Together, this gives you $\overline{FJ} \cong \overline{GH}$ and $\overline{FJ} \parallel \overline{GH}$. Using Theorem 8.9, *FGHJ* is a parallelogram.

Problem Solving Workshop for the lesson "Show that a Quadrilateral is a Parallelogram"

1. Slope of $\overline{AB} = \frac{3-5}{-3-2} = \frac{2}{5}$

Slope of $\overline{BC} = \frac{5-2}{2-5} = \frac{3}{-3} = -1$

Slope of $\overline{CD} = \frac{2-0}{5-0} = \frac{2}{5}$

Slope of $\overline{DA} = \frac{0-3}{0-(-3)} = -\frac{3}{3} = -1$

The slopes of $\overline{AB}$ and $\overline{CD}$ are equal, so $\overline{AB}$ and $\overline{CD}$ are parallel. The slopes of $\overline{BC}$ and $\overline{DA}$ are equal, so they are parallel. Because the quadrilateral *ABCD* has two pairs of parallel sides, it is a parallelogram.

2. Method 1: Show diagonals bisect each other.

The coordinates of the endpoints of diagonal $\overline{EG}$ are $E(-2, 1)$ and $G(1, 0)$.

Midpoint of $\overline{EG} = \left(\frac{-2+1}{2}, \frac{1+0}{2}\right) = \left(-\frac{1}{2}, \frac{1}{2}\right)$

The coordinates of the endpoints of diagonal $\overline{HF}$ are $H(-4, -2)$ and $F(3, 3)$.

Midpoint of $\overline{HF} = \left(\frac{-4+3}{2}, \frac{-2+3}{2}\right) = \left(-\frac{1}{2}, \frac{1}{2}\right)$

Because the midpoints of both diagonals are the same point, the diagonals bisect each other. So, *EFGH* is a parallelogram.

Method 2: Show both pairs of opposite sides are parallel.

Slope of $\overline{EF} = \frac{3-1}{3-(-2)} = \frac{2}{5}$

Slope of $\overline{GF} = \frac{3-0}{3-1} = \frac{3}{2}$

Slope of $\overline{HG} = \frac{0-(-2)}{1-(-4)} = \frac{2}{5}$

Slope of $\overline{EH} = \frac{1-(-2)}{-2-(-4)} = \frac{3}{2}$

Both pairs of opposite sides $\overline{EH}$ and $\overline{GF}$, and $\overline{HG}$ and $\overline{EF}$ have the same slope. So, $\overline{EH} \parallel \overline{GF}$ and $\overline{HG} \parallel \overline{EF}$. *EFGH* is a parallelogram.

3. Draw a line connecting Newton, Packard, Quarry, and Riverdale. Label the quadrilateral *NPQR*. Use the midpoint formula to find the midpoints of diagonals $\overline{NQ}$ and $\overline{RP}$.

The coordinates of the endpoints of $\overline{NQ}$ are $N(3, 4)$ and $Q(12, 3)$.

Midpoint of $\overline{NQ} = \left(\frac{3+12}{2}, \frac{4+3}{2}\right) = \left(\frac{15}{2}, \frac{7}{2}\right)$

The coordinates of the endpoints of $\overline{RP}$ are $R(5, 1)$ and $P(9, 6)$.

Midpoint of $\overline{RP} = \left(\frac{5+9}{2}, \frac{1+6}{2}\right) = \left(\frac{14}{2}, \frac{7}{2}\right) = \left(7, \frac{7}{2}\right)$

The midpoints of the two diagonals are not the same point. The diagonals $\overline{NQ}$ and $\overline{RP}$ do not bisect each other. So, the four towns on the map do not form the vertices of a parallelogram.

4. a. Midpoint of $\overline{AC} = \left(\frac{1+7}{2}, \frac{0+2}{2}\right) = \left(\frac{8}{2}, \frac{2}{2}\right) = (4, 1)$

Midpoint of $\overline{BD} = \left(\frac{5+3}{2}, \frac{0+2}{2}\right) = \left(\frac{8}{2}, \frac{2}{2}\right) = (4, 1)$

The midpoints of the two diagonals are the same. So $\overline{AC}$ and $\overline{BD}$ bisect each other. *ABCD* is a parallelogram.

b. Midpoint of $\overline{EG} = \left(\frac{3+9}{2}, \frac{4+5}{2}\right) = \left(\frac{12}{2}, \frac{9}{2}\right) = \left(6, \frac{9}{2}\right)$

Midpoint of $\overline{FH} = \left(\frac{6+6}{2}, \frac{8+0}{2}\right) = \left(\frac{12}{2}, \frac{8}{2}\right) = (6, 4)$

The midpoints of the two diagonals are not the same. So $\overline{EG}$ and $\overline{FH}$ do not bisect each other. *EFGH* is not a parallelogram.

c. Midpoint of $\overline{JL} = \left(\frac{-1+2}{2}, \frac{0+2}{2}\right), = \left(\frac{1}{2}, \frac{2}{2}\right) = \left(\frac{1}{2}, 1\right)$

Midpoint of $\overline{KM} = \left(\frac{2+(-1)}{2}, \frac{-2+4}{2}\right) = \left(\frac{1}{2}, \frac{2}{2}\right)$

$= \left(\frac{1}{2}, 1\right)$

The midpoints of the two diagonals are the same, so $\overline{JL}$ and $\overline{KM}$ bisect each other. *JKLM* is a parallelogram.

5. The student's error was making $\overline{PQ}$ and $\overline{QR}$ opposite sides when in fact they are adjacent sides.

$\overline{PQ}$ and $\overline{RS}$, and $\overline{QR}$ and $\overline{SP}$ are opposite sides.

$PQ = \sqrt{(3-2)^2 + (4-2)^2} = \sqrt{5}$

$RS = \sqrt{(6-5)^2 + (5-3)^2} = \sqrt{5}$

$QR = \sqrt{(6-3)^2 + (5-4)^2} = \sqrt{10}$

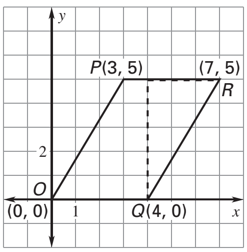
$SP = \sqrt{(5-2)^2 + (3-2)^2} = \sqrt{10}$

$\overline{PQ} \cong \overline{RS}$ and $\overline{QR} \cong \overline{SP}$, so *PQRS* is a parallelogram.

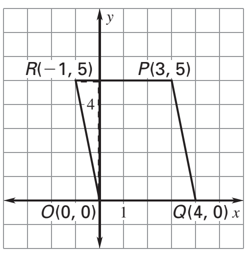
6. The possible coordinates of *R* are (7, 5), (−1, 5), (1, −5).

Sample answer:

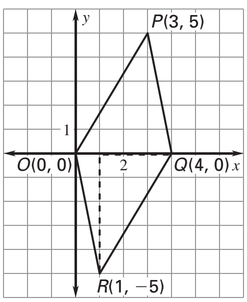
To find (7, 5), use the slope from point *O* to point *P*. To locate *R*, start at *Q* and use the slope and length of $\overline{OP}$.



To find (−1, 5), use the slope from point *Q* to point *P*. To locate *R*, start at *O* and use the slope and length of $\overline{QP}$.



To find (1, −5), use the slope from point *P* to point *O*. To locate *R*, start at *Q* and use the slope and length of $\overline{PO}$.



Mixed Review of Problem Solving for the lessons "Find Angle Measures in Polygons", "Use Properties of Parallelograms", and "Show that a Quadrilateral is a Parallelogram"

1. a. The polygon has 5 sides. It is a pentagon.

b. $(n - 2) \cdot 180° = (5 - 2) \cdot 180° = 540°$

c. The sum of the exterior angles of a polygon is always 360°.

2. The sum of the measures of the interior angles of the house is $(5 - 3) \cdot 180° = 540°$ because there are 5 sides. To find $m\angle A$ and $m\angle C$, subtract 270° from 540° and then divide by 2.

3. Diagonals in a parallelogram bisect each other, so the lengths of the halves of the diagonals are equal.

$12x + 1 = 49$ and $8y + 4 = 36$

$12x = 48$ $\quad 8y = 32$

$x = 4$ $\quad y = 4$

4. $157° + 128° + 115° + 162° + 169° + 131° + 155° + 168° + x° + 2x° = (n - 2) \cdot 180°$

$$1185 + 3x = (10 - 2) \cdot 180$$

$$3x = 255$$

$$x = 85$$

5. In a parallelogram, consecutive angles are supplementary. Solve $x° + (3x - 12)° = 180°$ for *x*. Use the value of *x* to find the degree measure of the two consecutive angles. In a parallelogram the angle opposite each of these known angles has the same measure.

6. a. $\overline{HG}$ and $\overline{EF}$ are congruent and parallel, so the quadrilateral is a parallelogram. As the binoculars are moved, the shape of the parallelogram changes but both pairs of opposite angles remain congruent. As long as the angles keep their congruency, $\overline{EF}$ and $\overline{GH}$ and $\overline{FG}$ and $\overline{GH}$ remain parallel.

b. As $m\angle E$ changes from 55° to 50°, $m\angle G$ will change from 55° to 50°. This means $m\angle H$ and $m\angle F$ change from 135° to 140° because consecutive angles in a parallelogram are supplementary.

7. a. Slope of $\overline{MN} = \frac{4-1}{3-(-8)} = \frac{3}{11}$

Slope of $\overline{NP} = \frac{-1-4}{7-3} = -\frac{5}{4}$

Slope of $\overline{PQ} = \frac{-4-(-1)}{-4-7} = \frac{-3}{-11} = \frac{3}{11}$

Slope of $\overline{QM} = \frac{1-(-4)}{8-(-4)} = -\frac{5}{4}$

$\overline{MN}$ and $\overline{PQ}$ and $\overline{NP}$ and $\overline{QM}$ have the same slope so they are parallel. By definition, quadrilateral *MNPQ* is a parallelogram.

b. $MN = \sqrt{(3-(-8))^2 + (4-1)^2} = \sqrt{130}$

$NP = \sqrt{(7-3)^2 + (-1-4)^2} = \sqrt{41}$

$PQ = \sqrt{(-4-7)^2 + (-4-(-1))^2} = \sqrt{130}$

$QM = \sqrt{(-8-(-4))^2 + (1-(-4))^2} = \sqrt{41}$

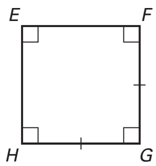
$\overline{MN} \cong \overline{PQ}$ and $\overline{NP} \cong \overline{QM}$. So *MNPQ* is a parallelogram because both pairs of opposite sides are congruent.

8.

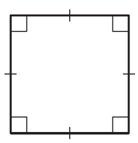
Statements	Reasons
1. $\overline{BX} \parallel \overline{DY}$	**1.** Lines ⊥ to a Transversal Theorem
2. *ABCD* is a parallelogram. $\overline{BX} \perp \overline{AC}$ and $\overline{DY} \perp \overline{AC}$.	**2.** Given
3. $\angle BXA$ and $\angle DYC$ are right angles.	**3.** Definition of perpendicular lines
4. $\angle BXA \cong \angle DYC$	**4.** Right Angles Congruence Theorem
5. $\angle BAX \cong \angle DCY$	**5.** Alternate Interior Angles Theorem
6. $\overline{AB} \cong \overline{CD}$	**6.** Theorem 8.3
7. $\triangle BXA \cong \triangle DYC$	**7.** AAS Congruence Theorem
8. $\overline{BX} \cong \overline{DY}$	**8.** Corr. parts of ≅ ⚠ are ≅.
9. *XBYD* is a ▱.	**9.** Theorem 8.9

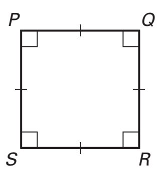
Lesson 8.4 Properties of Rhombuses, Rectangles, and Squares

Guided Practice for the lesson "Properties of Rhombuses, Rectangles, and Squares"

1. 

If rectangle *EFGH* is a square, then all four sides are congruent. So, $\overline{FG} \cong \overline{GH}$ if *EFGH* is a square. Because not all rectangles are squares, the statement is sometimes true.

2. 
The quadrilateral has four congruent sides and four congruent angles. So, the quadrilateral is a rhombus and a rectangle. By the Square Corollary, the quadrilateral is a square.

3. 

The square is a parallelogram, rhombus, and rectangle. Opposite pairs of sides are parallel and all four sides are congruent. All angles are right angles. Diagonals are congruent and bisect each other. Diagonals are perpendicular and each diagonal bisects a pair of opposite angles.

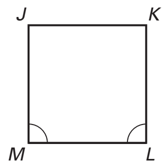
4. Yes. A parallelogram is a rectangle iff its diagonals are congruent, therefore the diagonals of a rectangle are congruent. If the lengths of the diagonals are found to be the same, the boards will form a rectangle.

Exercises for the lesson "Properties of Rhombuses, Rectangles, and Squares"

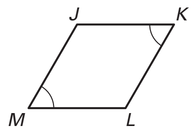
Skill Practice

1. Another name for an equilateral rectangle is a square.

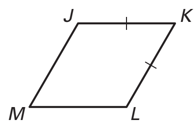
2. Yes. The diagonals of the figure are perpendicular so the figure must be a rhombus.

3. 

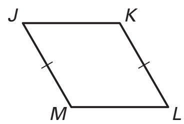
Sometimes; if rhombus *JKLM* is a square, then all four angles will be right angles and congruent.

4. 

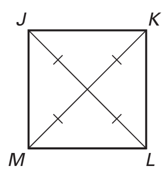
Always; opposite angles in a rhombus are always congruent.

5. 

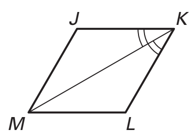
Always; all four sides in a rhombus are congruent.

6. 

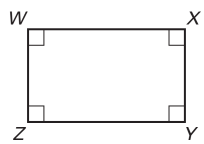
Always; all four sides in a rhombus are congruent.

7. 

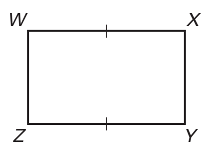
Sometimes; if rhombus *JKLM* is also a square, the diagonals are congruent because only diagonals of rectangles are congruent.

8. 

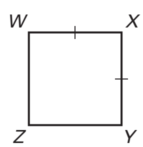
Always; the diagonals in a rhombus bisect opposite angles.

9. 

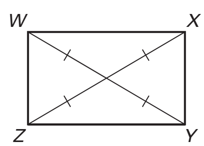
Always; a rectangle has four right angles and right angles are congruent.

10. 

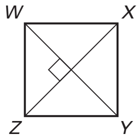
Always; opposite sides of a rectangle are congruent.

11. 

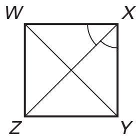
Sometimes; if rectangle *WXYZ* is also a square then $WX \cong XY$.

12. 

Always; the diagonals of a rectangle are congruent.

13. 

Sometimes; if $WXYZ$ is also a rhombus then $\overline{XY} \perp \overline{XZ}$.

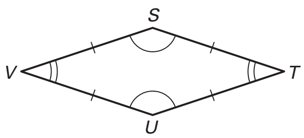
14. 

Sometimes; if $WXYZ$ is also a rhombus then $\angle WXZ \cong \angle YXZ$.

15. The quadrilateral is a square because all four sides and angles are congruent.

16. Both pairs of opposite sides are congruent. Because consecutive angles are supplementary, all four angles are right angles. So the quadrilateral is a rectangle.

17. The fourth angle measure is 40°, meaning that both pairs of opposite sides are parallel. So, the figure is a parallelogram with two consecutive sides congruent. But this is only possible if the remaining two sides are also congruent, so the quadrilateral is a rhombus.

18. 

Rhombus $STUV$ has four congruent sides. Both pairs of opposite sides and angles are congruent. The diagonals $\overline{SU}$ and $\overline{TV}$ bisect one another, are perpendicular to each other, and they bisect opposite angles. Because rhombus $STUV$ is a parallelogram, by definition, both pairs of opposite sides are parallel.

19. rectangle, square

20. square

21. rhombus, square

22. parallelogram, rectangle, rhombus, square

23. parallelogram, rectangle, rhombus, square

24. rhombus, square

25. The quadrilateral is a rectangle but it is not a rhombus. Angles $\angle PSQ$ and $\angle QSR$ are not necessarily congruent. They are, however, complementary so their sum is 90°.

$(7x - 4)^\circ + (3x + 14)^\circ = 90^\circ$

$10x + 10 = 90$

$10x = 80$

$x = 8$

26. The quadrilateral is a rhombus because all four sides are congruent.

$x^\circ + 104^\circ = 180^\circ$

$x = 76$

$3y = y + 8$

$2y = 8$

$y = 4$

27. The quadrilateral is a rectangle because all four angles are right angles.

$5x - 9 = x + 31$

$4x - 9 = 31$

$4x = 40$

$x = 10$

$4y + 5 = 2y + 35$

$2y + 5 = 35$

$2y = 30$

$y = 15$

28. The quadrilateral is a square because all four angles are right angles and the diagonals are perpendicular.

$5x = 3x + 18$

$2x = 18$

$x = 9$

$2y = 10$

$y = 5$

29. The quadrilateral is a parallelogram because both pairs of opposite sides are congruent, so $m\angle EFG = m\angle EHG$.

$(5x - 6)^\circ = (4x + 7)^\circ$

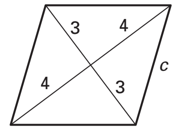
$x - 6 = 7$

$x = 13$

$2y + 1 = y + 3$

$y + 1 = 3$

$y = 2$

30. 

The diagonals bisect each other and are perpendicular. Four right triangles are formed by the diagonals. Each triangle has leg lengths of 3 and 4 and a hypotenuse of length c. Using the Pythagorean Theorem:

$c^2 = 3^2 + 4^2$

$c = \sqrt{9 + 16}$

$c = 5$

The perimeter is $4 \cdot 5 = 20$ inches.

31. C; $\frac{AC}{CD} = \frac{FH}{HJ}$

$\frac{5}{4} = \frac{2 \cdot FM}{HJ}$

$5(HJ) = 4(2 \cdot 5)$

$5(HJ) = 40$

$HJ = 8$

32. Because $\angle DAC \cong \angle BAC$, $m\angle DAC = 53^\circ$.

33. The diagonals of a rhombus are perpendicular. So, $m\angle AED = 90^\circ$.

34. $m\angle ADC + m\angle BAD = 180^\circ$

$m\angle ADC + 2 \cdot m\angle BAC = 180^\circ$

$m\angle ADC + 2 \cdot 53^\circ = 180^\circ$

$m\angle ADC + 106^\circ = 180^\circ$

$m\angle ADC = 74^\circ$

35. $DB = 2 \cdot DE = 2 \cdot 8 = 16$

36. The diagonals form the right triangle AED and bisect $\angle D$.

$\cos 37^\circ = \frac{8}{x}$

$x = \frac{8}{\cos 37^\circ}$

$x \approx 10$

$(DE)^2 + (AE)^2 = (AD)^2$

$8^2 + (AE)^2 \approx (10)^2$

$(AE)^2 \approx 36$

$AE \approx 6$

37. $AC = 2 \cdot AE \approx 2 \cdot 6 \approx 12$

38. $m\angle SRT + m\angle PTS + m\angle TSR = 180°$

$m\angle SRT + 34° + 90° = 180°$

$m\angle SRT + 124° = 180°$

$m\angle SRT = 56°$

39. $\angle PTS \cong \angle PRQ \cong \angle RQP \cong \angle PST$

$m\angle PQR + m\angle QPR + m\angle PRQ = 180°$

$34° + m\angle QPR + 34° = 180°$

$m\angle QPR + 68° = 180°$

$m\angle QPR = 112°$

40. $QP = \frac{1}{2} \cdot QS = \frac{1}{2}(10) = 5$

41. $RP = QP = 5$

42. $m\angle PST + m\angle PSR = 90°$

$34° + m\angle PSR = 90°$

$m\angle PSR = 56°$

$\sin(\angle PSR) = \frac{QR}{QS}$

$\sin 56° = \frac{QR}{10}$

$10(\sin 56°) = QR$

$8.3 \approx QR$

43. $\sin(\angle PQR) = \frac{RS}{QS}$

$\sin 34° = \frac{RS}{10}$

$10(\sin 34°) = RS$

$5.6 \approx RS$

44. The diagonals are $\perp$. $m\angle MKN = 90°$.

45. Diagonals bisect opposite angles.

$m\angle LMK = \frac{1}{2} \cdot m\angle LMN = \frac{1}{2} \cdot 90° = 45°$

46. Diagonals bisect opposite angles.

$m\angle LPK = \frac{1}{2} \cdot \angle LPN = \frac{1}{2} \cdot 90° = 45°$

47. Diagonals bisect each other.

$KN = LK = 1$

48. Diagonals have equal lengths.

$MP = 2 \cdot LK = 2 \cdot 1 = 2$

49. $\triangle LPN$ is a right triangle with side lengths x and hypotenuse $LN = 2$. Using the Pythagorean Theorem:

$x^2 + x^2 = (LN)^2$

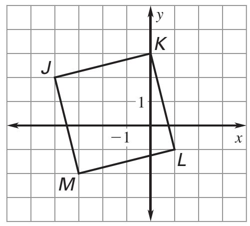
$2x^2 = 4$

$x^2 = 2$

$x = \sqrt{2}$

The length of LP is $\sqrt{2}$.

50.



Slope of $\overline{JK} = \frac{3 - 2}{0 - (-4)} = \frac{1}{4}$

Slope of $\overline{JM} = \frac{-2 - 2}{-3 - (-4)}$

$= \frac{-4}{1} = -4$

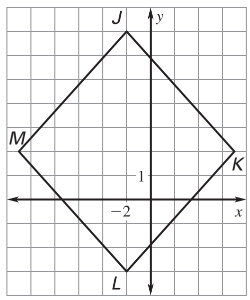
Product of slopes $= \left(\frac{1}{4}\right)(-4) = -1$

$\overline{JK} = \sqrt{(0 - 4)^2 + (3 - 2)^2} = \sqrt{17}$

$\overline{JM} = \sqrt{(-3 - (-4))^2 + (-2 - 2)^2} = \sqrt{17}$

Because $JKLM$ is a parallelogram, its opposite sides are congruent. So, $\overline{JK} \cong \overline{ML}$ and $\overline{JM} \cong \overline{KL}$. Because $\overline{JK} \cong \overline{JM}$, $\overline{JK} \cong \overline{ML} \cong \overline{JM} \cong \overline{KL}$. $\overline{JK}$ and $\overline{JM}$ are perpendicular lines because the product of their slopes is -1. So, $m\angle KJM = 90°$. Because opposite angles of a parallelogram are congruent and consecutive angles are supplementary, all four angles are right angles. So, the parallelogram is a square. The perimeter of the square is four times one side length, or $4\sqrt{17}$.

51.



Slope of $\overline{JK} = \frac{2 - 7}{7 - (-2)} = \frac{-5}{9}$

$= -\frac{5}{9}$

Slope of $\overline{KL} = \frac{-3 - 2}{-2 - 7} = \frac{-5}{-9}$

$= \frac{5}{9}$

Product of slopes $= \left(-\frac{5}{9}\right)\left(\frac{5}{9}\right) = -\frac{25}{81} \neq -1$

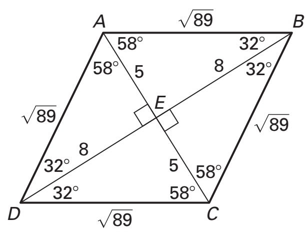
$JK = \sqrt{(7 - (-2))^2 + (2 - 7)^2} = \sqrt{106}$

$KL = \sqrt{(-2 - 7)^2 + (-3 - 2)^2} = \sqrt{106}$

Because $JKLM$ is a parallelogram, $\overline{JM} \cong \overline{KL}$ and $\overline{JK} \cong \overline{ML}$. Because $\overline{JK} \cong \overline{KL}$, $\overline{JM} \cong \overline{JK} \cong \overline{KL} \cong \overline{ML}$. $\overline{JK}$ and $\overline{KL}$ are not perpendicular because the product of their slopes is not -1. So, there are not right angles. The parallelogram is a rhombus. The perimeter of the rhombus is four times one side length, or $4\sqrt{106}$.

52. Not all rhombuses are similar. Two rhombuses do not have to have the same angle measures. All squares are similar. Their angle measures are 90°, and the ratio of the lengths of their sides are equal.

53.



The diagonals of a rhombus bisect each other and intersect at a right angle. So, $m\angle AEB = m\angle AED = m\angle DEC = m\angle BEC = 90°$. $AE = CE = 5$, and $DE = BE = 8$.

$\triangle ABE$ is a right triangle. Use the Pythagorean Theorem to find AB.

$(AB)^2 = (AE)^2 + (BE)^2$

$(AB)^2 = 5^2 + 8^2$

$AB = \sqrt{89}$

$ABCD$ is a rhombus, so $AB = BC = CD = DA = \sqrt{89}$.

Because $\triangle ABE$ is a right triangle, use the inverse tangent ratio to find $m\angle EAB$.

$\tan(EAB)° = \frac{\text{opp.}}{\text{adj.}} = \frac{8}{5} = 1.6$

$\angle EAB = \tan^{-1}(1.6) \approx 58°$

Diagonals of a rhombus bisect congruent opposite angles, so $m\angle EAB = m\angle EAD = m\angle ECB = m\angle ECD = 58°$. Use the fact that the sum of the measure of the interior angles of a triangle is 180° to find $m\angle EBA$.

$m\angle EBA + m\angle EAB + m\angle AEB = 180°$

$m\angle EBA + 58° + 90° = 180°$

$m\angle EBA = 32°$

Diagonals of a rhombus bisect congruent opposite angles, so $m\angle EBA = m\angle EBC = m\angle EDC = m\angle EDA = 32°$.

Problem Solving

54. a. $HBDF$ is a rhombus because all four sides are congruent. $ACEG$ is a rectangle because all four angles are right angles.

b. Because $ACEG$ is a rectangle, the lengths of $\overline{AE}$ and $\overline{GC}$ are congruent. The lengths of $\overline{AJ}$, $\overline{JE}$, $\overline{CJ}$, and $\overline{JG}$ are congruent because the diagonals of a rectangle bisect each other.

55. You can measure the diagonals of the square. If the diagonals are the same length, the quadrilateral patio is a square.

56. $ABCD$ is a rhombus so $\overline{AB} \cong \overline{BC}$. $ABCD$ is a parallelogram so its diagonals $\overline{AC}$ and $\overline{BD}$ bisect each other. So, $\overline{AX} \cong \overline{CX}$ and $\overline{BX} \cong \overline{BX}$. The SSS Congruence Postulate then proves that $\triangle AXB \cong \triangle CXB$. Because correspnding parts of congruent triangles are congruent, $\angle AXB \cong \angle CXB$. Because $\overline{AC}$ and $\overline{BD}$ intersect to form congruent adjacent angles, $\overline{AC} \perp \overline{BD}$.

57. If a quadrilateral is a rhombus, then it has four congruent sides. The conditional statement is true because a rhombus is a parallelogram with four congruent sides. If a quadrilateral has four congruent sides, then it is rhombus. The converse is true because a quadrilateral with four congruent sides is also a parallelogram with four congruent sides making it a rhombus.

58. If a quadrilateral is a rectangle, then it has four right angles. The conditional statement is true by the definition of a rectangle.

If a quadrilateral has four right angles, then it is a rectangle. The converse is true because both pairs of opposite angles are congruent, so the rectangle is a parallelogram. A parallelogram with 4 right angles is a rectangle.

59. If a quadrilateral is a square, then it is a rhombus and a rectangle. The conditional statement is true because a square is a parallelogram with four right angles (so it is a rectangle) and four congruent sides (so it is a rhombus).

If a quadrilateral is a rhombus and a rectangle, then it is a square. The converse is true because a rhombus has four congruent sides and a rectangle has four right angles. By definition, a parallelogram that has four congruent sides and four right angles is a square.

60.

Statements	Reasons
1. $PQRS$ is a parallelogram, $\overline{PR}$ bisects $\angle SPQ$ and $\angle QRS$, $\overline{SQ}$ bisects $\angle PSR$ and $\angle RQP$.	**1.** Given
2. $\angle QPT \cong \angle SPT$ $\angle PST \cong \angle RST$ $\angle SRT \cong \angle QRT$ $\angle RQT \cong \angle PQT$	**2.** Definition of angle bisector
3. $\overline{PR} \cong \overline{PR}$, $\overline{QS} \cong \overline{QS}$	**3.** Reflexive Property of Congruence
4. $\triangle PRQ \cong \triangle PRS$ $\triangle PQS \cong \triangle RQS$	**4.** ASA Congruence Postulate
5. $\overline{RS} \cong \overline{RQ}$, $\overline{PQ} \cong \overline{RQ}$ $\overline{SR} \cong \overline{PS}$, $\overline{PQ} \cong \overline{PS}$	**5.** Corresponding parts of congruent triangles are congruent.
6. $\overline{PQ} \cong \overline{RQ} \cong \overline{SR} \cong \overline{PS}$	**6.** Transitive Property of Congruence
7. $PQRS$ is a rhombus.	**7.** Definition of a rhombus

61.

Statements	Reasons
1. $WXYZ$ is a rhombus.	**1.** Given
2. $\overline{WX} \cong \overline{XY} \cong \overline{YZ} \cong \overline{ZW}$	**2.** A quadrilateral is a rhombus if and only if it has four congruent sides.
3. $\overline{WY} \cong \overline{WY}$, $\overline{XZ} \cong \overline{XZ}$	**3.** Reflexive Property of Congruence
4. $\triangle WYX \cong \triangle WYZ$ $\triangle WZX \cong \triangle YZX$	**4.** SSS Congruence Postulate
5. $\angle ZWY \cong \angle XWY$ $\angle ZYW \cong \angle XYW$ $\angle WZX \cong \angle YZX$ $\angle WXZ \cong \angle YXZ$	**5.** Corresponding parts of congruent triangles are congruent.
6. $\overline{WY}$ bisects $\angle ZWX$ and $\angle XYZ$; $\overline{ZX}$ bisects $\angle WZY$ and $\angle YXW$.	**6.** Definition of an angle bisector

62. **a.** It is given that $\overline{AB} \parallel \overline{CD}$ and $\overline{DB}$ bisects $\angle ADC$. $\angle ABD \cong \angle CDB$ using the Alternate Interior Angles Theorem. Because $\angle ABD \cong \angle CDB$ and $\angle ADB \cong \angle CDB$, $\angle ABD \cong \angle ADB$ using the Transitive Property of Congruence. $\overline{AB} \cong \overline{AD}$ using the Converse of Base Angles Theorem.

b. If $\overline{AD} \parallel \overline{BC}$, then the quadrilateral is a parallelogram by definition. Using the fact that opposite sides of a parallelogram are congruent along with the fact that $\overline{AB} \cong \overline{AD}$ means all four sides of the parallelogram are congruent. So, $ABCD$ is a rhombus.

63. *Sample answer:* Let rectangle $ABCD$ have vertices $(0, 0)$, $(a, 0)$, (a, b), and $(0, b)$, respectively. The diagonal $\overline{AC}$ has a length of $\sqrt{a^2 + b^2}$ and diagonal $\overline{BD}$ has a length of $\sqrt{a^2 + b^2}$. So, $AC = BD = \sqrt{a^2 + b^2}$.

64. The diagonals of a ▱ bisect each other. So, $OD = OG$ and $OH = OF$. $OD + OG = DG$ and $OH + OF = HF$ the $OD + OG = OH + OF$ by substitution. $OD + OD = OH + OH$ and $OG + OG = OF + OF$ again by substitution giving $OD = OH$ and $OG = OF$. By the Transitive Property of Congruence, $OD = OF = OG = OH$.

Find the y-coordinate of point D.

$OF = \sqrt{(0 - b)^2 + (0 - 0)^2} = \sqrt{b^2} = b$

Because $OF = OD$, $OD = b$. Let $D = (a, y)$.

$$b = \sqrt{(a - 0)^2 + (y - 0)^2}$$
$$b = \sqrt{a^2 + y^2}$$
$$b^2 = a^2 + y^2$$
$$b^2 - a^2 = y^2$$
$$\sqrt{b^2 - a^2} = y$$
$$D\left(a, \sqrt{b^2 - a^2}\right)$$

Find the coordinates of H and G.

Because $OF = OH$, $OH = b$, and OH lies on the x-axis, $H = (-b, 0)$. Because $OD = OG$, $OG = b$, and G is in quadrant III, the coordinates must be negative. So, $G = \left(-a, -\sqrt{b^2 - a^2}\right)$.

Find and compare the slopes $\overline{OF}$ and $\overline{GF}$.

$$\text{Slope of } \overline{DF} = \frac{\sqrt{b^2 - a^2} - 0}{a - b} = \frac{\sqrt{b^2 - a^2}}{a - b}$$

$$\text{Slope of } \overline{GF} = \frac{-\sqrt{b^2 - a^2} - 0}{-a - b} = \frac{\sqrt{b^2 - a^2}}{a + b}$$

$$\text{Product of slopes of } \overline{DF} \text{ and } \overline{GF} = \frac{\sqrt{b^2 - a^2}}{a - b} \cdot \frac{\sqrt{b^2 - a^2}}{a + b} = \frac{b^2 - a^2}{(a - b)(a + b)} = \frac{(b + a)(b - a)}{(a - b)(a + b)} = \frac{(b - a)}{(a - b)} = \frac{(-1)(a - b)}{a - b} = -1$$

So, $\overline{DF} \perp \overline{GF}$.

Find and compare the slopes of $\overline{DH}$ and $\overline{GH}$.

$$\text{Slope of } \overline{DH} = \frac{\sqrt{b^2 - a^2} - 0}{a - (-b)} = \frac{\sqrt{b^2 - a^2}}{a + b}$$

$$\text{Slope of } \overline{GH} = \frac{-\sqrt{b^2 - a^2} - 0}{-a - (-b)} = \frac{\sqrt{b^2 - a^2}}{a - b}$$

$$\text{Product of slopes of } \overline{DH} \text{ and } \overline{GH} = \frac{\sqrt{b^2 - a^2}}{a + b} \cdot \frac{\sqrt{b^2 - a^2}}{a - b} = \frac{b^2 - a^2}{(a + b)(a - b)} = \frac{(b + a)(b - a)}{(a + b)(a - b)} = \frac{(b - a)}{(a - b)} = \frac{(-1)(a - b)}{a - b} = -1$$

So, $\overline{DH} \perp \overline{GH}$. $ABCD$ is a parallelogram with four right angles, or it is a rectangle.

Quiz for the lessons "Show that a Quadrilateral is a Parallelogram" and "Properties of Rhombuses, Rectangles, and Squares"

1. $5x + 3 = 7x - 5$
$-2x = -8$
$x = 4$

2. $(3x - 13)° = (x + 19)°$
$2x = 32$
$x = 16$

3. $3x = 5x - 48$
$-2x = -48$
$x = 24$

4. Because the diagonals are perpendicular and bisect each other, the quadrilateral is a rhombus and a rectangle. So, the quadrilateral is a square.

5. The quadrilateral is a rhombus because a parallelogram is a rhombus iff each diagonal bisects opposite angles.

6. The quadrilateral has four right angles. The quadrilateral is a rectangle.

Lesson 8.5 Use Properties of Trapezoids and Kites

Investigating Geometry Activity for the lesson "Use Properties of Trapezoids and Kites"

STEP 5

The length of EF is always equal to $\frac{AB + DC}{2}$.

1. The length of the midsegment of a trapezoid is always equal to one-half the sum of the lengths of the two parallel sides.

2. If the midsegment is equidistant from each side at two points, it must be parallel to both.

3. It divides two sides of the polygon into congruent segments. The length of the midsegment of a triangle is half of the length of the side parallel to it. The length of the midsegment of a trapezoid is one-half the sum of the lengths of the two parallel sides.

Guided Practice for the lesson "Use Properties of Trapezoids and Kites"

1. Slope of $\overline{RS} = \frac{5-3}{4-0} = \frac{2}{4} = \frac{1}{2}$

Slope of $\overline{OT} = \frac{2-0}{4-0} = \frac{2}{4} = \frac{1}{2}$

The slopes of $\overline{RS}$ and $\overline{OT}$ are the same, so $\overline{RS} \parallel \overline{OT}$.

Slope of $\overline{ST} = \frac{2-5}{4-4} = \frac{3}{0}$, which is undefined

Slope of $\overline{OR} = \frac{3-0}{0-0} = \frac{3}{0}$, which is undefined

The slopes of $\overline{ST}$ and $\overline{OR}$ are the same, so $\overline{ST} \parallel \overline{OR}$.

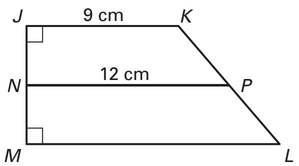
Because both pairs of opposite sides are parallel, quadrilateral $ORST$ is a parallelogram.

2. $\angle R$ and $\angle O$ and $\angle S$ and $\angle T$ are supplementary angles by the Consecutive Interior Angles Theorem.

3. A trapezoid is isosceles if its diagonals are congruent.

4. By the Consecutive Interior Angles Theorem, $m\angle EFG + m\angle FGH = 180°$. $m\angle EFG = 180° - 110° = 70°$. Because the base angles are congruent, trapezoid $EFGH$ is isosceles.

5.



$NP = \frac{JK + ML}{2}$

$12 = \frac{9 + ML}{2}$

$24 = 9 + ML$

$15 = ML$

The length of $\overline{NP}$ is one half the sum of the two parallel sides. So the length of $\overline{ML}$ is 15 cm.

6. $3x° + 75° + 90° + 120° = 360°$

$3x + 285 = 360$

$3x = 75$

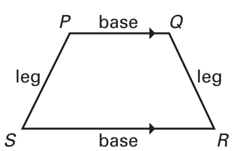
$x = 25$

The value of x is 25, so $(3x)°$ is 75°. The congruent angles are both 75°.

Exercises for the lesson "Use Properties of Trapezoids and Kites"

Skill Practice

1.



The bases are sides $\overline{PQ}$ and $\overline{RS}$. The nonparallel sides $\overline{PS}$ and $\overline{QR}$ are the legs of the trapezoid.

2. A trapezoid has exactly one pair of parallel sides and at most one pair of congruent opposite sides.

A kite has two pairs of consecutive congruent sides and one pair of opposite congruent angles.

3. Slope of $\overline{AB} = \frac{4-4}{4-0} = \frac{0}{4} = 0$

Slope of $\overline{CD} = \frac{1-(-2)}{2-8} = \frac{-3}{6} = -\frac{1}{2}$

The slopes of $\overline{AB}$ and $\overline{CD}$ are not the same, so $\overline{AB}$ is not parallel to $\overline{CD}$.

Slope of $\overline{BC} = \frac{-2-4}{8-4} = \frac{-6}{4} = -\frac{3}{2}$

Slope of $\overline{DA} = \frac{4-1}{0-2} = -\frac{3}{2}$

The slopes of $\overline{BC}$ and $\overline{DA}$ are the same, so $\overline{BC} \parallel \overline{DA}$.

Quadrilateral $ABCD$ has exactly one pair of parallel sides. $ABCD$ is a trapezoid.

4. Slope of $\overline{AB} = \frac{3-0}{2-(-5)} = \frac{3}{7}$

Slope of $\overline{CD} = \frac{-2-1}{-2-3} = \frac{-3}{-5} = \frac{3}{5}$

The slopes of $\overline{AB}$ and $\overline{CD}$ are not the same, so $\overline{AB}$ is not parallel to $\overline{CD}$.

Slope of $\overline{BC} = \frac{1-3}{3-2} = \frac{-2}{1} = -2$

Slope of $\overline{DA} = \frac{0-(-2)}{-5-(-2)} = -\frac{2}{3}$

The slopes of $\overline{BC}$ and $\overline{DA}$ are not the same, so $\overline{BC}$ is not parallel to $\overline{DA}$.

The quadrilateral $ABCD$ is not a trapezoid because it does not have exactly one pair of parallel sides.

5. Slope of $\overline{AB} = \frac{1-1}{6-2} = \frac{0}{4} = 0$

Slope of $\overline{CD} = \frac{-4-(-3)}{-1-3} = \frac{-1}{-4} = \frac{1}{4}$

The slopes of $\overline{AB}$ and $\overline{CD}$ are not the same, so $\overline{AB}$ is not parallel to $\overline{CD}$.

Slope of $\overline{BC} = \frac{-3-1}{3-6} = \frac{-4}{-3} = \frac{4}{3}$

Slope of $\overline{DA} = \frac{1-(-4)}{2-(-1)} = \frac{5}{3}$

The slopes of $\overline{BC}$ and $\overline{DA}$ are not the same, so $\overline{BC}$ is not parallel to $\overline{DA}$.

The quadrilateral $ABCD$ is not a trapezoid because it does not have exactly one pair of parallel sides.

6. Slope of $\overline{AB} = \frac{1-3}{-1(-3)} = \frac{-2}{2} = -1$

Slope of $\overline{CD} = \frac{0-1}{-3-(-4)} = \frac{-1}{1} = -1$

The slopes of $\overline{AB}$ and $\overline{CD}$ are the same, so $\overline{AB} \parallel \overline{CD}$.

Slope of $\overline{BC} = \frac{-4-1}{1-(-1)} = \frac{-5}{2} = -\frac{5}{2}$

Slope of $\overline{DA} = \frac{0-3}{-3-(-3)} = \frac{-3}{0}$, which is undefined

The slopes of $\overline{BC}$ and $\overline{DA}$ are not the same, so $\overline{BC}$ is not parallel to $\overline{DA}$.

The quadrilateral *ABCD* has exactly one pair of parallel sides. *ABCD* is a trapezoid.

7. $m\angle L = m\angle K = 50°$, $m\angle J = 180° - 50° = 130°$, $m\angle M = m\angle J = 130°$

8. $m\angle L = m\angle K = 100°$, $m\angle J = 180° - 100° = 80°$, $m\angle M = m\angle J = 80°$

9. $m\angle J = m\angle K = 118°$, $m\angle L = 180° - 118° = 62°$, $m\angle M = m\angle L = 62°$

10. Both pairs of base angles are congruent, so the quadrilateral is an isoceles trapezoid by Theorem 8.14.

11. Because there are exactly two right angles, there is exactly one pair of parallel sides. So, the quadrilateral is a trapezoid.

12. Not a trapezoid; $\angle J$ and $\angle M$, and $\angle K$ and $\angle L$ are supplementary by the Consecutive Interior Angles Theorem. Because both pairs of opposite angles are congruent, *JKLM* is a parallelogram.

13. $MN = \frac{1}{2}(10 + 18) = \frac{1}{2}(28) = 14$

14. $MN = \frac{1}{2}(21 + 25) = \frac{1}{2}(46) = 23$

15. $MN = \frac{1}{2}(57 + 76) = \frac{1}{2}(133) = 66.5$

16. D; Not all trapezoids are isosceles. So the legs of a trapezoid are not always congruent.

17. There is only one pair of congruent opposite angles in a kite. These angles are the two that join the non-congruent sides. So, $m\angle A = 360° - 120° - 120° - 50° = 70°$.

18. $m\angle E + m\angle G + m\angle H + m\angle F = 360°$

$m\angle G + m\angle G + 100° + 140° = 360°$

$2(m\angle G) + 140° = 360°$

$m\angle G = 110°$

19. $m\angle G + m\angle F + m\angle H + m\angle E = 360°$

$m\angle G + 110° + 110° + 60° = 360°$

$m\angle G + 280° = 360°$

$m\angle G = 80°$

20. $m\angle E + m\angle G + m\angle F + m\angle H = 360°$

$m\angle G + m\angle G + 150° + 90° = 360°$

$2(m\angle G) + 240° = 360°$

$m\angle G = 60°$

21. $XY = WX = \sqrt{3^2 + 3^2} = \sqrt{18} = 3\sqrt{2}$

$WZ = YZ = \sqrt{3^2 + 5^2} = \sqrt{34}$

22. $WZ = WX = \sqrt{4^2 + 6^2} = \sqrt{52} = 2\sqrt{13}$

$XY = YZ = \sqrt{6^2 + 12^2} = \sqrt{180} = 6\sqrt{5}$

23. $WX = WZ = \sqrt{19^2 + 10^2} = \sqrt{461}$

$XY = YZ = \sqrt{5^2 + 10^2} = \sqrt{125} = 5\sqrt{5}$

24. The length of the midsegment of a trapezoid is not the difference in lengths of the two parallel sides. It is one-half the sum of the two parallel sides.

$MN = \frac{1}{2}(DC + AB)$

$8 = \frac{1}{2}(DC + 14)$

$16 = DC + 14$

$2 = DC$

25. $7 = \frac{1}{2}(2x + 10)$

$14 = 2x + 10$

$4 = 2x$

$2 = x$

26. $12.5 = \frac{1}{2}[(3x + 1) + 15]$

$25 = 3x + 16$

$9 = 3x$

$3 = x$

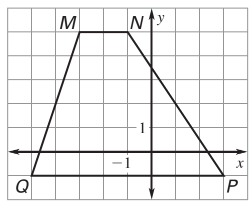
27. $18.7 = \frac{1}{2}[5x + (12x - 1.7)]$

$37.4 = 17x - 1.7$

$39.1 = 17x$

$2.3 = x$

28.

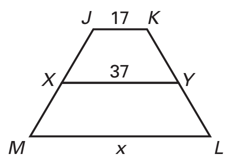


$MP = \sqrt{(3 - (-3))^2 + (-1 - 5)^2} = \sqrt{72} = 6\sqrt{2}$

$NQ = \sqrt{(-1 - (-5))^2 + (5 - (-1))^2} = \sqrt{52} = 2\sqrt{13}$

The lengths of the diagonals $\overline{MP}$ and $\overline{NQ}$ are not congruent, so trapezoid *MNPQ* is not isosceles.

29.



$XY = \frac{1}{2}(JK + LM)$

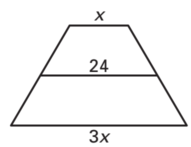
$37 = \frac{1}{2}(17 + x)$

$74 = 17 + x$

$57 = x$

The length of $\overline{LM}$ is 57.

30.



$24 = \frac{1}{2}(x + 3x)$

$48 = 4x$

$12 = x$ so $3x = 36$

The lengths of the bases are 12 and 36.

31. A; $RS : PQ = 5 : 1$

$MN = \frac{1}{2}(RS + PQ)$

$= \frac{1}{2}(5PQ + PQ)$

$= \frac{1}{2}(6PQ)$

$= 3PQ$

$MN : RS$

$3PQ : 5PQ$

$3 : 5$

32. $7x - 6 = \frac{1}{2}(36 + x^2)$

$14x - 12 = 36 + x^2$

$0 = x^2 - 14x + 48$

$0 = (x - 6)(x - 8)$

$x - 6 = 0$ or $x - 8 = 0$

$x = 6$ or $x = 8$

The possible values of x are 6 and 8.

If $x = 6$:

midsegment $= 7x - 6$

$= 7(6) - 6$

$= 36$

If $x = 8$:

midsegment $= 7x - 6$

$= 7(8) - 6$

$= 50$

If $x = 6$, the length of either base and of the midsegment is 36. So, $x = 6$ is rejected. The length of the midsegment is 50.

33. *Sample answer:* A kite or a quadrilateral that is not a parallelogram or a trapezoid will not have a pair of opposite sides parallel. So, no consecutive angles are supplementary. So, the measure of an interior angle could be greater than 180°.

Problem Solving

34. $HC = \frac{1}{2}(AB + GD) = \frac{1}{2}(13.9 + 50.5) = \frac{1}{2}(64.4) = 32.2$

$GD = \frac{1}{2}(HC + FE)$

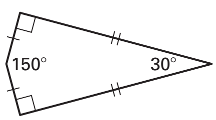
$50.5 = \frac{1}{2}(32.2 + FE)$

$101 = 32.2 + FE$

$68.8 = FE$

The length of HC is 32.2 centimeters and the length of FE is 68.8 centimeters.

35. *Sample answer:*



36. **a.** The quadrilaterals are a kite and a trapezoid.

b. The length of $\overline{BF}$ increases. $m\angle BAF$ and $m\angle BCF$ both increase. $m\angle ABC$ and $m\angle CFA$ both decrease.

c. $m\angle DEF = m\angle CFE = 65°$,
$m\angle FCD = 180° - 65° = 115°$,
$m\angle CDE = m\angle FCD = 115°$

The trapezoid is isosceles, so both pairs of base angles are congruent.

37.

Statements	Reasons
1. $ABCD$ is an isosceles trapezoid with $\overline{AB} \cong \overline{CD}$ and $BC \parallel AD$.	**1.** Given
2. $\overline{EC} \parallel \overline{AB}$	**2.** Given
3. $ABCD$ is a ▱.	**3.** Definition of a parallelogram
4. $\overline{AB} \cong \overline{CE}$	**4.** Opposite sides of a ▱ are ≅.
5. $\overline{CE} \cong CD$	**5.** Transitive Property of ∠ Congruence
6. $\triangle CDE$ is isosceles.	**6.** Definition of isosceles triangle
7. $\angle D \cong \angle DEC$	**7.** Base Angles Theorem
8. $\angle DEC \cong \angle A$	**8.** Corresponding Angles Congruence Postulate
9. $\angle D \cong \angle A$	**9.** Transitive Property of Equality
10. $m\angle B + m\angle A = 180°$, $m\angle BCD + m\angle D = 180°$	**10.** Consecutive Interior Angles Theorem
11. $m\angle B + m\angle A = m\angle BCD + m\angle D$	**11.** Transitive Property of ∠ Equality
12. $m\angle B + m\angle D = m\angle BCD + m\angle D$	**12.** Substitution Property of Equality
13. $m\angle B = \angle BCD$	**13.** Substitution Property of Equality
14. $\angle B \cong \angle BCD$	**14.** Definition of ∠ Congruence

38.

Statements	Reasons
1. $EFGH$ is a trapezoid, $\overline{FG} \parallel \overline{EH}$, $\angle E \cong \angle H$, $\overline{JG} \parallel \overline{EF}$	**1.** Given
2. $EFGJ$ is a parallelogram.	**2.** Definition of a parallelogram
3. $\overline{EF} \cong \overline{JG}$	**3.** Opposite sides in a parallelogram are congruent.
4. $\angle FEJ \cong \angle GJH$	**4.** Corresponding Angles Postulate
5. $\triangle GJH$ is isosceles.	**5.** Converse of the Base Angles Theorem
6. $\overline{JG} \cong \overline{GH}$	**6.** Base Angles Theorem
7. $\overline{EF} \cong GH$	**7.** Transitive Property of Congruence
8. $EFGH$ is an isosceles trapezoid.	**8.** Definition of an isosceles trapezoid

39.

Statements	Reasons
1. $JKLM$ is an isosceles trapezoid, $\overline{KL} \parallel \overline{JM}$. $\overline{JK} \cong \overline{LM}$	**1.** Given
2. $\angle JKL \cong \angle MLK$	**2.** Base angles in an isosceles trapezoid are congruent.
3. $\overline{KL} \cong \overline{KL}$	**3.** Reflexive Property of Congruence
4. $\triangle LKJ \cong \triangle KLM$	**4.** SAS Congruence Postulate
5. $\overline{JL} \cong \overline{KM}$	**5.** Corresponding parts of congruent triangles are congruent

40. By the Midsegment Theorem, $BG = \frac{1}{2}CD$ and $GE = \frac{1}{2}AF$. $BG + GE = \frac{1}{2}CD + \frac{1}{2}AF$, which implies $BE = \frac{CD + AF}{2}$. The midsegment $\overline{BE}$ is parallel to $\overline{CD}$ and $\overline{AF}$ because $\overline{BE}$, $\overline{BG}$, and $\overline{GE}$ all lie on the same line.

41.

Statements	Reasons
1. $\overline{AB} \cong \overline{CB}$, $\overline{AD} \cong \overline{CD}$	**1.** Given
2. $\overline{BD} \cong \overline{BD}$	**2.** Reflexive Property of Congruence
3. $\triangle BCD \cong \triangle BAD$	**3.** SSS Congruence Postulate
4. $\angle CBE \cong \angle ABE$	**4.** Corresponding parts of congruent triangles are congruent.
5. $\overline{BE} \cong BE$	**5.** Reflexive Property of Congruence
6. $\triangle BAE \cong \triangle BCE$	**6.** SAS Congruence Postulate
7. $\angle BEC \cong \angle BEA$	**7.** Corresponding parts of congruent triangles are congruent.
8. $\angle BEC$ and $\angle BEA$ are a linear pair.	**8.** Definition of a linear pair
9. $\overline{AC} \perp \overline{BD}$	**9.** If two lines intersect and form a linear pair of congruent angles, the lines are perpendicular.

42. Draw $\overline{FH}$. $\overline{FH} \cong \overline{FH}$ by the Reflexive Property of Congruence. Because $\overline{EF} \cong \overline{FG}$ and $\overline{GH} \cong \overline{EH}$, $\triangle FGH \cong \triangle FEH$ by the SSS Congruence Postulate. Corresponding parts of congruent triangles are congruent, so $\angle E \cong \angle G$. If the assumption that $\angle F \cong \angle H$, then both pairs of opposite sides of $EFGH$ are congruent. $EFGH$ is a parallelogram. Because this contradicts the definition of a kite, $\angle F \not\cong \angle H$.

43. If the diagonals of a trapezoid are congruent, then the trapezoid is isosceles. It is given that $JKLM$ is a trapezoid with $\overline{KM} \cong \overline{JL}$. Draw $\overline{KP}$ perpendicular to $\overline{JM}$ at point P and draw $\overline{LQ}$ perpendicular to $\overline{JM}$ at point Q. Because $\triangle LQJ$ and $\triangle KPM$ are right triangles, they are congruent by the HL Congruence Theorem. Using corresponding parts of congruent triangles are congruent, $\angle LJM \cong \angle KMJ$. Using the Reflexive Property of Congruence, $\overline{JM} \cong \overline{JM}$. $\triangle LJM \cong \triangle KMJ$ by the SAS Congruence Postulate. Using corresponding parts of congruent triangles are congruent, $\overline{KJ} \cong \overline{LM}$. So, trapezoid $JKLM$ is isosceles.

Lesson Identify Special Quadrilaterals

Guided Practice for the lesson "Identify Special Quadrilaterals"

1. Parallelogram, rectangle: Both pairs of opposite sides are congruent.

Rhombus, square: All sides are congruent.

Trapezoid: One pair of opposite sides are congruent.

2. kite; There are two pairs of consecutive congruent sides.

3. trapezoid; There is exactly one pair of parallel sides. Because the diagonals do not bisect each, it is not a parallelogram.

4. quadrilateral; There are no parallel sides, one pair of congruent sides and one bisected diagonal. Not enough information to further classify the quadrilateral.

5. It is possible that $MNPQ$ could be a rectangle or a square because you don't know the relationship between $\overline{MQ}$ and $\overline{NP}$.

Exercises for the lesson "Identify Special Quadrilaterals"

Skill Practice

1. A quadrilateral that has exactly one pair of parallel sides and diagonals that are congruent is an isosceles trapezoid.

2. You can prove all four sides of the parallelogram are congruent. You can also prove that the diagonals of the parallelogram are perpendicular. Proving the diagonals bisect opposite angles can also show that the parallelogram is a rhombus.

Property	▱	Rectangle	Rhombus
3. All sides are ≅.			X
4. Both pairs of opp. sides are ≅.	X	X	X
5. Both pairs of opp. sides are ∥.	X	X	X
6. Exactly 1 pair of opp. sides are ∥.			
7. All ∠s are ≅.		X	

8. Exactly 1 pair of opp. ∠s are ≅.			
9. Diagonals are ⊥.			X
10. Diagonals are ≅.		X	
11. Diagonals bisect each other.	X	X	X

Property	Square	Kite	Trapezoid
3. All sides are ≅.	X		
4. Both pairs of opp. sides are ≅.	X		
5. Both pairs of opp. sides are ∥.	X		
6. Exactly 1 pair of opp. sides are ∥.			X
7. All ∠s are ≅.	X		
8. Exactly 1 pair of opp. ∠s are ≅.		X	
9. Diagonals are ⊥.	X	X	
10. Diagonals are ≅.	X		
11. Diagonals bisect each other.	X		

12. Because $\angle D$ and $\angle C$ are not supplementary, $\overline{AD}$ is not parallel to $\overline{BC}$. So, $ABCD$ is not a parallelogram. Because $m\angle A = 121°$, $ABCD$ is not a kite. $ABCD$ is a trapezoid because $\overline{AB} \parallel \overline{CD}$.

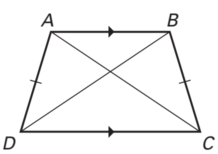
13. A; Rectangle

14. Because all 4 angles are right angles, the quadrilateral is a rectangle.

15. $\overline{PS} \perp \overline{SR}$ and $\overline{QR} \perp \overline{SR}$ so $\overline{PS} \parallel \overline{QR}$. Because there is exactly one pair of parallel sides, the quadrilateral is a trapezoid.

16. There are two sets of consecutive congruent sides, so the quadrilateral is a kite.

17.



$\overline{AC} \cong \overline{BD}$

Isosceles trapezoid; An isosceles trapezoid has exactly one pair of congruent sides and congruent diagonals.

18. No; squares, rhombuses, rectangles, and kites all have perpendicular diagonals.

19. No; because $m\angle F = 109°$, $\angle E$ is not congruent to $\angle F$. So, $EFGH$ is not an isosceles trapezoid.

20. No; it is not known whether the diagonals are perpendicular or whether all four side lengths are equal. So, the quadrilateral can only be classified as a rectangle.

21. $PQ = \sqrt{(1-1)^2 + (2-0)^2} = \sqrt{4} = 2$

$QR = \sqrt{(6-1)^2 + (5-2)^2} = \sqrt{5^2 + 3^2} = \sqrt{34}$

$RS = \sqrt{(3-6)^2 + (0-5)^2} = \sqrt{(-3)^2 + (-5)^2} = \sqrt{34}$

$PS = \sqrt{(1-3)^2 + (0-0)^2} = \sqrt{(-2)^2} = 2$

Kite; $\overline{PQ} \cong \overline{PS}$ and $\overline{QR} \cong \overline{RS}$

22. Slope of $\overline{PQ} = \frac{1-1}{6-2} = \frac{0}{4} = 0$

Slope of $\overline{QR} = \frac{8-1}{5-6} = \frac{7}{-1} = -7$

Slope of $\overline{RS} = \frac{8-8}{3-5} = \frac{0}{-2} = 0$

Slope of $\overline{PS} = \frac{8-1}{3-2} = \frac{7}{1} = 7$

$PQ = \sqrt{(6-2)^2 + (1-1)^2} = \sqrt{16} = 4$

$QR = \sqrt{(5-6)^2 + (8-1)^2}$

$= \sqrt{(-1)^2 + 7^2} = \sqrt{50} = 5\sqrt{2}$

$RS = \sqrt{(3-5)^2 + (8-8)^2} = \sqrt{4} = 2$

$PS = \sqrt{(3-2)^2 + (8-1)^2} = \sqrt{1^2 + 7^2} = \sqrt{50} = 5\sqrt{2}$

Isosceles trapezoid; $\overline{PQ} \parallel \overline{RS}$, and $\overline{QR}$ and $\overline{PS}$ are congruent but not parallel.

23. $PQ = \sqrt{(6-2)^2 + (9-7)^2} = \sqrt{4^2 + 2^2} = \sqrt{20} = 2\sqrt{5}$

$QR = \sqrt{(9-6)^2 + (3-9)^2}$

$= \sqrt{3^2 + (-6)^2} = \sqrt{45} = 3\sqrt{5}$

$RS = \sqrt{(5-9)^2 + (1-3)^2}$

$= \sqrt{(-4)^2 + (-2)^2} = \sqrt{20} = 2\sqrt{5}$

$SP = \sqrt{(2-5)^2 + (7-1)^2}$

$= \sqrt{(-3)^2 + 6^2} = \sqrt{45} = 3\sqrt{5}$

$PR = \sqrt{(9-2)^2 + (3-7)^2} = \sqrt{7^2 + (-4)^2} = \sqrt{65}$

$QS = \sqrt{(5-6)^2 + (1-9)^2} = \sqrt{(-1)^2 + (-8)^2} = \sqrt{65}$

Rectangle; because both pairs of opposite sides and diagonals are congruent, $PQRS$ is a rectangle.

24. $PQ = \sqrt{(5-1)^2 + (8-7)^2} = \sqrt{4^2 + 1^2} = \sqrt{17}$

$QR = \sqrt{(6-5)^2 + (2-8)^2} = \sqrt{1^2 + (-6)^2} = \sqrt{37}$

$RS = \sqrt{(2-6)^2 + (1-2)^2} = \sqrt{(-4)^2 + (-1)^2} = \sqrt{17}$

$SP = \sqrt{(1-2)^2 + (7-1)^2} = \sqrt{(-1)^2 + 6^2} = \sqrt{37}$

$PR = \sqrt{(6-1)^2 + (2-7)^2}$

$= \sqrt{5^2 + (-5)^2} = \sqrt{50} = 5\sqrt{2}$

$QS = \sqrt{(5-2)^2 + (8-1)^2} = \sqrt{3^2 + 7^2} = \sqrt{58}$

Parallelogram; both pairs of opposite sides are congruent. Because the diagonals are not congruent, $PQRS$ is a parallelogram.

25. **a.** Rhombus, square, kite

b. Parallelogram, rectangle, trapezoid; two consecutive pairs of sides are always congruent and one pair of opposite angles remain congruent.

26. Show any two consecutive sides are congruent.

Sample answer: $\overline{AB} \cong \overline{BC}$

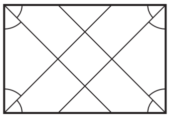
27. Show $\angle B \cong \angle A$ or $\angle C \cong \angle D$ and $\overline{AB} \parallel \overline{CD}$.

28. Show $\overline{DV} \cong \overline{BU}$. So, diagonals bisect each other.

29. No; if $m\angle JKL = m\angle KJM = 90°$, $JKLM$ would be a rectangle.

30. Yes; $JKLM$ has one pair of parallel sides and a pair of congruent base angles. By Theorem 8.15, $JKLM$ is an isosceles trapezoid.

31. Yes; $JKLM$ has one pair of non-congruent parallel sides with congruent diagonals. By Theorem 8.16, $JKLM$ is an isosceles trapezoid.

32. 

Square; when the rectanlge's angles are bisected, the resulting angle measures are 45°. The triangles created all have angle measures 45°-45°-90° and are similar. So, the quadrilateral has four right angles since each is one of a pair of vertical angles where the other angle is a right angle. Pairs of angle bisectors are parallel since they are perpendicular to the same line (one of the other angle bisectors). Therefore, the quadrilateral is a parallelogram, making its opposite sides congruent. Consecutive sides of the quadrilateral can be shown congruent using congruent triangles and the Subtraction Property of Equality. Therefore, the quadrilateral has four congruent sides and four right angles, which makes it a square.

Problem Solving

33. There is exactly one pair of parallel sides. So, the quadrilateral is a trapezoid.

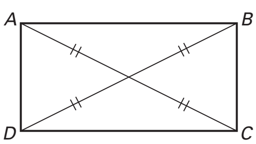
34. There is exactly one pair of opposite congruent angles and two pairs of consecutive congruent sides. So, the quadrilateral is a kite.

35. Both pairs of opposite sides are congruent. So, the quadrilateral is a parallelogram.

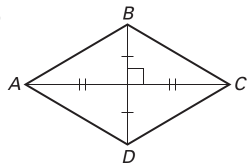
36. **a.** There is only one pair of parallel sides. So, this part of the pyramid is a trapezoid.

b. There are two pairs of parallel sides and 4 congruent angles. So, this part of the pyramid is a rectangle.

37. The consecutive angles of a parallelogram are supplementary. If one angle is a right angle, then each interior angle is 90°. So, the parallelogram is a rectangle by definition.

38. **a.** 

Because the diagonals bisect each other, $ABCD$ is a parallelogram. The diagonals are congruent, so $ABCD$ is a square or a rectangle. Because the diagonals are not perpendicular, $ABCD$ is a rectangle.

b. 

Because the diagonals bisect each other, $ABCD$ is a parallelogram. The diagonals are perpendicular, so the quadrilateral is a square or a rhombus. Because the diagonals are not congruent, $ABCD$ is a rhombus.

39. **a.** $\overline{QV} \cong \overline{UV} \cong \overline{RS} \cong \overline{ST}$ and $\angle V \cong \angle S$ because all sides and all interior angles of a regular hexagon are congruent. So, $\triangle QVU$ and $\triangle RST$ are isosceles. By the SAS Congruence Postulate, $\triangle QVU \cong \triangle RST$.

b. All sides in a regular hexagon are congruent, so $\overline{QR} = \overline{UT}$. Because corresponding parts of congruent triangles are congruent, $\overline{QU} \cong \overline{RT}$.

c. Because $\angle Q \cong \angle R \cong \angle T \cong \angle U$ and $\angle VUQ \cong \angle VQU \cong \angle STR \cong \angle SRT$, $\angle UQR \cong \angle QRT \cong \angle RTU \cong \angle TUQ$ by the Angle Addition Postulate.

The measure of each interior angle of a regular hexagon is $\frac{(n-2) \cdot 180}{6} = \frac{(6-2) \cdot 180}{6} = 120°$.

Find the sum of the interior angle measures of $\triangle QUV$:

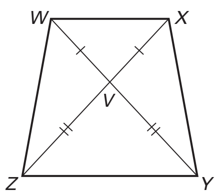
$$m\angle QVU + m\angle VQU + m\angle VUQ = 180°$$
$$120° + 2(m\angle VQU) = 180°$$
$$2m\angle VQU = 60°$$
$$m\angle VQU = 30°$$

Find $m\angle UQR$: $m\angle Q = m\angle VQU + m\angle UQR$

$$120° = 30° + m\angle UQR$$
$$90° = m\angle UQR$$

Because $\angle UQR \cong \angle QRT \cong \angle RTU \cong \angle TUQ$, $m\angle UQR = m\angle QRT = m\angle RTU = m\angle TUQ = 90°$.

d. The quadrilateral is a rectangle because it has two pairs of opposite congruent sides and four right angles.

40. 

The quadrilateral is an isosceles trapezoid. Show $\overline{WX} \parallel \overline{ZY}$ by showing $\triangle WVX \sim \triangle YVZ$ which leads to $\angle XWV \cong \angle ZYV$ and parallel sides. Now show base angles $\angle ZWX \cong \angle YXW$ using $\triangle ZVW \cong \triangle YVX$ and $\angle XWV \cong \angle WXV$.

41. Square; $PQRS$ is a square with E, F, G, and H midpoints of the square. Using the definition of a square and the definition of midpoint, $\overline{FQ} \cong \overline{QG} \cong \overline{GR} \cong \overline{RH} \cong \overline{HS} \cong \overline{SE} \cong \overline{PE} \cong \overline{PF}$. Using the definition of a square, $\angle P \cong \angle Q \cong \angle R \cong \angle S$. Using the SAS Congruence Theorem, $\triangle EPF \cong \triangle FQG \cong \triangle GRH \cong \triangle HSE$. Using corresponding parts of congruent triangles are congruent, $\overline{EF} \cong \overline{FG} \cong \overline{GH} \cong \overline{HE}$. Because the base angles of all four triangles measure 45°, $m\angle EFG = m\angle FGH = m\angle GHE = m\angle HEF = 90°$ since each of these angles along with two 45° angles form a line. By definition, $PQRS$ is a square.

42. Rhombus; $\overline{JK} \cong \overline{LM}$ and E, F, G, and H are the midpoints of $\overline{JL}$, $\overline{KL}$, $\overline{KM}$, and $\overline{JM}$, respectively. Using the definition of midsegment, $\overline{FG}$ and $\overline{EH}$ are parallel to $\overline{LM}$ and half its length. This makes $\overline{FG} \parallel \overline{EH}$ and $\overline{FG} \cong \overline{EH}$. Using the definition of midsegment, $\overline{GH}$ and $\overline{FE}$ are parallel to $\overline{JK}$ and half its length. This makes $\overline{GH} \parallel \overline{FE}$ and $\overline{GH} \cong \overline{FE}$. Because $\overline{JK} \cong \overline{LM}$, $\overline{FG} \cong \overline{EH} \cong \overline{GH} \cong \overline{FE}$ by the Transitive Property of Congruence. By definition, $EFGH$ is a rhombus.

Quiz for the lessons "Use Properties of Trapezoids and Kites" and "Identify Special Quadrilaterals"

1. $\angle D \cong \angle A$, so $m\angle D = 55°$.

$\angle A$ and $\angle B$ are supplementary, so $m\angle B = 180° - 55° = 125°$.

$\angle B \cong \angle C$, so $m\angle C = 125°$.

2. $\angle B \cong \angle C$, so $m\angle C = 48°$.

$\angle B$ and $\angle A$ are supplementary, so $m\angle A = 180° - 48° = 132°$.

$\angle A \cong \angle D$, so $m\angle D = 132°$.

3. $\angle A \cong \angle B$, so $m\angle B = 110°$.

$\angle A$ and $\angle D$ are supplementary, so $m\angle D = 180° - 110° = 70°$.

$\angle D \cong \angle C$ so $m\angle C = 70°$.

4. rectangle, square

5. Consecutive sides are congruent and both pairs of opposite angles are congruent, so $EFGH$ is a rhombus.

Mixed Review of Problem Solving for the lessons "Properties of Rhombuses, Rectangles, and Squares", "Use Properties of Trapezoids and Kites", and "Identify Special Quadrilaterals"

1. a. There is exactly one pair of parallel sides.

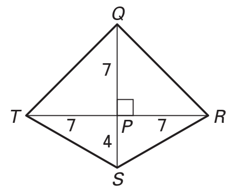
b. Yes; the trapezoid has a pair of congruent base angles.

2. The diagonals are congruent and bisect each other. So, the quadrilateral is a square or a rectangle. Because the diagonals are perpendicular, $JKLM$ is a square.

3. a. A kite has exactly one pair of opposite congruent angles. Because $\angle TQR \not\cong \angle RST$, $\angle QTS \not\cong \angle QRS$.

$$m\angle TQR + m\angle QTS + m\angle RST + m\angle QRS = 360°$$
$$102° + 125° + 2(m\angle QTS) = 360°$$
$$227° + 2(m\angle QTS) = 360°$$
$$2(m\angle QTS) = 133°$$
$$m\angle QTS = 66.5°$$

b. $TP = RP = \frac{1}{2}TR$, $TP = RP = QP = 7$



Using the Pythagorean Theorem, find QR.

$QR = \sqrt{QP^2 + RP^2} = \sqrt{7^2 + 7^2} \approx 10$

$\triangle QPS \cong \triangle QPR$ by the SAS Congruence Postulate. So $QR = QT \approx 10$ feet.

Using the Pythagorean Theorem, find RS.

$RS = \sqrt{RP^2 + SP^2} = \sqrt{7^2 + 4^2} \approx 8$

$\triangle SPR \cong \triangle SPT$ by the SAS Congruence Postulate. So $RS = ST \approx 8$ feet.

4. The midsegment is one-half the sum of the length of the bases.

Midsegment $= \frac{1}{2}(48 + 24) = \frac{1}{2}(72) = 36$

The midsegment of trapezoid $ABCD$ is 36 inches.

5. If $WZ = 20$, $WY = 20 \cdot 2 = 40$. Because the rhombuses are similar, corresponding parts are proportional.

$$\frac{QR}{QS} = \frac{WX}{WY}$$

$$\frac{20}{32} = \frac{WX}{40}$$

$32(WX) = 800$

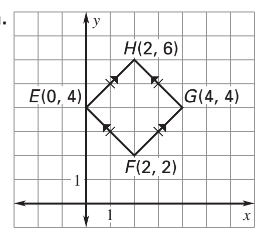
$WX = 25$

The length of WX is 25.

6. a. $MNPQ$ could be a rectangle, square, or isosceles trapezoid because the diagonals of these quadrilaterals are congruent.

b. For a rectangle you need to know that opposite sides are congruent. For a square you need to know that opposite sides are congruent and that consecutive sides are congruent. For an isosceles trapezoid you need to know that only one pair of opposite sides are parallel.

7. a.



Find the length of $\overline{EF}$.

$EF = \sqrt{(0-2)^2 + (4-2)^2} = 2\sqrt{2}$

$EFGH$ is a rhombus. Because $\overline{HG} \cong \overline{GF} \cong \overline{FE} \cong \overline{EH}$, $HG = GF = FE = EH = 2\sqrt{2}$.

Find the slope of $\overline{FG}$.

Slope of $\overline{FG} = \frac{4-2}{4-2} = \frac{2}{2} = 1$

Because $EFGH$ is a rhombus, $\overline{EH} \parallel \overline{FG}$. So, the slope of $\overline{EH}$ is 1.

$H(2, 6)$ is the only location where $HG = GF = FE = EH$, $\overline{HG} \parallel \overline{FE}$, and $\overline{EH} \parallel \overline{FG}$.

b. The coordinates of H could be (2, 10) or (2, 14). Both points allow $EFGH$ to meet the definition of a kite. The points lie on the line with equation $x = 2$, excluding (2, 6) and (2, 2).

Chapter Review for the chapter "Quadrilaterals"

1. The midsegment of a trapezoid is parallel to the bases.

2. A diagonal of a polygon is a segment whose endpoints are nonconsecutive vertices.

3. Show the trapezoid has a pair of congruent base angles. Show the diagonals of the trapezoid are congruent.

4. C. Rhombus; because both pairs of opposite sides are parallel and all four sides are congruent.

5. A. Square; there are four right angles and four congruent sides.

6. B. Parallelogram; both pairs of opposite sides are congruent.

7. $(n-2) \cdot 180° = 3960°$

$n - 2 = 22$

$n = 24$

The polygon has 24 sides. It is a 24-gon. The measure of each interior angle is $\frac{3960°}{24} = 165°$.

8. $x° + 120° + 97° + 130° + 150° + 90° = (n-2) \cdot 180°$

$x + 120 + 97 + 130 + 150 + 90 = (6-2) \cdot 180$

$x + 587 = 720$

$x = 133$

9.

$x° + 160° + 2x° + 125° + 110° + 112° + 147° = (n-2) \cdot 180°$

$x + 160 + 2x + 125 + 110 + 112 + 147 = (7-2) \cdot 180$

$3x + 654 = 900$

$3x = 246$

$x = 82$

10. $8x° + 5x° + 5x° = 360°$

$18x = 360$

$x = 20$

11. The measure of one exterior angle is $\frac{360°}{9} = 40°$.

12. $m = 10$ and $n - 3 = 8$

$n = 11$

13. $c + 5 = 11$ and $d + 4 = 14$

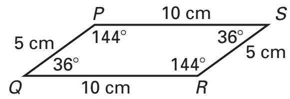
$c = 6$ $\quad$ $d = 10$

14. $a - 10 = 18$ and $(b + 16)° = 103°$

$a = 28$ $\quad$ $b = 87$

15. $m\angle QRS = 180° - m\angle PQR = 180° - 136° = 144°$

Opposite sides and opposite angles of a parallelogram are congruent.



16. $\overline{EF} \cong \overline{GH}$, $\overline{FG} \cong \overline{EH}$

Perimeter $= EF + GH + FG + EH$

$= 2(EF) + 2(FG)$

$16 = 2(5) + 2(FG)$

$6 = 2(FG)$

$3 = FG$

The length of $\overline{GH}$ is 5 inches. The length of $\overline{FG}$ and $\overline{EH}$ is 3 inches.

17. Consecutive angles of a parallelogram are supplementary.

$m\angle J + m\angle M = 180°$

$5x + 4x = 180$

$9x = 180$

$x = 20$

$m\angle J = 5x = 5(20) = 100°$

$m\angle M = 4x = 4(20) = 80°$

18. $2x + 4 = x + 9$

$x + 4 = 9$

$x = 5$

19. $5x - 4 = 3x + 2$

$2x - 4 = 2$

$2x = 6$

$x = 3$

20. Both pairs of opposite sides are parallel and the diagonals are perpendicular. So the quadrilateral is a rhombus. $y = 21$ because diagonals of a rhombus bisect opposite angles. $x° + y° = 90°$. So $x = 90 - 21 = 69$.

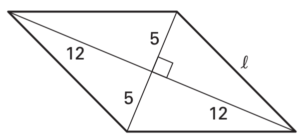
21. All four angles are right angles, so the quadrilateral is a rectangle.

$4x - 5 = 3x + 4$ $\quad$ $6y - 10 = 4y$

$x - 5 = 4$ $\quad$ $2y - 10 = 0$

$x = 9$ $\quad$ $y = 5$

22.



ℓ = length of one side

$= \sqrt{5^2 + 12^2}$

$= \sqrt{169}$

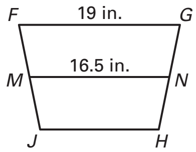
$= 13$

The length of one side is 13 centimeters.

23. $m\angle G = m\angle F = 79°$

$m\angle J = 180° - m\angle F = 180° - 79° = 101°$

$m\angle H = m\angle J = 101°$

24. 

$$MN = \frac{1}{2}(FG + JH)$$
$$16.5 = \frac{1}{2}(19 + JH)$$
$$33 = 19 + JH$$
$$14 = JH$$

The length of $\overline{JH}$ is 14 inches.

25. All four sides of the quadrilateral are congruent, so it is a rhombus. You do not know the angle measures, so it cannot be determined if it is a square.

26. $\angle E$ and $\angle H$ are supplementary, so $\overline{EF} \parallel \overline{HG}$, but $\angle G$ and $\angle H$ are *not* supplementary, so $\overline{EH}$ is *not* parallel to $\overline{FG}$. Since *EFGH* has exactly one pair of parallel sides, it is a trapezoid.

27. Because both pairs of opposite sides are congruent, the quadrilateral is a parallelogram. You do not know if the angles are right angles. So it cannot be classified as a rectangle.

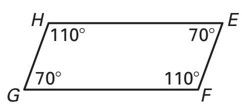
28. The quadrilateral has three right angles. Because the sum of the measures of the interior angles is 360°, the fourth angle is a right angle. So the quadrilateral is a rectangle. Because consecutive sides of the rectangle are congruent, the rectangle is a square.

Chapter Test for the chapter "Quadrilaterals"

1. $x° + 103° + 122° + 98° + 99° = (n - 2) \cdot 180°$
$$x + 103 + 122 + 98 + 99 = (5 - 2) \cdot 180$$
$$x + 422 = 540$$
$$x = 118$$

2. $5x° + 170° + 90° + 166° +$
$$150° + 143° + 112° + 94° = (n - 2) \cdot 180°$$
$$5x + 170 + 90 + 166 + 150 + 143 + 112 + 94 = (8 - 2) \cdot 180$$
$$5x + 925 = 1080$$
$$5x = 155$$
$$x = 31$$

3. $x° + 59° + 47° + 36° + 65° + 82° = 360°$
$$x + 59 + 47 + 36 + 65 + 82 = 360$$
$$x + 289 = 360$$
$$x = 71$$

4. 

$$m\angle F = m\angle G + 40°$$
$$m\angle F = m\angle H$$
$$m\angle E = m\angle G$$

$180° = m\angle F + m\angle G$

$180° = m\angle G + 40° + m\angle G$

$140° = 2m\angle G$

$70° = m\angle G$

So, $m\angle G = 70° = m\angle E$.

$m\angle F = m\angle G + 40° = 70° + 40° = 110°$

So, $m\angle F = 110° = m\angle H$.

5. No; you need to know that consecutive angles are supplementary or that both pairs of opposite angles are congruent.

6. Because the diagonals bisect each other, the quadrilateral is a parallelogram.

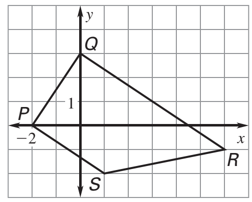
7. You need to know that one pair of sides is congruent and parallel. The figure could be an isosceles trapezoid.

8. Only rhombuses and squares are equilateral quadrilaterals.

9. Only rectangles and squares have four interior right angles.

10. Only rectangles and squares have congruent diagonals.

11. Parallelograms, rectangles, rhombuses, and squares have opposite sides that are parallel.

12. 

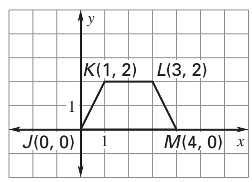
Slope of $\overline{PQ} = \frac{3 - 0}{0 - (-2)} = \frac{3}{2}$

Slope of $\overline{QR} = \frac{-1 - 3}{6 - 0} = \frac{-4}{6} = -\frac{2}{3}$

Slope of $\overline{RS} = \frac{-2 - (-1)}{1 - 6} = \frac{-1}{-5} = \frac{1}{5}$

Slope of $\overline{PS} = \frac{0 - (-2)}{-2 - 1} = -\frac{2}{3}$

Sides $\overline{QR}$ and $\overline{PS}$ have the same slope so they are parallel. Because there is exactly one pair of parallel sides, *PQRS* is a trapezoid.

13. a. *Sample answer:* 

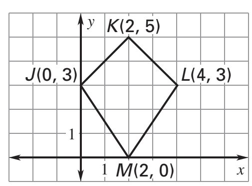
$JK = \sqrt{(1 - 0)^2 + (2 - 0)^2} = \sqrt{1^2 + 2^2} = \sqrt{5}$

$LM = \sqrt{(4 - 3)^2 + (0 - 2)^2} = \sqrt{1^2 + (-2)^2} = \sqrt{5}$

Slope of $\overline{KL} = \frac{2 - 2}{3 - 1} = \frac{0}{2} = 0$

Slope of $\overline{JM} = \frac{0 - 0}{4 - 0} = \frac{0}{4} = 0$

Opposite sides $\overline{KJ}$ and $\overline{LM}$ are congruent and opposite sides $\overline{KL}$ and $\overline{JM}$ are parallel. So *JKLM* is an isosceles trapezoid.

b. *Sample answer:* 

$JK = \sqrt{(2 - 0)^2 + (5 - 3)^2} = \sqrt{2^2 + 2^2} = \sqrt{8} = 2\sqrt{2}$

$KL = \sqrt{(4 - 2)^2 + (3 - 5)^2}$

$= \sqrt{2^2 + (-2)^2} = \sqrt{8} = 2\sqrt{2}$

$LM = \sqrt{(2 - 4)^2 + (0 - 3)^2}$

$= \sqrt{(-2)^2 + (-3)^2} = \sqrt{13}$

$JM = \sqrt{(2 - 0)^2 + (0 - 3)^2} = \sqrt{2^2 + (-3)^2} = \sqrt{13}$

Consecutive sides $\overline{JK}$ and $\overline{KL}$ and $\overline{LM}$ and $\overline{JM}$ are congruent. So *JKLM* is a kite.

c. *JKLM* could be a parallelogram, trapezoid, or rectangle.

14. Trapezoid; exactly one pair of parallel sides are parallel.

15. Rhombus; by the Triangle Sum Theorem, Linear Pair Postulate, and the Vertical Angle Theorem, the diagonals $\overline{EG}$ and $\overline{HF}$ are perpendicular.

16. Kite; The diagonal forms similar triangles by the SAS Postulate, so there are two pairs of consecutive congruent sides.

17. $\text{midsegment} = \frac{1}{2}(WX + YZ)$

$$2.75 = \frac{1}{2}(WX + 4.25)$$
$$5.5 = WX + 4.25$$
$$1.25 = WX$$

The length of $\overline{WX}$ is 1.25 centimeters.

18. $RS + TU + ST + RU = 42$ $\qquad ST = RS + 3$

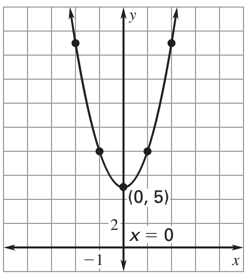
$$RS + RS + ST + ST = 42 \qquad = 9 + 3$$
$$2RS + 2ST = 42 \qquad = 12$$
$$2RS + 2(RS + 3) = 42$$
$$4RS + 6 = 42$$
$$4RS = 36$$
$$RS = 9$$

The length of $\overline{RS}$ is 9 centimeters and the length of $\overline{ST}$ is 12 centimeters.

Chapter Algebra Review for the chapter "Quadrilaterals"

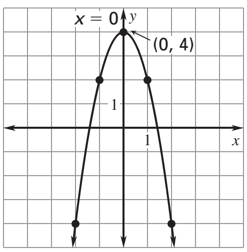
1. $y = 3x^2 + 5$

x	−2	−1	0	1	2
y	17	8	5	8	17



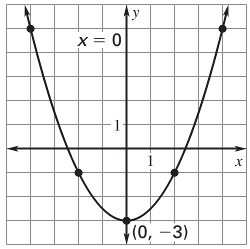
2. $y = -2x^2 + 4$

x	−2	−1	0	1	2
y	−4	2	4	2	−4



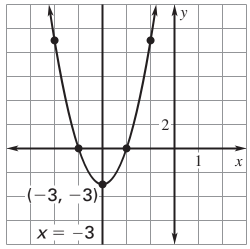
3. $y = 0.5x^2 - 3$

x	−4	−2	0	2	4
y	5	−1	−3	−1	5



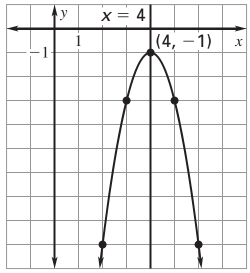
4. $y = 3(x + 3)^2 - 3$

x	−5	−4	−3	−2	−1
y	9	0	−3	0	9



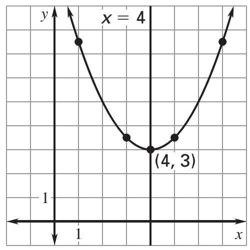
5. $y = -2(x - 4)^2 - 1$

x	2	3	4	5	6
y	−9	−3	−1	−3	−9



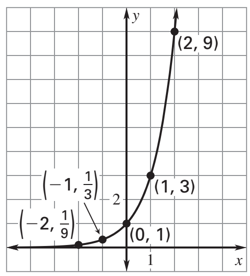
6. $y = \frac{1}{2}(x - 4)^2 + 3$

x	1	3	4	5	7
y	7.5	3.5	3	3.5	7.5



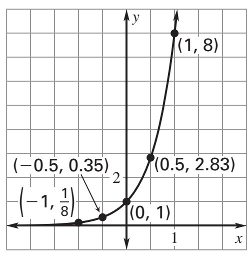
7. $y = 3^x$

x	−2	−1	0	1	2
y	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9



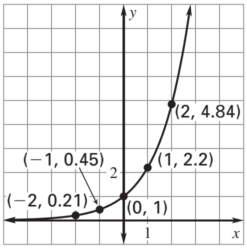
8. $y = 8^x$

x	−1	−0.5	0	0.5	1
y	$\frac{1}{8}$	0.35	1	2.83	8



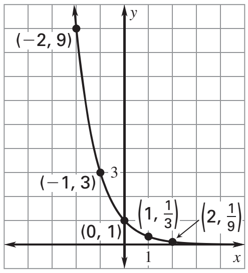
9. $y = 2.2^x$

x	−2	−1	0	1	2
y	0.21	0.45	1	2.2	4.84



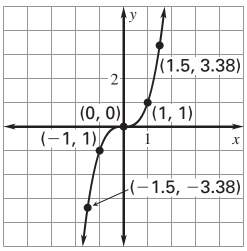
10. $y = \left(\frac{1}{3}\right)^x$

x	−2	−1	0	1	2
y	9	3	1	$\frac{1}{3}$	$\frac{1}{9}$



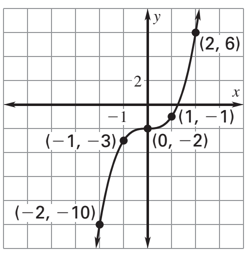
11. $y = x^3$

x	−1.5	−1	0	1	1.5
y	−3.38	−1	0	1	3.38



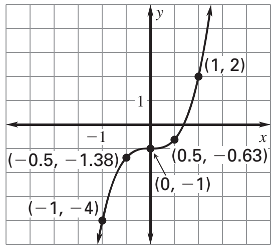
12. $y = x^3 - 2$

x	−2	−1	0	1	2
y	−10	−3	−2	−1	6



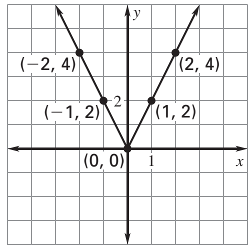
13. $y = 3x^3 - 1$

x	−1	−0.5	0	0.5	1
y	−4	−1.38	−1	−0.63	2



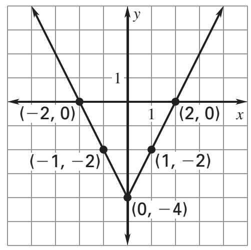
14. $y = 2|x|$

x	−2	−1	0	1	2
y	4	2	0	2	4



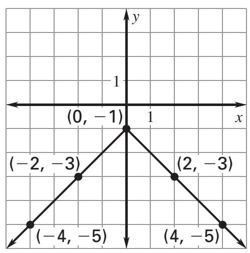
15. $y = 2|x| - 4$

x	−2	−1	0	1	2
y	0	−2	−4	−2	0



16. $y = -|x| - 1$

x	−4	−2	0	2	4
y	−5	−3	−1	−3	−5



Extra Practice

For the chapter "Quadrilaterals"

1. Quadrilateral; $(4 - 2) \cdot 180° = 360°$

$x° + 59° + 128° + 61° = 360°$

$x = 112$

2. Pentagon; $(5 - 2) \cdot 180° = 540°$

$x° + 137° + 82° + 140° + 91° = 540°$

$x = 90$

3. Heptagon; $(7 - 2) \cdot 180° = 900°$

$x° + 154° + 115° + 122° + 149° + 153° + 90° = 900°$

$x = 117$

4. $x° + 146° + 136° = 360°$

$x = 78$

5. $x° + 46° + 94° + 35° + (180° - 148°) + 85° = 360°$

$x = 68$

6. Pentagon; $(5 - 2) \cdot 180° = 540°$

$x° + 101° + 107° + x° + 100° = 540°$

$2x = 232$

$x = 116$

7. $(6 - 2) \cdot 180° = 720°$

Interior angle: $\frac{720°}{6} = 120°$

Exterior angle: $\frac{360°}{6} = 60°$

The measure of an interior angle of a regular hexagon is 120°. The measure of an exterior angle of a regular hexagon is 60°.

8. $(9 - 2) \cdot 180° = 1260°$

Interior angle: $\frac{1260°}{9} = 140°$

Exterior angle: $\frac{360°}{9} = 40°$

The measure of an interior angle of a regular 9-gon is 140°. The measure of an exterior angle of a regular 9-gon is 40°.

9. $(17 - 2) \cdot 180° = 2700°$

Interior angle: $\frac{2700°}{17} \approx 158.8°$

Exterior angle: $\frac{360°}{17} \approx 21.2°$

The measure of an interior angle of a regular 17-gon is about 158.8°. The measure of an exterior angle of a regular 17-gon is about 21.2°.

10. $a = 7, b = 12$

11. $2a + 4 = 14$

$2a = 10$

$a = 5$

$b + 1 = 6$

$b = 5$

So, $a = 5$ and $b = 5$.

12. $a = 18$

$\frac{2}{3}a = b$

$\frac{2}{3}(18) = b$

$12 = b$

So, $a = 18$ and $b = 12$.

13. $b = 63$

$a = 180 - 63$

$a = 117$

So, $a = 117$ and $b = 63$.

14. $a° + 3a° = 180°$

$4a = 180$

$a = 45$

$b = 3a$

$b = 3(45)$

$b = 135$

So, $a = 45$ and $b = 135$.

15. $a = 7$

$2b + 4 = b + 7$

$b = 3$

So, $a = 7$ and $b = 3$.

16. $\angle WXV \cong \angle YZV$

17. $\angle ZWV \cong \angle XYV$

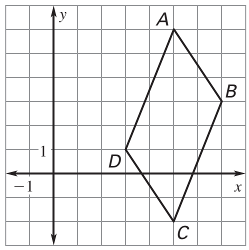
18. $\angle WVX \cong \angle YVZ$

19. $WV = YV$

20. $WZ = YX$

21. $2 \cdot ZV = ZX$

22.



$AB = \sqrt{(7-5)^2 + (3-6)^2} = \sqrt{13}$

$CD = \sqrt{(3-5)^2 + (1-(-2))^2} = \sqrt{13}$

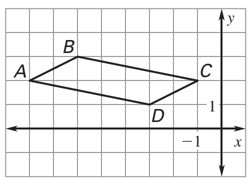
So, $\overline{AB} \cong \overline{CD}$.

Slope of $\overline{AB} = \frac{3-6}{7-5} = -\frac{3}{2}$

Slope of $\overline{CD} = \frac{1-(-2)}{3-5} = -\frac{3}{2}$

Slopes are equal, so $\overline{AB} \parallel \overline{CD}$. $\overline{AB} \cong \overline{CD}$ and $\overline{AB} \parallel \overline{CD}$, so $ABCD$ is a parallelogram.

23.



$AB = \sqrt{(-6-(-8))^2 + (3-2)^2} = \sqrt{5}$

$CD = \sqrt{(-3-(-1))^2 + (1-2)^2} = \sqrt{5}$

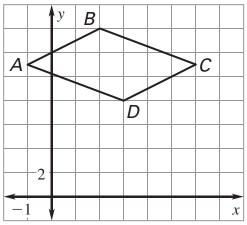
So, $\overline{AB} \cong \overline{CD}$.

Slope of $\overline{AB} = \frac{3-2}{-6-(-8)} = \frac{1}{2}$

Slope of $\overline{CD} = \frac{1-2}{-3-(-1)} = \frac{1}{2}$

Slopes are equal, so $\overline{AB} \parallel \overline{CD}$.

$\overline{AB} \cong \overline{CD}$ and $\overline{AB} \parallel \overline{CD}$, so $ABCD$ is a parallelogram.

24. 

$AB = \sqrt{(2 - (-1))^2 + (14 - 11)^2} = \sqrt{18} = 3\sqrt{2}$

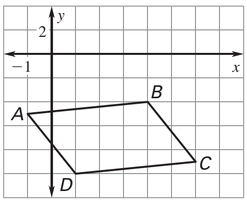
$CD = \sqrt{(3 - 6)^2 + (8 - 11)^2} = \sqrt{18} = 3\sqrt{2}$

So, $\overline{AB} \cong$ CD.

Slope of $\overline{AB} = \dfrac{14 - 11}{2 - (-1)} = 1$

Slope of $\overline{CD} = \dfrac{8 - 11}{3 - 6} = 1$

Slopes are equal, so $\overline{AB} \parallel \overline{CD}$. $\overline{AB} \cong \overline{CD}$ and $\overline{AB} \parallel \overline{CD}$, so *ABCD* is a parallelogram.

25. 

$AB = \sqrt{(4 - (-1))^2 + (-4 - (-5))^2} = \sqrt{26}$

$CD = \sqrt{(1 - 6)^2 + (-10 - (-9))^2} = \sqrt{26}$

So, $\overline{AB} \cong \overline{CD}$.

Slope of $\overline{AB} = \dfrac{-4 - (-5)}{4 - (-1)} = \dfrac{1}{5}$

Slope of $\overline{CD} = \dfrac{-10 - (-9)}{1 - 6} = \dfrac{1}{5}$

Slopes are equal, so $\overline{AB} \parallel \overline{CD}$. $\overline{AB} \cong \overline{CD}$ and $\overline{AB} \parallel \overline{CD}$, so *ABCD* is a parallelogram.

26. Draw $\overline{PR}$ to form $\triangle PQR$ and $\triangle RSP$. Show that $\triangle PQR \cong \triangle RSP$. Then show that $\angle QPR \cong \angle SRP$ and $\angle QRP \cong \angle SPR$. Use the Alternate Interior Angles Converse to show that $\overline{PS} \parallel \overline{RQ}$ and $\overline{PQ} \parallel \overline{RS}$. Then by definition, *PQRS* is a parallelogram.

27. Show that $\triangle PQR \cong \triangle RSP$ by the AAS Congruence Theorem. Show that $\angle QPR \cong \angle SRP$, $\overline{PQ} \cong \overline{RS}$ and $\overline{QR} \cong \overline{SP}$. Use the Alternate Interior Angles Converse to show that $\overline{PS} \parallel \overline{RQ}$ and $\overline{PQ} \parallel \overline{RS}$. Then by definition, *PQRS* is a parallelogram.

28. $\angle PTQ \cong \angle RTS$ because they are vertical angles. Show that $\triangle PTQ \cong \triangle RTS$ by the AAS Congruence Theorem. Show that $\overline{PT} \cong \overline{RT}$ and $\angle PTS \cong \angle RTQ$. Use the SAS Congruence Postulate to show that $\triangle PTS \cong \triangle RTQ$. Show that $\angle TRQ \cong \angle TPS$. Finally show that $\overline{PS} \parallel \overline{RQ}$ and $\overline{PQ} \parallel \overline{RS}$ using the Alternate Interior Angles Converse. Then by definition, *PQRS* is a parallelogram.

29. Square; because both pairs of opposite angles of the quadrilateral are congruent, *ABCD* is a parallelogram (Theorem 8.8); because each diagonal bisects a pair of opposite angles, *ABCD* is a rhombus (Theorem 8.12); because the diagonals of parallelogram *ABCD* are congruent, it is a rectangle (Theorem 8.13); because *ABCD* is a rhombus and a rectangle, it is a square (Square Corollary).

30. Rhombus; because *PQRS* has a pair of opposite sides that are congruent and parallel, it is a parallelogram (Theorem 8.9); by the Triangle Sum Theorem and the definition of perpendicular, the diagonals are perpendicular; because *PQRS* is a parallelogram with perpendicular diagonals, it is a rhombus (Theorem 8.11).

31. Rectangle; because opposite sides are congruent, *VWXY* is a parallelogram (Theorem 8.7); because its diagonals are congruent, *VWXY* is a rectangle (Theorem 8.13).

32. The diagonals of a rhombus are perpendicular, so $m\angle LQM = 90°$.

$$m\angle LMQ + m\angle QLM + m\angle LQM = 180°$$
$$m\angle LMQ + 30° + 90° = 180°$$
$$m\angle LMQ = 60°$$

33. The diagonals of a rhombus are perpendicular, so $m\angle LQM = 90°$.

34. The four sides of a rhombus are congruent, so $MN = 5$.

35. $x = \frac{1}{2}(19 + 31)$

$x = 25$

36. $34 = \frac{1}{2}(x + 43)$

$68 = x + 43$

$25 = x$

37. $0.5 = \frac{1}{2}(0.6 + x)$

$1 = 0.6 + x$

$0.4 = x$

38. $m\angle V = m\angle S = 75°$

39. $m\angle S = m\angle V$

$$m\angle S + m\angle T + m\angle V + m\angle R = 360°$$
$$m\angle V + 104° + m\angle V + 60° = 360°$$
$$2(m\angle V) + 164° = 360°$$
$$2(m\angle V) = 196°$$
$$m\angle V = 98°$$

40. $m\angle T = m\angle R = 90°$

$$m\angle V + m\angle R + m\angle S + m\angle T = 360°$$
$$m\angle V + 90° + 80° + 90° = 360°$$
$$m\angle V = 100°$$

41. The diagonals bisect each other. By Theorem 8.10, *ABCD* is a parallelogram.

42. $m\angle A + m\angle B + m\angle C + m\angle D = 360°$

$$119° + m\angle B + 51° + 61° = 360°$$
$$m\angle B = 129°$$

Because $m\angle A + m\angle D = 180°$ and $m\angle B + m\angle C = 180°$, by the Consecutive Interior Angles Converse, $\overline{AB} \parallel \overline{CD}$. So, *ABCD* is a trapezoid.

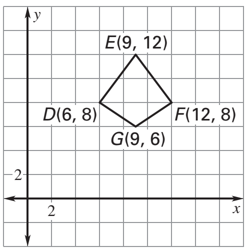
43. Because one pair of opposite sides, $\overline{AD}$ and $\overline{BC}$, are congruent and parallel, $ABCD$ is a parallelogram by Theorem 8.9. Because the diagonals of the parallelogram are perpendicular, by Theorem 8.11 $ABCD$ is a rhombus.

44. Because the diagonals $\overline{AC}$ and $\overline{BD}$ bisect each other, $ABCD$ is a parallelogram. $ABCD$ is also a rectangle because its diagonals are congruent and a rhombus because its diagonals are perpendicular. Because $ABCD$ is a rectangle and a rhombus, $ABCD$ is a square.

45. Because $\overline{AB} \parallel \overline{CD}$, $m\angle D + m\angle A = 180°$ and $m\angle B + m\angle C = 180°$. So, $m\angle D = 65°$ and $m\angle B = 115°$. Because the base angles are congruent and $ABCD$ has one pair of parallel opposite sides, $ABCD$ is an isosceles trapezoid.

46. By the Alternate Interior Angles Converse, one pair of opposite sides is parallel, and that pair is congruent, so $ABCD$ is a parallelogram (Theorem 8.9); by the ASA Congruence Postulate, $\triangle GAD \cong \triangle GDA \cong \triangle GBC \cong \triangle GCB$; by the definition of congruence, $AG = DG = CG = BG$; the diagonals of parallelogram $ABCD$ are congruent, so it is a rectangle (Theorem 8.13); the diagonals are not perpendicular, so $ABCD$ is not a rhombus (Theorem 8.11); rectangle $ABCD$ is not a rhombus, so it is not a square (Square Corollary).

47.



$DE = \sqrt{(9-6)^2 + (12-8)^2} = 5$

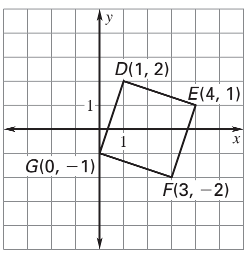
$EF = \sqrt{(12-9)^2 + (8-12)^2} = 5$

$FG = \sqrt{(9-12)^2 + (6-8)^2} = \sqrt{13}$

$GD = \sqrt{(6-9)^2 + (8-6)^2} = \sqrt{13}$

$DEFG$ is a kite because two pairs of consecutive sides are congruent and the opposite sides are not congruent.

48.



$DE = \sqrt{(4-1)^2 + (1-2)^2} = \sqrt{10}$

$EF = \sqrt{(3-4)^2 + (-2-1)^2} = \sqrt{10}$

$FG = \sqrt{(0-3)^2 + (-1-(-2))^2} = \sqrt{10}$

$GD = \sqrt{(1-0)^2 + (2-(-1))^2} = \sqrt{10}$

Slope of $\overline{DE} = \frac{1-2}{4-1} = -\frac{1}{3}$

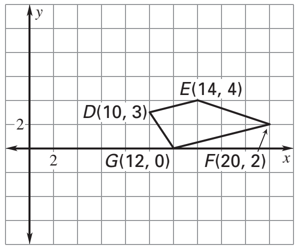
Slope of $\overline{EF} = \frac{-2-1}{3-4} = 3$

Slope of $\overline{FG} = \frac{-1-(-2)}{0-3} = -\frac{1}{3}$

Slope of $\overline{GD} = \frac{2-(-1)}{1-0} = 3$

So, $\angle D$, $\angle E$, $\angle F$, and $\angle G$ are right angles because the segments that form them are perpendicular. $DEFG$ is a square because the four sides are congruent and the four angles are right angles.

49.



Slope of $\overline{DE} = \frac{4-3}{14-10} = \frac{1}{4}$

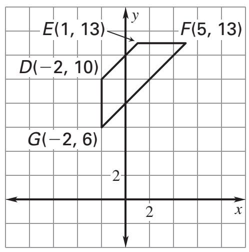
Slope of $\overline{EF} = \frac{2-4}{20-14} = -\frac{1}{3}$

Slope of $\overline{FG} = \frac{0-2}{12-20} = \frac{1}{4}$

Slope of $\overline{GD} = \frac{3-0}{10-12} = -\frac{3}{2}$

So, $\overline{DE} \parallel \overline{FG}$. $DEFG$ is a trapezoid because it has one pair of parallel sides.

50.



Slope of $\overline{DE} = \frac{13-10}{1-(-2)} = 1$

Slope of $\overline{EF} = \frac{13-13}{5-1} = 0$

Slope of $\overline{FG} = \frac{13-6}{5-(-2)} = 1$

Slope of $\overline{GD} = \frac{10-6}{-2-(-2)} = \frac{4}{0} =$ undefined

So, $\overline{DE} \parallel \overline{FG}$.

$EF = \sqrt{(5-1)^2 + (13-13)^2} = 4$

$GD = \sqrt{(-2-(-2))^2 + (10-6)^2} = 4$

So, $\overline{EF} \cong \overline{GD}$. $DEFG$ is an isosceles trapezoid because it has one pair of parallel sides and one pair of non-parallel congruent sides.