Chapter 7 Right Triangles and Trigonometry

Prerequisite Skills for the chapter "Right Triangles and Trigonometry"

1. The triangle is an equilateral triangle.

2. The triangle is a right triangle.

3. The triangle is an acute triangle.

4. The triangle is an obtuse isosceles triangle.

5. $\sqrt{45} = \sqrt{9 \cdot 5} = 3\sqrt{5}$

6. $(3\sqrt{7})^2 = 3^2(\sqrt{7})^2 = 9(7) = 63$

7. $\sqrt{3} \cdot \sqrt{5} = \sqrt{15}$

8. $\frac{7}{\sqrt{2}} = \frac{7 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{7\sqrt{2}}{2}$

9. $\frac{3}{x} = \frac{12}{16}$

$12x = 48$

$x = 4$

10. $\frac{2}{3} = \frac{x}{18}$

$3x = 36$

$x = 12$

11. $\frac{x + 5}{4} = \frac{1}{2}$

$2(x + 5) = 4$

$2x + 10 = 4$

$2x = -6$

$x = -3$

12. $\frac{x + 4}{x - 4} = \frac{6}{5}$

$5(x + 4) = 6(x - 4)$

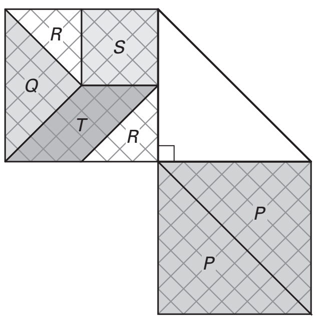
$5x + 20 = 6x - 24$

$44 = x$

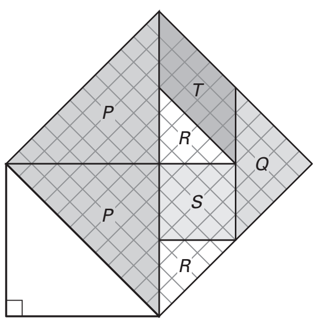
Lesson 7.1 Apply the Pythagorean Theorem

Investigating Geometry Activity for the lesson "Apply the Pythagorean Theorem"

Step 3:



Step 4:



1. $P = 36 \text{ units}^2$

$Q = 18 \text{ units}^2$

$R = 9 \text{ units}^2$

$S = 18 \text{ units}^2$

$T = 18 \text{ units}^2$

$$\begin{aligned} \text{Sum of areas} &= 2P + Q + 2R + S + T \\ &= 2(36) + 18 + 2(9) + 18 + 18 \\ &= 144 \text{ units}^2 \end{aligned}$$

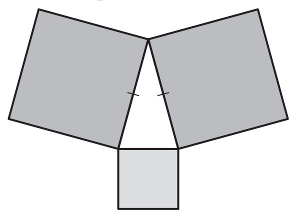
The side lengths of the squares are equal to the lengths of the two legs of Triangle P.

2. Area of square in Step 4 $= 2P + Q + 2R + S + T = 144 \text{ units}^2$. The side length of this square is equal to the length of the hypotenuse of Triangle P.

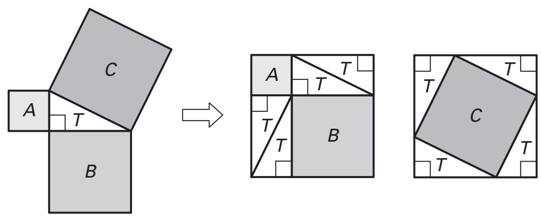
3. The sum of the areas of the two squares in Step 3 is equal to the area of the square in Step 4. So, the sum of the squares of the legs of a right triangle is equal to the square of the hypotenuse.

4. The legs of Triangle P are congruent, and they meet to form a right angle. The conjecture in Exercise 3 is not true for all isosceles triangles.

For example:



The conjecture is true for all right triangles. In the figures below, you can see that $A + B + 4T = C + 4T$. So, $A + B = C$.



Guided Practice for the lesson "Apply the Pythagorean Theorem"

1. The unknown side is a leg.

$5^2 = x^2 + 3^2$

$5^2 - 3^2 = x^2$

$25 - 9 = x^2$

$16 = x^2$

$4 = x$

2. The unknown side is a hypotenuse.

$x^2 = 6^2 + 4^2$

$x^2 = 36 + 16$

$x^2 = 52$

$x = 2\sqrt{13}$

3. (Length of ladder)^2 = (Distance from house)^2 + (Height of ladder)^2

$x^2 = 6^2 + 23^2$

$x^2 = 36 + 529 = 565$

$x = \sqrt{565} \approx 23.8$

The length of the ladder is about 23.8 feet.

4. The Pythagorean Theorem is only true for right triangles.

5. The base of each right triangle is $\left(\frac{1}{2}\right)(30) = 15$.

$18^2 = 15^2 + h^2$

$324 = 225 + h^2$

$99 = h^2$

$3\sqrt{11} = h$

$\text{Area} = \frac{1}{2}(\text{base})(\text{height})$

$= \frac{1}{2}(30)(3\sqrt{11})$

$= 45\sqrt{11}$

≈ 149.25

The area of the triangle is about 149.25 square feet.

6. The base of each right triangle is 10 m.

$26^2 = 10^2 + h^2$

$676 = 100 + h^2$

$576 = h^2$

$24 = h$

$\text{Area} = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(20)(24) = 240$

The area of the triangle is 240 square meters.

7. Using the Pythagorean Theorem:

$x^2 = 9^2 + 12^2$

$x^2 = 81 + 144$

$x^2 = 225$

$x = 15$

Using a Pythagorean triple: A common triple is 3, 4, 5. Multiply each number by 3.

$3 \cdot 3 = 9$

$4 \cdot 3 = 12$

$5 \cdot 3 = 15$

So, the length of the hypotenuse is 15 inches.

8. Using the Pythagorean Theorem:

$x^2 = 14^2 + 48^2$

$x^2 = 196 + 2304$

$x^2 = 2500$

$x = 50$

Using a Pythagorean triple: a common triple is 7, 24, 25. Multiply each number by 2.

$7 \cdot 2 = 14$

$24 \cdot 2 = 48$

$25 \cdot 2 = 50$

So, the length of the hypotenuse is 50 centimeters.

Exercises for the lesson "Apply the Pythagorean Theorem"

Skill Practice

1. A set of three positive integers a, b, and c that satisfies the equation $c^2 = a^2 + b^2$ is called a Pythagorean triple.

2. In order to use the Pythagorean Theorem, you must have the lengths of two of the sides of a right triangle.

3. $x^2 = 50^2 + 120^2$

$x^2 = 2500 + 14{,}400$

$x^2 = 16{,}900$

$x = 130$

4. $x^2 = 33^2 + 56^2$

$x^2 = 1089 + 3136$

$x^2 = 4225$

$x = 65$

5. $x^2 = 40^2 + 42^2$

$x^2 = 1600 + 1764$

$x^2 = 3364$

$x = 58$

6. In the Pythagorean Theorem, b and c were substituted incorrectly.

$a^2 + b^2 = c^2$

$10^2 + 24^2 = 26^2$

7. It is not algebraically correct to simplify $7^2 + 24^2$ as $(7 + 24)^2$. The solution should be:

$x^2 = 7^2 + 24^2$

$x^2 = 49 + 576$

$x^2 = 625$

$x = 25$

8. $16.7^2 = 8.9^2 + h^2$

$278.89 = 79.21 + h^2$

$199.68 = h^2$

$14.13 \approx h$

The height of the fire escape landing is about 14.13 feet.

9. $13.4^2 = 9.8^2 + h^2$

$179.56 = 96.04 + h^2$

$83.52 = h^2$

$9.14 \approx h$

The height of the backboard frame is about 9.14 inches.

10. $5.7^2 = 4.9^2 + b^2$

$32.49 = 24.01 + b^2$

$8.48 = b^2$

$2.9 \approx b$

The base of the frame is about 2.9 feet.

11. The base of each right triangle is $\frac{1}{2}(16) = 8$ meters.

$17^2 = 8^2 + h^2$

$289 = 64 + h^2$

$225 = h^2$

$15 = h$

$\text{Area} = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(16)(15) = 120$

The area of the triangle is 120 square meters.

12. The base of each right triangle is $\frac{1}{2}(32) = 16$ feet.

$20^2 = 16^2 + h^2$

$400 = 256 + h^2$

$144 = h^2$

$12 = h$

Area $= \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(32)(12) = 192$

The area of the triangle is 192 square feet.

13. The base of each right triangle is 6 centimeters.

$10^2 = 6^2 + h^2$

$100 = 36 + h^2$

$64 = h^2$

$8 = h$

Area $= \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(12)(8) = 48$

The area of the triangle is 48 square centimeters.

14. A common triple is 7, 24, 25. Multiply each length by 3.

$7 \cdot 3 = 21$

$24 \cdot 3 = 72$

$25 \cdot 3 = 75$

So, $x = 75$.

15. A common triple is 3, 4, 5. Multiply each length by 10.

$3 \cdot 10 = 30$

$4 \cdot 10 = 40$

$5 \cdot 10 = 50$

So, $x = 40$.

16. A common triple is 8, 15, 17. Multiply each length by 4.

$8 \cdot 4 = 32$

$15 \cdot 4 = 60$

$17 \cdot 4 = 68$

So, $x = 32$.

17. B;

$c^2 = a^2 + b^2$

$c^2 = 8^2 + 15^2$

$c^2 = 64 + 225$

$c^2 = 289$

$c = 17$

The length of the hypotenuse is 17 inches.

18. 24 and 51; A common Pythagorean triple is 8, 15, 17. Multiply the lengths by 3.

$8 \cdot 3 = 24$

$15 \cdot 3 = 45$

$17 \cdot 3 = 51$

The missing side is a leg with length of 45.

19. 20 and 25; A common Pythagorean triple is 3, 4, 5. Multiply the lengths by 5.

$3 \cdot 5 = 15$

$4 \cdot 5 = 20$

$5 \cdot 5 = 25$

The missing side is a leg with length of 15.

20. 28 and 96; A common Pythagorean triple is 7, 24, 25. Multiply the lengths by 4.

$7 \cdot 4 = 28$

$24 \cdot 4 = 96$

$25 \cdot 4 = 100$

The missing side is a hypotenuse with length of 100.

21. 20 and 48; A common Pythagorean triple is 5, 12, 13. Multiply the lengths by 4.

$5 \cdot 4 = 20$

$12 \cdot 4 = 48$

$13 \cdot 4 = 52$

The missing side is a hypotenuse with length of 52.

22. 75 and 85; A common Pythagorean triple is 8, 15, 17. Multiply the lengths by 5.

$8 \cdot 5 = 40$

$15 \cdot 5 = 75$

$17 \cdot 5 = 85$

The missing side is a leg with length of 40.

23. 72 and 75; A common Pythagorean triple is 7, 24, 25. Multiply the lengths by 3.

$7 \cdot 3 = 21$

$24 \cdot 3 = 72$

$25 \cdot 3 = 75$

The missing side is a leg with length of 21.

24. $x^2 = 6^2 + 3^2$

$x^2 = 36 + 9$

$x^2 = 45$

$x = 3\sqrt{5}$

25. $x^2 = 11^2 + 11^2$

$x^2 = 121 + 121$

$x^2 = 242$

$x = 11\sqrt{2}$

26. Let h represent the height.

$5^2 = 3^2 + h^2$

$25 = 9 + h^2$

$16 = h^2$

$4 = h$

$x^2 = 4^2 + 7^2$

$x^2 = 16 + 49$

$x^2 = 65$

$x = \sqrt{65}$

27. A;

$39^2 = 15^2 + b^2$

$1521 = 225 + b^2$

$1296 = b^2$

$36 = b$

Area $= \frac{1}{2}$(base)(height) $= \frac{1}{2}(15)(36) = 270$

The area of the triangle is 270 square feet.

28. $(4x - 4)^2 = (2x)^2 + (2x + 4)^2$

$16x^2 - 32x + 16 = 4x^2 + 4x^2 + 16x + 16$

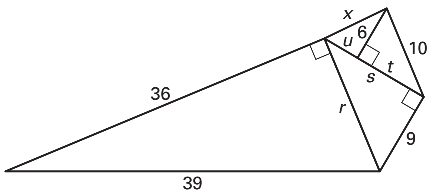
$8x^2 - 48x = 0$

$8x(x - 6) = 0$

$x = 0 \quad \text{or} \quad x = 6$

Because x cannot be zero, the value of x is 6.

29.



$39^2 = 36^2 + r^2$ $\qquad$ $15^2 = 9^2 + s^2$

$1521 = 1296 + r^2$ $\qquad$ $225 = 81 + s^2$

$225 = r^2$ $\qquad$ $144 = s^2$

$15 = r$ $\qquad$ $12 = s$

$10^2 = 6^2 + t^2$

$100 = 36 + t^2$

$64 = t^2$

$8 = t$

$u = s - t = 12 - 8 = 4$

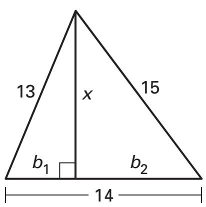
$x^2 = 6^2 + 4^2$

$x^2 = 36 + 16$

$x^2 = 52$

$x = 2\sqrt{13}$

30.



$b_1 + b_2 = 14$

$(14 - b_2)^2 + x^2 = 13^2$

and $b_2^{\,2} + x^2 = 15^2$

$(14 - b_2)^2 + x^2 = 13^2$

$196 - 28b_2 + b_2^{\,2} + x^2 = 169$

$-28b_2 + b_2^{\,2} + x^2 = -27$

$28b_2 - b_2^{\,2} - x^2 = 27$

Solve the system of equations.

$$\begin{array}{rl} 28b_2 - b_2^{\,2} - x^2 = & 27 \\ b_2^{\,2} + x^2 = & 225 \\ \hline 28b_2 \qquad\quad = & 252 \\ b_2 = & 9 \end{array}$$

So, $b_1 = 14 - 9 = 5$.

$13^2 = 5^2 + x^2$

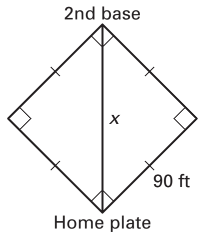
$169 = 25 + x^2$

$144 = x^2$

$12 = x$

Problem Solving

31.



$x^2 = 90^2 + 90^2$

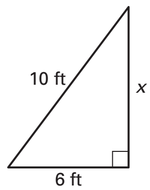
$x^2 = 8100 + 8100$

$x^2 = 16{,}200$

$x \approx 127.3$

A ball thrown from home plate to second base must go about 127.3 feet.

32.



$10^2 = 6^2 + x^2$

$100 = 36 + x^2$

$64 = x^2$

$8 = x$

The balloon is 8 feet above the ground.

33. The hypotenuse is 65, because it must be the longest of the three sides.

34. a. $80^2 = 35^2 + x^2$

$6400 = 1225 + x^2$

$5175 = x^2$

$71.9 \approx x$

$P \approx 35 + 80 + 71.9 \approx 186.9$

The perimeter of the field is about 186.9 feet.

b. $P \div 10 =$ Number of dogwoods to plant

$186.9 \div 10 = 18.7$

You will need about 19 dogwood seedlings.

c.

Number of dogwoods $\times$ cost of dogwoods $=$ total cost

19 $\times$ 12 $=$ total cost

228 $=$ total cost

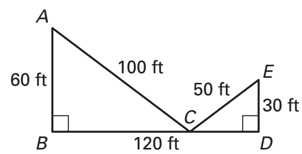
The trees will cost \$228.

35. a–b.

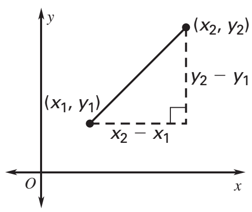
BC	*AC*	*CE*	*AC* + *CE*
10	$\sqrt{10^2 + 60^2} \approx 60.8$	$\sqrt{110^2 + 30^2} \approx 114$	174.8
20	$\sqrt{20^2 + 60^2} \approx 63.2$	$\sqrt{100^2 + 30^2} \approx 104.4$	167.6
30	$\sqrt{30^2 + 60^2} \approx 67.1$	$\sqrt{90^2 + 30^2} \approx 94.9$	162
40	$\sqrt{40^2 + 60^2} \approx 72.1$	$\sqrt{80^2 + 30^2} \approx 85.4$	157.5
50	$\sqrt{50^2 + 60^2} \approx 78.1$	$\sqrt{70^2 + 30^2} \approx 76.2$	154.3
60	$\sqrt{60^2 + 60^2} \approx 84.9$	$\sqrt{60^2 + 30^2} \approx 67.1$	152
70	$\sqrt{70^2 + 60^2} \approx 92.2$	$\sqrt{50^2 + 30^2} \approx 58.3$	150.5
80	$\sqrt{80^2 + 60^2} = 100$	$\sqrt{40^2 + 30^2} = 50$	150
90	$\sqrt{90^2 + 60^2} \approx 108.2$	$\sqrt{30^2 + 30^2} \approx 42.4$	150.6
100	$\sqrt{100^2 + 60^2} \approx 116.6$	$\sqrt{20^2 + 30^2} \approx 36.1$	152.7
110	$\sqrt{110^2 + 60^2} \approx 125.3$	$\sqrt{10^2 + 30^2} \approx 31.6$	156.9
120	$\sqrt{120^2 + 60^2} \approx 134.2$	$\sqrt{0^2 + 30^2} = 30$	164.2

b. The shortest distance that you must travel is 150 feet.

c.



36.



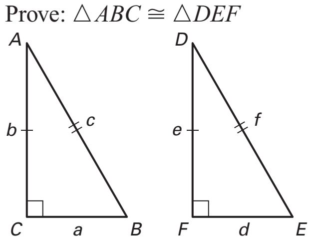
By the Pythagorean Theorem,

$(x_2 - x_1)^2 + (y_2 - y_1)^2 = d^2$

So, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

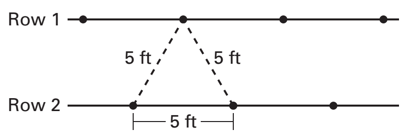
37. Given: $\triangle ABC$ and $\triangle DEF$ are right triangles; $b = e$, $c = f$

Prove: $\triangle ABC \cong \triangle DEF$



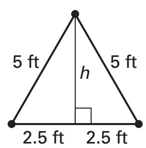
Statements	Reasons
1. $\triangle ABC$ and $\triangle DEF$ are right triangles.	**1.** Given
2. $c^2 = a^2 + b^2$, $f^2 = d^2 + e^2$	**2.** Pythagorean Theorem
3. $b = e$, $c = f$	**3.** Given
4. $f^2 = a^2 + e^2$	**4.** Substitution Property of Equality
5. $a^2 + e^2 = d^2 + e^2$	**5.** Substitution Property of Equality
6. $a^2 = d^2$	**6.** Subtraction Property of Equality
7. $a = d$	**7.** Definition of square roots
8. $\overline{BC} \cong \overline{EF}$	**8.** Definition of congruent segments
9. $\angle ACB \cong \angle DFE$	**9.** Right Angles Congruence Theorem
10. $\triangle ABC \cong \triangle DEF$	**10.** SAS Congruence Postulate

38.



If the trees are staggered so that trees in row 2 are halfway between the trees in row 1, at the minimum distance of 5 feet, an equilateral triangle is formed with each leg having a length of 5 feet.

Bisect the base to form two right triangles.



$$5^2 = 2.5^2 + h^2$$
$$25 = 6.25 + h^2$$
$$18.75 = h^2$$
$$4.3 \approx h$$

The minimum distance between the rows is about 4.3 feet.

Lesson 7.2 Use the Converse of the Pythagorean Theorem

Investigating Geometry Activity for the lesson "Use the Converse of the Pythagorean Theorem"

1. If a triangle is a right triangle, then the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.

If the square of the length of the longest side of a triangle is equal to the sum of the squares of the lengths of the other two sides, then the triangle is a right triangle.

2. The converse of the Pythagorean Theorem is true because an obtuse or acute triangle will not produce the same equivalency in the squares of the side lengths.

3. Let C be the largest angle in $\triangle ABC$. If $m\angle C < 90°$, then $AB^2 < AC^2 + CB^2$. If $m\angle C = 90°$, then $AB^2 = AC^2 + CB^2$. If $m\angle C > 90°$, then $AB^2 > AC^2 + CB^2$.
4. If $AB^2 > AC^2 + CB^2$, then the triangle is an *obtuse* triangle.
5. If $AB^2 < AC^2 + CB^2$, then the triangle is an *acute* triangle.
6. If $AB^2 = AC^2 + CB^2$, then the triangle is a *right* triangle.

Guided Practice for the lesson "Use the Converse of the Pythagorean Theorem"

1. $8^2 \stackrel{?}{=} 4^2 + (4\sqrt{3})^2$

   $64 \stackrel{?}{=} 16 + 48$

   $64 = 64$ ✓

   The triangle is a right triangle.
2. $14^2 \stackrel{?}{=} 10^2 + 11^2$

   $196 \stackrel{?}{=} 100 + 121$

   $196 \neq 221$

   The triangle is not a right triangle.
3. $(\sqrt{61})^2 \stackrel{?}{=} 5^2 + 6^2$

   $61 \stackrel{?}{=} 25 + 36$

   $61 = 61$ ✓

   The triangle is a right triangle.
4. $3 + 4 = 7$ $\quad$ $3 + 6 = 9$ $\quad$ $4 + 6 = 10$

   $7 > 6$ $\quad$ $9 > 4$ $\quad$ $10 > 3$

   The side lengths 3, 4, and 6 can form a triangle.

   $c^2 \ \underline{\ ?\ }\ a^2 + b^2$

   $6^2 \ \underline{\ ?\ }\ 3^2 + 4^2$

   $36 \ \underline{\ ?\ }\ 9 + 16$

   $36 > 25$

   The triangle is obtuse.
5. No, triangles with side lengths 2, 3, and 4 could not be used to verify that you have perpendicular lines, because the side lengths do not form a right triangle.

   $4^2 \neq 2^2 + 3^2$

Exercises for the lesson "Use the Converse of the Pythagorean Theorem"

Skill Practice

1. The longest side of a right triangle is called a hypotenuse.
2. The side lengths of a triangle can be used to classify a triangle as acute, obtuse, or right by comparing the square of the length of the longest side to the sum of the squares of the lengths of the two other sides. If $c^2 = a^2 + b^2$, the triangle is a right triangle. If $c^2 > a^2 + b^2$, the triangle is an obtuse triangle. If $c^2 < a^2 + b^2$, the triangle is an acute triangle.
3. $97^2 \stackrel{?}{=} 65^2 + 72^2$

   $9409 \stackrel{?}{=} 4225 + 5184$

   $9409 = 9409$ ✓

   The triangle is a right triangle.
4. $23^2 \stackrel{?}{=} 11.4^2 + 21.2^2$

   $529 \stackrel{?}{=} 129.96 + 449.44$

   $529 \neq 579.4$

   The triangle is not a right triangle.
5. $(3\sqrt{5})^2 \stackrel{?}{=} 2^2 + 6^2$

   $45 \stackrel{?}{=} 4 + 36$

   $45 \neq 40$

   The triangle is not a right triangle.
6. $(4\sqrt{19})^2 \stackrel{?}{=} 10^2 + 14^2$

   $304 \stackrel{?}{=} 100 + 196$

   $304 \neq 296$

   The triangle is not a right triangle.
7. $(\sqrt{26})^2 \stackrel{?}{=} 1^2 + 5^2$

   $26 \stackrel{?}{=} 1 + 25$

   $26 = 26$ ✓

   The triangle is a right triangle.
8. $89^2 \stackrel{?}{=} 39^2 + 80^2$

   $7921 \stackrel{?}{=} 1521 + 6400$

   $7921 = 7921$ ✓

   The triangle is a right triangle.
9. $15^2 \stackrel{?}{=} 9^2 + 12^2$

   $225 \stackrel{?}{=} 81 + 144$

   $225 = 225$ ✓

   The triangle is a right triangle.
10. $15^2 \stackrel{?}{=} 9^2 + 10^2$

    $225 \stackrel{?}{=} 81 + 100$

    $225 \neq 181$

    The triangle is not a right triangle.
11. $60^2 \stackrel{?}{=} 36^2 + 48^2$

    $3600 \stackrel{?}{=} 1296 + 2304$

    $3600 = 3600$ ✓

    The triangle is a right triangle.
12. $(2\sqrt{34})^2 \stackrel{?}{=} 6^2 + 10^2$

    $136 \stackrel{?}{=} 36 + 100$

    $136 = 136$ ✓

    The triangle is a right triangle.
13. $(7\sqrt{5})^2 \stackrel{?}{=} 7^2 + 14^2$

    $245 \stackrel{?}{=} 49 + 196$

    $245 = 245$ ✓

    The triangle is a right triangle.
14. $20^2 \stackrel{?}{=} 10^2 + 12^2$

    $400 \stackrel{?}{=} 100 + 144$

    $400 \neq 244$

    The triangle is not a right triangle.

15. 10, 11, and 14

$10 + 11 = 21 \quad 10 + 14 = 28 \quad 11 + 14 = 25$

$21 > 14 \quad 28 > 11 \quad 25 > 10$

The segment lengths form a triangle.

$c^2 \underline{\ ?\ } a^2 + b^2$

$14^2 \underline{\ ?\ } 10^2 + 11^2$

$196 \underline{\ ?\ } 100 + 121$

$196 < 221$

The triangle is an acute triangle.

16. 10, 15, and $5\sqrt{13}$

$10 + 15 = 25 \quad 10 + 5\sqrt{13} \approx 28 \quad 15 + 5\sqrt{13} \approx 33$

$25 > 5\sqrt{13} \quad 28 > 15 \quad 33 > 10$

The segment lengths form a triangle.

$c^2 \underline{\ ?\ } a^2 + b^2$

$(5\sqrt{13})^2 \underline{\ ?\ } 10^2 + 15^2$

$325 \underline{\ ?\ } 100 + 225$

$325 = 325$

The triangle is a right triangle.

17. 24, 30, and $6\sqrt{43}$

$24 + 30 = 54 \quad 24 + 6\sqrt{43} \approx 63.3 \quad 30 + 6\sqrt{43} \approx 69.3$

$54 > 6\sqrt{43} \quad 63.3 > 30 \quad 69.3 > 24$

The segment lengths form a triangle.

$c^2 \underline{\ ?\ } a^2 + b^2$

$(6\sqrt{43})^2 \underline{\ ?\ } 24^2 + 30^2$

$1548 \underline{\ ?\ } 576 + 900$

$1548 > 1476$

The triangle is an obtuse triangle.

18. 5, 6, and 7

$5 + 6 = 11 \quad 5 + 7 = 12 \quad 6 + 7 = 13$

$11 > 7 \quad 12 > 6 \quad 13 > 5$

The segment lengths form a triangle.

$c^2 \underline{\ ?\ } a^2 + b^2$

$7^2 \underline{\ ?\ } 5^2 + 6^2$

$49 \underline{\ ?\ } 25 + 36$

$49 < 61$

The triangle is an acute triangle.

19. 12, 16, and 20

$12 + 16 = 28 \quad 12 + 20 = 32 \quad 16 + 20 = 36$

$28 > 20 \quad 32 > 16 \quad 36 > 12$

The segment lengths form a triangle.

$c^2 \underline{\ ?\ } a^2 + b^2$

$20^2 \underline{\ ?\ } 12^2 + 16^2$

$400 \underline{\ ?\ } 144 + 256$

$400 = 400$

The triangle is a right triangle.

20. 8, 10, and 12

$8 + 10 = 18 \quad 8 + 12 = 20 \quad 10 + 12 = 22$

$18 > 12 \quad 20 > 10 \quad 22 > 8$

The segment lengths form a triangle.

$c^2 \underline{\ ?\ } a^2 + b^2$

$12^2 \underline{\ ?\ } 8^2 + 10^2$

$144 \underline{\ ?\ } 64 + 100$

$144 < 164$

The triangle is an acute triangle.

21. 15, 20, and 36

$15 + 20 = 35 \quad 15 + 36 = 51 \quad 20 + 36 = 56$

$35 < 36 \quad 51 > 20 \quad 56 > 15$

The segment lengths do not form a triangle.

22. 6, 8, and 10

$6 + 8 = 14 \quad 6 + 10 = 16 \quad 8 + 10 = 18$

$14 > 10 \quad 16 > 8 \quad 18 > 6$

The segment lengths form a triangle.

$c^2 \underline{\ ?\ } a^2 + b^2$

$10^2 \underline{\ ?\ } 6^2 + 8^2$

$100 \underline{\ ?\ } 36 + 64$

$100 = 100$

The triangle is a right triangle.

23. 8.2, 4.1, and 12.2

$8.2 + 4.1 = 12.3 \quad 8.2 + 12.2 = 20.4 \quad 4.1 + 12.2 = 16.3$

$12.3 > 12.2 \quad 20.4 > 4.1 \quad 16.3 > 8.2$

The segment lengths form a triangle.

$c^2 \underline{\ ?\ } a^2 + b^2$

$12.2^2 \underline{\ ?\ } 8.2^2 + 4.1^2$

$148.84 \underline{\ ?\ } 67.24 + 16.81$

$148.84 > 84.05$

The triangle is an obtuse triangle.

24. B; 10, 24, and 28

$28^2 \stackrel{?}{=} 10^2 + 24^2$

$784 \stackrel{?}{=} 100 + 576$

$784 \neq 676$

25. C; 4, 7, and 9

$c^2 \underline{\ ?\ } a^2 + b^2$

$9^2 \underline{\ ?\ } 4^2 + 7^2$

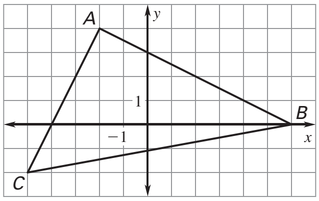
$81 \underline{\ ?\ } 16 + 49$

$81 > 65$

The triangle is an obtuse scalene triangle.

26. Multiplying all of the lengths of a Pythagorean triple by a constant will produce another Pythagorean triple.

27.



$AB = \sqrt{(6 - (-2))^2 + (0 - 4)^2} = 4\sqrt{5}$

$BC = \sqrt{(-5 - 6)^2 + (-2 - 0)^2} = 5\sqrt{5}$

$AC = \sqrt{(-5 - (-2))^2 + (-2 + 4)^2} = \sqrt{13}$

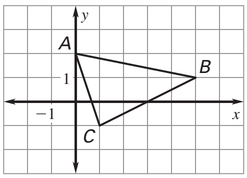
$(5\sqrt{5})^2 \underline{\ ?\ } (4\sqrt{5})^2 + (\sqrt{13})^2$

$125 \underline{\ ?\ } 80 + 13$

$125 > 93$

The triangle is an obtuse triangle.

28.



$AB = \sqrt{(5 - 0)^2 + (1 - 2)^2} = \sqrt{26}$

$BC = \sqrt{(1 - 5)^2 + (-1 - 1)^2} = 2\sqrt{5}$

$AC = \sqrt{(1 - 0)^2 + (-1 - 2)^2} = \sqrt{10}$

$(\sqrt{26})^2 \underline{\ ?\ } (2\sqrt{5})^2 + (\sqrt{10})^2$

$26 \underline{\ ?\ } 20 + 10$

$26 < 30$

The triangle is an acute triangle.

29. $(13x)^2 \underline{\ ?\ } (5x)^2 + (12x)^2$

$169x^2 \underline{\ ?\ } 25x^2 + 144x^2$

$169x^2 = 169x^2$

The triangle is a right triangle.

30. $BC^2 \underline{\ ?\ } AC^2 + AB^2$ $\qquad EF^2 \underline{\ ?\ } DF^2 + DE^2$

$(4\sqrt{10})^2 \underline{\ ?\ } 4^2 + 12^2$ $\qquad (2\sqrt{96})^2 \underline{\ ?\ } 8^2 + 18^2$

$160 \underline{\ ?\ } 16 + 144$ $\qquad 384 \underline{\ ?\ } 64 + 324$

$160 = 160$ $\qquad 384 < 388$

Triangle ABC is a right triangle. Triangle DEF is an acute triangle. So, $m\angle A > m\angle D$.

31. When $m\angle A = 90°$:

$m\angle A + m\angle B + m\angle C = 180°$

$90° + m\angle B + m\angle C = 180°$

$m\angle B + m\angle C = 90°$

When $m\angle D < 90°$:

$m\angle D + m\angle E + m\angle F = 180°$

$m\angle E + m\angle F = 180° - m\angle D$

$m\angle E + m\angle F > 180° - 90° = 90°$

So, $m\angle B + m\angle C < m\angle E + m\angle F$.

32. 6, 8, and x

$6 + 8 = 14$ $\qquad 6 + x > 8$ $\qquad 8 + x > 6$

$14 > x$ $\qquad x > 2$ $\qquad x > -2$

$x^2 = 6^2 + 8^2$

$x^2 = 36 + 64$

$x^2 = 100$

$x = 10$

Right triangle: $x = 10$

Acute triangle: $2 < x < 10$

Obtuse triangle: $10 < x < 14$

33. $20^2 < x^2 + (x + 4)^2$

$400 < x^2 + x^2 + 8x + 16$

$0 < 2x^2 + 8x - 384$

$0 < 2(x^2 + 4x - 192)$

$0 < 2(x - 12)(x + 16)$

$x > 12$ and $x > -16$

For 20 to be the longest side, $x + 4 < 20$, so $x < 16$. The triangle is an acute triangle when $12 < x < 16$.

34. $(6x - 1)^2 > (4x + 6)^2 + (2x + 1)^2$

$36x^2 - 12x + 1 > 16x^2 + 48x + 36 + 4x^2 + 4x + 1$

$16x^2 - 64x - 36 > 0$

$4(4x^2 - 16x - 9) > 0$

$4(2x + 1)(2x - 9) > 0$

$2x + 1 > 0$ and $2x - 9 > 0$

$x > -\frac{1}{2}$ and $x > \frac{9}{2}$

The triangle is an obtuse triangle when $x > \frac{9}{2}$.

Problem Solving

35. $c^2 = 8^2 + 10^2$

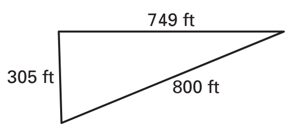
$c^2 = 64 + 100$

$c^2 = 164$

$c \approx 12.8$

To be certain that the corners are 90°, make sure that the diagonals of the rectangular frame measure 12.8 inches.

36.



$800^2 \stackrel{?}{=} 749^2 + 305^2$

$640{,}000 \stackrel{?}{=} 561{,}001 + 93{,}025$

$640{,}000 \neq 654{,}026$

The triangle is not a right triangle. So, you do not live directly north of the library.

37. a. Because $\triangle BCD$ is a right triangle:

$13^2 = 12^2 + BC^2$

$169 = 144 + BC^2$

$25 = BC^2$

$5 = BC$

b. $BC^2 \stackrel{?}{=} AB^2 + AC^2$

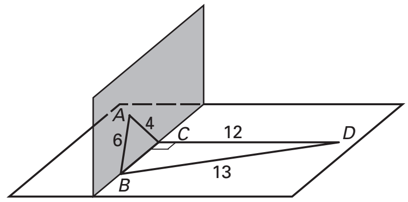
$5^2 \stackrel{?}{=} 4^2 + 3^2$

$25 \stackrel{?}{=} 16 + 9$

$25 = 25$ ✓

So, $\triangle ABC$ is a right triangle.

c. *Sample answer:*



38. $9.25^2 \stackrel{?}{=} 7^2 + 5^2$

$85.5625 \stackrel{?}{=} 49 + 25$

$85.5625 \neq 74$

The pole is not perpendicular to the ground. To make it perpendicular, tighten the rope to a length of $\sqrt{74}$ or about 8 feet 7 inches. Then tie another rope of the same length at the same point on the pole, and stake it in the ground 5 feet from the pole in another direction.

39. a. $20^2 \stackrel{?}{=} 12^2 + 16^2$

$400 \stackrel{?}{=} 144 + 256$

$400 = 400$ ✓

The triangle is a right triangle because the square of the length of the longest side is equal to the sum of the squares of the lengths of the other two sides.

b. $18^2 \stackrel{?}{=} 9^2 + 12^2$

$324 \stackrel{?}{=} 81 + 144$

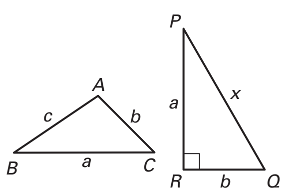
$324 \neq 225$

The triangle is not a right triangle because the square of the length of the longest side does not equal the sum of the squares of the lengths of the other two sides.

c. No; the second corner does not form a right triangle, so the frame is not sound.

40. Given: In $\triangle ABC$, $c^2 < a^2 + b^2$ where c is the length of the longest side.

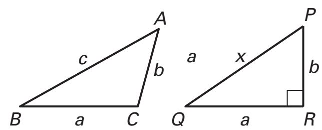
Prove: $\triangle ABC$ is an acute triangle.



Statements	Reasons
1. In $\triangle ABC$, $c^2 < a^2 + b^2$ where c is the length of the longest side. In $\triangle PQR$, $\angle R$ is a right angle.	**1.** Given
2. $a^2 + b^2 = x^2$	**2.** Pythagorean Theorem
3. $c^2 < x^2$	**3.** Substitution Property
4. $c < x$	**4.** A property of square roots
5. $m\angle R = 90°$	**5.** Definition of right angle
6. $m\angle C < m\angle R$	**6.** Converse of the Hinge Theorem
7. $m\angle C < 90°$	**7.** Substitution Property
8. $\angle C$ is an acute angle.	**8.** Definition of an acute angle
9. $\triangle ABC$ is an acute triangle.	**9.** Definition of an acute triangle

41. Given: In $\triangle ABC$, $c^2 > a^2 + b^2$ where c is the length of the longest side.

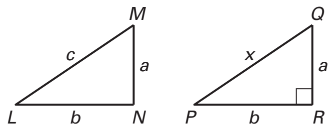
Prove: $\triangle ABC$ is an obtuse triangle.



Statements	Reasons
1. In $\triangle ABC$, $c^2 > a^2 + b^2$ where c is the length of the longest side. In $\triangle PQR$, $\angle R$ is a right angle.	**1.** Given
2. $a^2 + b^2 = x^2$	**2.** Pythagorean Theorem
3. $c^2 > x^2$	**3.** Substitution Property
4. $c > x$	**4.** A property of square roots
5. $m\angle R = 90°$	**5.** Definition of right angle
6. $m\angle C > m\angle R$	**6.** Converse of the Hinge Theorem
7. $m\angle C > 90°$	**7.** Substitution Property
8. $\angle C$ is an obtuse angle.	**8.** Definition of an obtuse angle
9. $\triangle ABC$ is an obtuse triangle.	**9.** Definition of an obtuse triangle

42. Given: In $\triangle LMN$, $\overline{LM}$ is the longest side, and $c^2 = a^2 + b^2$.

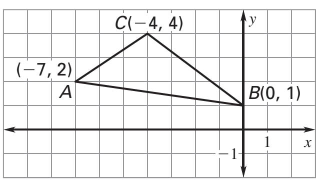
Prove: $\triangle LMN$ is a right triangle.



Statements	Reasons
1. In $\triangle LMN$, $c^2 = a^2 + b^2$ where c is the longest side. In $\triangle PQR$, $\angle R$ is a right angle.	**1.** Given
2. $a^2 + b^2 = x^2$	**2.** Pythagorean Theorem
3. $c^2 = x^2$	**3.** Substitution Property
4. $c = x$	**4.** A property of square roots
5. $m\angle R = 90°$	**5.** Definition of right angle
6. $m\angle N = m\angle R$	**6.** Converse of the Hinge Theorem
7. $m\angle N = 90°$	**7.** Substitution Property
8. $\angle N$ is an right angle.	**8.** Definition of a right angle
9. $\triangle LMN$ is a right triangle.	**9.** Definition of a right triangle

43. Because $\angle ACB$ and $\angle DCE$ are vertical angles, they are congruent. Also $\frac{AC}{CD} = \frac{BC}{CE}$. So, $\triangle ABC \sim \triangle DEC$. Because $15^2 = 12^2 + 9^2$, $\angle A$ is a right angle. So, $\angle D$ is also a right angle.

44. a.



b. Slope of $\overline{AC}$: $m = \frac{4-2}{-4-(-7)} = \frac{2}{3}$

Slope of $\overline{AB}$: $m = \frac{1-2}{0-(-7)} = \frac{-1}{7} = -\frac{1}{7}$

Slope of $\overline{BC}$: $m = \frac{1-4}{0-(-4)} = \frac{-3}{4} = -\frac{3}{4}$

Because none of the slopes are negative reciprocals, the triangle is not a right triangle.

c. $AC = \sqrt{(-4-(-7))^2 + (4-2)^2} = \sqrt{13}$

$AB = \sqrt{(0-(-7))^2 + (1-2)^2} = 5\sqrt{2}$

$BC = \sqrt{(0-(-4))^2 + (1-4)^2} = 5$

$c^2 \stackrel{?}{=} a^2 + b^2$

$(5\sqrt{2})^2 \stackrel{?}{=} 5^2 + (\sqrt{13})^2$

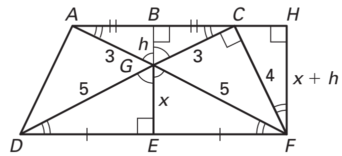
$50 \stackrel{?}{=} 25 + 13$

$50 \neq 38$

By the Converse of the Pythagorean Theorem, the triangle is not a right triangle.

d. The answers in (b) and (c) are the same.

45.



$\triangle DGF$ is isosceles, so $\angle GDE \cong \angle GFE$ and $\angle DGE \cong \angle FGE$.

$\angle AGD \cong \angle CGF$ by Vertical Angles Congruent Theorem.

$\angle BGC \cong \angle BGA$ by Congruent Supplements Theorem.

So, by AA Similarity Postulate, $\triangle GEF \sim \triangle GBC$.

$\frac{x}{h} = \frac{5}{3} \rightarrow x = \frac{5}{3}h$

Because $3^2 + 4^2 = 5^2$, $\triangle GCF$ is a right triangle with $m\angle GCF = 90°$. So, $\angle HCF \cong \angle BGC$ and $\triangle CHF \sim \triangle GBC$.

$$\frac{\frac{y}{2}}{x+h} = \frac{3}{4}$$

$$\frac{y}{2} = \frac{3}{4}(x+h)$$

$$\frac{y}{2} = \frac{3}{4}\left(\frac{5}{3}h + h\right)$$

$$\frac{y}{2} = 2h$$

Using Pythagorean Theorem:

$$h^2 + \left(\frac{y}{2}\right)^2 = 3^2$$

$$h^2 + (2h)^2 = 3^2$$

$$5h^2 = 9$$

$$h = \frac{3\sqrt{5}}{5}$$

$$x = \frac{5}{3}\left(\frac{3\sqrt{5}}{5}\right) = \sqrt{5}$$

$$y = 4\left(\frac{3\sqrt{5}}{5}\right) = \frac{12\sqrt{5}}{5}$$

Quiz for the lessons "Apply the Pythagorean Theorem" and "Use the Converse of the Pythagorean Theorem"

1. $9^2 = 3^2 + x^2$

$81 = 9 + x^2$

$72 = x^2$

$6\sqrt{2} = x$

2. $x^2 + 10^2 = 18^2$

$x^2 + 100 = 324$

$x^2 = 224$

$x = 4\sqrt{14}$

3. $x^2 = 4^2 + 14^2$

$x^2 = 16 + 196$

$x^2 = 212$

$x = 2\sqrt{53}$

4. $9^2 \ \underline{?}\ 6^2 + 7^2$

$81 \ \underline{?}\ 36 + 49$

$81 < 85$

The triangle is an acute triangle.

5. $16^2 \ \underline{?}\ 10^2 + 12^2$

$256 \ \underline{?}\ 100 + 144$

$256 > 244$

The triangle is an obtuse triangle.

6. $(8\sqrt{6})^2 \ \underline{?}\ 8^2 + 16^2$

$64(6) \ \underline{?}\ 64 + 256$

$384 > 320$

The triangle is an obtuse triangle.

7. $29^2 \ \underline{?}\ 20^2 + 21^2$

$841 \ \underline{?}\ 400 + 441$

$841 = 841$

The triangle is a right triangle.

8. $(\sqrt{73})^2 \ \underline{?}\ 3^2 + 8^2$

$73 \ \underline{?}\ 9 + 64$

$73 = 73$

The triangle is a right triangle.

9. $12^2 \ \underline{?}\ 8^2 + 10^2$

$144 \ \underline{?}\ 64 + 100$

$144 > 164$

The triangle is an acute triangle.

Lesson 7.3 Use Similar Right Triangles

Investigating Geometry Activity for the lesson "Use Similar Right Triangles"

1. The two smaller triangles have different side lengths than the larger one, but the angle measurements are the same.

$m\angle 2 = m\angle 5 = m\angle 8 = 90°$

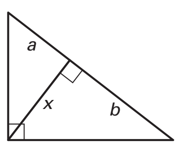
$m\angle 3 = m\angle 6 = m\angle 9$

$m\angle 1 = m\angle 4 = m\angle 7$

2. Because $m\angle 1 = m\angle 7$, $m\angle 3 = m\angle 9$, and $m\angle 2 = m\angle 8$, the green triangle is similar to the red triangle.

3. Because $m\angle 8 = m\angle 5$, $m\angle 9 = m\angle 6$, and $m\angle 7 = m\angle 4$, the red triangle is similar to the blue triangle.

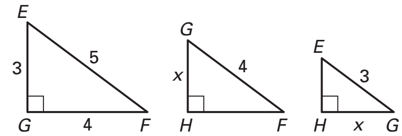
4.



$\frac{a}{x} = \frac{x}{b}$

Guided Practice for the lesson "Use Similar Right Triangles"

1.



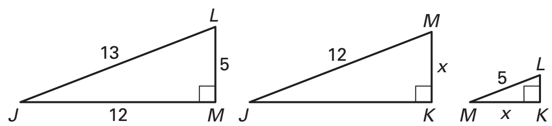
$\triangle EFG \sim \triangle GFH \sim \triangle EGH$

$\frac{GH}{EG} = \frac{GF}{EF}$

$\frac{x}{3} = \frac{4}{5}$

$x = \frac{12}{5} = 2.4$

2.



$\triangle JLM \sim \triangle JMK \sim \triangle MLK$

$\frac{MK}{LM} = \frac{MJ}{LJ}$

$\frac{x}{5} = \frac{12}{13}$

$x = \frac{60}{13} \approx 4.62$

3. In Example 3, Theorem 7.7 was used to solve for y. The length of the leg y is the geometric mean of the length of the hypotenuse of $\triangle RPQ$ and the length of the segment of the hypotenuse that is adjacent to y.

4. The wall is 19.5 feet high and Mary's eye level from the ground is 5.5 feet, so $w = 19.5 - 5.5 = 14$ feet.

$\frac{14}{d} = \frac{d}{5.5}$

$(14)(5.5) = d^2$

$77 = d^2$

$8.77 \approx d$

Mary would have to stand about 8.77 feet from the wall.

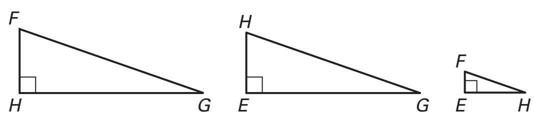
Exercises for the lesson "Use Similar Right Triangles"

Skill Practice

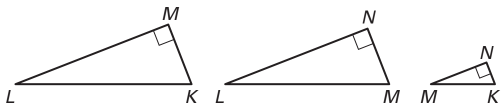
1. Two triangles are *similar* if their corresponding angles are congruent and their corresponding side lengths are proportional.

2. *Sample answer:* When two similar triangles share a common side length, and you write a proportion of side lengths that includes the common length twice, this length is the geometric mean of the other two lengths in the proportion.

3.



$\triangle FGH \sim \triangle HGE \sim \triangle FHE$

4. 

$\triangle LMK \sim \triangle LNM \sim \triangle MNK$

5. $\dfrac{x}{76} = \dfrac{76}{107.5}$

$107.5x = 76^2$

$x = \dfrac{5776}{107.5}$

$x \approx 53.7$

The length of the altitude is about 53.7 feet.

6. $\dfrac{x}{23} = \dfrac{12.8}{26.6}$

$26.6x = 12.8(23)$

$x = \dfrac{294.4}{26.6}$

$x \approx 11.1$

The length of the altitude is about 11.1 feet.

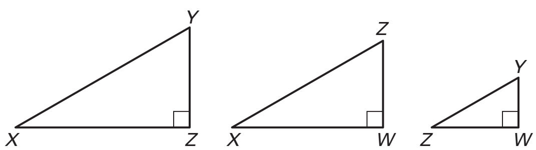
7. $\dfrac{x}{4.6} = \dfrac{3.5}{5.8}$

$5.8x = 4.6(10)$

$x = \dfrac{16.1}{5.8}$

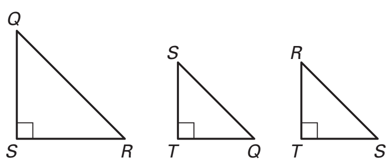
$x \approx 2.8$

The length of the altitude is about 6.7 feet.

8. 

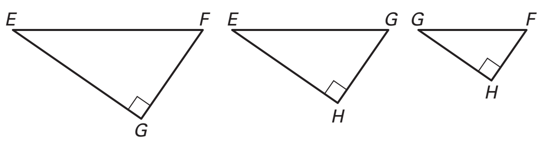
$\triangle XYZ \sim \triangle XZW \sim \triangle ZYW$

$\dfrac{XW}{ZW} = \dfrac{ZW}{YW}$

9. 

$\triangle QRS \sim \triangle SQT \sim \triangle RST$

$\dfrac{QR}{SR} = \dfrac{SQ}{TQ}$

10. 

$\triangle EFG \sim \triangle EGH \sim \triangle GFH$

$\dfrac{EF}{EG} = \dfrac{EG}{EH}$

11. The length z is the geometric mean of the length of the hypotenuse, $w + v$, and the segment adjacent to the leg of length z, which is v, not w.

$\dfrac{v}{z} = \dfrac{z}{w + v}$

12. The length d is the geometric mean of the lengths of the two segments of the hypotenuse, not the two segments whose lengths are e and f.

$\dfrac{e}{d} = \dfrac{d}{g}$

13. $\dfrac{5}{x} = \dfrac{x}{4 + 5}$

$x^2 = 5(9)$

$x^2 = 45$

$x = \sqrt{45} \approx 6.7$

14. $\dfrac{y}{18} = \dfrac{18}{12}$

$12y = 18^2$

$y = \dfrac{324}{12} = 27$

15. $\dfrac{z}{27} = \dfrac{27}{16}$

$16z = 27^2$

$z = \dfrac{729}{16} \approx 45.6$

16. $\dfrac{4}{x} = \dfrac{x}{9}$

$4(9) = x^2$

$36 = x^2$

$6 = x$

17. $\dfrac{5}{y} = \dfrac{y}{8}$

$5(8) = y^2$

$40 = y^2$

$6.3 \approx y$

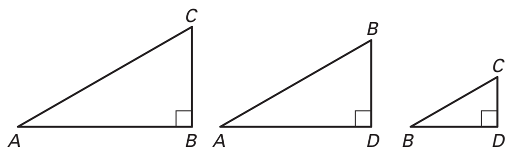
18. $\dfrac{8}{x} = \dfrac{x}{(8 - 2)}$

$8(8 - 2) = x^2$

$48 = x^2$

$6.9 \approx x$

19. C;



$\dfrac{CA}{BA} = \dfrac{BA}{CA}$ is incorrect; it should be $\dfrac{CA}{BA} = \dfrac{BA}{DA}$.

20. C;

$\dfrac{AC}{BC} = \dfrac{BC}{DC}$

$\dfrac{36}{18} = \dfrac{18}{DC}$

$36DC = 18^2$

$DC = \dfrac{324}{36} = 9$

So, $AD = 36 - 9 = 27$.

21. $\dfrac{a + 5}{12} = \dfrac{12}{18}$

$18(a + 5) = 12^2$

$18a + 90 = 144$

$18a = 54$

$a = 3$

22. $\frac{b+3}{6} = \frac{6}{8}$

$8(b+3) = 6^2$

$8b + 24 = 36$

$8b = 12$

$b = \frac{12}{8} = \frac{3}{2}$

23. $\frac{x}{12} = \frac{12}{16}$

$16x = 12^2$

$x = \frac{144}{16} = 9$

$\frac{x}{y} = \frac{y}{16+x}$

$\frac{9}{y} = \frac{y}{16+9}$

$9(16+9) = y^2$

$9(25) = y^2$

$225 = y^2$

$15 = y$

$\frac{z}{16+x} = \frac{12}{y}$

$\frac{z}{16+9} = \frac{12}{15}$

$15z = 12(16+9)$

$15z = 12(25)$

$15z = 300$

$z = 20$

$x = 9, y = 15, z = 20$

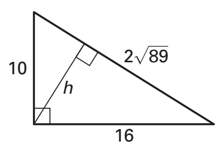
24. $c^2 \stackrel{?}{=} a^2 + b^2$

$(2\sqrt{89})^2 \stackrel{?}{=} 10^2 + 16^2$

$356 \stackrel{?}{=} 100 + 256$

$356 = 356$ ✓

The triangle is a right triangle.



$\frac{h}{10} = \frac{16}{2\sqrt{89}}$

$2\sqrt{89}h = 10(16)$

$h = \frac{160}{2\sqrt{89}}$

$h \approx 8.5$

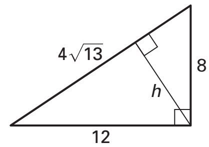
25. $c^2 \stackrel{?}{=} a^2 + b^2$

$(4\sqrt{13})^2 \stackrel{?}{=} 8^2 + 12^2$

$208 \stackrel{?}{=} 64 + 144$

$208 = 208$ ✓

The triangle is a right triangle.



$\frac{h}{8} = \frac{12}{4\sqrt{13}}$

$4\sqrt{13}h = 8(12)$

$h = \frac{96}{4\sqrt{13}}$

$h \approx 6.7$

26. $c^2 \stackrel{?}{=} a^2 + b^2$

$(4\sqrt{33})^2 \stackrel{?}{=} 14^2 + 18^2$

$528 \stackrel{?}{=} 196 + 324$

$528 \neq 520$

The triangle is not a right triangle.

27. $AC^2 = AB^2 + BC^2$

$AC^2 = 20^2 + 15^2$

$AC^2 = 400 + 225$

$AC^2 = 625$

$AC = 25$

Using Theorem 7.7:

$\frac{AC}{BC} = \frac{BC}{DC}$

$\frac{25}{15} = \frac{15}{DC}$

$25(DC) = 15^2$

$DC = \frac{225}{25} = 9$

So, $AD = 25 - 9 = 16$.

Using Theorem 7.6:

$\frac{DC}{BD} = \frac{BD}{AD}$

$\frac{9}{BD} = \frac{BD}{16}$

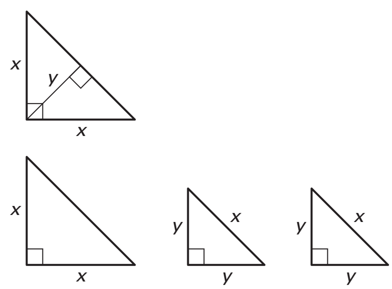
$\frac{9}{BD} = \frac{BD}{16}$

$9(16) = (BD)^2$

$144 = (BD)^2$

$12 = BD$

28.



The two smaller triangles are congruent isosceles triangles. Because they are similar to the larger triangle, and the larger triangle is isosceles, the smaller triangles must also be isosceles.

Problem Solving

29. $c^2 = a^2 + b^2$

$c^2 = 1.5^2 + 1.5^2$

$c^2 = 2.25 + 2.25$

$c^2 = 4.5$

$c \approx 2.12$

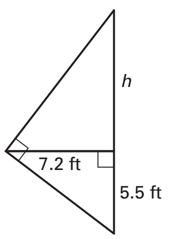
$\frac{x}{1.5} = \frac{1.5}{2.12}$

$2.12x = (1.5)^2$

$x = \frac{2.25}{2.12} \approx 1.1$

The height of the roof is about 1.1 feet.

30.



$\frac{h}{7.2} = \frac{7.2}{5.5}$

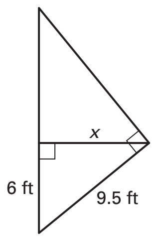
$5.5h = 7.2^2$

$h = \frac{51.84}{5.5} \approx 9.4$

Height of monument $= h + 5.5 \approx 9.4 + 5.5 = 14.9$

The monument is about 15 feet high.

31.



$9.5^2 = 6^2 + x^2$

$90.25 = 36 + x^2$

$54.25 = x^2$

$\sqrt{54.25} = x$

Find h.

$\frac{h}{x} = \frac{x}{6}$

$\frac{h}{\sqrt{54.25}} = \frac{\sqrt{54.25}}{6}$

$6h = \left(\sqrt{54.25}\right)^2$

$6h = 54.25$

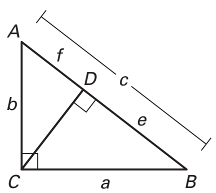
$h \approx 9$

Height of monument $= h + 6 \approx 9 + 6 = 15$

The monument is about 15 feet high. This is the same answer as in Exercise 30. The similar triangles on Paul's side of the monument are different than those on your side, but the hypotenuses of the large triangles, which represent the monument's height, are the same.

32. Given: In $\triangle ABC$, $\angle BCA$ is a right angle.

Prove: $c^2 = a^2 + b^2$



Statements	Reasons
1. Draw $\triangle ABC$. $\angle BCA$ is a right angle.	**1.** Given
2. Draw a perpendicular from C to $\overline{AB}$.	**2.** Perpendicular Postulate
3. $\frac{c}{a} = \frac{a}{e}$ and $\frac{c}{b} = \frac{b}{f}$	**3.** Geometric Mean (Leg) Theorem
4. $ce = a^2$ and $cf = b^2$	**4.** Cross Products Property
5. $ce + b^2 = a^2 + b^2$	**5.** Addition Property of Equality
6. $ce + cf = a^2 + b^2$	**6.** Substitution Property of Equality
7. $c(e + f) = a^2 + b^2$	**7.** Distributive Property
8. $e + f = c$	**8.** Segment Addition Postulate
9. $c \cdot c = a^2 + b^2$	**9.** Substitution Property of Equality
10. $c^2 = a^2 + b^2$	**10.** Simplify.

33. a. Every triangle has three altitudes. Triangle EGF has the altitude $\overline{FH}$ which is perpendicular to $\overline{EG}$ and crosses through vertex F. It also has the altitudes $\overline{EF}$ and $\overline{FG}$ which lie on the triangle. Both of these segments cross through a vertex and are perpendicular to the side opposite the vertex.

b. $\frac{HG}{FH} = \frac{FH}{EH}$

$\frac{7}{FH} = \frac{FH}{5}$

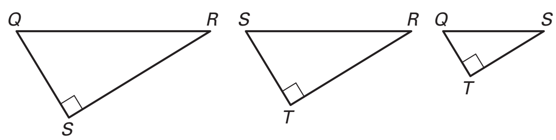
$35 = FH^2$

$\sqrt{35} = FH$

c. Area $= \frac{1}{2}$(base)(height)

$A = \frac{1}{2}(5 + 7)\left(\sqrt{35}\right) \approx 35.5$

34. a.



After sketching the three triangles so that the corresponding angles and sides have the same orientation, notice which segments are the short legs, long legs, and hypotenuses of the triangles and label them accordingly.

b. $\triangle QRS \sim \triangle SRT \sim \triangle QST$

c. Segment RS is the geometric mean of $\overline{RT}$ and $\overline{RQ}$, because $\frac{RT}{RS} = \frac{RS}{RQ}$.

35. Prove: $\triangle CBD \sim \triangle ABC$, $\triangle ACD \sim \triangle ABC$, and $\triangle CBD \sim \triangle ACD$

Statements	Reasons
1. $\triangle ABC$ is a right triangle. $\angle ACB$ is a right angle. $\overline{CD}$ is an altitude.	**1.** Given
2. $\angle CDB$ is a right angle.	**2.** Definition of altitude
3. $\angle ACB \cong \angle CDB$	**3.** Right Angles Congruence Theorem
4. $\angle B \cong \angle B$	**4.** Reflexive Property of Congruence
5. $\triangle CBD \sim \triangle ABC$	**5.** AA Similarity Postulate
6. $\angle A \cong \angle A$	**6.** Reflexive Property of Congruence
7. $\triangle ACD \sim \triangle ABC$	**7.** AA Similarity Postulate
8. $\angle ACD$ and $\angle A$ are complementary.	**8.** Corollary to the triangle Sum Theorem
9. $\angle A$ and $\angle B$ are complementary.	**9.** Corollary to the triangle Sum Theorem
10. $\angle ACD \cong \angle B$	**10.** Congruent Complements Theorem
11. $\angle CDA \cong \angle CDB$	**11.** Right Angles Congruence Theorem
12. $\triangle CBD \sim \triangle ACD$	**12.** AA Similarity Postulate

36. Prove: $\frac{BD}{CD} = \frac{CD}{AD}$

Statements	Reasons
1. $\triangle ABC$ is a right triangle.	**1.** Given
2. $\overline{CD} \perp \overline{AB}$	**2.** Given
3. $\triangle ABC \sim \triangle ACD \sim \triangle CBD$	**3.** Theorem 7.5
4. $\frac{BD}{CD} = \frac{CD}{AD}$	**4.** Definition of similar triangles

37. Prove: $\frac{AB}{CB} = \frac{CB}{DB}, \frac{AB}{AC} = \frac{AC}{AD}$

Statements	Reasons
1. $\triangle ABC$ is a right triangle.	**1.** Given
2. $\overline{CD} \perp \overline{AB}$	**2.** Given
3. $\triangle ABC \sim \triangle CBD$	**3.** Theorem 7.5
4. $\frac{AB}{CB} = \frac{CB}{DB}$	**4.** Definition of similar triangles
5. $\triangle ABC \sim \triangle ACD$	**5.** Theorem 7.5
6. $\frac{AB}{AC} = \frac{AC}{AD}$	**6.** Definition of similar triangles

38. a. $a = 10, b = 15$

$$\frac{2ab}{a+b} = \frac{2(10)(15)}{(10+15)} = \frac{300}{25} = 12$$

The harmonic mean of 10 and 15 is 12.

b. $a = 6, b = 14$

$$\frac{2ab}{a+b} = \frac{2(6)(14)}{6+14} = \frac{168}{20} = 8.4$$

The harmonic mean of 6 and 14 is 8.4.

c. Strings whose lengths have the ratio 4 : 6 : 12 will have lengths $4k$, $6k$, and $12k$, for any constant k. The strings will sound harmonious if $6k = \frac{2(4k)(12k)}{4k + 12k}$.

$$\frac{2(4k)(12k)}{4k + 12k} = \frac{96k^2}{16k} = 6k$$

So, $6k$ is the harmonic mean of $4k$ and $12k$, and the strings will sound harmonious.

Lesson 7.4 Special Right Triangles

Guided Practice for the lesson "Special Right Triangles"

1. By the Base Angles Theorem and the Corollary to the Triangle Sum Theorem, the triangle is a 45°-45°-90° triangle.

$$\text{hypotenuse} = \text{leg} \cdot \sqrt{2}$$
$$2\sqrt{2} = x \cdot \sqrt{2}$$
$$2 = x$$

2. By the Base Angles Theorem and the Corollary to the Triangle Sum Theorem, the triangle is 45°-45°-90° triangle.

$$\text{hypotenuse} = \text{leg} \cdot \sqrt{2}$$
$$y = \sqrt{2} \cdot \sqrt{2}$$
$$y = \sqrt{4}$$
$$y = 2$$

3. By the Base Angles Theorem and the Corollary to the Triangle Sum Theorem, the two triangles are 45°-45°-90° triangles.

$$\text{hypotenuse} = \text{leg} \cdot \sqrt{2}$$
$$d = 8\sqrt{2} \approx 11.31$$

4. $\text{hypotenuse} = \text{leg} \cdot \sqrt{2}$

$$6 = x \cdot \sqrt{2}$$
$$\frac{6}{\sqrt{2}} = x$$
$$\frac{6}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = x$$
$$3\sqrt{2} = x$$

The length of the hypotenuse is $3\sqrt{2}$.

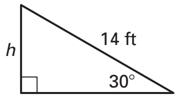
5. $\text{longer leg} = \text{shorter leg} \cdot \sqrt{3}$

$$x = \sqrt{3} \cdot \sqrt{3}$$
$$x = 3$$

6. The equilateral triangle has an altitude that forms the longer leg of two 30°-60°-90° triangles.

longer leg = shorter leg • $\sqrt{3}$

$h = 2\sqrt{3} \approx 3.46$

7. 

hypotenuse = 2 • shorter leg

$14 = 2 \cdot h$

$7 = h$

The height of the body of the dump truck is 7 feet.

8. In a 30°-60°-90° triangle, the shorter side of the triangle is opposite the 30° angle. The longer side of the triangle is opposite the 60° angle.

Exercises for the lesson "Special Right Triangles"

Skill Practice

1. A triangle with two congruent sides and a right angle is called an *isosceles right triangle*.

2. The Corollary to the Triangle Sum Theorem requires that the acute angles of a right triangle are complementary. Because the triangle is isosceles, its base angles are congruent. Half of 90° is 45°, so each of the acute angles measures 45°.

3. hypotenuse = leg • $\sqrt{2}$

$x = 7\sqrt{2}$

4. hypotenuse = leg • $\sqrt{2}$

$x = 5\sqrt{2} \cdot \sqrt{2}$

$x = 5 \cdot 2 = 10$

5. hypotenuse = leg • $\sqrt{2}$

$3\sqrt{2} = x \cdot \sqrt{2}$

$3 = x$

6. C;

hypotenuse = leg • $\sqrt{2}$

$7 = AC \cdot \sqrt{2}$

$\frac{7}{\sqrt{2}} = AC$

$\frac{7\sqrt{2}}{2} = AC$

7. hypotenuse = leg • $\sqrt{2}$

$2\sqrt{2} = x\sqrt{2}$

$2 = x$

The corner triangles have leg lengths of 2 inches. The overall side length of the tile is $2 \cdot 2 = 4$ inches.

8. hypotenuse = 2 • shorter leg

$y = 2 \cdot 9$

$y = 18$

longer leg = shorter leg • $\sqrt{3}$

$x = 9\sqrt{3}$

9. hypotenuse = 2 • shorter leg

$y = 2x$

longer leg = shorter leg • $\sqrt{3}$

$3\sqrt{3} = x\sqrt{3}$

$3 = x$

$y = 2(3) = 6$

10. hypotenuse = 2 • shorter leg

$12\sqrt{3} = 2y$

$6\sqrt{3} = y$

longer leg = shorter leg • $\sqrt{3}$

$x = 6\sqrt{3} \cdot \sqrt{3}$

$x = 6(3)$

$x = 18$

11. $a = b$

hypotenuse = leg • $\sqrt{2}$

$c = a\sqrt{2}$

a	7	11	$\frac{10}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = 5\sqrt{2}$	6	$\sqrt{5}$
b	7	11	$5\sqrt{2}$	6	$\sqrt{5}$
c	$7\sqrt{2}$	$11\sqrt{2}$	10	$6\sqrt{2}$	$\sqrt{5} \cdot \sqrt{2} = \sqrt{10}$

12. hypotenuse = 2 • shorter leg

$f = 2 \cdot d \rightarrow d = \frac{f}{2}$

longer leg = shorter leg • $\sqrt{3}$

$e = d \cdot \sqrt{3} \rightarrow d = \frac{e}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{e\sqrt{3}}{3}$

d	5	$\frac{14}{2} = 7$	$\frac{8\sqrt{3} \cdot \sqrt{3}}{3} = 8$
e	$5\sqrt{3}$	$7\sqrt{3}$	$8\sqrt{3}$
f	$2 \cdot 5 = 10$	14	$2 \cdot 8 = 16$

d	$\frac{18\sqrt{3}}{2} = 9\sqrt{3}$	$\frac{12\sqrt{3}}{3} = 4\sqrt{3}$
e	$9\sqrt{3} \cdot \sqrt{3} = 27$	12
f	$18\sqrt{3}$	$2 \cdot 4\sqrt{3} = 8\sqrt{3}$

13. hypotenuse = 2 • shorter leg

$15 = 2y$

$\frac{15}{2} = y$

longer leg = shorter leg • $\sqrt{3}$

$x = \frac{15}{2} \cdot \sqrt{3}$

$x = \frac{15\sqrt{3}}{2}$

14. hypotenuse = leg • $\sqrt{2}$

$\sqrt{6} = m\sqrt{2}$

$\frac{\sqrt{6}}{\sqrt{2}} = m$

$\sqrt{3} = m$

$n = m = \sqrt{3}$

15. hypotenuse $=$ 2 • shorter leg

$24 = 2p$

$12 = p$

longer leg $=$ shorter leg • $\sqrt{3}$

$q = p \cdot \sqrt{3}$

$q = 12\sqrt{3}$

16. The altitude s forms the longer leg of two 30°-60°-90° triangles.

hypotenuse $=$ 2 • shorter leg

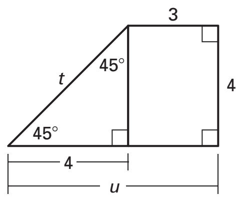
$r = 2\left(\frac{18}{2}\right)$

$r = 18$

longer leg $=$ shorter leg • $\sqrt{3}$

$s = 9\sqrt{3}$

17.



The triangle formed is a 45°-45°-90° triangle, so each leg has a length of 4.

$u = 4 + 3 = 7$

hypotenuse $=$ leg • $\sqrt{2}$

$t = 4\sqrt{2}$

18. The upper triangle is a 30°-60°-90° triangle.

longer leg $=$ shorter leg • $\sqrt{3}$

$9\sqrt{3} = e\sqrt{3}$

$9 = e$

hypotenuse $=$ 2 • shorter leg

$f = 2(9)$

$f = 18$

The lower triangle is a 45°-45°-90° triangle.

hypotenuse $=$ leg • $\sqrt{2}$

$f = g\sqrt{2}$

$18 = g\sqrt{2}$

$\frac{18}{\sqrt{2}} = g$

$\frac{18}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = g$

$9\sqrt{2} = g$

19. C; $\frac{5}{2}, \frac{5\sqrt{3}}{2}, 10$

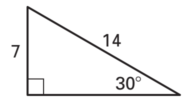
hypotenuse $\stackrel{?}{=}$ 2 • shorter leg

$10 \stackrel{?}{=} 2 \cdot \frac{5}{2}$

$10 \neq 5$

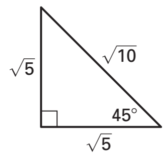
20. The formula for the longer leg was used instead of the hypotenuse formula. The correct solution is:

hypotenuse $=$ 2 • shorter leg $= 2 \cdot 7 = 14$



21. The length of the hypotenuse was incorrectly calculated as leg • leg • $\sqrt{2} = \sqrt{5} \cdot \sqrt{5} \cdot \sqrt{2} = 5\sqrt{2}$. The correct solution is:

hypotenuse $=$ leg • $\sqrt{2} = \sqrt{5} \cdot \sqrt{2} = \sqrt{10}$



22. Abigail's method does work.

longer leg $=$ shorter leg • $\sqrt{3}$

$9 = x\sqrt{3}$

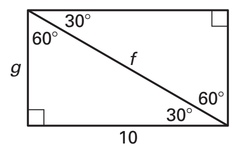
$9\sqrt{3} = x\sqrt{3} \cdot \sqrt{3}$

$9\sqrt{3} = x \cdot 3$

$3\sqrt{3} = x$

Her method is algebraically correct, so the equation simplifies to the same answer found in Example 5.

23.



longer leg $=$ shorter leg • $\sqrt{3}$

$10 = g\sqrt{3}$

$\frac{10}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = g$

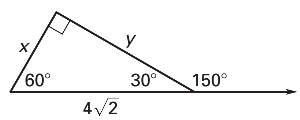
$\frac{10\sqrt{3}}{3} = g$

hypotenuse $=$ 2 • shorter leg

$f = 2\left(\frac{10\sqrt{3}}{3}\right)$

$f = \frac{20\sqrt{3}}{3}$

24.



hypotenuse $=$ 2 • shorter leg

$4\sqrt{2} = 2x$

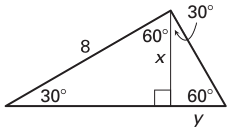
$2\sqrt{2} = x$

longer leg $=$ shorter leg • $\sqrt{3}$

$y = x \cdot \sqrt{3}$

$y = 2\sqrt{2} \cdot \sqrt{3}$

$y = 2\sqrt{6}$

25. 

First triangle:

hypotenuse $= 2 \cdot$ shorter leg

$$8 = 2x$$
$$4 = x$$

Second triangle:

longer leg $=$ shorter leg $\cdot \sqrt{3}$

$$x = y\sqrt{3}$$
$$4 = y\sqrt{3}$$
$$\frac{4}{\sqrt{3}} = y$$
$$\frac{4\sqrt{3}}{3} = y$$

26. $CB = \sqrt{(3-(-3))^2 + (-1-(-1))^2}$

$$= \sqrt{6^2 + 0^2} = \sqrt{36} = 6$$

hypotenuse $= 2 \cdot$ shorter leg

$$CB = 2AB$$
$$6 = 2AB$$
$$3 = AB$$

longer leg $=$ shorter leg $\cdot \sqrt{3}$

$$AC = AB \cdot \sqrt{3}$$
$$AC = 3\sqrt{3}$$

Let (x, y) represent the coordinates of A.

$$AB = \sqrt{(x-3)^2 + (y-(-1))^2}$$
$$3 = \sqrt{(x-3)^2 + (y+1)^2}$$
$$9 = (x-3)^2 + (y+1)^2$$
$$9 - (x-3)^2 = (y+1)^2$$

$$AC = \sqrt{(x-(-3))^2 + (y-(-1))^2}$$
$$3\sqrt{3} = \sqrt{(x+3)^2 + (y+1)^2}$$
$$27 = (x+3)^2 + (y+1)^2$$
$$27 - (x+3)^2 = (y+1)^2$$

$$9 - (x-3)^2 = 27 - (x+3)^2$$
$$9 - x^2 + 6x - 9 = 27 - x^2 - 6x - 9$$
$$12x = 18$$
$$x = \frac{3}{2}$$

$$9 - \left(\frac{3}{2} - 3\right)^2 = (y+1)^2$$
$$9 - \left(-\frac{3}{2}\right)^2 = y^2 + 2y + 1$$
$$9 - \frac{9}{4} = y^2 + 2y + 1$$
$$0 = y^2 + 2y - \frac{23}{4}$$
$$0 = 4y^2 + 8y - 23$$

$$y = \frac{-8 \pm \sqrt{8^2 - 4(4)(-23)}}{2(4)}$$
$$= \frac{-8 \pm \sqrt{432}}{8}$$
$$= -1 \pm \frac{3\sqrt{3}}{2}$$

Point A lies in the first quadrant, so its y-coordinate is positive.

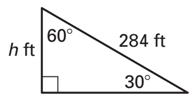
$$A\left(\frac{3}{2}, -1 + \frac{3\sqrt{3}}{2}\right)$$

Problem Solving

27. hypotenuse $= 2 \cdot$ shorter leg

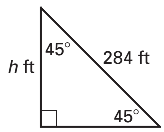
$$11 = 2h$$
$$\frac{11}{2} = h$$
$$5.5 = h$$

The height of the ramp is 5 feet 6 inches.

28. 

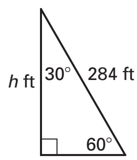
hypotenuse $= 2 \cdot$ shorter leg

$$284 = 2 \cdot h$$
$$142 = h$$



hypotenuse $=$ leg $\cdot \sqrt{2}$

$$284 = h \cdot \sqrt{2}$$
$$\frac{284}{\sqrt{2}} = h$$
$$142\sqrt{2} = h$$



hypotenuse $= 2 \cdot$ shorter leg

$$284 = 2 \cdot s$$
$$142 = s$$

longer leg $=$ shorter leg $\cdot \sqrt{3}$

$$h = 142\sqrt{3}$$
$$h \approx 246$$

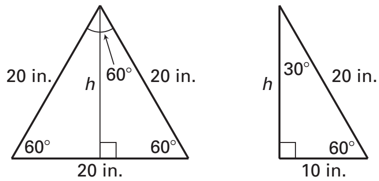
When the angle is 30°, the seagull rises 142 feet; when the angle is 45°, the seagull rises about 200 feet 10 inches; when the angle is 60°, the seagull rises about 246 feet.

29. You could show that all isosceles right triangles are similar to each other by showing that the corresponding angles are congruent. They are all 45°-45°-90° triangles.

You could also show that the corresponding side lengths are always proportional, because in an isosceles right triangle, the side lengths are always x, x, and $x\sqrt{2}$. For example, let 1, 1, $\sqrt{2}$, and 2, 2, $2\sqrt{2}$, be the side lengths of two isosceles right triangles. The ratios of corresponding side lengths are $\frac{2}{1} = 2$, $\frac{2}{1} = 2$, and $\frac{2\sqrt{2}}{\sqrt{2}} = 2$.

30. Because $\triangle DEF$ is a 45°-45°-90° triangle, $DF = FE$. By the Pythagorean Theorem, $DF^2 + FE^2 = DE^2$. This equation can be written as $2DF^2 = DE^2$ or $2FE^2 = DE^2$. By the Property of Squares, $\sqrt{2}DF = DE$ and $\sqrt{2}FE = DE$. So, the hypotenuse is equal to $\sqrt{2}$ times the length of one leg of the 45°-45°-90° triangle.

31.



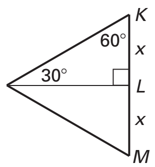
The height h divides the equilateral triangle into two 30°-60°-90° triangles.

longer leg = shorter leg • $\sqrt{3}$

$h = 10\sqrt{3} \approx 17.3$

The height of the equilateral triangle is about 17.3 inches.

32.

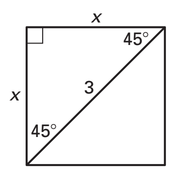


Construct $\triangle JML$ congruent to $\triangle JKL$. Because they are congruent, $m\angle KJL = m\angle LJM = 30°$, so $m\angle KJM = m\angle KJL + m\angle LJM = 30° + 30° = 60°$.

Because $\triangle JML \cong \triangle JKL$, $m\angle M = m\angle K = 60°$. So, all three angles of $\triangle JKM$ measure 60°, and the triangle is equilateral. So, all of its side lengths are $x + x = 2x$. It is given that the shorter leg of $\triangle JKL$ is x. So, $\triangle JKL$'s hypotenuse is two times the length of its shorter side. By the Pythagorean Theorem, $JL^2 + x^2 = (2x)^2$, and $JL^2 = 4x^2 - x^2 = 3x^2$. This simplifies to $JL = \sqrt{3}x$, showing that the longer leg is equal to $\sqrt{3}$ times the length of the shorter leg.

33. a. The large orange triangles are 45°-45°-90° triangles, because they are isosceles right triangles. The smaller blue triangles are also 45°-45°-90° triangles, because the 45° angles of the orange triangle are complementary with the acute angles of the blue triangle.

b.



hypotenuse = leg • $\sqrt{2}$

$3 = x \cdot \sqrt{2}$

$\frac{3}{\sqrt{2}} = x$

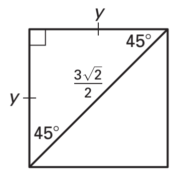
$\frac{3}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = x$

$\frac{3\sqrt{2}}{2} = x$

$2.12 \approx x$

The square of fabric for the large orange triangles should be about 2.12 inches by 2.12 inches.

c.



hypotenuse = leg • $\sqrt{2}$

$\frac{3\sqrt{2}}{2} = y \cdot \sqrt{2}$

$\frac{3}{2} = y$

$1.5 = y$

The square of fabric for the small blue triangles should be 1.5 inches by 1.5 inches.

34. a. The first triangle is a 45°-45°-90° triangle, because the legs are both 1.

hypotenuse = leg • $\sqrt{2}$

$r = 1 \cdot \sqrt{2}$

$r = \sqrt{2}$

The second triangle is a right triangle with legs 1 and $r = \sqrt{2}$. Using the Pythagorean Theorem,

$s^2 = 1^2 + r^2$

$s^2 = 1 + (\sqrt{2})^2$

$s^2 = 1 + 2$

$s^2 = 3$

$s = \sqrt{3}$.

This process can be duplicated to find the unknown lengths of the remaining right triangles.

$t^2 = 1^2 + s^2$ | $u^2 = 1^2 + t^2$

$t^2 = 1 + (\sqrt{3})^2$ | $u^2 = 1 + 2^2$

$t^2 = 1 + 3$ | $u^2 = 5$

$t^2 = 4$ | $u = \sqrt{5}$

$t = 2$

$v^2 = 1^2 + u^2$ | $w^2 = 1^2 + v^2$

$v^2 = 1 + (\sqrt{5})^2$ | $w^2 = 1^2 + (\sqrt{6})^2$

$v^2 = 1 + 5$ | $w^2 = 1 + 6$

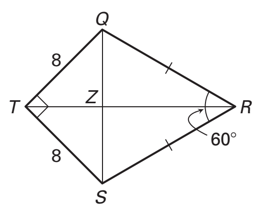
$v^2 = 6$ | $w^2 = 7$

$v = \sqrt{6}$ | $w = \sqrt{7}$

b. The only triangle that is a 45°-45°-90° triangle is the first one, because it is the only one whose legs are equal in length.

c. The third triangle, with sides 1, $\sqrt{3}$, 2, is the only 30°-60°-90° triangle. It is the only one whose side lengths satisfy the 30°-60°-90° Triangle Theorem.

35. a.

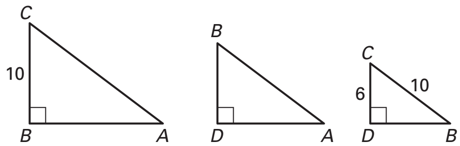


b. $QR \cong SR$, $QT \cong ST$, and TR is shared by the two triangles so $\triangle RQT \cong \triangle RST$ by the Side-Side-Side Congruence Postulate.

c. RT is longer than QS. ZR is the longer leg of a 30°-60°-90° triangle, so $ZR > ZS$. From part (b), $\triangle RQT \cong \triangle RST$, so $\angle QTR \cong \angle STR$. Because $m\angle QTS = 90°$, $m\angle QTR = m\angle STR = 45°$. Because $\triangle QTS$ is an isosceles right triangle, its base angles measure 45°. So, $\triangle QZT$ and $\triangle TZS$ are congruent right isosceles triangles, and $QZ = TZ = ZS$. So, $ZR + TZ > ZS + QZ$, and $RT > QS$.

Quiz for the lessons "Use Similar Right Triangles" and "Special Right Triangles"

1.



$\frac{AC}{BC} = \frac{BC}{CD}$

BC is the geometric mean of AC and CD.

2. $BC^2 = CD^2 + BD^2$

$10^2 = 6^2 + BD^2$

$100 = 36 + BD^2$

$64 = BD^2$

$8 = BD$

$\frac{AD}{BD} = \frac{BD}{CD}$

$\frac{AD}{8} = \frac{8}{6}$

$6 \cdot AD = 8 \cdot 8$

$AD = \frac{64}{6}$

$AD = \frac{32}{3} \approx 10.67$

$\frac{AB}{BD} = \frac{BC}{CD}$

$\frac{AB}{8} = \frac{10}{6}$

$6 \cdot AB = 8 \cdot 10$

$6 \cdot AB = 80$

$AB = \frac{80}{6} = \frac{40}{3} \approx 13.3$

3. hypotenuse $=$ leg $\cdot \sqrt{2}$

$x = 8\sqrt{2}$

4. hypotenuse $= \sqrt{2}$

$10 = y \cdot \sqrt{2}$

$\frac{10}{\sqrt{2}} = y$

$\frac{10\sqrt{2}}{2} = y$

$5\sqrt{2} = y$

5. longer leg $=$ shorter leg $\cdot \sqrt{3}$

$3\sqrt{2} = a\sqrt{3}$

$\frac{3\sqrt{2}}{\sqrt{3}} = a$

$\frac{3\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = a$

$\sqrt{6} = a$

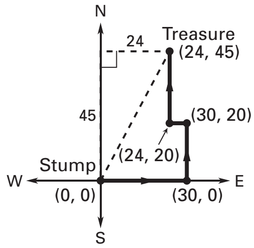
hypotenuse $= 2 \cdot$ shorter leg

$b = 2a$

$b = 2\sqrt{6}$

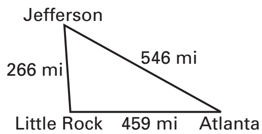
Mixed Review of Problem Solving for the lessons "Apply the Pythagorean Theorem", "Use the Converse of the Pythagorean Theorem", "Use Similar Right Triangles", and "Special Right Triangles"

1.



If the stump is at point (0, 0), the hidden treasure is at point (24, 45). The distance to the treasure from the stump is $\sqrt{45^2 + 24^2} = \sqrt{2601} = 51$ paces.

2. a.



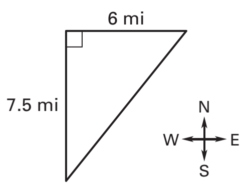
b. $546^2 \underline{\ ?\ } 266^2 + 459^2$

$298{,}116 \underline{\ ?\ } 70{,}756 + 210{,}681$

$298{,}116 > 281{,}437$

The pushpins do not form a right triangle. They form an obtuse scalene triangle, because none of the side lengths are equal, and the square of the longest side is greater than the sum of the squares of the other two sides.

3.



Bob's distance $= (4\text{mi/h})(1.5\text{ h}) = 6$ mi

John's distance $= (5\text{ mi/h})(1.5\text{ h}) = 7.5$ mi

Because Bob ran east and John ran south, their paths are legs of a right triangle. The distance between them at 11:30 A.M. was $\sqrt{6^2 + 7.5^2} = \sqrt{92.25} \approx 9.6$ miles.

4. $8 + 6 > x$ $\quad$ $6 + x > 8$ $\quad$ $8 + x > 6$

$14 > x$ $\quad$ $x > 2$ $\quad$ $x > -2$

So, the figure is a triangle when $2 < x < 14$.

a. $x^2 = 6^2 + 8^2$

$x^2 = 36 + 64$

$x^2 = 100$

$x = 10$

Using the Pythagorean Theorem, the triangle is a right triangle when $x = 10$.

b. $x^2 > 6^2 + 8^2$

$x^2 > 36 + 64$

$x^2 > 100$

$x > 10$

When $x > 10$, $\angle 1$ can be obtuse, because the square of the length of the longest side is greater than the sum of the squares of the lengths of the other two sides. But to satisfy the Triangle Inequality Theorem, $2 < x < 14$. So, $\angle 1$ is obtuse when $10 < x < 14$.

c. $x^2 < 6^2 + 8^2$

$x^2 < 36 + 64$

$x^2 < 100$

$x < 10$

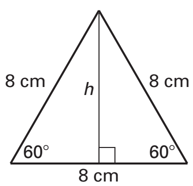
When $x < 10$, $\angle 1$ can be acute, because the square of the length of the longest side of the triangle is less than the sum of the squares of the lengths of the other two sides. But to satisfy the Triangle Inequality Theorem, $2 < x < 14$. So, $\angle 1$ is acute when $2 < x < 10$.

d. The triangle is isosceles when $x = 6$ or $x = 8$, because in either case, two of the sides of the triangle are congruent.

e. No triangle is possible when $x \le 2$ or $x \ge 14$, because the values would not satisfy the Triangle Inequality Theorem.

5. a. The purple triangle has ten marble holes. It is an equilateral triangle, because each side of the triangle has four equally-spaced marble holes.

b.



The height h forms the longer leg of two 30°-60°-90° triangles.

longer leg = shorter leg $\cdot \sqrt{3}$

$$h = \left(\frac{8}{2}\right)\sqrt{3} = 4\sqrt{3}$$

$$A = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(8)\left(4\sqrt{3}\right) = 16\sqrt{3} \approx 27.7$$

The area of the purple triangle is about 27.7 square centimeters.

c. There are 61 marble holes in the center hexagon. The purple triangle has about $\frac{1}{6}$ the area of the center hexagon.

d. $$\frac{\text{area of center hexagon}}{\text{area of purple triangle}} = \frac{\text{\# marble holes in hexagon}}{\text{\# marble holes in triangle}}$$

$$\frac{A}{27.7} = \frac{61}{10}$$

$$10A = 61(27.7)$$

$$A = \frac{1689.7}{10} = 168.97$$

Using marble holes as an estimate, the area of the hexagon is about 169 square centimeters.

6. a. $x^2 = 3^2 + 2^2$

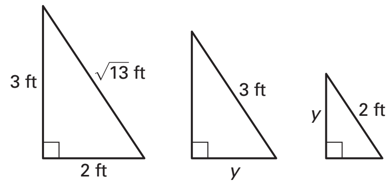
$x^2 = 9 + 4$

$x^2 = 13$

$x = \sqrt{13} \approx 3.6$

The length of the plywood is about 3 feet 7 inches.

b.



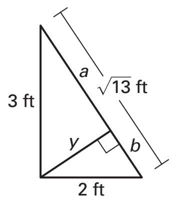
$$\frac{y}{3} = \frac{2}{\sqrt{13}}$$

$$y\sqrt{13} = 2 \cdot 3$$

$$y = \frac{6}{\sqrt{13}} = \frac{6\sqrt{13}}{13} \approx 1.66$$

The length of the support is about 1 foot 8 inches.

c.



Using the Geometric Mean (Leg) Theorem:

$$\frac{\sqrt{13}}{2} = \frac{2}{a}$$

$$\sqrt{13}a = 4$$

$$a = \frac{4}{\sqrt{13}} = \frac{4\sqrt{13}}{13} \approx 1.1$$

So, the support attaches about 1.1 feet up from the bottom of the plywood.

Lesson 7.5 Apply the Tangent Ratio

Activity for the lesson "Apply the Tangent Ratio"

Step 1

Check student's work.

Step 2

Sample answer:

Triangle	Adjacent Leg	Opposite Leg	$\frac{\text{Opposite Leg}}{\text{Adjacent Leg}}$
$\triangle ABC$	5 cm	2.9 cm	0.58
$\triangle ADE$	10 cm	5.8 cm	0.58
$\triangle AFG$	15 cm	8.7 cm	0.58

Step 3

The proportions $\frac{BC}{DE} = \frac{AC}{AE}$ and $\frac{BC}{AC} = \frac{DE}{AE}$ are true because $\triangle ABC \sim \triangle ADE$. The two triangles are similar because their corresponding angles are congruent.

Step 4

Sample answer: The ratio of the lengths of the legs in a right triangle is constant for a given angle measure; answers will vary.

Guided Practice for the lesson "Apply the Tangent Ratio"

1. $\tan J = \frac{\text{opp. } \angle J}{\text{adj. to } \angle J} = \frac{24}{32} = \frac{3}{4} = 0.75$

$\tan K = \frac{\text{opp. } \angle K}{\text{adj. to } \angle K} = \frac{32}{24} = \frac{4}{3} \approx 1.3333$

2. $\tan J = \frac{\text{opp. } \angle J}{\text{adj. to } \angle J} = \frac{8}{15} \approx 0.5333$

$\tan K = \frac{\text{opp. } \angle K}{\text{adj. to } \angle K} = \frac{15}{8} = 1.875$

3. $\tan 61° = \frac{\text{opp.}}{\text{adj.}}$

$\tan 61° = \frac{22}{x}$

$x \cdot \tan 61° = 22$

$x = \frac{22}{\tan 61°}$

$x \approx \frac{22}{1.8040} \approx 12.2$

4. $\tan 56° = \frac{\text{opp.}}{\text{adj.}}$

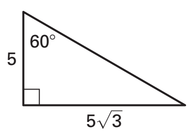
$\tan 56° = \frac{x}{13}$

$13 \cdot \tan 56° = x$

$13(1.4826) \approx x$

$19.3 \approx x$

5.



longer leg $=$ shorter leg $\cdot \sqrt{3}$

$x = 5\sqrt{3}$

$\tan 60° = \frac{\text{opp.}}{\text{adj.}} = \frac{5\sqrt{3}}{5} = \sqrt{3}$

Exercises for the lesson "Apply the Tangent Ratio"

Skill Practice

1. The tangent ratio compares the length of *the leg opposite the angle* to the length of *the leg adjacent to the angle.*

2. All right triangles with an acute angle measuring $n°$ will have the same ratio of the leg opposite the angle to the leg adjacent to the angle because tangent $n°$ is a constant. So, the triangles must be similar.

3. $\tan A = \frac{24}{7} \approx 3.4286$

$\tan B = \frac{7}{24} \approx 0.2917$

4. $\tan A = \frac{35}{12} \approx 2.9167$

$\tan B = \frac{12}{35} \approx 0.3429$

5. $\tan A = \frac{48}{20} = \frac{12}{5} = 2.4$

$\tan B = \frac{20}{48} = \frac{5}{12} \approx 0.4167$

6. $\tan 41° = \frac{12}{x}$

$x \cdot \tan 41° = 12$

$x = \frac{12}{\tan 41°}$

$x \approx \frac{12}{0.8693} \approx 13.8$

7. $\tan 27° = \frac{x}{15}$

$15 \cdot \tan 27° \approx x$

$15 \cdot 0.5095 \approx x$

$7.6 \approx x$

8. $\tan 58° = \frac{22}{x}$

$x \cdot \tan 58° = 22$

$x = \frac{22}{\tan 58°}$

$x \approx \frac{22}{1.6003} \approx 13.7$

9. $\tan 45° = \frac{x}{6}$

$t \cdot \tan 45° = x$

$6 \cdot 1 = x$

$6 = x$

Using the 45°-45°-90° Theorem:

hypotenuse $=$ leg $\cdot \sqrt{2}$

$6\sqrt{2} = x \cdot \sqrt{2}$

$6 = x$

The results are the same using either method.

10. $\tan 30° = \frac{x}{10\sqrt{3}}$

$10\sqrt{3} \cdot \tan 30° = x$

$10\sqrt{3} \cdot 0.5774 \approx x$

$10 \approx x$

Using the 30°-60°-90° Theorem:

longer leg $=$ shorter leg $\cdot \sqrt{3}$

$10\sqrt{3} = x\sqrt{3}$

$10 = x$

The results are the same using either method.

11. $\tan 60° = \frac{x}{4}$

$4 \cdot \tan 60° = x$

$4 \cdot 1.7321 \approx x$

$6.9 \approx x$

Using the 30°-60°-90° Theorem:

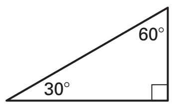
longer leg $=$ shorter leg $\cdot \sqrt{3}$

$x = 4\sqrt{3}$

$x \approx 6.9$

The results are the same using either method.

12. $\tan 30° = \dfrac{\text{opp.}}{\text{adj.}}$

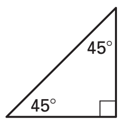


$\tan 30° = \dfrac{\cancel{\text{shorter leg}}}{\cancel{\text{longer leg}}}$

$\tan 30° = \dfrac{\text{shorter leg}}{\text{shorter leg} \cdot \sqrt{3}}$

$= \dfrac{1}{\sqrt{3}} = \dfrac{\sqrt{3}}{3} \approx 0.5774$

$\tan 45° = \dfrac{\text{opp.}}{\text{adj.}}$



$\tan 45° = \dfrac{\text{leg}}{\text{leg}} = 1$

13. The tangent was incorrectly written as the ratio of the adjacent leg, 18, to the hypotenuse, 82. The tangent of D is the ratio of the length of the opposite side to the length of the adjacent side. It should be $\tan D = \dfrac{80}{18} = \dfrac{40}{9}$.

14. The error is that the triangle is not a right triangle. If it were, the other two angles would be complementary, but they are not. So, using the tangent ratio is not possible.

15. In order to use the tangent ratio, you must know that the triangle is a right triangle, and you must know either the measurement of the acute angle and one of the legs, or the lengths of the two legs.

16. C;

$\tan 40° = \dfrac{20}{x}$

$x \cdot \tan 40° = 20$

$x = \dfrac{20}{\tan 40°}$

17. C;

$\tan 32° = \dfrac{x}{12}$

$12 \cdot \tan 32° = x$

$12 \cdot 0.6249 \approx x$

$7.5 \approx x$

18. $\tan 25° = \dfrac{8}{x}$

$x \cdot \tan 25° = 8$

$x = \dfrac{8}{\tan 25°} = \dfrac{8}{0.4663} \approx 17.2$

The other acute angle is $90° - 25° = 65°$.

$\tan 65° = \dfrac{x}{8}$

$\tan 65° \stackrel{?}{=} \dfrac{17.2}{8}$

$2.1445 \approx 2.15$ ✓

19. $\tan 40° = \dfrac{13}{x}$

$x \cdot \tan 40° = 13$

$x = \dfrac{13}{\tan 40°} \approx \dfrac{13}{0.8391} \approx 15.5$

The other acute angle is $90° - 40° = 50°$.

$\tan 50° = \dfrac{x}{13}$

$\tan 50° \stackrel{?}{=} \dfrac{15.5}{13}$

$1.1923 \approx 1.19$ ✓

20. $\tan 65° = \dfrac{x}{9}$

$9 \cdot \tan 65° = x$

$9 \cdot 2.1445 \approx x$

$19.3 \approx x$

The other acute angle is $90° - 65° = 25°$.

$\tan 25° = \dfrac{a}{x}$

$\tan 25° \stackrel{?}{=} \dfrac{a}{19.3}$

$0.4663 = 0.466$ ✓

21. $\tan 38° = \dfrac{11}{x}$

$x \cdot \tan 38° = 11$

$x = \dfrac{11}{\tan 38°}$

$\approx \dfrac{11}{0.7813} \approx 14.1$

Area $= \dfrac{1}{2}$(base)(height)

$= \dfrac{1}{2}(11)(14.1)$

≈ 77.6

22. $\tan 55° = \dfrac{16}{x}$

$x \cdot \tan 55° = 16$

$x = \dfrac{16}{\tan 55°}$

$\approx \dfrac{16}{1.4281} \approx 11.2$

Area $= \dfrac{1}{2}$(base)(height)

$\approx \dfrac{1}{2}(11.2)(16)$

≈ 89.6

23. $\tan 22° = \frac{7}{x}$

$x \cdot \tan 22° = 7$

$x = \frac{7}{\tan 22°}$

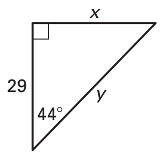
$\approx \frac{7}{0.4040} \approx 17.3$

Area $= \frac{1}{2}$(base)(height)

$= \frac{1}{2}(17.3)(7)$

≈ 60.6

24. $\tan 44° = \frac{x}{29}$



$29 \cdot \tan 44° = x$

$29 \cdot (0.9657) \approx x$

$28 \approx x$

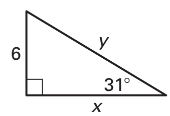
$y^2 \approx 28^2 + 29^2$

$y^2 \approx 1625$

$y \approx 40.3$

Perimeter $\approx 28 + 29 + 40.3 = 97.3$

25. $\tan 31° = \frac{6}{x}$



$x \cdot \tan 31° = 6$

$x = \frac{6}{\tan 31°}$

$\approx \frac{6}{0.6009} \approx 10$

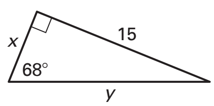
$y^2 \approx 6^2 + 10^2$

$y^2 \approx 136$

$y \approx 11.7$

Perimeter $\approx 6 + 10 + 11.7 = 27.7$

26. $\tan 68° = \frac{15}{x}$



$x \cdot \tan 68° = 15$

$x = \frac{5}{\tan 68°}$

$\approx \frac{15}{2.4751} \approx 6.1$

$y^2 \approx 15^2 + 6.1^2$

$y^2 \approx 262.21$

$y \approx 16.2$

Perimeter $\approx 15 + 6.1 + 16.2 = 37.3$

27. $y = \frac{1}{2} \cdot 120 = 60$

$\tan 42° = \frac{z}{60}$

$60 \cdot \tan 42° = z$

$60 \cdot 0.9004 \approx z$

$54.0 \approx z$

28. The first triangle is a 30°-60°-90° triangle.

hypotenuse $= 2 \cdot$ shorter leg

$150 = 2y$

$75 = y$

$\tan 40° = \frac{y}{z}$

$\tan 40° = \frac{75}{z}$

$z \cdot \tan 40° = 75$

$z = \frac{75}{\tan 40°} \approx \frac{15}{0.8391} \approx 89.4$

29. $\tan 45° = \frac{y}{82}$

$82 \cdot \tan 45° = y$

$82 = y$

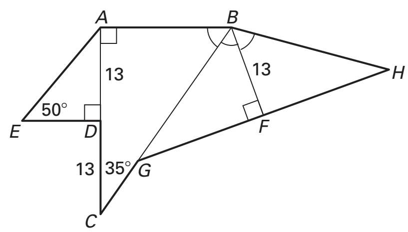
$\tan 28° = \frac{y}{z}$

$\tan 28° = \frac{82}{z}$

$z \cdot \tan 28° = 82$

$z = \frac{82}{\tan 28°} \approx \frac{82}{0.5317} \approx 154.2$

30.



$\tan 35° = \frac{AB}{26}$

$26 \cdot \tan 35° = AB$

$18.2 \approx AB$

$\tan 50° = \frac{13}{ED}$

$ED \cdot \tan 50° = 13$

$ED = \frac{13}{\tan 50°} \approx 10.9$

$AE^2 = ED^2 + AD^2$

$AE^2 \approx (10.9)^2 + 13^2$

$AE^2 \approx 287.81$

$AE \approx 17.0$

$BC^2 = AC^2 + AB^2$

$BC^2 \approx 26^2 + 18.2^2$

$BC^2 \approx 1007.24$

$BC \approx 31.7$

$m\angle GBF = m\angle ABG = 90° - 35° = 55°$

$\tan 55° = \frac{FG}{13}$

$13 \cdot \tan 55° = FG$

$18.6 \approx FG$

$BG^2 = BF^2 + FG^2$

$BG^2 \approx 13^2 + 18.6^2$

$BG^2 \approx 514.96$

$BG \approx 22.7$

$CG = BC - BG \approx 31.7 - 22.7 \approx 9$

By the ASA Congruence Postulate, $\triangle BFG \cong \triangle BFH$. So, $FH = FG \approx 18.6$.

Because $\triangle BFG \cong \triangle BFH$, $BH = BG \approx 22.7$.

$$\begin{aligned}\text{Perimeter} &= AB + AE + ED + DC \\ &\quad + CG + GF + FH + BH \\ &\approx 18.2 + 17 + 10.9 + 13 \\ &\quad + 9 + 18.6 + 18.6 + 22.7 \\ &\approx 128\end{aligned}$$

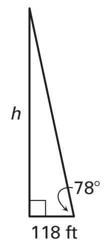
Problem Solving

31. $\tan 78° = \frac{h}{118}$

$118 \cdot \tan 78° = h$

$555 \approx h$

The height of the Washington Monument is about 555 feet.



32. $\tan 52° = \frac{h}{121}$

$121 \cdot \tan 52° \approx h$

$155 \approx h$

The height of the roller coaster is about 155 feet.

33. $\tan 50° = \frac{x}{14}$

$14 \cdot \tan 50° = x$

$16.7 \approx x$

The distance from one end of the class to the other is about $16.7 \cdot 2 = 33.4$ feet.

34. a. $\tan 50° = \frac{x}{14}$

$14 \tan 50° = x$

$16.7 \approx x$

The last student is about 16.7 feet from the center of the row.

b. $\tan 60° = \frac{x}{14}$

$14 \cdot \tan 60° = x$

$24.2 \approx x$

The end of the camera range is about 24.2 feet from the center of the row.

c. The length of the empty space is about $24.2 - 16.7 = 7.5$ feet.

d. Number of students = Length of empty space ÷ Space needed per student

$= 7.5 \text{ feet} \div 2 \text{ feet}$

$= 3.75$

Because each student needs 2 feet of space, no more than 3 students can fit into the extra $7\frac{1}{2}$ feet of space.

35. $\tan A = \frac{a}{b}$; $\tan B = \frac{b}{a}$

The tangent of one of the acute angles is the reciprocal of the tangent of the other acute angle.

$\angle A$ and $\angle B$ are complementary angles.

36. $\tan 1° = \frac{h}{20}$

$20 \cdot \tan 1° = h$

$0.3491 \approx h$

$(0.3491 \text{ ft})\left(\frac{12 \text{ in.}}{1 \text{ ft}}\right) \approx 4 \text{ in.}$

The height of the "E" is about 4 inches.

37. a. Maximum height: 30 in. = 2.5 ft

$\tan 5° = \frac{2.5}{\ell}$

$\ell \cdot \tan 5° = 2.5$

$\ell = \frac{2.5}{\tan 5°} \approx 29$

The maximum horizontal length of one ramp is about 29 feet.

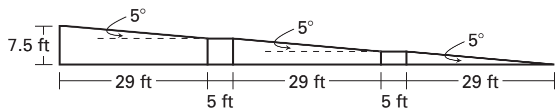
b. Total height = Number of ramps × 2.5 feet

$7.5 \text{ ft} = r \cdot 2.5 \text{ ft}$

$\frac{7.5}{2.5} = r$

$3 = r$

The least number of ramps needed is 3, with 2 landings.



c. Total length of base $= 29 + 5 + 29 + 5 + 29 = 97$

The total length of the base of the system of ramps and landings is about 97 feet.

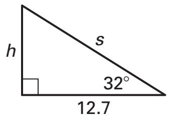
38. $C = 2\pi r$

$80 = 2\pi r$

$\frac{80}{2\pi} = r$

$12.7 \approx r$

The radius of the base of the cone is about 12.7 feet.



$$\tan 32° \approx \frac{h}{12.7}$$

$$12.7 \tan 32° \approx h$$

$$7.9 \approx h$$

The height of the cone is about 7.9 feet.

$$s^2 = h^2 + r^2$$

$$s^2 \approx 7.9^2 + 12.7^2$$

$$s^2 \approx 223.7$$

$$s \approx 15$$

The length s of the cone-shaped pile is about 15 feet.

Lesson 7.6 Apply the Sine and Cosine Ratios

Guided Practice for the lesson "Apply the Sine and Cosine Ratios"

1. $\sin X = \frac{\text{opp. } \angle X}{\text{hyp.}} = \frac{ZY}{XY} = \frac{8}{17} \approx 0.4706$

$\sin Y = \frac{\text{opp. } \angle Y}{\text{hyp.}} = \frac{XZ}{XY} = \frac{15}{17} \approx 0.8824$

2. $\sin X = \frac{\text{opp. } \angle X}{\text{hyp.}} = \frac{YZ}{XY} = \frac{15}{25} = \frac{3}{5} = 0.6$

$\sin Y = \frac{\text{opp. } \angle Y}{\text{hyp.}} = \frac{XZ}{XY} = \frac{20}{25} = \frac{4}{5} = 0.8$

3. $RS^2 = RT^2 + TS^2$

$RS^2 = 9^2 + 12^2$

$RS^2 = 225$

$RS = 15$

$\cos R = \frac{\text{adj. to } \angle R}{\text{hyp.}} = \frac{RT}{RS} = \frac{9}{15} = \frac{3}{5} = 0.6$

$\cos S = \frac{\text{adj. to } \angle S}{\text{hyp.}} = \frac{TS}{RS} = \frac{12}{15} = \frac{4}{5} = 0.8$

4. $SR^2 = ST^2 + TR^2$

$SR^2 = 16^2 + 30^2$

$SR^2 = 1156$

$SR = 34$

$\cos R = \frac{\text{adj. to } \angle R}{\text{hyp.}} = \frac{TR}{SR} = \frac{30}{34} = \frac{15}{17} \approx 0.8824$

$\cos S = \frac{\text{adj. to } \angle S}{\text{hyp.}} = \frac{ST}{SR} = \frac{16}{34} = \frac{8}{17} \approx 0.4706$

5. $\cos 35° = \frac{\text{adj.}}{\text{hyp.}}$

$\cos 35° \approx \frac{\text{adj}}{19.2}$

$19.2 \bullet \cos 35° \approx \text{adj.}$

$19.2 \bullet 0.8192 \approx \text{adj.}$

$15.7 \approx \text{adj.}$

The length of the other leg of the triangle formed is about 15.7 feet.

6. $\sin 28° = \frac{\text{opp.}}{\text{hyp.}}$

$\sin 28° = \frac{1200}{x}$

$x \bullet \sin 28° = 1200$

$x = \frac{1200}{\sin 28°}$

$x \approx \frac{1200}{0.4695} \approx 2555.9$

You would ski about 2556 meters down the mountain.

7. $\sin 35° = \frac{\text{opp.}}{\text{hyp.}}$

$\sin 35° = \frac{x}{14}$

$14 \bullet \sin 35° = x$

$8.0 \approx x$

The height of the ramp is about 8 feet.

$\cos 35° = \frac{\text{adj.}}{\text{hyp.}}$

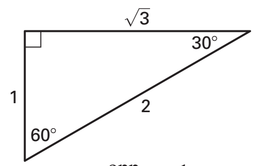
$\cos 35° = \frac{y}{14}$

$14 \bullet \cos 35° = y$

$11.5 \approx y$

The length of the base is about 11.5 feet.

8. Using the 30°-60°-90° Triangle Theorem, the side lengths are 1, $\sqrt{3}$, and 2.



$\sin 30° = \frac{\text{opp.}}{\text{hyp.}} = \frac{1}{2} = 0.5$

$\cos 30° = \frac{\text{adj}}{\text{hyp.}} = \frac{\sqrt{3}}{2} \approx 0.8660$

Exercises for the lesson "Apply the Sine and Cosine Ratios"

Skill Practice

1. The sine ratio compares the length of *the opposite side* to the length of *the hypotenuse.*

2. The side of a right triangle that is adjacent to an acute angle is the leg that is next to the angle. The hypotenuse is the longest side of the right triangle, and is the side opposite the right angle.

3. $\sin D = \frac{12}{15} = \frac{4}{5} = 0.8$

$\sin E = \frac{9}{15} = \frac{3}{5} = 0.6$

4. $\sin D = \frac{35}{37} \approx 0.9459$

$\sin E = \frac{12}{37} \approx 0.3243$

5. $\sin D = \frac{28}{53} \approx 0.5283$

$\sin E = \frac{45}{53} \approx 0.8491$

6. The student used the adjacent side over the hypotenuse rather than the opposite side over the hypotenuse. The correct statement for the sine of the angle is $\sin A = \frac{12}{13}$.

7. $\cos X = \frac{27}{45} = \frac{3}{5} = 0.6$

$\cos Y = \frac{36}{45} = \frac{4}{5} = 0.8$

8. $\cos X = \frac{15}{17} \approx 0.8824$

$\cos Y = \frac{8}{17} \approx 0.4706$

9. $\cos X = \frac{13}{26} = \frac{1}{2} = 0.5$

$\cos Y = \frac{13\sqrt{3}}{26} = \frac{\sqrt{3}}{2} \approx 0.8660$

10. $\sin 32° = \frac{x}{18}$

$18 \cdot \sin 32° = x$

$9.5 \approx x$

$\cos 32° = \frac{y}{18}$

$18 \cdot \cos 32° = y$

$15.3 \approx y$

11. $\cos 48° = \frac{10}{a}$

$a \cdot \cos 48° = 10$

$a = \frac{10}{\cos 48°} \approx 14.9$

$\sin 48° = \frac{b}{a}$

$a \cdot \sin 48° = b$

$(14.9) \cdot \sin 48° \approx b$

$11.1 \approx b$

12. $\sin 71° = \frac{5}{v}$

$v \cdot \sin 71° = 5$

$v = \frac{5}{\sin 71°} \approx 5.3$

$\cos 71° = \frac{w}{v}$

$v \cdot \cos 71° = w$

$(5.3) \cos 71° \approx w$

$1.7 \approx w$

13. $\cos 43° = \frac{r}{26}$

$26 \cos 43° = r$

$19.0 \approx r$

$\sin 43° = \frac{s}{26}$

$26 \cdot \sin 43° = s$

$17.7 \approx s$

14. $\sin 64° = \frac{p}{34}$

$34 \cdot \sin 64° = p$

$30.6 \approx p$

$\cos 64° = \frac{q}{34}$

$34 \cdot \cos 64° = q$

$14.9 \approx q$

15. $\sin 50° = \frac{8}{n}$

$n \cdot \sin 50° = 8$

$n = \frac{8}{\sin 50°} \approx 10.4$

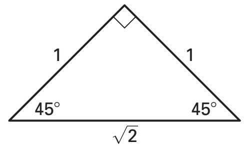
$\cos 50° = \frac{m}{n}$

$n \cdot \cos 50° = m$

$(10.4) \cos 50° \approx m$

$6.7 \approx m$

16. Using the 45°-45°-90° Triangle Theorem, the side lengths are 1, 1, and $\sqrt{2}$.



$\sin 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \approx 0.7071$

$\cos 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \approx 0.7071$

17. In order to use the sine or cosine ratio, you must know that the triangle is a right triangle, and you must know either the lengths of two of its sides, or the measurement of an acute angle and the length of the leg opposite the angle (for sine) or adjacent to the angle (for cosine), or the measurement of an acute angle and the length of the hypotenuse.

18. C;

$\sin 29° = \frac{10}{PQ}$

$PQ \cdot \sin 29° = 10$

$PQ = \frac{10}{\sin 29°}$

19. $\sin 42° = \frac{2}{x}$

$x \cdot \sin 42° = 2$

$x = \frac{2}{\sin 42°}$

$x \approx 3.0$

20. $\sin 53° = \frac{11}{x}$

$x \cdot \sin 53° = 11$

$x = \frac{11}{\sin 53°}$

$x \approx 13.8$

21. $\cos 39° = \frac{x}{26}$

$26 \cdot \cos 39° = x$

$20.2 \approx x$

22. $14^2 = (7\sqrt{3})^2 + ZY^2$

$196 = 147 + ZY^2$

$49 = ZY^2$

$7 = ZY$

$\sin X = \frac{7}{14} = \frac{1}{2} = 0.5$

$\cos X = \frac{7\sqrt{3}}{14} = \frac{\sqrt{3}}{2} \approx 0.8660$

23. $XY^2 = 4^2 + (8\sqrt{2})^2$

$XY^2 = 16 + 128$

$XY^2 = 144$

$XY = 12$

$\sin X = \frac{8\sqrt{2}}{12} = \frac{2\sqrt{2}}{3} \approx 0.9428$

$\cos X = \frac{4}{12} = \frac{1}{3} \approx 0.3333$

24. $XY^2 = 35^2 + 12^2$

$XY^2 = 1225 + 144$

$XY^2 = 1369$

$XY = 37$

$\sin X = \frac{12}{37} \approx 0.3243$

$\cos X = \frac{35}{37} \approx 0.9459$

25. $(3\sqrt{5})^2 = YZ^2 + 6^2$

$45 = YZ^2 + 36$

$9 = YZ^2$

$3 = YZ$

$\sin X = \frac{3}{3\sqrt{5}} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5} \approx 0.4472$

$\cos X = \frac{6}{3\sqrt{5}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5} \approx 0.8944$

26. $XY^2 = 16^2 + 30^2$

$XY^2 = 256 + 900$

$XY^2 = 1156$

$XY = 34$

$\sin X = \frac{16}{34} = \frac{8}{17} \approx 0.4706$

$\cos X = \frac{30}{34} = \frac{15}{17} \approx 0.8824$

27. $65^2 = 56^2 + XZ^2$

$4225 = 3136 + XZ^2$

$1089 = XZ^2$

$33 = XZ$

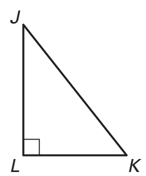
$\sin X = \frac{56}{65} \approx 0.8615$

$\cos X = \frac{33}{65} \approx 0.5077$

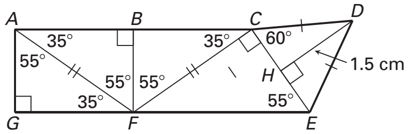
28. If you know two side lengths of a right triangle, you could find the sine, cosine, or tangent of an acute angle and use a calculator to find its angle measure.

29. D:

Because $0° < m\angle K < 90°$, $0 < \sin J < 1$. So, $\sin J = 1.1$ cannot be true.



30.



$\sin 60° = \frac{1.5}{CD}$

$CD \cdot \sin 60° = 1.5$

$CD = \frac{1.5}{\sin 60°} \approx 1.7$

$DE = CD \approx 1.7$

$\triangle CDE$ is an equilateral triangle because $m\angle DCE = m\angle CDE = m\angle DEC = 60°$. So, $CE = CD = DE \approx 1.7$.

$\cos 55° \approx \frac{1.7}{FE}$ $\qquad$ $\sin 55° \approx \frac{FC}{3}$

$FE \cdot \cos 55° \approx 1.7$ $\qquad$ $3 \cdot \sin 55° \approx FC$

$FE \approx \frac{1.7}{\cos 55°} \approx 3.0$ $\qquad$ $2.5 \approx FC$

$\sin 55° = \frac{BC}{2.5}$

$2.5 \sin 55° = BC$

$2.0 \approx BC$

Because $\triangle ABF \cong \triangle CBF \cong \triangle FGA$, $AB = GF = BC \approx 2$.

$\cos 55° = \frac{AG}{2.5}$

$2.5 \cdot \cos 55° = AG$

$1.4 \approx AG$

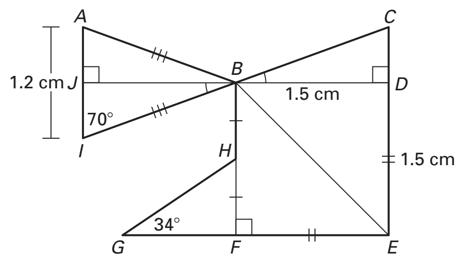
$P = AB + BC + CD + DE + EF + FG + AG$

$\approx 2 + 2 + 1.7 + 1.7 + 3 + 2 + 1.4$

≈ 13.8

The perimeter is about 14 centimeters.

31.



$\overline{JB}$ bisects $\overline{AI}$, so $\overline{AJ} = \overline{JI} = \frac{1}{2}AI = \frac{1}{2}(1.2) = 0.6$.

$\cos 70° = \frac{0.6}{IB}$

$IB \cdot \cos 70° = 0.6$

$IB = \frac{0.6}{\cos 70°} \approx 1.8$

$AB = IB \approx 1.8$

$m\angle CBD = m\angle IBJ = 20°$

$\cos 20° = \frac{1.5}{BC}$ $\qquad$ $\sin 20° \approx \frac{CD}{1.6}$

$BC \cdot \cos 20° = 1.5$ $\qquad$ $1.6 \cdot \sin 20° \approx CD$

$BC = \frac{1.5}{\cos 20°} \approx 1.6$ $\qquad$ $0.55 \approx CD$

From the diagram, $FE = DE = 1.5$. $\triangle FBE$ is an isosceles right triangle, congruent to $\triangle DEB$. So, $BF = FE = 1.5$. $\overline{BH} \cong \overline{FH}$, so $BH = FH = \frac{1}{2}(BF) = \frac{1}{2}(1.5) = 0.75$.

$\sin 34° = \frac{0.75}{GH}$ $\qquad$ $\cos 34° \approx \frac{GF}{1.3}$

$GH \cdot \sin 34° = 0.75$ $\qquad$ $1.3 \cdot \cos 34° = GF$

$$GH = \frac{0.75}{\sin 34°} \approx 1.3 \qquad\qquad 1.1 \approx GF$$

$$\begin{aligned} P &= AB + BC + CD + DE + EF \\ &\quad + FG + GH + HB + BI + IA \\ &\approx 1.8 + 1.6 + 0.55 + 1.5 + 1.5 \\ &\quad + 1.1 + 1.3 + 0.75 + 1.8 + 1.2 \\ &\approx 13.1 \end{aligned}$$

The perimeter is about 13 centimeters.

32. a.
$$\begin{aligned} \frac{\sin A}{\cos A} &= \frac{\frac{\text{opp. } \angle A}{\text{hyp.}}}{\frac{\text{adj. to } \angle A}{\text{hyp.}}} \\ &= \frac{\text{opp. } \angle A}{\text{hyp.}} \cdot \frac{\text{hyp.}}{\text{adj. to } \angle A} \\ &= \frac{\text{opp. } \angle A}{\text{adj. to } \angle A} = \tan A \end{aligned}$$

b.
$$\begin{aligned} (\sin A)^2 + (\cos A)^2 &= \left(\frac{\text{opp. } \angle A}{\text{hyp.}}\right)^2 + \left(\frac{\text{adj. to } \angle A}{\text{hyp.}}\right)^2 \\ &= \frac{(\text{opp. } \angle A)^2 + (\text{adj. } \angle A)^2}{\text{hyp.}^2} \\ &= \frac{\text{hyp.}^2}{\text{hyp.}^2} = 1 \end{aligned}$$

Problem Solving

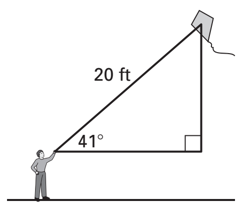
33.
$$\begin{aligned} \sin 31° &= \frac{19}{y} \\ y \cdot \sin 31° &= 19 \\ y &= \frac{19}{\sin 31°} \approx 36.9 \end{aligned}$$

The length of the ramp is about 36.9 feet.

34.
$$\begin{aligned} \cos 27° &= \frac{h}{18} \\ 18 \cdot \cos 27° &= h \\ 16 &\approx h \end{aligned}$$

The bleachers cover a horizontal distance of about 16 feet.

35. a.



b.
$$\begin{aligned} \sin 41° &= \frac{h}{20} \\ 20 \cdot \sin 41° &= h \\ 13.1 &\approx h \end{aligned}$$

The spool is 5 feet off the ground, so the height of the kite is $13.1 + 5 = 18.1$ feet. The higher the spool, the higher the kite will be. The lower the spool, the lower the kite will be.

36. a.
$$\begin{aligned} \sin 70° &= \frac{x}{38} \\ 38 \cdot \sin 70° &= x \\ 35.7 &\approx x \end{aligned}$$

The window is about 35.7 feet high.

b. Distance from top of bush to window $= 35.7 - 6 = 29.7$. Yes, the banner will fit above the bush.

c. The cosine ratio should be used because you are finding the side adjacent to the acute angle.

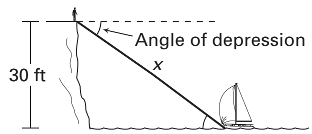
37. By the definition of the sine ratio, $\sin C = \frac{h}{a}$, so $h = a \sin C$. Substitute the expression $(a \sin C)$ for h in the area formula for a triangle.

Area $= \frac{1}{2}bh = \frac{1}{2}b(a \sin C) = \frac{1}{2}ab \sin C$

If $a = 4$, $b = 7$, and $m\angle C = 40°$, then the area is $\frac{1}{2}ab \sin C = \frac{1}{2}(4)(7)(\sin 40°) \approx 9$ square units.

38. Answers will vary; The sine is the same for each triangle because the measurement of the angle does not change.

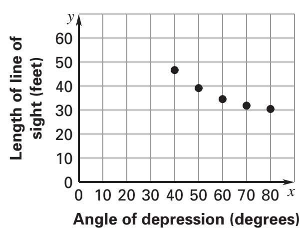
39. a.



b.

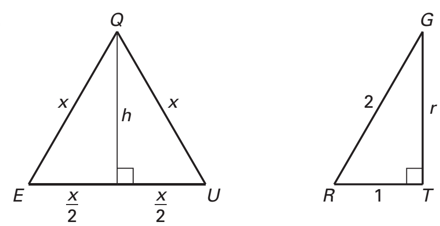
Angle of depression	Length of line of sight
40°	$\frac{30}{\sin 40°} \approx 46.7$ ft
50°	$\frac{30}{\sin 50°} \approx 39.2$ ft
60°	$\frac{30}{\sin 60°} \approx 34.6$ ft
70°	$\frac{30}{\sin 70°} \approx 31.9$ ft
80°	$\frac{30}{\sin 80°} \approx 30.5$ ft

c.



d. Judging from the graph, a good prediction for the line of sight when the angle of depression is 30° would be about 60 feet.

40.



$$\begin{aligned} h^2 + \left(\frac{x}{2}\right)^2 &= x^2 \\ h^2 + \frac{x^2}{4} &= x^2 \\ h^2 &= \frac{3x^2}{4} \end{aligned} \qquad\qquad \begin{aligned} r^2 + 1^2 &= 2^3 \\ r^2 &= 3 \\ r &= \sqrt{3} \end{aligned}$$

$\sin E = \frac{h}{x}$ $\qquad$ $\cos G = \frac{r}{2}$

$= \frac{\frac{x\sqrt{3}}{2}}{x}$ $\qquad$ $= \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{3}}{2}$

So, $\sin E = \cos G$.

41. a.

$\angle A$	$\angle B$	**sin *A***	**cos *A***	**sin *B***	**cos *B***
45°	45°	0.7071	0.7071	0.7071	0.7071
30°	60°	0.5	0.8660	0.8660	0.5
34°	56°	0.5592	0.8290	0.8290	0.5592
17°	73°	0.2924	0.9563	0.9563	0.2924

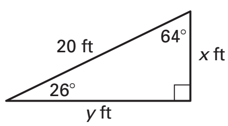
b. The sine of each acute angle equals the cosine of its complement, and vice versa.

c. If you have two complementary angles, the sine of one equals the cosine of the other.

d. The conjecture from part (c) is true for any right triangle. In $\triangle ABC$, if $\angle C$ is a right angle, then $\sin A = \frac{a}{c}$ and $\cos B = \frac{a}{c}$, and $\sin B = \frac{b}{c}$ and $\cos A = \frac{b}{c}$.

Problem Solving Workshop for the lesson "Apply the Sine and Cosine Ratios"

1. $\cos 64° = \frac{x}{20}$



$20 \cdot \cos 64° = x$

$8.8 \approx x$

The height of the ramp is about 8.8 feet.

$20^2 \approx (8.8)^2 + y^2$

$400 \approx 77.44 + y^2$

$322.56 \approx y^2$

$18.0 \approx y$

The length of the base is about 18 feet.

2. $\tan 35° = \frac{6}{x}$

$x \cdot \tan 35° = 6$

$x = \frac{6}{\tan 35°}$

$x \approx 8.6$

The distance from the swimmer to the base of the lifeguard chair is about 8.6 feet.

$y^2 \approx 6^2 + 8.6^2$

$y^2 \approx 36 + 73.96$

$y^2 \approx 109.96$

$y \approx 10.5$

The distance from the lifeguard to the swimmer is about 10.5 feet.

3. $17^2 = 9.5^2 + x^2$

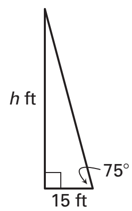
$\tan 34° = \frac{9.5}{x}$

$\cos 34° = \frac{x}{17}$

4. The Pythagorean Theorem would be used to find the length of an unknown side if the lengths of the other two sides are known. Trigonometric ratios would be used if the measurement of one of the acute angles is known along with the length of one of the sides.

5. The cosine ratio compares the side adjacent to the angle and the hypotenuse. The student used the tangent ratio which compares the opposite side and the adjacent side. The correct statement is $\cos A = \frac{7}{25}$.

6. a.



Use the tangent ratio to find the height of the tree.

$\tan 75° = \frac{h}{15}$

$15 \cdot \tan 75° = h$

$56 \approx h$

The height of the tree is about 56 feet.

b. To solve this problem using similar triangles, you would need to know the height and shadow length of another object nearby. Because the sun's rays are parallel, the angle of elevation will be 75°, so the triangles will be similar.

c. The sine ratio cannot be used to find the height of the tree, because the sine ratio compares the opposite side to the hypotenuse, which is unknown.

Lesson 7.7 Solve Right Triangles

Guided Practice for the lesson "Solve Right Triangles"

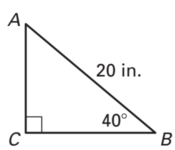
1. $\tan C = \frac{20}{15} = \frac{4}{3} = 1.33$

$m\angle C = \tan^{-1} 1.33 \approx 53.1$

2. $\sin D = 0.54$

$m\angle D = \sin^{-1} 0.54 \approx 32.7°$

3.



$90° + 40° + m\angle A = 180°$

$m\angle A = 50°$

$\sin 40° = \frac{AC}{20}$ $\qquad$ $\cos 40° = \frac{BC}{20}$

$20 \cdot \sin 40° = AC$ $\quad$ $20 \cdot \cos 40° = BC$

$20 \cdot 0.6428 \approx AC$ $\quad$ $20 \cdot 0.7660 \approx BC$

$12.9 \approx AC$ $\quad$ $15.3 \approx BC$

The angle measures are 40°, 50°, and 90°. The side lengths are 20 inches, about 12.9 inches, and about 15.3 inches.

4. $\sin x° = \frac{2}{20} = \frac{1}{10} = 0.1$

$x = \sin^{-1} 0.1 \approx 5.739$

The rake is about 5.7°, so it is not within the suggested range of 5° or less.

Exercises for the lesson "Solve Right Triangles"

Skill Practice

1. To solve a right triangle means to find the measures of all of its *sides* and *angles*.

2. A trigonometric ratio is used to find a side length of a right triangle when one side length and the measure of one acute angle are known. The Pythagorean Theorem is used to find a side length if two side lengths are known.

3. $\tan A = \frac{12}{18} = \frac{2}{3} \approx 0.6667$

$m\angle A = \tan^{-1} 0.6667 \approx 33.7°$

4. $\tan A = \frac{10}{22} = \frac{5}{11} \approx 0.4545$

$m\angle A = \tan^{-1} 0.4545 \approx 24.4°$

5. $\tan A = \frac{14}{4} = \frac{7}{2} = 3.5$

$m\angle A = \tan^{-1} 3.5 \approx 74.1°$

6. $\sin A = \frac{5}{11} \approx 0.4545$

$m\angle A = \sin^{-1} 0.4545 \approx 27.0°$

7. $\cos A = \frac{6}{10} = \frac{3}{5} = 0.6$

$m\angle A = \cos^{-1} 0.6 \approx 53.1°$

8. $\cos A = \frac{7}{12} \approx 0.5833$

$m\angle A = \cos^{-1} 0.5833 \approx 54.3°$

9. B;

Because $\tan J = \frac{KL}{JL}$, $\tan^{-1} \frac{KL}{JL} = m\angle J$.

10. $90° + 40° + m\angle K = 180°$

$m\angle K = 50°$

$\sin 40° = \frac{KL}{8}$ $\quad$ $\cos 40° = \frac{LM}{8}$

$8 \cdot \sin 40° = KL$ $\quad$ $8 \cdot \cos 40° = LM$

$8 \cdot 0.6428 \approx KL$ $\quad$ $8 \cdot 0.7660 \approx LM$

$5.1 \approx KL$ $\quad$ $6.1 \approx LM$

The angle measures are 40°, 50°, and 90°. The side lengths are 8, about 5.1, and about 6.1.

11. $90° + 65° + m\angle N = 180°$

$m\angle N = 25°$

$\tan 65° = \frac{NP}{10}$ $\quad$ $\cos 65° = \frac{10}{NQ}$

$10 \cdot \tan 65° = NP$ $\quad$ $NQ \cdot \cos 65° = 10$

$10 \cdot 2.1445 \approx NP$ $\quad$ $NQ = \frac{10}{\cos 65°}$

$21.4 \approx NP$ $\quad$ $NQ \approx \frac{10}{0.4226}$

$NQ \approx 23.7$

The angle measures are 25°, 65°, and 90°. The side lengths are 10, about 21.4, and about 23.7.

12. $90° + 57° + m\angle T = 180°$

$m\angle T = 33°$

$\tan 57° = \frac{15}{RS}$ $\quad$ $\sin 57° = \frac{15}{RT}$

$RS \cdot \tan 57° = 15$ $\quad$ $RT \cdot \sin 57° = 15$

$RS = \frac{15}{\tan 57°}$ $\quad$ $RT = \frac{15}{\sin 57°}$

$RS \approx \frac{15}{1.5399}$ $\quad$ $RT \approx \frac{15}{0.8387}$

$RS \approx 9.7$ $\quad$ $RT \approx 17.9$

The angle measures are 33°, 57°, and 90°. The side lengths are 15, about 9.7, and about 17.9.

13. $AC^2 = 9^2 + 12^2$

$AC^2 = 81 + 144$

$AC^2 = 225$

$AC = 15$

$\tan A = \frac{9}{12} = \frac{3}{4} = 0.75$

$m\angle A = \tan^{-1} 0.75 \approx 36.9°$

$\tan C = \frac{12}{9} = \frac{4}{3} \approx 1.3333$

$m\angle C \approx \tan^{-1} 1.3333 \approx 53.1$

The angle measures are 90°, about 36.9°, and about 53.1°. The side lengths are 9, 12, and 15.

14. $9^2 = 3^2 + EF^2$

$81 = 9 + EF^2$

$72 = EF^2$

$8.5 \approx EF$

$\cos D = \frac{3}{9} = \frac{1}{3} \approx 0.3333$

$m\angle D \approx \cos^{-1} 0.3333 \approx 70.5°$

$\sin F = \frac{3}{9} = \frac{1}{3} \approx 0.3333$

$m\angle F \approx \sin^{-1} 0.3333 \approx 19.5°$

The angle measures are about 19.5°, about 70.5°, and 90°. The side lengths are 3, 9, and about 8.5.

15. $16^2 = 14^2 + HJ^2$

$256 = 196 + HJ^2$

$60 = HJ^2$

$7.7 \approx HJ$

$\cos G = \frac{14}{16} = \frac{7}{8} = 0.875$

$m\angle G = \cos^{-1} 0.875 \approx 29.0°$

$\sin J = \frac{14}{16} = \frac{7}{8} = 0.875$

$m\angle J = \sin^{-1} 0.875 \approx 61.0°$

The angle measures are about 29.0°, about 61.0°, and 90°. The side lengths are 14, 16, and about 7.7.

16. $90° + 43.6° + m\angle A = 180°$

$m\angle A = 46.4°$

$\cos 43.6° = \frac{5.2}{AB}$ $\qquad$ $\tan 43.6° = \frac{AC}{5.2}$

$AB \cdot \cos 43.6° = 5.2$ $\qquad$ $5.2 \cdot \tan 43.6° = AC$

$AB = \frac{5.2}{\cos 43.6°}$ $\qquad$ $5.2 \cdot 0.9523 \approx AC$

$AB \approx \frac{5.2}{0.7242} \approx 7.2$ $\qquad$ $5.0 \approx AC$

The angle measures are 43.6°, 46.4°, and 90°. The side lengths are about 5.0, 5.2, and about 7.2.

17. $ED^2 = \left(\frac{8}{3}\right)^2 + \left(\frac{14}{3}\right)^2$

$ED^2 = \frac{64}{9} + \frac{196}{9}$

$ED^2 = \frac{260}{9}$

$ED = \frac{2\sqrt{65}}{3}$

$\tan E = \frac{\frac{14}{3}}{\frac{8}{3}} = \frac{7}{4} = 1.75$

$m\angle E = \tan^{-1} 1.75 \approx 60.3°$

$\tan D = \frac{\frac{8}{3}}{\frac{14}{3}} = \frac{4}{7} \approx 0.5714$

$m\angle D \approx \tan^{-1} 0.5714 \approx 29.7°$

The angle measures are about 29.7°, about 60.3°, and 90°. The side lengths are $\frac{8}{3}$, $\frac{14}{3}$, and $\frac{2\sqrt{65}}{3}$.

18. $90° + 29.9° + m\angle H = 180°$

$m\angle H = 60.1°$

$\cos 29.9° = \frac{GJ}{10\frac{7}{8}}$ $\qquad$ $\sin 29.9° = \frac{JH}{10\frac{7}{8}}$

$10\frac{7}{8} \cdot \cos 29.9° = GJ$ $\qquad$ $10\frac{7}{8} \cdot \sin 29.9° = JH$

$10.875 \cdot 0.8669 \approx GJ$ $\qquad$ $10.875 \cdot 0.4985 \approx JH$

$9.4 \approx GJ$ $\qquad$ $5.4 \approx JH$

The angle measures are 29.9°, 60.1°, and 90°. The side lengths are about 5.4, about 9.4, and $10\frac{7}{8}$.

19. The student incorrectly used WY, the length of the opposite side, in place of the length of the hypotenuse in the inverse sine ratio. The correct statement is $\sin^{-1} \frac{7}{WX} = 36°$.

20. The student incorrectly used the term $\cos^{-1}$ when using an inverse tangent ratio. The correct statement is $\tan^{-1} \frac{8}{15} = m\angle T$.

21. $\sin A = 0.5$

$m\angle A = \sin^{-1} 0.5 = 30°$

22. $\sin A = 0.75$

$m\angle A = \sin^{-1} 0.75 \approx 48.6°$

23. $\cos A = 0.33$

$m\angle A = \cos^{-1} 0.33 \approx 70.7°$

24. $\cos A = 0.64$

$m\angle A = \cos^{-1} 0.64 \approx 50.2°$

25. $\tan A = 1.0$

$m\angle A = \tan^{-1} 1.0 = 45°$

26. $\tan A = 0.28$

$m\angle A = \tan^{-1} 0.28 \approx 15.6°$

27. $\sin A = 0.19$

$m\angle A = \sin^{-1} 0.19 \approx 11.0°$

28. $\cos A = 0.81$

$m\angle A = \cos^{-1} 0.81 \approx 35.9°$

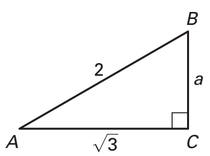
29. B; You need to know at least one side length to solve a right triangle.

30. The statement $\tan^{-1} x = \frac{1}{\tan x}$ is incorrect because, when used with trigonometric ratios, the "-1" superscript does not indicate the reciprocal of the ratio. The expression $\tan^{-1} x$ represents the angle that has a tangent of x.

31. $\sin A = \frac{1}{2}\sqrt{2} = \frac{1}{\sqrt{2}} = \frac{\text{opp.}}{\text{hyp.}}$

In a 45°-45°-90° triangle, the length of the hypotenuse equals $\sqrt{2}$ times the length of a leg. So, $m\angle A = 45°$.

$\sin B = \frac{1}{2}\sqrt{3} = \frac{\sqrt{3}}{2} = \frac{\text{opp.}}{\text{hyp.}}$



$a^2 + (\sqrt{3})^2 = 2^2$

$a^2 = 4 - 3 = 1$

$a = 1$

In a 30°-60°-90° triangle, the hypotenuse is twice as long as the shorter leg, and the longer leg is $\sqrt{3}$ times as long as the shorter leg. So, $m\angle B = 60°$.

32. a. 0° to 10° have nearly the same sine and tangent values.

b. The angle 89° has the greatest difference in its sine and tangent values.

c. The angle 60° has a tangent value that is double its sine value.

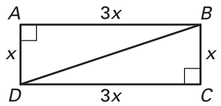
d. $\sin 2x$ is not equal to $2 \sin x$.

For example:

$2 \sin 34° \stackrel{?}{=} \sin 68°$

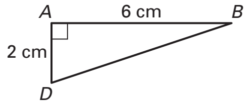
$2(0.5592) \stackrel{?}{=} 0.9272$

$1.1184 \neq 0.9272$

33. 

$$\text{Perimeter} = 2(3x) + 2(x)$$
$$16 = 6x + 2x$$
$$16 = 8x$$
$$2 = x$$

$$BD^2 = 2^2 + 6^2$$
$$BD^2 = 40$$
$$BD = 2\sqrt{10} \approx 6.3$$

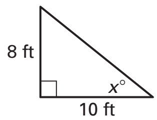


$\tan D = \frac{6}{2} = 3$ $\qquad$ $\tan B = \frac{2}{6} = \frac{1}{3} \approx 0.3333$

$m\angle D = \tan^{-1} 3 \approx 71.6°$ $\qquad$ $m\angle B \approx \tan^{-1} 0.3333 \approx 18.4°$

The side lengths of $\triangle ABD$ are 2 centimeters, 6 centimeters, and about 6.3 centimeters. The angle measures are about 18.4°, about 71.6°, and 90°.

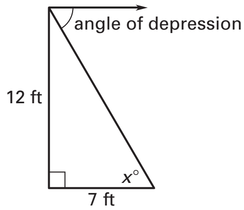
Problem Solving

34. 

$$\tan x° = \frac{8}{10} = \frac{4}{5} = 0.8$$
$$\tan^{-1} 0.8 = x°$$
$$38.7 \approx x$$

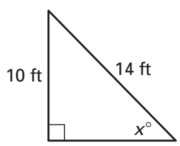
The angle of elevation of the kick is about 38.7°.

35. The angle of depression is congruent to the angle of elevation from the duck to the top of the bridge.



$$\tan x° = \frac{12}{7} \approx 1.7143$$
$$\tan^{-1} 1.7143 \approx x°$$
$$59.7 \approx x$$

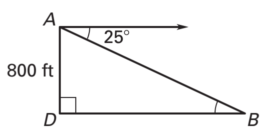
The angle of depression is about 59.7°.

36. 

$$\sin x° = \frac{10}{14} = \frac{5}{7} \approx 0.7143$$
$$\sin^{-1} 0.7143 \approx x°$$
$$45.6 \approx x$$

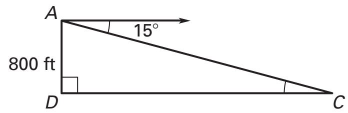
The body of the dump truck has been elevated to about 45.6°. Because this is less than 55°, the clay will not pour out easily.

37. The best expression to choose would be $\tan^{-1} \frac{BC}{AC}$ because the lengths BC and AC are known. The other expressions would require an extra step of calculating AB.

38. a. 

$$\tan 25° = \frac{800}{BD}$$
$$BD \cdot \tan 25° = 800$$
$$BD = \frac{800}{\tan 25°} \approx 1715.6$$

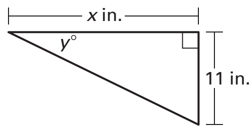
The hiker at B is about 1716 feet from the base of the plateau.

b. 

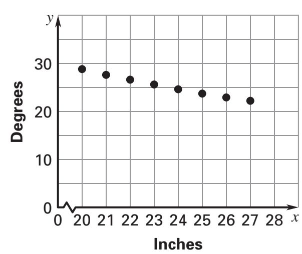
$$\tan 15° = \frac{800}{CD}$$
$$CD \cdot \tan 15° = 800$$
$$CD = \frac{800}{\tan 15°} \approx 2985.6$$

The hiker at C is about 2986 feet from the base of the plateau.

c. The hikers are about $2986 - 1716 = 1270$ feet apart. This is the length of $\overline{BC}$ in the diagram.

39. a. 

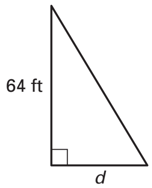
x	$\tan y° = \frac{11}{x}$	$y° = \tan^{-1} \frac{11}{x}$
20	$\frac{11}{20} = 0.55$	28.8°
21	$\frac{11}{21} \approx 0.5238$	27.6°
22	$\frac{11}{22} = 0.5$	26.6°
23	$\frac{11}{23} \approx 0.4783$	25.6°
24	$\frac{11}{24} \approx 0.4583$	24.6°
25	$\frac{11}{25} = 0.44$	23.7°
26	$\frac{11}{26} \approx 0.4231$	22.9°
27	$\frac{11}{27} \approx 0.4074$	22.2°

b. 

c. As the length of the rack increases, the angle needed decreases in size.

40. Answers will vary.

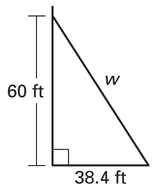
41. a.



$d = (0.60)(64) = 38.4$

The tower and the ground connection wire are 38.4 feet apart.

b.



$w^2 = 60^2 + 38.4^2$

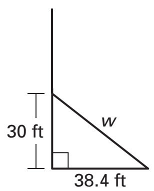
$w^2 = 3600 + 1474.56$

$w^2 = 5074.56$

$w \approx 71.2$

The length of the guy wire is about 71.2 feet.

c.



$w^2 = 30^2 + 38.4^2$

$w^2 = 900 + 1474.56$

$w^2 = 2374.56$

$w \approx 48.7$

The length of the guy wire is about 48.7 feet.

d. $\tan x \approx \frac{60}{38.4} = 1.5625$

$x \approx \tan^{-1} 1.5625 \approx 57.4°$

When the guy wire is attached 60 feet high, the angle of elevation is about 57.4°.

$\tan x \approx \frac{30}{38.4} \approx 0.7813$

$x \approx \tan^{-1} 0.7813 \approx 38.0°$

When the guy wire is attached 30 feet high, the angle of elevation is about 38°.

The two triangles are neither congruent nor similar because their angles are not congruent.

e. The tangent ratio and inverse tangent ratio were used because the opposite and adjacent sides were known.

42.

Statements	Reasons
1. $\triangle ABC$ with altitude $\overline{CD}$	**1.** Given
2. $\sin A = \frac{CD}{b}$ and $\sin B = \frac{CD}{a}$	**2.** Definition of sine ratio
3. $CD = b \sin A$ and $CD = a \sin B$	**3.** Multiplication Property of Equality
4. $b \sin A = a \sin B$	**4.** Substitution Property of Equality
5. $\frac{\sin A}{a} = \frac{\sin B}{b}$	**5.** Division Property of Equality

Quiz for the lessons "Apply the Tangent Ratio", "Apply the Sine and Cosine Ratios", and "Solve Right Triangles"

1. $\tan 65° = \frac{18}{x}$

$x \cdot \tan 65° = 18$

$x = \frac{18}{\tan 65°}$

$x \approx \frac{18}{2.1445}$

$x \approx 8.4$

2. $\sin 36° = \frac{x}{11}$

$11 \cdot \sin 36° = x$

$11 \cdot 0.5878 \approx x$

$6.5 \approx x$

3. $\cos 57° = \frac{14}{x}$

$x \cdot \cos 57° = 14$

$x = \frac{14}{\cos 57°}$

$x \approx \frac{14}{0.5446}$

$x \approx 25.7$

4. $AC^2 = 5^2 + 13^2$

$AC^2 = 25 + 169$

$AC^2 = 194$

$AC \approx 13.9$

$\tan A = \frac{5}{13} \approx 0.3846$ $\quad$ $\tan C = \frac{13}{5} = 2.6$

$m\angle A \approx \tan^{-1} 0.3846$ $\quad$ $m\angle C = \tan^{-1} 2.6$

$m\angle A \approx 21.0°$ $\quad$ $m\angle C \approx 69.0°$

The angle measures are about 21°, about 69°, and 90°. The side lengths are 5, 13, and about 13.9.

5. $17^2 = 10^2 + EF^2$

$289 = 100 + EF^2$

$189 = EF^2$

$13.7 \approx EF$

$\cos D = \frac{10}{17} \approx 0.5882$ $\quad$ $\sin F = \frac{10}{17} \approx 0.5882$

$m\angle D \approx \cos^{-1} 0.5882$ $\quad$ $m\angle D \approx \sin^{-1} 0.5882$

$m\angle D \approx 54.0°$ $\quad$ $m\angle D \approx 36.0°$

The angle measures are about 36°, about 54°, and 90°. The side lengths are 10, about 13.7, and 17.

6. $90° + 28.9° + m\angle G = 180°$

$m\angle G = 61.1°$

$\tan 28.9 = \frac{GH}{13}$

$13 \cdot \tan 28.9 = GH$

$13 \cdot 0.5520 \approx GH$

$7.2 \approx GH$

$$\cos 28.9 = \frac{13}{GJ}$$

$$GJ \cdot \cos 28.9 = 13$$

$$GJ = \frac{13}{\cos 28.9}$$

$$GJ \approx \frac{13}{0.8755}$$

$$GJ \approx 14.8$$

The angle measures are 28.9°, 61.1°, and 90°. The side lengths are about 7.2, 13, and about 14.8.

Extension for the extension "Solve Right Triangles"

1. $85° + 29° + m\angle C = 180°$

$$m\angle C = 66°$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} \qquad \frac{\sin C}{c} = \frac{\sin B}{b}$$

$$\frac{\sin 29°}{a} = \frac{\sin 85°}{9} \qquad \frac{\sin 66°}{c} = \frac{\sin 85°}{9}$$

$$a = \frac{9 \cdot \sin 29°}{\sin 85°} \qquad c = \frac{9 \cdot \sin 66°}{\sin 85°}$$

$$a \approx 4.4 \qquad c \approx 8.3$$

The angle measures are 29°, 66°, and 85°. The side lengths are about 4.4, about 8.3, and 9.

2. $70° + 81° + m\angle A = 180°$

$$m\angle A = 29°$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} \qquad \frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\frac{\sin 29°}{10} = \frac{\sin 70°}{b} \qquad \frac{\sin 29°}{10} = \frac{\sin 81°}{c}$$

$$b = \frac{10 \cdot \sin 70°}{\sin 29°} \qquad c = \frac{10 \cdot \sin 81°}{\sin 29°}$$

$$b \approx 19.4 \qquad c \approx 20.4$$

The angle measures are 29°, 70°, and 81°. The side lengths are 10, about 19.4, and about 20.4.

3. $\frac{\sin C}{c} = \frac{\sin A}{a}$

$$\frac{\sin C}{17} = \frac{\sin 51°}{18}$$

$$\sin C = 17 \cdot \frac{\sin 51°}{18}$$

$$\sin C \approx 0.7340$$

$$m\angle C \approx \sin^{-1} 0.7340 \approx 47.2°$$

$$51° + 47.2° + m\angle B = 180°$$

$$m\angle B \approx 81.8°$$

$$\sin \frac{A}{a} = \sin \frac{B}{b}$$

$$\frac{\sin 51°}{18} = \frac{\sin 81.8°}{b}$$

$$b = \frac{18 \cdot \sin 81.8°}{\sin 51°}$$

$$b \approx 22.9$$

The angle measures are about 81.8°, about 47.2°, and 51°. The side lengths are 17, 18, and about 22.9.

4. $a^2 = b^2 + c^2 - 2bc \cos A$

$$4^2 = 6^2 + 5^2 - 2(6)(5) \cos A$$

$$0.75 = \cos A$$

$$m\angle A = \cos^{-1} 0.75 \approx 41.4°$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$6^2 = 4^2 + 5^2 - 2(4)(5) \cos B$$

$$0.125 = \cos B$$

$$m\angle B = \cos^{-1} 0.125 \approx 82.8°$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$5^2 = 4^2 + 6^2 - 2(4)(6) \cos C$$

$$0.5625 = \cos C$$

$$m\angle C = \cos^{-1} 0.5625 \approx 55.8°$$

The angle measures are about 41.4°, about 55.8°, and about 82.8°.

5. $a^2 = b^2 + c^2 - 2bc \cos A$

$$23^2 = 27^2 + 16^2 - 2(27)(16) \cos A$$

$$0.5278 \approx \cos A$$

$$m\angle A \approx \cos^{-1} 0.5278 \approx 58.1°$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$27^2 = 23^2 + 16^2 - 2(23)(16) \cos B$$

$$0.0761 \approx \cos B$$

$$m\angle B \approx \cos^{-1} 0.0761 \approx 85.6°$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$16^2 = 23^2 + 27^2 - 2(23)(27) \cos C$$

$$0.8068 \approx \cos C$$

$$m\angle C \approx \cos^{-1} 0.8068 \approx 36.2°$$

The angle measures are about 36.2°, about 58.1°, and about 85.6°.

6. $b^2 = a^2 + c^2 - 2ac \cos B$

$$b^2 = 45^2 + 43^2 - 2(45)(43) \cos 88°$$

$$b^2 \approx 3738.9$$

$$b \approx 61.1$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$45^2 \approx 61.1^2 + 43^2 - 2(61.1)(43) \cos A$$

$$0.6770 \approx \cos A$$

$$m\angle A \approx \cos^{-1} 0.6770 \approx 47.4°$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

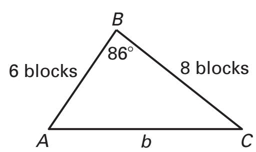
$$43^2 \approx 45^2 + 61.1^2 - 2(45)(61.1) \cos C$$

$$0.7109 \approx \cos C$$

$$m\angle C \approx \cos^{-1} 0.7109 \approx 44.7°$$

The angle measures are about 44.7°, about 47.4°, and 88°. The side lengths are 45, about 61.1, and 43.

7.



$b^2 = a^2 + c^2 - 2ac \cos B$

$b^2 = 8^2 + 6^2 - 2(8)(6) \cos 86°$

$b^2 \approx 93.3$

$b \approx 9.7$

The distance from the zoo to the movie theater is about 9.7 blocks.

Mixed Review of Problem Solving for the lessons "Apply the Tangent Ratio", "Apply the Sine and Cosine Ratios", and "Solve Right Triangles"

1. a. $\sin A = \dfrac{\text{opp.}}{\text{hyp.}}$

$\sin 60° = \dfrac{h}{10.9}$

$10.9 \cdot \sin 60° = h$

$9.4 \approx h$

The greatest height the arm can reach is $3.6 + 9.4 = 13$ meters.

b. $\sin 60° = \dfrac{h}{16.4}$

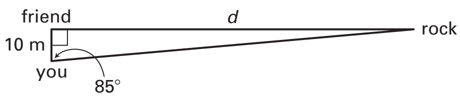
$16.4 \cdot \sin 60° = h$

$14.2 \approx h$

The greatest height the arm can reach when extended is $3.6 + 14.2 = 17.8$ meters.

c. The difference between the two heights the arm can reach above the ground is $17.8 - 13 = 4.8$ meters.

2. a. Top view:



b. Approximate the distance d by using a tangent ratio.

$\tan 85° = \dfrac{d}{10}$

$10 \cdot \tan 85° = d$

$114.3 \approx d$

The distance across the canyon is about 114.3 meters.

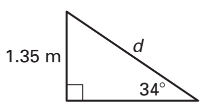
c. $\tan 87° = \dfrac{d}{10}$

$10 \cdot \tan 87° = d$

$190.8 \approx d$

Your estimate is $190.8 - 114.3 = 76.5$ meters off of the actual distance.

3. The height of the rim above your eye is about $3.05 - 1.7 = 1.35$ meters. So, use a sine ratio to find the distance d in the diagram below.



$\sin 34° = \dfrac{1.35}{d}$

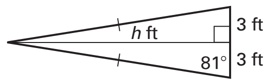
$d \cdot \sin 34° = 1.35$

$d = \dfrac{1.35}{\sin 34°}$

$d \approx 2.4$

The distance from you to the rim is about 2.4 meters.

4.



$\tan 81° = \dfrac{h}{3}$

$3 \cdot \tan 81° = h$

$18.9 \approx h$

The height of the isosceles triangle is about 18.9 feet.

5. a. Find x by using the Pythagorean Theorem.

$DC^2 = ED^2 + EC^2$

$18^2 = 9^2 + x^2$

$324 = 81 + x^2$

$243 = x^2$

$15.6 \approx x$

b. $EB = x + 2x = 3x \approx 3(15.6) = 46.8$

Because the opposite and adjacent sides are known, $m\angle ABC$ can be found by using an inverse tangent ratio.

$m\angle ABC \approx \tan^{-1} \dfrac{9}{46.8} \approx \tan^{-1} 0.1923 \approx 10.9°$

c. A different method for finding x could be using an inverse cosine ratio to find $m\angle EDC$, then using a tangent ratio for $\angle EDC$ to find x.

A different method for finding $m\angle ABC$ could be finding EB, then using the Pythagorean Theorem to find AB, then using a sine ratio to find $m\angle ABC$.

6. $\cos 52° = \dfrac{h}{14}$

$14 \cdot \cos 52° = h$

$8.6 \approx h$

The height of the staircase is about 8.6 feet.

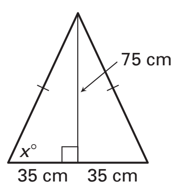
$\sin 52° = \dfrac{b}{14}$

$14 \cdot \sin 52° = b$

$11.0 \approx b$

The base of the staircase is about 11 feet.

7.



$\tan^{-1} \dfrac{75}{35} = x°$

$65° \approx x$

Chapter Review for the chapter "Right Triangles and Trigonometry"

1. A Pythagorean triple is a set of three positive integers a, b, and c that satisfy the equation $c^2 = a^2 + b^2$.

2. To solve a right triangle means to find all three angle measures and all three side lengths. In order to solve a right triangle you need to know two of the side lengths, or one side length and the measure of one acute angle.

3. An angle of elevation is the angle that your line of sight makes with a line drawn horizontally when you are looking up at a point in the distance. An angle of depression is the angle that your line of sight makes with a line drawn horizontally when you are looking down at a point in the distance.

4. $x^2 = 12^2 + 16^2$

$x^2 = 144 + 256$

$x^2 = 400$

$x = 20$

5. $x^2 = 6^2 + 10^2$

$x^2 = 36 + 100$

$x^2 = 136$

$x = 2\sqrt{34} \approx 11.7$

6. $(\sqrt{369})^2 = 12^2 + x^2$

$369 = 144 + x^2$

$225 = x^2$

$15 = x$

7. 6, 8, 9

$9^2 \underline{\ ?\ } 6^2 + 8^2$

$81 \underline{\ ?\ } 36 + 64$

$81 < 100$

The triangle is acute.

8. 4, 2, 5

$5^2 \underline{\ ?\ } 2^2 + 4^2$

$25 \underline{\ ?\ } 4 + 16$

$25 > 20$

The triangle is obtuse.

9. $10, 2\sqrt{2}, 6\sqrt{3}$

$(6\sqrt{3})^2 \underline{\ ?\ } 10^2 + (2\sqrt{2})^2$

$108 \underline{\ ?\ } 100 + 8$

$108 = 108$

The triangle is a right triangle.

10. 15, 20, 15

$20^2 \underline{\ ?\ } 15^2 + 15^2$

$400 \underline{\ ?\ } 225 + 225$

$400 < 450$

The triangle is acute.

11. $3, 3, 3\sqrt{2}$

$(3\sqrt{2})^2 \underline{\ ?\ } 3^2 + 3^2$

$18 \underline{\ ?\ } 9 + 9$

$18 = 18$

The triangle is a right triangle.

12. $13, 18, 3\sqrt{55}$

$(3\sqrt{55})^2 \underline{\ ?\ } 13^2 + 18^2$

$495 \underline{\ ?\ } 169 + 324$

$495 > 493$

The triangle is obtuse.

13. $\frac{x}{9} = \frac{9}{6}$

$6x = 9 \cdot 9$

$x = \frac{81}{6} = \frac{27}{2} = 13.5$

14. $\frac{4}{x} = \frac{x}{9}$

$x^2 = 4 \cdot 9$

$x^2 = 36$

$x = 6$

15. $\frac{4}{x} = \frac{x}{4 + 6}$

$x^2 = 4 \cdot 10$

$x^2 = 40$

$x = 2\sqrt{10} \approx 6.3$

16. $\frac{2}{x} = \frac{x}{5 + 2}$

$x^2 = 2 \cdot 7$

$x^2 = 14$

$x = \sqrt{14} \approx 3.7$

17. $\frac{16}{12} = \frac{12}{x}$

$16x = 12 \cdot 12$

$x = \frac{144}{16} = 9$

18. $\frac{x}{20} = \frac{20}{25}$

$25x = 20 \cdot 20$

$x = \frac{400}{25} = 16$

19. The triangle is an isosceles right triangle, with angles 45°-45°-90°.

hypotenuse $=$ leg $\cdot \sqrt{2}$

$x = 6\sqrt{2}$

20. By the Triangle Sum Theorem, the triangle is a 30°-60°-90° triangle.

hypotenuse $= 2 \cdot$ shorter leg

$14 = 2x$

$7 = x$

21. By the Triangle Sum Theorem, the triangle is a 30°-60°-90° triangle.

hypotenuse $= 2 \cdot$ shorter leg

$x = 2 \cdot 8\sqrt{3}$

$x = 16\sqrt{3}$

22. $\tan 75° = \frac{x}{4}$

$4 \cdot \tan 75° = x$

$14.9 \approx x$

The tree is about 15 feet tall.

23. $\tan 55° = \frac{x}{4}$

$4 \cdot \tan 55° = x$

$5.7 \approx x$

The tree is about 6 feet tall.

24. $\tan 54° = \frac{x}{32}$

$32 \cdot \tan 54° = x$

$44.0 \approx x$

25. $\tan 25° = \frac{x}{20}$

$20 \cdot \tan 25° = x$

$9.3 \approx x$

26. $\tan 38° = \frac{10}{x}$

$x \cdot \tan 38° = 10$

$x = \frac{10}{\tan 38°}$

$x \approx 12.8$

27. $\sin X = \frac{3}{5} = 0.6$

$\cos X = \frac{4}{5} = 0.8$

28. $\sin X = \frac{7}{\sqrt{149}} = \frac{7\sqrt{149}}{149} \approx 0.5735$

$\cos X = \frac{10}{\sqrt{149}} = \frac{10\sqrt{149}}{149} \approx 0.8192$

29. $\sin X = \frac{55}{73} \approx 0.7534$

$\cos X = \frac{48}{73} \approx 0.6575$

30. $15^2 = 10^2 + BC^2$

$225 = 100 + BC^2$

$125 = BC^2$

$11.2 \approx BC$

$\cos A = \frac{10}{15} \approx 0.6667$ $\quad$ $\sin B = \frac{10}{15} \approx 0.6667$

$m\angle A \approx \cos^{-1} 0.6667$ $\quad$ $m\angle B \approx \sin^{-1} 0.6667$

$m\angle A \approx 48.2°$ $\quad$ $m\angle B \approx 41.8°$

The angle measures are about 41.8°, about 48.2°, and 90°. The side lengths are 10, about 11.2, and 15.

31. $90° + 37° + m\angle L = 180°$

$m\angle L = 53°$

$\tan 37° = \frac{ML}{6}$

$6 \cdot \tan 37° = ML$

$4.5 \approx ML$

$\cos 37° = \frac{6}{NL}$

$NL \cdot \cos 37° = 6$

$NL = \frac{6}{\cos 37°}$

$NL \approx 7.5$

The angle measures are 37°, 53°, and 90°. The side lengths are about 4.5, 6, and about 7.5.

32. $25^2 = 18^2 + XY^2$

$625 = 324 + XY^2$

$301 = XY^2$

$17.3 \approx XY$

$m\angle X = \sin^{-1} \frac{18}{25}$ $\quad$ $m\angle Z = \cos^{-1} \frac{18}{25}$

$m\angle X \approx 46.1°$ $\quad$ $m\angle Z \approx 43.9°$

The angle measures are about 43.9°, about 46.1°, and 90°. The side lengths are 18, about 17.3, and 25.

33. $90° + 40° + m\angle GED = 180°$

$m\angle GED = 50°$

Because EG forms the altitude of right $\triangle DEF$, $\triangle DGE \sim \triangle EGF \sim \triangle DEF$. So, $\angle GEF \cong \angle GDE$ and $m\angle GEF = 40°$, and $\angle EFG \cong \angle GED$ and $m\angle EFG = 50°$.

$\sin 40° = \frac{EG}{10}$

$10 \cdot \sin 40° = EG$

$6.4 \approx EG$

$\cos 40° = \frac{10}{DF}$

$DF \cdot \cos 40° = 10$

$DF = \frac{10}{\cos 40°}$

$DF \approx 13.1$

$\tan 40° = \frac{EF}{10}$

$10 \cdot \tan 40° = EF$

$8.4 \approx EF$

Chapter Test for the chapter "Right Triangles and Trigonometry"

1. $20^2 = 12^2 + x^2$

$400 = 144 + x^2$

$256 = x^2$

$16 = x$

2. $x^2 = 9^2 + 13^2$

$x^2 = 81 + 169$

$x^2 = 250$

$x = 5\sqrt{10}$

3. $21^2 = 15^2 + x^2$

$441 = 225 + x^2$

$216 = x^2$

$6\sqrt{6} = x$

4. $5, 15, 5\sqrt{10}$

$(5\sqrt{10})^2 \ \underline{\ ?\ }\ 5^2 + 15^2$

$250 \ \underline{\ ?\ }\ 25 + 225$

$250 = 250$

The triangle is a right triangle.

5. 4.3, 6.7, 8.2

$8.2^2 \ \underline{\ ?\ }\ 4.3^2 + 6.7^2$

$67.24 \ \underline{\ ?\ }\ 18.49 + 44.89$

$67.24 > 63.38$

The triangle is obtuse.

6. 5, 7, 8

$8^2 \ \underline{\ ?\ }\ 5^2 + 7^2$

$64 \ \underline{\ ?\ }\ 25 + 49$

$64 < 74$

The triangle is acute.

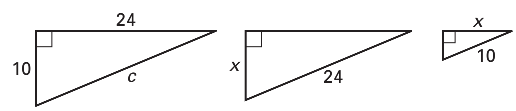
7. $\frac{5}{x} = \frac{x}{20}$

$5 \cdot 20 = x^2$

$100 = x^2$

$10 = x$

8.



$c^2 = 10^2 + 24^2$

$c^2 = 100 + 576$

$c^2 = 676$

$c = 26$

$\frac{24}{x} = \frac{26}{10}$

$26x = 240$

$x = \frac{240}{26} \approx 9.2$

9. $\frac{12 + 3}{x} = \frac{x}{12}$

$15 \cdot 12 = x^2$

$180 = x^2$

$13.4 \approx x$

10. hypotenuse $=$ 2 • shorter leg

$y = 2 \cdot 4$

$y = 8$

longer leg = shorter leg • $\sqrt{3}$

$x = 4\sqrt{3}$

11. leg = leg

$x = 24$

hypotenuse = leg • $\sqrt{2}$

$y = 24\sqrt{2}$

12. hypotenuse = 2 • shorter leg

$7\sqrt{3} = 2x$

$\frac{7\sqrt{3}}{2} = x$

longer leg = shorter leg • $\sqrt{3}$

$y = x \cdot \sqrt{3}$

$y = \left(\frac{7\sqrt{3}}{2}\right)(\sqrt{3}) = \frac{21}{2}$

13. $AC^2 = 5^2 + 11^2$

$AC^2 = 25 + 121$

$AC^2 = 146$

$AC \approx 12.1$

$m\angle A = \tan^{-1}\frac{5}{11} \approx 24.4°$

$m\angle A = \tan^{-1}\frac{11}{5} \approx 65.6°$

The angle measures are about 24.4°, about 65.6°, and 90°. The side lengths are 5, 11, and about 12.1.

14. $9.2^2 = 5.4^2 + EF^2$

$84.64 = 29.16 + EF^2$

$55.48 = EF^2$

$7.4 \approx EF$

$m\angle D = \cos^{-1}\frac{5.4}{9.2} \approx 54.1°$

$m\angle F = \sin^{-1}\frac{5.4}{9.2} \approx 35.9°$

The angle measures are about 35.9°, about 54.1°, and 90°. The side lengths are 5.4, about 7.4, and 9.2.

15. $90° + 53.2° + m\angle G = 180°$

$m\angle G = 36.8°$

$\tan 53.2° = \frac{GH}{14}$

$14 \cdot \tan 53.2° = GH$

$18.7 \approx GH$

$\cos 53.2° = \frac{14}{GJ}$

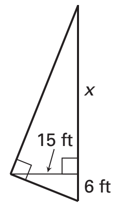
$GJ \cdot \cos 53.2° = 14$

$GJ = \frac{14}{\cos 53.2°}$

$GJ \approx 23.4$

The angle measures are 36.8°, 53.2°, and 90°. The side lengths are 14, about 18.7, and about 23.4.

16.



$\frac{x}{15} = \frac{15}{6}$

$6x = 15^2$

$x = \frac{225}{6} = 37.5$

The height of the flagpole is about 37.5 + 6 = 43.5 feet.

17. $\sin a° = \frac{750}{2000} = 0.375$

$a = \sin^{-1} 0.375 \approx 22.0°$

The angle of elevation of the hill is about 22°.

Chapter Algebra Review for the chapter "Right Triangles and Trigonometry"

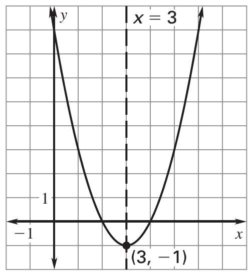
1. $y = x^2 - 6x + 8$

$a = 1 > 0 \rightarrow$ graph opens upward

Vertex: $x = -\frac{b}{2a} = -\frac{(-6)}{2(1)} = 3$

$y = (3)^2 - 6(3) + 8 = -1$

Axis of symmetry: $x = 3$



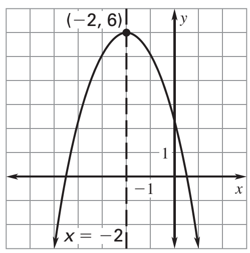
2. $y = -x^2 - 4x + 2$

$a = -1 < 0 \rightarrow$ graph opens downward

Vertex: $x = -\frac{b}{2a} = -\frac{(-4)}{2(-1)} = -2$

$y = -(-2)^2 - 4(-2) + 2 = 6$

Axis of symmetry: $x = -2$



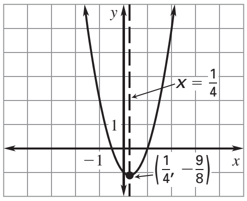
3. $y = 2x^2 - x - 1$

$a = 2 > 0 \rightarrow$ graph opens upward

Vertex: $x = -\frac{b}{2a} = -\frac{(-1)}{2(2)} = \frac{1}{4}$

$y = 2\left(\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right) - 1 = -\frac{9}{8}$

Axis of symmetry: $x = \frac{1}{4}$



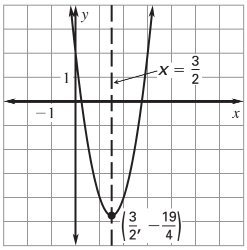
4. $y = 3x^2 - 9x + 2$

$a = 3 > 0 \rightarrow$ graph opens upward

Vertex: $x = -\frac{b}{2a} = -\frac{(-9)}{2(3)} = \frac{3}{2}$

$y = 3\left(\frac{3}{2}\right)^2 - 9\left(\frac{3}{2}\right) + 2 = -\frac{19}{4}$

Axis of symmetry: $x = \frac{3}{2}$



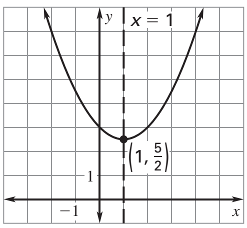
5. $y = \frac{1}{2}x^2 - x + 3$

$a = \frac{1}{2} > 0 \rightarrow$ graph opens upward

Vertex: $x = -\frac{b}{2a} = -\frac{(-1)}{2\left(\frac{1}{2}\right)} = 1$

$y = \left(\frac{1}{2}\right)(1)^2 - (1) + 3 = \frac{5}{2}$

Axis of symmetry: $x = 1$



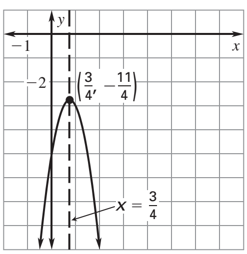
6. $y = -4x^2 + 6x - 5$

$a = -4 < 0 \rightarrow$ graph opens downward

Vertex: $x = -\frac{b}{2a} = -\frac{6}{2(-4)} = \frac{3}{4}$

$y = -4\left(\frac{3}{4}\right)^2 + 6\left(\frac{3}{4}\right) - 5 = -\frac{11}{4}$

Axis of symmetry: $x = \frac{3}{4}$

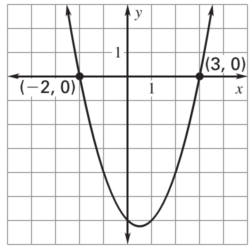


7. $x^2 = x + 6$

$x^2 - x - 6 = 0$

Graph $y = x^2 - x - 6$.

The x-intercepts are -2 and 3, so the solutions of $x^2 = x + 6$ are -2 and 3.



Check:

$(-2)^2 \stackrel{?}{=} (-2) + 6$ $\qquad$ $3^2 \stackrel{?}{=} 3 + 6$

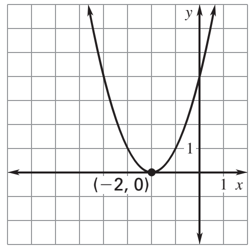
$4 = 4$ ✓ $\qquad$ $9 = 9$ ✓

8. $4x + 4 = -x^2$

$x^2 + 4x + 4 = 0$

Graph $y = x^2 + 4x + 4$.

The x-intercept is -2, so the solution of $4x + 4 = -x^2$ is -2.



Check:

$4(-2) + 4 \stackrel{?}{=} -(-2)^2$

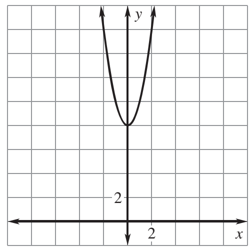
$-4 = -4$ ✓

9. $2x^2 = -8$

$2x^2 + 8 = 0$

Graph $y = 2x^2 + 8$.

There are no x-intercepts, so there are no solutions of $2x^2 = -8$.

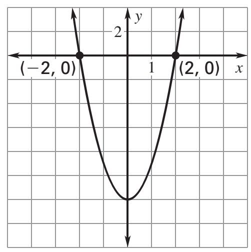


10. $3x^2 + 2 = 14$

$3x^2 - 12 = 0$

Graph $y = 3x^2 - 12$.

The x-intercepts are 2 and -2, so the solutions of $3x^2 + 2 = 14$ are 2 and -2.



Check:

$3(2)^2 + 2 \stackrel{?}{=} 14$ $\qquad$ $3(-2)^2 + 2 \stackrel{?}{=} 14$

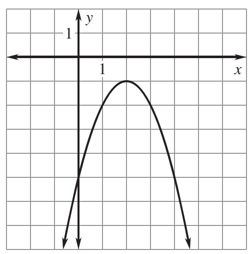
$12 + 2 \stackrel{?}{=} 14$ $\qquad$ $12 + 2 \stackrel{?}{=} 14$

$14 = 14$ ✓ $\qquad$ $14 = 14$ ✓

11. $-x^2 + 4x - 5 = 0$

Graph $y = -x^2 + 4x - 5$.

There are no x-intercepts, so there are no solutions of $-x^2 + 4x - 5 = 0$.

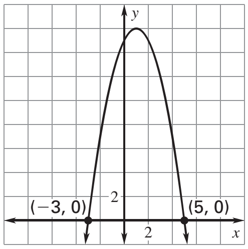


12. $2x - x^2 = -15$

$-x^2 + 2x + 15 = 0$

Graph $y = -x^2 + 2x + 15$.

The x-intercepts are -3 and 5, so the solutions of $2x - x^2 = -15$ are -3 and 5.



Check:

$2(-3) - (-3)^2 \stackrel{?}{=} -15$ $\qquad$ $2(5) - (5)^2 \stackrel{?}{=} -15$

$-6 - 9 \stackrel{?}{=} -15$ $\qquad$ $10 - 25 \stackrel{?}{=} -15$

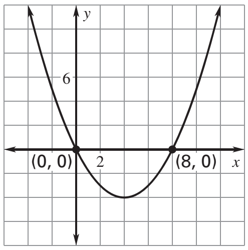
$-15 = -15$ ✓ $\qquad$ $-15 = -15$ ✓

13. $\frac{1}{4}x^2 = 2x$

$\frac{1}{4}x^2 - 2x = 0$

Graph $y = \frac{1}{4}x^2 - 2x$.

The x-intercepts are 0 and 8, so the solutions of $\frac{1}{4}x^2 = 2x$ are 0 and 8.



Check:

$\frac{1}{4}(0)^2 \stackrel{?}{=} 2(0)$ $\qquad$ $\frac{1}{4}(8)^2 \stackrel{?}{=} 2(8)$

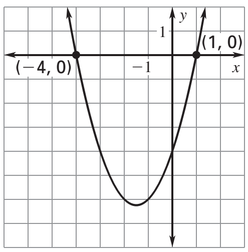
$0 = 0$ ✓ $\qquad$ $16 = 16$ ✓

14. $x^2 + 3x = 4$

$x^2 + 3x - 4 = 0$

Graph $y = x^2 + 3x - 4$.

The x-intercepts are 1 and -4, so the solutions of $x^2 + 3x = 4$ are 1 and -4.



Check:

$(1)^2 + 3(1) \stackrel{?}{=} 4$ $\qquad$ $(-4)^2 + 3(-4) \stackrel{?}{=} 4$

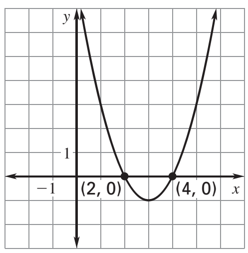
$4 = 4$ ✓ $\qquad$ $4 = 4$ ✓

15. $x^2 + 8 = 6x$

$x^2 - 6x + 8 = 0$

Graph $y = x^2 - 6x + 8$.

The x-intercepts are 2 and 4, so the solutions of $x^2 + 8 = 6x$ are 2 and 4.



Check:

$(2)^2 + 8 \stackrel{?}{=} 6(2)$ $\qquad$ $(4)^2 + 8 \stackrel{?}{=} 6(4)$

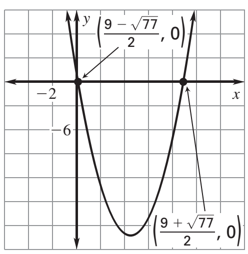
$12 = 12$ ✓ $\qquad$ $24 = 24$ ✓

16. $x^2 = 9x - 1$

$x^2 - 9x + 1 = 0$

Graph $x^2 - 9x + 1 = 0$.

The x-intercepts are $\frac{9 \pm \sqrt{77}}{2}$, so the solutions of $x^2 = 9x - 1$ are $\frac{9 \pm \sqrt{77}}{2}$.



Check:

$\left(\frac{9 + \sqrt{77}}{2}\right)^2 \stackrel{?}{=} 9\left(\frac{9 + \sqrt{77}}{2}\right) - 1$

$\frac{158 + 18\sqrt{77}}{4} \stackrel{?}{=} \frac{81 + 9\sqrt{77}}{2} - 1$

$158 + 18\sqrt{77} \stackrel{?}{=} 2(81 + 9\sqrt{77}) - 4$

$158 + 18\sqrt{77} = 158 + 18\sqrt{77}$ ✓

$\left(\frac{9 - \sqrt{77}}{2}\right)^2 \stackrel{?}{=} 9\left(\frac{9 - \sqrt{77}}{2}\right) - 1$

$\frac{158 - 18\sqrt{77}}{4} \stackrel{?}{=} \frac{81 - 9\sqrt{77}}{2} - 1$

$158 - 18\sqrt{77} \stackrel{?}{=} 2(81 - 9\sqrt{77}) - 4$

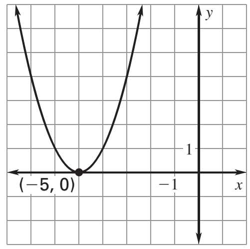
$158 - 18\sqrt{77} = 158 - 18\sqrt{77}$ ✓

17. $-25 = x^2 + 10x$

$0 = x^2 + 10x + 25$

Graph $y = x^2 + 10x + 25$.

The x-intercept is -5, so the solution of $-25 = x^2 + 10x$ is -5.



Check:

$-25 \stackrel{?}{=} (-5)^2 + 10(-5)$

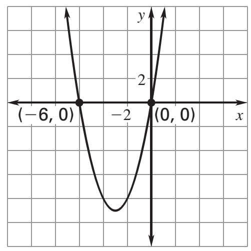
$-25 \stackrel{?}{=} 25 - 50$

$-25 = -25$ ✓

18. $x^2 + 6x = 0$

Graph $y = x^2 + 6x$.

The x-intercepts are -6 and 0, so the solutions of $x^2 + 6x = 0$ are -6 and 0.



Check:

$(0)^2 + 6(0) \stackrel{?}{=} 0$ $\qquad$ $(-6)^2 + 6(-6) \stackrel{?}{=} 0$

$0 = 0$ ✓ $\qquad$ $36 - 36 = 0$ ✓

Extra Practice

For the chapter "Right Triangles and Trigonometry"

1. A Pythagorean triple is 7, 24, 25. Notice that if you multiply the integers of the Pythagorean triple by 2, you get the lengths of the legs of the triangle: $7 \cdot 2 = 14$ and $24 \cdot 2 = 48$. So, the length of the hypotenuse is $25 \cdot 2 = 50$.

2. $(\text{hypotenuse})^2 = (\text{leg})^2 + (\text{leg})^2$

$51^2 = x^2 + 24^2$

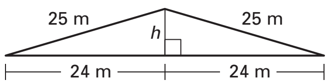
$2601 = x^2 + 576$

$2025 = x^2$

$45 = x$

The length of the leg is 45.

3. A Pythagorean triple is 5, 12, 13. Notice that if you multiply 12 and 13 by 12, you get the lengths of the longer leg and the hypotenuse of the triangle: $12 \cdot 12 = 144$ and $13 \cdot 12 = 156$. So, the length of the shorter leg is $5 \cdot 12 = 60$.

4. 

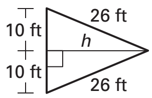
$24^2 + h^2 = 25^2$

$h^2 = 49$

$h = 7$

$A = \frac{1}{2}bh = \frac{1}{2}(48)(7) = 168$

The area is 168 m^2.

5. 

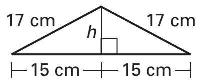
$10^2 + h^2 = 26^2$

$h^2 = 576$

$h = 24$

$A = \frac{1}{2}bh = \frac{1}{2}(20)(24) = 240$

The area is 240 ft^2.

6. 

$15^2 + h^2 = 17^2$

$h^2 = 64$

$h = 8$

$A = \frac{1}{2}bh = \frac{1}{2}(30)(8) = 120$

The area is 120 cm^2.

7. $40^2 \stackrel{?}{=} 24^2 + 32^2$

$1600 \stackrel{?}{=} 576 + 1024$

$1600 = 1600$ ✓

The triangle is a right triangle.

8. $75^2 \stackrel{?}{=} 21^2 + 72^2$

$5625 \stackrel{?}{=} 441 + 5184$

$5625 = 5625$ ✓

The triangle is a right triangle.

9. $27^2 \stackrel{?}{=} 11^2 + 25^2$

$729 \stackrel{?}{=} 121 + 625$

$729 \neq 746$

The triangle is not a right triangle.

10. $13^2 \stackrel{?}{=} 7^2 + 11^2$

$169 \stackrel{?}{=} 49 + 121$

$169 \neq 170$

The triangle is not a right triangle.

11. $(5\sqrt{26})^2 \stackrel{?}{=} 17^2 + 19^2$

$25 \cdot 26 \stackrel{?}{=} 289 + 361$

$650 = 650$ ✓

The triangle is a right triangle.

12. $(\sqrt{181})^2 \stackrel{?}{=} 9^2 + 10^2$

$181 \stackrel{?}{=} 81 + 100$

$181 = 181$ ✓

The triangle is a right triangle.

13. $14 + 21 = 35$ $\quad$ $14 + 25 = 39$

$35 > 25$ $\quad$ $39 > 21$

$21 + 25 = 46$

$46 > 14$

The segment lengths form a triangle.

$25^2 \underline{\ ?\ } 14^2 + 21^2$

$625 \underline{\ ?\ } 196 + 441$

$625 < 637$

The segment lengths 14, 21, and 25 form an acute triangle.

14. $32 + 60 = 92$ $\quad$ $32 + 68 = 100$

$92 > 68$ $\quad$ $100 > 60$

$60 + 68 = 128$

$128 > 32$

The segment lengths form a triangle.

$68^2 \underline{\ ?\ } 32^2 + 60^2$

$4624 \underline{\ ?\ } 1024 + 3600$

$4624 = 4624$

The segment lengths 32, 60, and 68 form a right triangle.

15. $11 + 19 = 30$

$30 < 32$

The segment lengths do not form a triangle.

16. $3\sqrt{11} \approx 9.95$

$3 + 9 = 12$ $\quad$ $3 + 3\sqrt{11} \approx 12.95$

$12 > 3\sqrt{11}$ $\quad$ $12.95 > 9$

$9 + 3\sqrt{11} = 18.95$

$18.95 > 3$

The segment lengths form a triangle.

$(3\sqrt{11})^2 \underline{\ ?\ } 3^2 + 9^2$

$9 \cdot 11 \underline{\ ?\ } 9 + 81$

$99 > 90$

The segment lengths 3, 9, and $3\sqrt{11}$ form an obtuse triangle.

17. $3\sqrt{40} \approx 18.97$

$12 + 15 = 27$ $\quad$ $12 + 3\sqrt{40} \approx 30.97$

$27 > 3\sqrt{40}$ $\quad$ $30.97 > 15$

$15 + 3\sqrt{40} \approx 33.97$

$33.97 > 12$

The segment lengths form a triangle.

$(3\sqrt{40})^2 \underline{\ ?\ } 12^2 + 15^2$

$9 \cdot 40 \underline{\ ?\ } 144 + 225$

$360 < 369$

The segment lengths 12, 15, and $3\sqrt{40}$ form an acute triangle.

18. $4\sqrt{21} \approx 18.33$

$4\sqrt{21} + 25 \approx 43.33$ $\quad$ $4\sqrt{21} + 31 \approx 49.33$

$43.33 > 31$ $\quad$ $49.33 > 25$

$25 + 31 = 56$

$56 > 4\sqrt{21}$

The segment lengths form a triangle.

$31^2 \underline{\ ?\ } (4\sqrt{21})^2 + 25^2$

$961 \underline{\ ?\ } 16 \cdot 21 + 625$

$961 \underline{\ ?\ } 336 + 625$

$961 = 961$

The segment lengths $4\sqrt{21}$, 25, and 31 form a right triangle.

19. $\triangle ABD \sim \triangle BCD \sim \triangle ACB$

$\frac{AB}{AD} = \frac{BC}{BD}$

20. $\triangle GHK \sim \triangle GJH \sim \triangle HJK$

$\frac{KJ}{HJ} = \frac{HJ}{JG}$

21. $\triangle PQR \sim \triangle PSQ \sim \triangle QSR$

$\frac{SR}{RQ} = \frac{RQ}{PR}$

22. $\frac{5}{5} = \frac{5}{x}$

$x = 5$

23. $\frac{1}{y} = \frac{y}{4}$

$4 = y^2$

$2 = y$

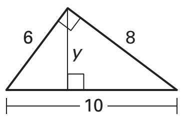
24. $\frac{x+3}{5} = \frac{5}{3}$

$3x + 9 = 25$

$3x = 16$

$x \approx 5.3$

25. Find the hypotenuse of the large triangle.



$h^2 = 6^2 + 8^2$

$h^2 = 36 + 64$

$h^2 = 100$

$h = 10$

$\frac{10}{6} = \frac{8}{y}$

$10y = 48$

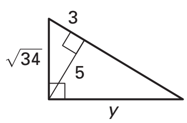
$y = 4.8$

26. $\frac{x}{9} = \frac{9}{7}$

$7x = 81$

$x \approx 11.6$

27. Find the hypotenuse of the smallest triangle.



$h^2 = 3^2 + 5^2$

$h^2 = 9 + 25$

$h^2 = 34$

$h = \sqrt{34}$

$\frac{y}{5} = \frac{\sqrt{34}}{3}$

$y \approx 9.7$

28. leg = leg

$x = 7$

hypotenuse = leg $\cdot \sqrt{2}$

$y = 7\sqrt{2}$

So, $x = 7$ and $y = 7\sqrt{2}$.

29. hypotenuse = 2 • shorter leg

$18 = 2 \cdot g$

$9 = g$

longer leg = shorter leg $\cdot \sqrt{3}$

$h = g\sqrt{3}$

$h = 9\sqrt{3}$

So, $g = 9$ and $h = 9\sqrt{3}$.

30. hypotenuse = leg $\cdot \sqrt{2}$

$9\sqrt{2} = a\sqrt{2}$

$9 = a$

So, $a = 9$ and $b = 9$.

31. longer leg = shorter leg $\cdot \sqrt{3}$

$m = 5\sqrt{3}$

hypotenuse = 2 • shorter leg

$n = 10$

So, $m = 5\sqrt{3}$ and $n = 10$.

32. hypotenuse = leg $\cdot \sqrt{2}$

$15 = s\sqrt{2}$

$\frac{15}{\sqrt{2}} = s$

$\frac{15\sqrt{2}}{2} = s$

So, $s = \frac{15\sqrt{2}}{2}$ and $t = \frac{15\sqrt{2}}{2}$.

33. longer leg = shorter leg $\cdot \sqrt{3}$

$10\sqrt{3} = w\sqrt{3}$

$10 = w$

hypotenuse = 2 • shorter leg

$v = 2w$

$v = 2(10)$

$v = 20$

So, $v = 20$ and $w = 10$.

34. $\tan A = \frac{\text{opp. } \angle A}{\text{adj. to } \angle A} = \frac{18}{27} = \frac{2}{3} \approx 0.6667$

$\tan B = \frac{\text{opp. } \angle B}{\text{adj. to } \angle B} = \frac{27}{18} = \frac{3}{2} = 1.5000$

35. $\tan A = \frac{\text{opp. } \angle A}{\text{adj. to } \angle A} = \frac{60}{100} = \frac{3}{5} = 0.6000$

$\tan B = \frac{\text{opp. } \angle B}{\text{adj. to } \angle B} = \frac{100}{60} = \frac{5}{3} \approx 1.6667$

36. $\tan A = \frac{\text{opp. } \angle A}{\text{adj. to } \angle A} = \frac{24}{56} = \frac{3}{7} \approx 0.4286$

$\tan B = \frac{\text{opp. } \angle B}{\text{adj. to } \angle B} = \frac{56}{24} = \frac{7}{3} \approx 2.3333$

37. $\tan 27° = \frac{x}{12}$

$12 \cdot \tan 27° = x$

$6.1 \approx x$

Check: $\tan 63° \stackrel{?}{=} \frac{12}{6.1}$

$1.9626 \approx 1.97$ ✓

38. $\tan 69° = \frac{25}{x}$

$x \cdot \tan 69° = 25$

$x = \frac{25}{\tan 69°}$

$x \approx 9.6$

Check: $\tan 21° \stackrel{?}{=} \frac{9.6}{25}$

$0.3839 \approx 0.384$

39. $\tan 41° = \frac{x}{19}$

$19 \cdot \tan 41° = x$

$16.5 \approx x$

Check: $\tan 49° \stackrel{?}{=} \frac{19}{16.5}$

$1.1504 \approx 1.1515$

40. $\sin 44° = \frac{x}{14}$

$14 \cdot \sin 44° = x$

$9.7 \approx x$

$\cos 44° = \frac{y}{14}$

$14 \cdot \cos 44° = y$

$10.1 \approx y$

So, $x \approx 9.7$ and $y \approx 10.1$.

41. $\sin 32° = \frac{8}{y}$

$y \cdot \sin 32° = 8$

$y = \frac{8}{\sin 32°}$

$y \approx 15.1$

$\cos 32° \approx \frac{x}{15.1}$

$15.1 \cdot \cos 32° = x$

$12.8 \approx x$

So, $x \approx 12.8$ and $y \approx 15.1$.

42. $\cos 26° = \frac{17}{x}$

$x \cdot \cos 26° = 17$

$x = \frac{17}{\cos 26°}$

$x \approx 18.9$

$\sin 26° \approx \frac{y}{18.9}$

$18.9 \cdot \sin 26° = y$

$8.3 \approx y$

So, $x \approx 18.9$ and $y \approx 8.3$.

43. $\cos 77° = \frac{\sqrt{3}}{y}$

$y \cdot \cos 77° = \sqrt{3}$

$y = \frac{\sqrt{3}}{\cos 77°}$

$y \approx 7.7$

$\sin 77° \approx \frac{x}{7.7}$

$7.7 \cdot \sin 77° = x$

$7.5 \approx x$

So, $x \approx 7.5$ and $y \approx 7.7$.

44. $\cos 54° = \frac{5.7}{y}$

$y \cdot \cos 54° = 5.7$

$y = \frac{5.7}{\cos 54°}$

$y \approx 9.7$

$\sin 54° \approx \frac{x}{9.7}$

$9.7 \cdot \sin 54° = x$

$7.8 \approx x$

So, $x \approx 7.8$ and $y \approx 9.7$.

45. $\sin 14° = \frac{4}{y}$

$y \cdot \sin 14° = 4$

$y = \frac{4}{\sin 14°}$

$y \approx 16.5$

$\cos 14° \approx \frac{x}{16.5}$

$16.5 \cdot \cos 14° = x$

$16.0 \approx x$

So, $x \approx 16.0$ and $y \approx 16.5$.

46. $(DE)^2 = (DF)^2 + (EF)^2$

$(DE)^2 = 12^2 + 5^2$

$(DE)^2 = 144 + 25$

$(DE)^2 = 169$

$DE = 13$

$\tan D = \frac{5}{12}$

$m\angle D = \tan^{-1}\left(\frac{5}{12}\right) \approx 22.6°$

$180° = 90° + 22.6° + m\angle E$

$67.4° \approx m\angle E$

The side lengths are 5 units, 12 units, and 13 units. The angle measures are 90°, about 22.6°, and about 67.4°.

47. $(GH)^2 = (GJ)^2 + (JH)^2$

$(GH)^2 = 6^2 + 7^2$

$(GH)^2 = 36 + 49$

$(GH)^2 = 85$

$GH = \sqrt{85}$

$GH \approx 9.2$

$\tan G = \frac{7}{6}$

$m\angle G = \tan^{-1}\left(\frac{7}{6}\right) \approx 49.4°$

$180° = 90° + 49.4° + m\angle H$

$40.6° \approx m\angle H$

The side lengths are 6 units, 7 units, and about 9.2 units. The angle measures are 90°, about 49.4°, and about 40.6°.

48. $180° = 90° + 25° + m\angle B$

$65° = m\angle B$

$\sin 25° = \frac{AB}{25}$

$25 \cdot \sin 25° = AB$

$10.6 \approx AB$

$\cos 25° = \frac{AC}{25}$

$25 \cdot \cos 25° = AC$

$22.7 \approx AC$

The side lengths are about 10.6 units, about 22.7 units, and 25 units. The angle measures are 90°, 65°, and 25°.