Chapter 6 Similarity

Prerequisite Skills for the chapter "Similarity"

1. The alternate interior angles formed when a transversal intersects two *parallel* lines are congruent.

2. Two triangles are congruent if and only if their corresponding parts are *congruent*.

3. $\frac{9 \cdot 20}{15} = \frac{180}{15} = 12$

4. $\frac{15}{25} = \frac{3}{5}$

5. $\frac{3 + 4 + 5}{6 + 8 + 10} = \frac{12}{24} = \frac{1}{2}$

6. $\frac{2(3 + 8)}{77} = \frac{2(11)}{77} = \frac{2(11)}{7(11)} = \frac{2}{7}$

7. $P = 2\ell + 2w = 2(5) + 2(12) = 10 + 24 = 34$

The perimeter is 34 inches.

8. $P = 2\ell + 2w = 2(30) + 2(10) = 60 + 20 = 80$

The perimeter is 80 feet.

9. Find the width: $A = \ell w$

$56 = 8w$

$7 = w$

Find the perimeter: $P = 2\ell + 2w$

$= 2(8) + 2(7)$

$= 16 + 14$

$= 30$

The perimeter is 30 meters.

10. $y - 4 = 7(x + 2)$

$y - 4 = 7x + 14$

$y = 7x + 18$

The slope of a line parallel to $y - 4 = 7(x + 2)$ is 7.

Lesson 6.1 Use Similar Polygons

Guided Practice for the lesson "Use Similar Polygons"

1. $\angle J \cong \angle P, \angle K \cong \angle Q, \angle L \cong \angle R;$

$\frac{JK}{PQ} = \frac{KL}{QR} = \frac{LJ}{RP}$

2. $\frac{TQ}{DA} = \frac{5}{10} = \frac{1}{2}, \frac{QR}{AB} = \frac{6}{12} = \frac{1}{2}, \frac{TS}{DC} = \frac{8}{16} = \frac{1}{2}$

The scale factor of *QRST* to *ABCD* is $\frac{1}{2}$.

3. $\frac{DC}{TS} = \frac{BC}{RS}$

$\frac{16}{8} = \frac{x}{4}$

$64 = 8x$

$8 = x$

4. Because *ABCDE* is similar to *FGHJK*, the scale factor is the ratio of the lengths, $\frac{15}{10} = \frac{3}{2}$.

5. $\frac{FG}{AB} = \frac{KF}{EA}$

$\frac{15}{10} = \frac{18}{x}$

$180 = 15x$

$12 = x$

6. Perimeter of *FGHJK*: $15 + 9 + 12 + 15 + 18 = 69$ units

Use Theorem 6.1 to find the perimeter x of *ABCDE*.

$\frac{69}{x} = \frac{3}{2}$

$138 = 3x$

$46 = x$

The perimeter of *ABCDE* is 46.

7. Scale factor of $\triangle JKL$ to $\triangle EFG$:

$\frac{JL}{EG} = \frac{96}{80} = \frac{6}{5}$

Because the ratio of the lengths of the medians in similar triangles is equal to the scale factor, you can write the following proportion.

$\frac{KM}{FH} = \frac{6}{5}$

$\frac{x}{35} = \frac{6}{5}$

$x = 42$

The length of the median $\overline{KM}$ is 42.

Exercises for the lesson "Use Similar Polygons"

Skill Practice

1. Two polygons are similar if corresponding angles are *congruent* and corresponding side lengths are *proportional*.

2. Yes; no; If two polygons are congruent, then the corresponding angles are congruent and the corresponding side lengths are congruent. The ratio of the side lengths of congruent sides is 1 : 1, so the corresponding side lengths are proportional. So, two congruent polygons must be similar.

If two polygons are similar, then corresponding angles are congruent and corresponding side lengths are proportional. Because two proportional side lengths are not always congruent, two similar polygons are not always similar.

3. $\angle A \cong \angle L, \angle B \cong \angle M, \angle C \cong \angle N; \frac{AB}{LM} = \frac{BC}{MN} = \frac{CA}{NL}$

4. $\angle D \cong \angle P, \angle E \cong \angle Q, \angle F \cong \angle R, \angle G \cong \angle S;$

$\frac{DE}{PQ} = \frac{EF}{QR} = \frac{FG}{RS} = \frac{GD}{SP}$

5. $\angle H \cong \angle W, \angle J \cong \angle X, \angle K \cong \angle Y, \angle L \cong \angle Z;$

$\frac{HJ}{WX} = \frac{JK}{XY} = \frac{KL}{YZ} = \frac{LH}{ZW}$

6. D; $\triangle ABC \sim \triangle DEF$, so $\frac{AB}{DE} = \frac{BC}{EF}$.

The correct answer is D.

7. All angles are right angles, so corresponding angles are congruent.

$\frac{RS}{WX} = \frac{64}{32} = \frac{2}{1}$ $\quad$ $\frac{ST}{XY} = \frac{48}{24} = \frac{2}{1}$

$\frac{TU}{YZ} = \frac{64}{32} = \frac{2}{1}$ $\quad$ $\frac{UR}{ZW} = \frac{48}{24} = \frac{2}{1}$

The ratios are equal, so the corresponding side lengths are proportional. So, $RSTU \sim WXYZ$. The scale factor of $RSTU$ to $WXYZ$ is $\frac{2}{1}$.

8. You can see that $\angle C \cong \angle T$, $\angle D \cong \angle U$, $\angle E \cong \angle V$. So, corresponding angles are congruent.

$\frac{CD}{TU} = \frac{10}{8} = \frac{5}{4}$, $\quad$ $\frac{DE}{UV} = \frac{5}{4}$, $\quad$ $\frac{EC}{VT} = \frac{12}{9.6} = \frac{120}{96} = \frac{5}{4}$

The ratios are equal, so the corresponding side lengths are proportional. So, $\triangle CDE \sim \triangle TUV$. The scale factor of $\triangle CDE \sim \triangle TUV$ is $\frac{5}{4}$.

9. $\frac{JK}{EF} = \frac{20}{8} = \frac{5}{2}$

The scale factor of $JKLM$ to $EFGH$ is $\frac{5}{2}$.

10. Find x: $\frac{KL}{FG} = \frac{JK}{EF}$ $\qquad$ Find y: $\frac{MJ}{HE} = \frac{JK}{EF}$

$\frac{x}{11} = \frac{20}{8}$ $\qquad$ $\frac{30}{y} = \frac{20}{8}$

$8x = 220$ $\qquad$ $240 = 20y$

$x = 27.5$ $\qquad$ $12 = y$

Find z: $\angle J \cong \angle E$

$65 = z$

11. Perimeter of $EFGH$:
$EF + FG + GH + HE = 8 + 11 + 3 + 12 = 34$

Perimeter of $JKLM$: Use Theorem 6.1 to find the perimeter x.

$\frac{x}{34} = \frac{5}{2}$

$2x = 170$

$x = 85$

The periemter of $EFGH$ is 34 and the perimeter of $JKLM$ is 85.

12. Let x be the small sign's perimeter.

$\frac{60 \text{ in.}}{x \text{ in.}} = \frac{5}{3}$

$180 = 5x$

$36 = x$

The small sign's perimeter is 36 inches.

13. The scale factor was used incorrectly.

Scale factor of A to B: $\frac{10}{5} = \frac{2}{1}$

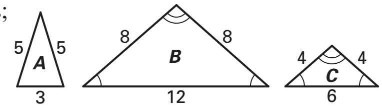
Perimeter of A: $10 + 12 + 6 = 28$

Perimeter of B: $\frac{28}{x} = \frac{2}{1}$

$x = 14$

The perimeter of B is 14.

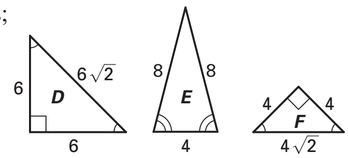
14. Sometimes;



$\triangle B \sim \triangle C$, but $\triangle A \nsim \triangle B$.

15. Always; The angles of all equilateral triangles are congruent, so corresponding angles are always congruent. Because the sides of an equilateral triangle are congruent, the ratios of corresponding side lengths of two equilateral triangles are always congruent. So, two equilateral triangles are always similar.

16. Sometimes;



$\triangle D \sim \triangle F$, but $\triangle D \nsim \triangle E$.

17. Never; A scalene triangle has no congruent sides and an isosceles triangle has at least two congruent sides. So, the ratios of corresponding side lengths of a scalene triangle and an isosceles triangle can never all be equal. So, a scalene triangle and an isosceles triangle are never similar.

18. $x:1$; The definition states that the "ratio of a to b is $a:b$. You can determine that the "ratio of b to a" is $b:a$. So, switch the order of the given ratio.

19. The special segment shown in blue is the altitude.

$\frac{27}{18} = \frac{x}{16}$

$432 = 18x$

$24 = x$

20. The special segment shown in blue is the median.

$\frac{18}{y} = \frac{16}{y - 1}$

$18y - 18 = 16y$

$2y = 18$

$y = 9$

21. $\frac{6}{8} = \frac{8}{x}$ $\qquad$ $\frac{6}{8} = \frac{10}{y}$

$6x = 64$ $\qquad$ $6y = 80$

$x = 10\frac{2}{3}$ $\qquad$ $y = 13\frac{1}{3}$

The other two sides of $\triangle RST$ are $10\frac{2}{3}$ inches and $13\frac{1}{3}$ inches.

22. Scale factor: $\frac{6}{8} = \frac{3}{4}$

$$\frac{4.8}{x} = \frac{3}{4}$$
$$19.2 = 3x$$
$$6.4 = x$$

The length of the corresponding altitude in $\triangle RST$ is 6.4 inches.

23. $\frac{BC}{EF} = \frac{19\frac{4}{5}}{9} = \frac{19.8}{9} = \frac{198}{90} = \frac{11}{5}$

The scale factor of $\triangle ABC$ to $\triangle DEF$ is $\frac{11}{5}$.

24. Find DE: $\frac{AB}{DE} = \frac{11}{5}$

$$\frac{22}{x} = \frac{11}{5}$$
$$110 = 11x$$
$$10 = x$$

Find AC: $\frac{AC}{DF} = \frac{11}{5}$

$$\frac{y}{10\frac{2}{5}} = \frac{11}{5}$$
$$5y = 114.4$$
$$y = 22.88$$

The length of $\overline{DE}$ is 10 and the length of $\overline{AC}$ is 22.88.

25. $\frac{x}{8} = \frac{11}{5}$

$$5x = 88$$
$$x = 17.6$$

The length of the altitude shown in $\triangle ABC$ is 17.6.

26. Area of $\triangle ABC$: $A = \frac{1}{2}bh$

$$= \frac{1}{2}(22.88)(17.6)$$
$$= 201.344$$

Area of $\triangle DEF$: $A = \frac{1}{2}bh$

$$= \frac{1}{2}\left(10\frac{2}{5}\right)(8)$$
$$= 41.6$$

The ratio of the area of $\triangle ABC$ to $\triangle DEF$ is $\frac{201.344}{41.6} = 4.84$, which is the square of the scale factor $\left(\frac{11^2}{5^2} = \frac{121}{25} = 4.84\right)$.

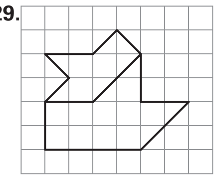
27. No; Because the triangles are similar, the angle measures are congruent. So, the extended ratio of the angle measures in $\triangle XYZ$ is $x : x + 30 : 3x$.

28. D; Other leg of $\triangle UVW$: $\frac{4}{x} = \frac{3}{4.5}$

$$3x = 18$$
$$x = 6$$

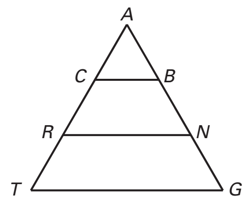
So, the legs of $\triangle UVW$ are 4.5 feet and 6 feet. The hypotenuse is the longest side, so it must be greater than 6 feet. The correct answer is D.

29.



30. Similarity is reflexive, symmetric, and transitive.

Sample answer:



Given: $\triangle RAN \sim \triangle TAG$
$\triangle TAG \sim \triangle CAB$

Reflexive: $\triangle RAN \sim \triangle RAN$

Symmetric: $\triangle RAN \sim \triangle TAG$, so $\triangle TAG \sim \triangle RAN$.

Transitive: $\triangle RAN \sim \triangle TAG$ and $\triangle TAG \sim \triangle CAB$, so $\triangle RAN \sim \triangle CAB$.

Problem Solving

31. $\frac{\text{length of court}}{\text{length of table}} = \frac{78 \text{ ft}}{9 \text{ ft}} = \frac{26}{3}$

$\frac{\text{width of court}}{\text{width of table}} = \frac{36 \text{ ft}}{5 \text{ ft}} = \frac{36}{5}$

The ratios are not equal, so the corresponding side lengths are not proportional, and the surfaces are not similar.

32. $\frac{\text{width of computer screen}}{\text{width of projected image}} = \frac{13.25 \text{ in.}}{53 \text{ in.}} = \frac{1}{4}$

$\frac{\text{height of computer screen}}{\text{height of projected image}} = \frac{10.6 \text{ in.}}{42.4 \text{ in.}} = \frac{1}{4}$

The ratios are equal, so the corresponding side lengths are proportional, and the surfaces are similar. The scale factor of the computer screen to the projected image is $\frac{1}{4}$.

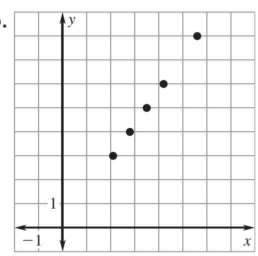
33. a.

$\frac{BC}{4} = \frac{7}{10}$ $\quad \frac{CD}{6} = \frac{7}{10}$ $\quad \frac{DE}{8} = \frac{7}{10}$ $\quad \frac{EA}{3} = \frac{7}{10}$

$BC = 2.8$ $\quad CD = 4.2$ $\quad DE = 5.6$ $\quad EA = 2.1$

	AB	BC	CD	DE	EA
Fig. 1	3.5	2.8	4.2	5.6	2.1
Fig. 2	5.0	4.0	6.0	8.0	3.0

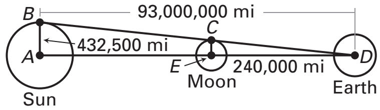
b.



Yes, the relationship is linear because the points lie in a line.

c. Because $\frac{x}{4} = \frac{7}{10}$, you know that $10x = 7y$. So, an equation is $y = \frac{10}{7}x$. The slope is $\frac{10}{7}$. The slope and the scale factor are the same.

34. a.



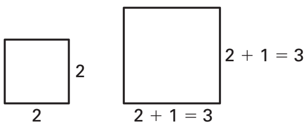
b. *Sample answer:* Because $\triangle BDA \sim \triangle CDE$, the sun's light that would normally reach Earth in $\triangle BDA$ is blocked by the moon, preventing the light from entering $\triangle CDE$.

c.
$$\frac{ED}{DA} = \frac{CE}{AB}$$
$$\frac{240{,}000}{93{,}000{,}000} = \frac{r}{432{,}500}$$
$$103{,}800{,}000{,}000 = 93{,}000{,}000r$$
$$1116.13 \approx r$$

The radius of the moon is about 1116 miles.

35. Yes; the images are similar if the original image is a square. The result will be a square, and all squares are similar.

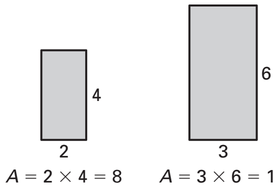
Sample answer:



The figures are similar squares with a scale factor of $\frac{2}{3}$.

36. The ratio of the areas of similar rectangles is the square of the scale factor.

Sample answer:



Scale factor: $\frac{3}{2}$

Ratio of Areas: $\frac{18}{8} = \frac{9}{4}$

$= \left(\frac{3}{2}\right)^2$

37. a. The two lines are parallel because they have the same slope.

b. $\angle BOA \cong \angle DOC$ by the Vertical Angles Theorem.
$\angle OBA \cong \angle ODC$ by the Alternate Interior Angles Theorem.
$\angle BAO \cong \angle DCO$ by the Alternate Interior Angles Theorem.

c. Coordinates of A: $(x, 0)$

$y = \frac{4}{3}x + 4$

$0 = \frac{4}{3}x + 4$

$-4 = \frac{4}{3}x$

$-3 = x$

Coordinates of C: $(x, 0)$

$y = \frac{4}{3}x - 8$

$0 = \frac{4}{3}x - 8$

$8 = \frac{4}{3}x$

$6 = x$

Coordinates of B: $(0, y)$

$y = \frac{4}{3}x + 4$

$y = \frac{4}{3}(0) + 4$

$y = 4$

Coordinates of D: $(0, y)$

$y = \frac{4}{3}x - 8$

$y = \frac{4}{3}(0) - 8$

$y = -8$

The coordinates of A, B, C, and D are $(-3, 0)$, $(0, 4)$, $(6, 0)$, and $(0, -8)$, respectively.

Lengths of sides of $\triangle AOB$:

$OA = |0 - (-3)| = 3$

$OB = |0 - 4| = 4$

$AB = \sqrt{(4 - 0)^2 + [0 - (-3)]^2} = \sqrt{25} = 5$

Lengths of sides of $\triangle COD$:

$OC = |0 - 6| = 6$

$OD = |0 - (-8)| = 8$

$CD = \sqrt{(-8 - 0)^2 + (0 - 6)^2} = \sqrt{100} = 10$

d. $\frac{OA}{OC} = \frac{3}{6} = \frac{1}{2}$ $\quad \frac{OB}{OD} = \frac{4}{8} = \frac{1}{2}$ $\quad \frac{AB}{CD} = \frac{5}{10} = \frac{1}{2}$

The ratios are equal, so the corresponding side lengths are proportional. Because corresponding angles are congruent and corresponding side lengths are proportional, $\triangle AOB \sim \triangle COD$.

38. Let $ABCD$ and $FGHJ$ be similar rectangles.

The scale factor of $ABCD$ to $FGHJ$ is $\frac{AB}{FG}$. Let $k = \frac{AB}{FG}$.

$$\text{So, } \frac{AB + BC + CD + DA}{FG + GH + HJ + JF} = \frac{kFG + kGH + kHJ + kJF}{FG + GH + HJ + JF}$$

$$= \frac{k\cancel{(FG + GH + HJ + JF)}}{\cancel{FG + GH + HJ + JF}}$$

$$= k$$

$$= \frac{AB}{FG}.$$

Because $ABCD \sim FGHJ$, you know that $\frac{AB}{FG} = \frac{BC}{GH} = \frac{CD}{HJ} = \frac{DA}{JF}$.

$$\text{So, } \frac{AB + BC + CD + DA}{FG + GH + HJ + JF} = \frac{AB}{FG} = \frac{BC}{GH} = \frac{CD}{HJ} = \frac{DA}{JF}.$$

39.
$$\frac{MS}{RQ} = \frac{LM}{MR}$$

$$\frac{x}{1} = \frac{1}{x - 1}$$

$$x(x - 1) = 1$$

$$x^2 - x - 1 = 0$$

Quadratic Formula:

$$\frac{-6 \pm \sqrt{b^2 - 4ac}}{2a} = \frac{1 \pm \sqrt{1 + 4}}{2} = \frac{1 \pm \sqrt{5}}{2}$$

You can disregard $\frac{1 - \sqrt{5}}{2}$ because it is a negative number.

The exact value of x is $\frac{1 + \sqrt{5}}{2}$.

$$\frac{MS}{LM} = \frac{\frac{1 + \sqrt{5}}{2}}{1} = \frac{1 + \sqrt{5}}{2}$$

So, $PLMS$ is a golden rectangle.

$$\frac{LM}{MR} = \frac{1}{x - 1}$$

$$= \frac{1}{\frac{1 + \sqrt{5}}{2} - 1}$$

$$= \frac{1}{\frac{\sqrt{5} - 1}{2}}$$

$$= \frac{2}{\sqrt{5} - 1} \cdot \frac{(\sqrt{5} + 1)}{(\sqrt{5} + 1)}$$

$$= \frac{2(\sqrt{5} + 1)}{4}$$

$$= \frac{1 + \sqrt{5}}{2}$$

So, $LMRQ$ is a golden rectangle.

Investigating Geometry Activity for the lesson "Relate Transformations and Similarity"

1–5. Sample answers are given.

1. $AB = \sqrt{10}$, $AC = \sqrt{10}$, $BC = 2\sqrt{2}$, $AD = 2\sqrt{10}$, $AE = 2\sqrt{10}$, $DE = 4\sqrt{2}$; $m\angle A \approx 53°$, $m\angle B = m\angle D \approx 63.5°$, $m\angle C = m\angle E \approx 63.5°$; the dilation does not preserve lengths but it does preserve angle measures.

2. The ratios are all equal to the scale factor, 2.

3. The center of dilation (0, 0) is mapped to $(k \cdot 0, k \cdot 0) = (2 \cdot 0, 2 \cdot 0) = (0, 0)$. The center of dilation is mapped to itself.

4. The lines are the same; the image of a line that passes through the center of dilation is the same as the preimage line; the center of dilation is mapped to itself, and any line that contains the origin is mapped to a line that contains the origin.

5. The lines are parallel; the image of a line that does not pass through the center of dilation is a line parallel to the preimage line; because $\angle C \cong \angle E$, the lines are parallel by the Corresponding Angles Converse.

Lesson 6.2 Relate Transformations and Similarity

Guided Practice for the lesson "Relate Transformations and Similarity"

1. The figures show a dilation with center B. The scale factor is $\frac{7}{3}$ because the ratio of BE to BC is 14:6, or 7:3.

2. The figures show a dilation with center D and a scale factor of $\frac{2}{3}$ because the ratio of DX to DA is 6:9, or 2:3. The figure is then reflected across the line passing through point D that is perpendicular to $\overline{DX}$.

3–6. Sample answers are given.

3. The red hexagon has all sides congruent, but the blue hexagon has 3 shorter sides and 3 longer sides, so ratios of corresponding side lengths are not constant.

4. The transformations are a dilation followed by a rotation of 30° about the center of the figures.

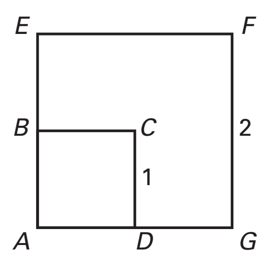
5. All angles are congruent, so angle measure is preserved, and all side lengths are congruent in each hexagon, so the ratio of any two corresponding side lengths is constant.

6. No; even though corresponding sides might be proportional, if corresponding angles are not congruent, the polygons are not similar.

Exercises for the lesson "Relate Transformations and Similarity"

Skill Practice

1. *Sample*:



The figure shows a dilation with center A and scale factor of 2:1.

2. *Sample answer:* A similarity transformation maps one figure onto a similar figure. The corresponding sides have lengths that are proportional and the corresponding angles have the same measure.

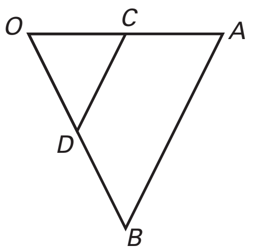
3. The figure shows a dilation with center at the intersection of the black lines and a scale factor of $\frac{3}{2}$.

4. The figure shows a dilation with center at the intersection of the black lines and a scale factor of $\frac{1}{2}$ because the ratio is 5:10, or 1:2.

5. The figure shows a dilation with center at the intersection of the black lines and a scale factor of $\frac{7}{4}$ because the ratio is 14:8, or 7:4.

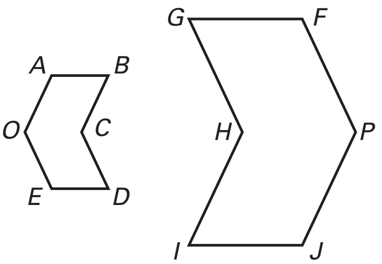
6. The function notation is for a dilation with scale factor 3. Corresponding sides will be proportional with a ratio of 3 to 1. Choosing a sample figure and drawing its image will show that the corresponding angles of the figures have the same measure.

7–9. Sample figures are given.



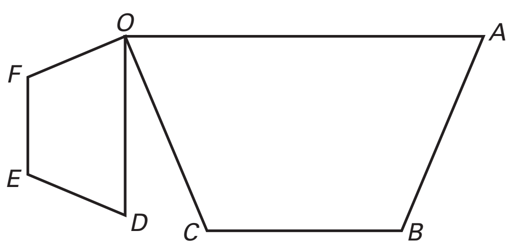
The figure shows a dilation with center O of $\triangle OAB$ onto $\triangle OCD$.

8.



The figure shows a dilation of hexagon *OABCDE* with center O followed by a reflection over the perpendicular bisector of the segment joining points O and P.

9.



The figure shows a dilation of quadrilateral *OABC* with center O followed by a 90° clockwise rotation around O onto quadrilateral *ODEF*.

10. The figures in answer choice C are exactly the same size, so no dilation has occurred. The correct answer is C.

11. In a dilation, the ratio of corresponding sides would be constant. This is not the case when comparing the red and blue figures. So the transformation does not involve a dilation.

12. The measure of each of the unknown sides of the red figure is $3\sqrt{2}$. The ratio of corresponding sides of the red figure to the blue figure is $3\sqrt{2}:3$, or $\sqrt{2}:1$. So the transformation does involve a dilation and the scale factor of the dilation is $\sqrt{2}$.

13. The first transformation is a dilation with center O and scale factor 2 because the ratio of corresponding sides is 2:1. The second transformation is a reflection in the x-axis because each ordered pair (x, y) in the image after the first transformation corresponds to the ordered pair $(x, -y)$ in the final image.

14. The first transformation is a dilation with center O and scale factor $\frac{1}{2}$ because the ratio of corresponding sides is 1:2. The second transformation is a rotation 90° clockwise around O because each ordered pair (x, y) in the image after the first transformation corresponds to the ordered pair $(y, -x)$ in the final image.

15. The first transformation is a dilation with center O and scale factor $\frac{3}{2}$ because the ratio of corresponding sides is 3:2. The second transformation is a reflection in the y-axis because each ordered pair (x, y) in the image after the first transformation corresponds to the ordered pair $(-x, y)$ in the final image.

16. The first transformation is a dilation with center O and scale factor $\frac{1}{3}$ because the ratio of corresponding sides is 1:3. The second transformation is a rotation 90° counterclockwise around O because each ordered pair (x, y) in the image after the first transformation corresponds to the ordered pair $(-y, x)$ in the final image.

17. *Sample answer*: The lengths are 8, 4, and 2; dilate the previous stage 3 times with scale factor $\frac{1}{2}$ using each corner of the triangle as a center to generate the next stage.

18–21. Check students' drawings.

18. The two circles share the same center point which will serve as the center of the dilation. The scale factor of the dilation is the ratio of the radii of the circles which in this case is $\frac{6}{2}$, or 3.

19. To find the center of dilation, draw a line through the endpoints of the radii that lie on the circles. Draw a line through the centers of the circles. The center of dilation is the intersection of the lines. The scale factor of the dilation is the ratio of the radii of the circles which is $\frac{3}{5}$.

20. To find the center of dilation, draw a line through the endpoints of the radii that lie on the circles. Draw a line through the centers of the circles. The center of dilation is the intersection of the lines. The scale factor of the dilation is the ratio of the radii of the circles which is $\frac{5}{4}$.

21. To find the center of dilation, draw a line through the endpoints of the radii that lie on the circles. Draw a line through the centers of the circles. The center of dilation is the intersection of the lines. The scale factor of the dilation is the ratio of the radii of the circles which is $\frac{1}{2}$.

22. a. *Sample answer*: Let x and y represent the length and width of the rectangle. The perimeter of the preimage would then be $x + x + y + y$, or $2x + 2y$. Each side of the image will be 4 times as long as the corresponding side of the preimage. So, the perimeter of the image would be $4x + 4x + 4y + 4y = 8x + 8y$, or $4(2x + 2y)$. Therefore the perimeter of the image is 4 times the perimeter of the preimage.

b. *Sample answer*: Let x and y represent the length and width of the rectangle. The area of the preimage would then be xy. The length and width of the image are 4 times the corresponding length and width of the preimage. So, the area of the image would be $(4x)(4y) = 16xy$. Therefore the area of the image is 16 times the area of the preimage.

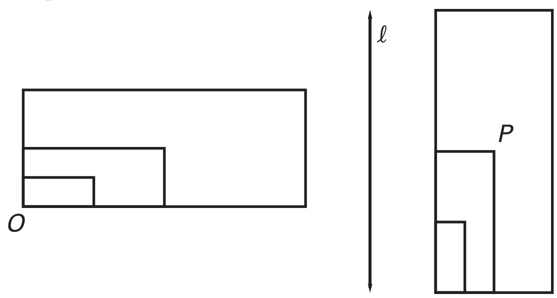
Problem Solving

23. *Sample answer*: Because the purses are similar, the designs of the purses should have the same shape but not the same size. Use a copy machine to enlarge the pattern from the smaller purse. For a purse twice as big, use a setting of 200% on the copy machine. Then transfer the pattern to the larger purse.

24. *Sample answer*: An overhead projector enlarges a figure onto a screen as a function of the distance from the projector to the screen. In place of the screen, affix a poster board. The pattern can be traced onto the board.

25. *Sample answer*: First dilate figure A so that it is congruent to figure B. Then rotate the image about the midpoint of the segment joining the tip of figure B and the tip of the image.

26. *Sample*:



27. The scale factor of the dilation is $\frac{10}{2.5} = 4$. Therefore, the height of the bug on the wall will be 4 times the height of the bug on the flashlight cap; $4 \cdot 2 \text{ cm} = 8 \text{ cm}$.

28. *Sample answer*: The figure shows a dilation with center at P. You want the distance of each bolt from P to be $\frac{1}{5}$ greater than the distance from the previous bolt, so the scale factor is $1 + \frac{1}{5} = \frac{6}{5}$.

29. *Sample answer*: Draw corresponding radii in the circles parallel to each other. Draw a line through the endpoints of the radii that lie on the circles. Draw a line through the centers of the circles. The center of dilation is the intersection of the lines. The scale factor of the dilation is the ratio of the radii of the circles.

30. a. *Sample answer*: The floor plan is a model. Each part of the actual house will copy the floor plan and be a dilation of the shape. The final size of every inch on the scale drawing will be 2 feet, or 24 inches, so the scale factor is 24 to 1.

b. *Sample answer*: Multiply each dimension on the floor plan by 24 to find the actual size in inches. Then divide the result by 12 to find the actual size in feet.

Length of living room:

$10 \cdot 24 = 240$ in.

$240 \div 12 = 20$ ft

Width of living area:

$5 \cdot 24 = 120$ in.

$120 \div 12 = 10$ ft

So the living room that is 10 inches by 5 inches on the floor plan is 20 feet by 10 feet.

c. For each room, multiply each dimension on the floor plan by 24 to find the actual size in inches and then divide by 12 to find the actual size in feet.

Bedroom 1:	15 ft by 10 ft
Storage:	5 ft by 10 ft
Den:	8 ft by 10 ft
Bedroom 2:	12 ft by 10 ft
Bath:	8 ft by 6 ft
Kitchen:	16 ft by 8 ft
Living:	20 ft by 10 ft

31. *Sample answer*: An isosceles right triangle with vertices (0, 0), (1, 0), and (1,1) is mapped to a triangle with vertices (0, 0), (2, 0), and (2, 5). The image triangle is not isosceles; its legs are not the same length and its acute angles do not have the same measure. Image side lengths are not in proportion to preimage side lengths, and angles are not preserved, so the transformation is not a similarity transformation.

32. a. The slope of an image side is the same as the slope of a corresponding preimage side.

Side 1:

Preimage vertices: (e, f) and (a, b)

Image vertices: (ke, kf) and (ka, kb)

Preimage slope: $\frac{b - f}{a - e}$

Image slope: $\frac{kb - kf}{ka - ke} = \frac{k(b - f)}{k(a - e)} = \frac{b - f}{a - e}$

Side 2:

Preimage vertices: (a, b) and (c, d)

Image vertices: (ka, kb) and (kc, kd)

Preimage slope: $\frac{d - a}{c - a}$

Image slope: $\frac{kd - kb}{kc - ka} = \frac{k(d - b)}{k(c - a)} = \frac{d - b}{c - a}$

Side 3:

Preimage vertices: (c, d) and (e, f)

Image vertices: (kc, kd) and (ke, kf)

Preimage slope: $\frac{f - d}{e - c}$

Image slope: $\frac{kf - kd}{ke - kc} = \frac{k(f - d)}{k(e - c)} = \frac{f - d}{e - c}$

b. *Sample answer*: The black ray that passes through (a, b) and (ka, kb) is a transversal intersecting parallel segments, so the angles marked are congruent. Therefore, their sums are also equal. Similar reasoning applies to the other angles of the triangle. So, angles are preserved under the dilation.

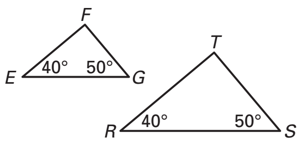
Investigating Geometry Activity for the lesson "Prove Triangle Similar by AA"

1. $m\angle C = m\angle F = m\angle J$; the third angles of the triangles are congruent because the sum of the measures of the angles of a triangle is 180°.
2. Dilations preserve angle measures, so corresponding angles of $\triangle DEF$ and $\triangle GHJ$ are congruent.
3. Check students' work. The ratios should be equal to the scale factor of the dilation.
4. $\angle A \cong \angle G$ and $\angle B \cong \angle H$ because dilations preserve angle measures, and $\overline{AB} \cong \overline{GH}$ because this was given in Step 1. So, $\triangle GHJ \cong \triangle ABC$ by ASA.
5. A dilation with scale factor $\frac{GO}{DO}$ and center O, followed by a translation a distance of GA maps $\triangle DEF$ onto $\triangle ABC$.

Lesson 6.3 Prove Triangles Similar by AA

Activity for the lesson "Prove Triangles Similar by AA"

Sample answer:



$180° = m\angle F + m\angle F + m\angle G$

$180° = 40° + m\angle F + 50°$

$90° = m\angle F$

$EF \approx 15$ mm

$FG \approx 13$ mm

$GE \approx 20$ mm

$180° = m\angle R + m\angle S + m\angle T$

$180° = 40° + m\angle S + 50°$

$90° = m\angle S$

$RS \approx 22.5$ mm

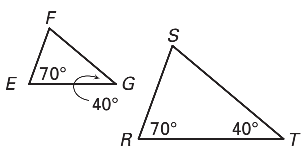
$ST \approx 19.5$ mm

$TR \approx 30$ mm

1. *Sample answer:*

   $\frac{EF}{RS} \approx \frac{15}{22.5} = \frac{2}{3}$ $\quad \frac{FG}{ST} \approx \frac{13}{19.5} = \frac{2}{3}$ $\quad \frac{GE}{TR} \approx \frac{20}{30} = \frac{2}{3}$

   Corresponding angles are congruent and corresponding side lengths are proportional, so the triangles are similar.
2. *Sample answer:*



$180° = m\angle E + m\angle F + m\angle G$

$180° = 70° + m\angle F + 40°$

$70° = m\angle F$

$EF \approx 18$ mm

$FG \approx 27$ mm

$GE \approx 27$ mm

$180° = m\angle R + m\angle S + m\angle T$

$180° = 70° + m\angle S + 40°$

$70° = m\angle S$

$RS \approx 30$ mm

$ST \approx 45$ mm

$TR \approx 45$ mm

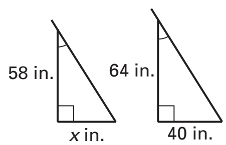
$\frac{EF}{RS} \approx \frac{18}{30} = \frac{3}{5}$ $\quad \frac{FG}{ST} \approx \frac{27}{45} = \frac{3}{5}$ $\quad \frac{GE}{TR} \approx \frac{27}{45} = \frac{3}{5}$

So, the triangles are similar.

Conjecture: Two triangles with two pairs of congruent corresponding angles are similar.

Guided Practice for the lesson "Prove Triangles Similar by AA"

1. Because $\triangle FGH$ and $\triangle QRS$ are equiangular, all angles measure 60°. So, all angles are congruent and $\triangle FGH \sim \triangle QRS$ by the AA Similarity Postulate.
2. Because they are both right angles, $\angle DFC$ and $\angle EFD$ are congruent. By the Triangle Sum Theorem, $32° + 90° + m\angle CDF = 180°$, so $m\angle CDF = 58°$. Therefore, $\angle CDF$ and $\angle DEF$ are congruent.

   So, $\triangle CDF \sim \triangle DEF$ by the AA Similarity Postulate.
3. Yes; if $\angle S \cong \angle T$ (or $\angle R \cong \angle U$), then the triangles are similar by the AA Similarity Postulate.
4. 

   $\frac{64 \text{ in.}}{58 \text{ in.}} = \frac{40 \text{ in.}}{x \text{ in.}}$

   $64x = 2320$

   $x = 36.25$

   The child's shadow is 36.25 inches long.
5. $\frac{\text{Tree height}}{\text{Your height}} = \frac{\text{length of tree shadow}}{\text{length of your shadow}}$

Exercises for the lesson "Prove Triangles Similar by AA"

Skill Practice

1. If two angles of one triangle are congruent to two angles of another triangle, then the triangles are *similar*.
2. No; the corresponding sides of two similar triangles are proportional, so they are not necessarily congruent.
3. $\triangle ABC \sim \triangle FED$ by the AA Similarity Postulate.

4. $\frac{BA}{EF} = \frac{AC}{FD} = \frac{CB}{DE}$ because the ratios of corresponding side lengths in similar triangles are equal.

5. $\frac{25}{15} = \frac{y}{12}$ because $\frac{AC}{FD} = \frac{BA}{EF}$.

6. $\frac{15}{25} = \frac{18}{x}$ because $\frac{FD}{AC} = \frac{DE}{CB}$.

7. $y = 20$; $\frac{25}{15} = \frac{y}{12}$

$15y = 300$

$y = 20$

8. $x = 30$; $\frac{15}{25} = \frac{18}{x}$

$15x = 450$

$x = 30$

9. Because they are both right angles, $\angle H$ and $\angle S$ are congruent.

By the Triangle Sum Theorem, $48° + 90° + m\angle F = 180°$, so $m\angle F = 42°$. Therefore, $\angle F$ and $\angle K$ are congruent. So, $\triangle FGH \sim \triangle KLJ$ by the AA Similarity Postulate

10. Because $m\angle YNM$ and $m\angle YZX$ both equal 45°, $\angle YNM \cong \angle YZX$. By the Vertical Angles Congruence Theorem, $\angle NYM \cong \angle ZYX$. So, $\triangle YNM \sim \triangle YZX$ by the AA Similarity Postulate.

11. By the Triangle Sum Theorem, $35° + 85° + m\angle R = 180°$ and $35° + 65° + m\angle V = 180°$. So, $m\angle R = 60°$ and $m\angle V = 80°$.

Corresponding angles are not congruent, so the triangles are not similar.

12. Because $m\angle EAC$ and $m\angle DBC$ both equal 65°, $\angle EAC \cong \angle DBC$. By the Reflexive Property, $\angle C \cong \angle C$.

So, $\triangle ACE \sim \triangle BCD$ by the AA Similarity Postulate.

13. By the Reflexive Property, $\angle Y \cong \angle Y$. By the Triangle Sum Theorem, $45° + 85° + m\angle YZX = 180°$, so $m\angle YZX = 50°$. Therefore, $\angle YZX$ and $\angle YWU$ are congruent.

So, $\triangle YZX \sim \triangle YWU$ by the AA Similarity Postulate.

14. By the Reflexive Property, $\angle N \cong \angle N$. By the Corresponding Angles Postulate, $\angle NMP \cong \angle NLQ$. So, $\triangle NMP \sim \triangle NLQ$ by the AA Similarity Postulate.

15. The AA Similarity Postulate is for triangles, not other polygons.

16. B; $\frac{24}{12} = \frac{p}{10}$

$240 = 12p$

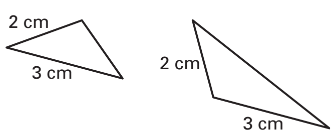
$20 = p$

The length of p is 20, so the correct answer is B.

17. The proportion is incorrect because 5 is not the length of the corresponding side of the larger triangle.

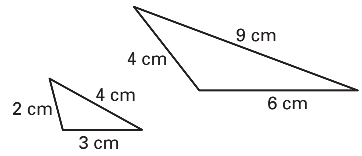
Sample answer: A correct proportion is $\frac{4}{6} = \frac{9}{x}$.

18. *Sample answer:*



The sketch shows that corresponding side lengths are not proportional.

19. *Sample answer:*



The sketch shows that corresponding side lengths are not proportional.

20. A; Find x: $\frac{CE}{AE} = \frac{DE}{BE}$

$\frac{3}{4} = \frac{5}{x}$

$3x = 20$

$x = \frac{20}{3}$

Find BD: $BD = BE + DE$

$= \frac{20}{3} + 5$

$= \frac{35}{3}$

Because $BD = \frac{35}{3}$, the correct answer is A.

21. length of $\overline{AB} = 4 - 0 = 4$

length of $\overline{AD} = 5 - 0 = 5$

length of $\overline{AC} = 8 - 0 = 8$

$\frac{AD}{AB} = \frac{AE}{AC}$

$\frac{5}{4} = \frac{AE}{8}$

$10 = AE$

The length of $\overline{AE}$ is 10, so the coordinates are $E(10, 0)$.

22. length of $\overline{AB} = 3 - 0 = 3$

length of $\overline{AD} = 7 - 0 = 7$

length of $\overline{AC} = 4 - 0 = 4$

$\frac{AD}{AB} = \frac{AE}{AC}$

$\frac{7}{3} = \frac{AE}{4}$

$AE = \frac{28}{3}$

The length of $\overline{AE}$ is $\frac{28}{3}$, so the coordinates are $E\left(\frac{28}{3}, 0\right)$.

23. length of $\overline{AB} = 1 - 0 = 1$

length of $\overline{AD} = 4 - 0 = 4$

length of $\overline{AC} = 6 - 0 = 6$

$\frac{AD}{AB} = \frac{AE}{AC}$

$\frac{4}{1} = \frac{AE}{6}$

$24 = AE$

The length of $\overline{AE}$ is 24, so the coordinates are $E(24, 0)$.

24. length of $\overline{AB} = 6 - 0 = 6$

length of $\overline{AD} = 9 - 0 = 9$

length of $\overline{AC} = 3 - 0 = 3$

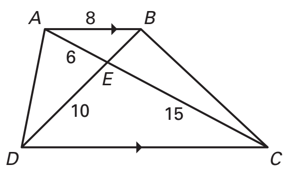
$$\frac{AD}{AB} = \frac{AE}{AC}$$

$$\frac{9}{6} = \frac{AE}{3}$$

$$\frac{9}{2} = AE$$

The length of $\overline{AE}$ is $\frac{9}{2}$, so the coordinates are $E\left(\frac{9}{2}, 0\right)$.

25. a.



b. *Sample answer:*

$\angle AEB \cong \angle CED$ by the Vertical Angles Congruence Theorem.

$\angle ABE \cong \angle CDE$ by the Alternate Interior Angles Theorem.

c. $\triangle AEB$ is similar to $\triangle CED$. $\triangle AEB \sim \triangle CED$ by the AA Similarity Postulate.

d. $\frac{BE}{DE} = \frac{AE}{CE}$ $\qquad$ $\frac{BA}{DC} = \frac{AE}{CE}$

$\frac{BE}{10} = \frac{6}{15}$ $\qquad$ $\frac{8}{DC} = \frac{6}{15}$

$BE = 4$ $\qquad$ $20 = DC$

26. Yes; Because $m\angle J$ and $m\angle X$ both equal 71°, $\angle J \cong \angle X$. By the Triangle Sum Theorem, $71° + 52° + m\angle L = 180°$, so $m\angle L = 57°$. Therefore, $\angle L$ and $\angle Z$ are congruent. So, $\triangle JKL \sim \triangle XYZ$ by the AA Similarity Postulate.

27. Yes; If $m\angle X = 90°$, $m\angle Y = 60°$, and $\triangle JKL$ contains a 60° angle, then the triangles are similar by the AA Similarity Postulate.

28. No; Because $m\angle J = 87°$, $\triangle XYZ$ needs to have an 87° angle in order for it to be possible that $\triangle JKL$ and $\triangle XYZ$ are similar. This is not possible because $m\angle Y = 94°$, and the sum of 94° and 87° is 181°, which contradicts the Triangle Sum Theorem.

29. No; By the Triangle Sum Theorem, $85° + m\angle L = 180°$, and $m\angle X + 80° = 180°$, so $m\angle L = 95°$ and $m\angle X = 100°$. So, $\triangle XYZ$ needs to have a 95° angle for it to be possible that $\triangle JKL$ and $\triangle XYZ$ are similar. This is not possible because $m\angle X = 100°$, and the sum of 95° and 100° is 195°, which contradicts the Triangle Sum Theorem.

30. Because $\angle P \cong \angle P$ by the Reflexive Property and $\angle PST \cong \angle R$ by the Corresponding Angles Postulate. $\triangle PST \sim \triangle PRQ$ by the AA Similarity Postulate.

Because the triangles are similar, you can set up the following proportion:

$$\frac{PT}{PQ} = \frac{PS}{PR}$$

$$\frac{PT}{PQ} = \frac{PS}{PS + SR}$$

$$\frac{x}{3x} = \frac{a}{a + \frac{8}{3}x}$$

$$x\left(a + \frac{8}{3}x\right) = 3ax$$

$$ax + \frac{8}{3}x^2 = 3ax$$

$$\frac{8}{3}x^2 = 2ax$$

$$\frac{4}{3}x = a$$

So, $PS = \frac{4}{3}x$.

Problem Solving

31. The triangles shown in the diagram are similar by the AA Similarity Postulate, so you can write the following proportion.

$$\frac{20 \text{ in.}}{d \text{ in.}} = \frac{26 \text{ in.}}{(66 - 26) \text{ in.}}$$

$$800 = 26d$$

$$30.8 \approx d$$

The distance between the puck and the wall when the opponent returns it is about 30.8 inches.

32. a. You can use the AA Similarity Postulate to show that the triangles are similar because you can show that two angles of $\triangle XYZ$ are congruent to two angles of $\triangle XVW$.

b. $\frac{WX}{ZX} = \frac{WV}{ZY}$

$$\frac{x \text{ m}}{6 \text{ m}} = \frac{104 \text{ m}}{8 \text{ m}}$$

$$8x = 624$$

$$x = 78$$

The width of the lake is 78 meters.

c. $\frac{XY}{VX} = \frac{ZY}{WV}$

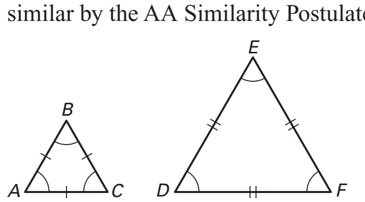
$$\frac{10 \text{ m}}{x \text{ m}} = \frac{8 \text{ m}}{104 \text{ m}}$$

$$1040 = 8x$$

$$130 = x$$

So, VX is 130 meters.

33. All equilateral triangles have the same angle measurements, 60°. So, all equilateral triangles are similar by the AA Similarity Postulate.



$m\angle A = m\angle B = m\angle C = m\angle D = m\angle E = m\angle F = 60°$, so $\angle A \cong \angle B \cong \angle C \cong \angle D \cong \angle E \cong \angle F$.

34. $\frac{f}{h} = \frac{n}{g}$

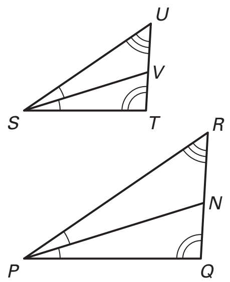
$\frac{8 \text{ cm}}{h \text{ m}} = \frac{3 \text{ cm}}{50 \text{ m}}$

$400 = 3h$

$133\frac{1}{3} = h$

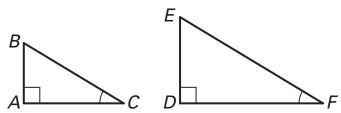
The blimp should fly at a height of $133\frac{1}{3}$ meters to take the photo.

35. *Sample answer:*



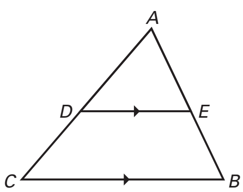
Angle bisectors $\overline{SV}$ and $\overline{PN}$ are corresponding lengths in similar triangles. So, $\frac{SV}{PN} = \frac{ST}{PQ}$ by the Corresponding Lengths Property on page 375.

36. *Sample answer:*



Because they are both right angles, $\angle A$ and $\angle D$ are congruent. The acute angles $\angle C$ and $\angle F$ are also congruent, so $\triangle ABC \sim \triangle DEF$ by the AA Similarity Postulate.

37. **a.** *Sample answer:*



b. *Sample answer:* $m\angle ADE \approx 47°$, $m\angle ACB \approx 47°$; $m\angle AED \approx 29°$, $m\angle ABC \approx 29°$;

So, $m\angle ADE = m\angle ACB$ and $m\angle AED = m\angle ABC$.

c. By the AA Similarity Postulate, $\triangle ADE \sim \triangle ACB$.

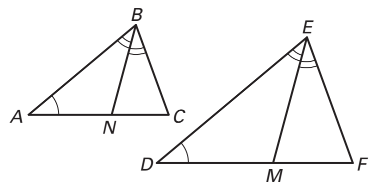
d. *Sample answer:* $AB = 3$ cm, $BC = 4$ cm, $AC = 2$ cm, $AD = 1$ cm, $DE = 2$ cm, $AE = 1.5$ cm;

$\frac{AD}{AC} = \frac{AE}{AB} = \frac{DE}{CB} = \frac{1}{2}$

e. The measures of the angles change, but the equalities remain the same. Yes; the triangles remain similar by the AA Similarity Postulate.

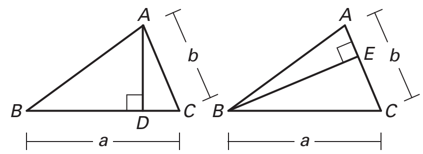
38. *Sample answer:* Given any two points on a line, you can draw similar triangles as shown in the diagram. Because the triangles are similar, the ratios of corresponding side lengths are the same. So, the ratio of the rise to the run is the same. Therefore, the slope is the same for any two points chosen on a line.

39. *Sample answer:*



Let $\triangle ABC \sim \triangle DEF$, let $\overline{BN}$ bisect $\angle ABC$ and let $\overline{EM}$ bisect $\angle DEF$. Because $\triangle ABC \sim \triangle DEF$, $\angle ABC \cong \angle DEF$ and $\angle A \cong \angle D$. Also, $\overline{BN}$ and $\overline{EM}$ bisect congruent angles, so $\angle ABN \cong \angle CBN \cong \angle DEM \cong \angle FEM$.
By the AA Similarity Postulate, $\triangle ABN \sim \triangle DEM$.
Therefore, $\frac{BN}{EM} = \frac{AB}{DE}$, where $\frac{AB}{DE}$ is the scale factor.

40. *Sample answer:*



Because $\angle ADC \cong \angle BEC$ and $\angle C \cong \angle C$, $\triangle ADC \sim \triangle BEC$ by the AA Similarity Postulate.

The ratio of the hypotenuses is $\frac{b}{a}$, so the ratio of the corresponding side lengths is also $\frac{b}{a}$. The altitudes are corresponding sides, so their lengths are in the ratio $\frac{b}{a}$.

Lesson 6.4 Prove Triangles Similar by SSS and SAS

Guided Practice for the lesson "Prove Triangles Similar by SSS and SAS"

1. Compare $\triangle LMN$ and $\triangle RST$:

Shortest sides	Longest sides	Remaining sides
$\frac{LM}{RS} = \frac{20}{24} = \frac{5}{6}$	$\frac{LN}{ST} = \frac{26}{33}$	$\frac{MN}{RT} = \frac{24}{30} = \frac{4}{5}$

The ratios are not all equal, so $\triangle LMN$ and $\triangle RST$ are not similar.

Compare $\triangle LMN$ and $\triangle XYZ$:

Shortest sides	Longest sides	Remaining sides
$\frac{LM}{YZ} = \frac{20}{30} = \frac{2}{3}$	$\frac{LN}{XY} = \frac{26}{39} = \frac{2}{3}$	$\frac{MN}{XZ} = \frac{24}{36} = \frac{2}{3}$

All of the ratios are equal, so $\triangle LMN \sim \triangle YZX$.

Because $\triangle LMN \sim \triangle XYZ$ and $\triangle LMN$ is not similar to $\triangle RST$, $\triangle XYZ$ is not similar to $\triangle RST$.

2. Scale factor: Longest sides Remaining sides

$\frac{24}{12} = \frac{2}{1}$ $\qquad \frac{33}{x} = \frac{2}{1}$ $\qquad \frac{30}{y} = \frac{2}{1}$

$33 = 2x \qquad 30 = 2y$

$16.5 = x \qquad 15 = y$

The lengths of the other sides are 16.5 and 15.

3. Both $\angle R$ and $\angle N$ are right angles, so $\angle R \cong \angle N$. Ratios of the lengths of the sides that include $\angle R$ and $\angle N$:

Shorter sides Longer sides

$\frac{SR}{PN} = \frac{24}{18} = \frac{4}{3}$ $\qquad \frac{RT}{NQ} = \frac{28}{21} = \frac{4}{3}$

The lengths of the sides that include $\angle R$ and $\angle N$ are proportional. So, $\triangle SRT \sim \triangle PNQ$ by the SAS Similarity Theorem.

4. *Sample answer:*
Ratios of the lengths of corresponding sides:

Shortest sides Longest sides Remaining sides

$\frac{XZ}{YZ} = \frac{12}{9} = \frac{4}{3}$ $\qquad \frac{XW}{XY} = \frac{20}{15} = \frac{4}{3}$ $\qquad \frac{WZ}{XZ} = \frac{16}{12} = \frac{4}{3}$

Corresponding side lengths are proportional, so $\triangle XZW \sim \triangle YZX$ by the SSS Similarity Theorem.

Exercises for the lesson "Prove Triangles Similar by SSS and SAS"

Skill Practice

1. Corresponding side lengths must be proportional, so $\frac{AC}{PX} = \frac{CB}{XQ} = \frac{AB}{PQ}$.

2. You would need to know that one pair of corresponding sides is congruent. You could then use the SAS Congruence Postulate.

3. Shortest sides Longest sides Remaining sides

$\frac{AC}{DF} = \frac{12}{8} = \frac{3}{2}$ $\qquad \frac{RC}{EF} = \frac{18}{12} = \frac{3}{2}$ $\qquad \frac{AB}{DE} = \frac{15}{10} = \frac{3}{2}$

All of the ratios are equal, so $\triangle ABC \sim \triangle DEF$ by the SSS Similarity Theorem. The scale factor of $\triangle ABC$ to $\triangle DEF$ is $\frac{3}{2}$.

4. Shortest sides Longest sides Remaining sides

$\frac{AB}{DE} = \frac{10}{25} = \frac{2}{5}$ $\qquad \frac{CA}{FD} = \frac{20}{50} = \frac{2}{5}$ $\qquad \frac{BC}{EF} = \frac{16}{40} = \frac{2}{5}$

All of the ratios are equal, so $\triangle ABC \sim \triangle DEF$ by the SSS Similarity Theorem. The scale factor of $\triangle ABC$ to $\triangle DEF$ is $\frac{2}{5}$.

5. Compare $\triangle ABC$ and $\triangle JKL$:

Shortest sides Longest sides Remaining sides

$\frac{AB}{JK} = \frac{7}{6}$ $\qquad \frac{AC}{JL} = \frac{12}{11}$ $\qquad \frac{BC}{KL} = \frac{8}{7}$

The ratios are not all equal, so $\triangle ABC$ and $\triangle JKL$ are not similar.

Compare $\triangle ABC$ and $\triangle RST$:

Shortest sides Longest sides Remaining sides

$\frac{AB}{RS} = \frac{7}{3.5} = \frac{2}{1}$ $\qquad \frac{AC}{RT} = \frac{12}{6} = \frac{2}{1}$ $\qquad \frac{BC}{ST} = \frac{8}{4} = \frac{2}{1}$

All of the ratios are equal, so $\triangle ABC \sim \triangle RST$.

6. Compare $\triangle ABC$ and $\triangle JKL$:

Shortest sides Longest sides Remaining sides

$\frac{BC}{KL} = \frac{14}{17.5} = \frac{4}{5}$ $\qquad \frac{AC}{JL} = \frac{20}{25} = \frac{4}{5}$ $\qquad \frac{AB}{JK} = \frac{16}{20} = \frac{4}{5}$

All of the ratios are equal, so $\triangle ABC \sim \triangle JKL$.

Compare $\triangle ABC$ and $\triangle RST$:

Shortest sides Longest sides Remaining sides

$\frac{BC}{ST} = \frac{14}{10.5} = \frac{4}{3}$ $\qquad \frac{AC}{RT} = \frac{20}{16} = \frac{5}{4}$ $\qquad \frac{AB}{RS} = \frac{16}{12} = \frac{4}{3}$

The ratios are not all equal, so $\triangle ABC$ and $\triangle RST$ are not similar.

7. Both $\angle W$ and $\angle D$ are right angles, so $\angle W \cong \angle D$. Ratios of the lengths of the sides that include $\angle W$ and $\angle D$:

Shorter sides Longer sides

$\frac{WY}{DE} = \frac{6}{9} = \frac{2}{3}$ $\qquad \frac{XW}{FD} = \frac{10}{15} = \frac{2}{3}$

The length of the sides that include $\angle W$ and $\angle D$ are proportional. So, by the SAS Similarity Theorem, $\triangle WXY \sim \triangle DFE$. The scale factor of Triangle B to Triangle A is $\frac{2}{3}$.

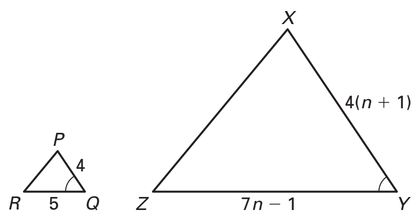
8. Both $m\angle L$ and $m\angle T = 112°$, so $\angle L \cong \angle T$. Ratios of the lengths of the sides that include $\angle L$ and $\angle T$:

Shorter sides Longer sides

$\frac{KL}{ST} = \frac{10}{8} = \frac{5}{4}$ $\qquad \frac{JL}{RT} = \frac{24}{18} = \frac{4}{3}$

Because the lengths of the sides that include $\angle L$ and $\angle T$ are not proportional, Triangle B is not similar to Triangle A.

9.



Find the value of n that makes corresponding side lengths proportional.

$$\frac{PQ}{XY} = \frac{QR}{YZ}$$

$$\frac{4}{4(n+1)} = \frac{5}{7n-1}$$

$$4(7n - 1) = 20(n + 1)$$

$$28n - 4 = 20n + 20$$

$$8n = 24$$

$n = 3$

10. Ratios of the lengths of the corresponding sides:

Shortest sides

$\frac{GH}{FH} = \frac{15}{15 + 5} = \frac{15}{20} = \frac{3}{4}$

Longest sides

$\frac{GJ}{FK} = \frac{18}{24} = \frac{3}{4}$

Remaining sides

$\frac{HJ}{HK} = \frac{16.5}{16.5 + 5.5} = \frac{16.5}{22} = \frac{3}{4}$

All the ratios are equal, so the corresponding side lengths are proportional. So, $\triangle GHJ \sim \triangle FHK$ by the SSS Similarity Theorem.

11. Ratios of the lengths of the corresponding sides:

Shorter sides

$\frac{BC}{CE} = \frac{21}{14} = \frac{3}{2}$

Longer sides

$\frac{AC}{CD} = \frac{27}{18} = \frac{3}{2}$

The corresponding side lengths are proportional. The included angles $\angle ACB$ and $\angle DCE$ are congruent because they are vertical angles. So, $\triangle ACB \sim \triangle DCE$ by the SAS Similarity Theorem.

12. Ratios of the lengths of the corresponding sides:

Shorter sides

$\frac{XY}{DJ} = \frac{21}{35} = \frac{3}{5}$

Longer sides

$\frac{XZ}{DG} = \frac{30}{50} = \frac{3}{5}$

The corresponding side lengths are proportional. Both $m\angle X$ and $m\angle L$ equal 47°, so $\angle X \cong \angle L$. So, $\triangle XYZ \sim \triangle DJG$ by the SAS Similarity Theorem.

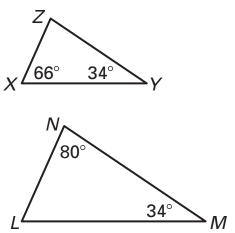
13. The student named the triangles incorrectly.

$\triangle ABC \sim \triangle RQP$ by the SAS Similarity Theorem

14. D; Because $\frac{MN}{MR} = \frac{MP}{MQ}$ and $\angle M \cong \angle M$,

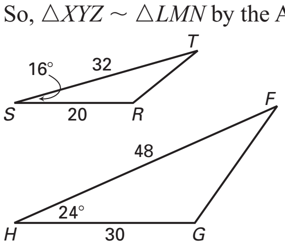
$\triangle MNP \sim \triangle MRQ$ by the SAS Similarity Theorem. The correct answer is D.

15.



Because $m\angle Y$ and $m\angle M$ both equal 34°, $\angle Y \cong \angle M$. By the Triangle Sum Theorem, $66° + 34° + m\angle Z = 180°$ so, $m\angle Z = 180°$. Therefore, $\angle Z$ and $\angle N$ are congruent. So, $\triangle XYZ \sim \triangle LMN$ by the AA Similarity Postulate.

16.



If $\triangle RST$ and $\triangle FGH$ were similar, then the scale factor would be $\frac{ST}{GH} = \frac{32}{30} = \frac{16}{15}$, $m\angle T = 24°$, and $m\angle R = 140°$.

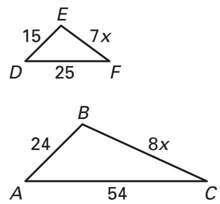
Find RT: $\frac{RT}{FH} = \frac{16}{15}$

$\frac{x}{48} = \frac{16}{15}$

$x = 51.2$

So, RT would be the longest side of $\triangle RST$, but this cannot be true because RT is not opposite the largest angle. So, the triangles cannot be similar.

17.



$\frac{AB}{DE} = \frac{24}{15} = \frac{8}{3}$ $\quad \frac{BC}{EF} = \frac{8x}{25}$ $\quad \frac{AC}{DF} = \frac{54}{7x}$

If $\triangle ABC$ and $\triangle DEF$ were similar, $\frac{8x}{25} = \frac{54}{7x}$

$56x^2 = 1350$

$x \approx 4.9.$

But $\frac{8x}{25}$ and $\frac{54}{7x}$ do not equal $\frac{8}{3}$ when $x \approx 4.9$. So, the triangles cannot be similar.

18. Because $\angle LSN$ and $\angle QRN$ are supplementary, $\overleftrightarrow{LM} \parallel \overleftrightarrow{PQ}$ by the Consecutive Interior Angles Converse. So $m\angle NLM = m\angle NQP = 53°$ by the Alternate Interior Angles Theorem.

19. Because $\angle LSN$ and $\angle QRN$ are supplementary, $\overleftrightarrow{LM} \parallel \overleftrightarrow{PQ}$ by the Consecutive Interior Angles Converse. So $m\angle QPN = m\angle LMN = 45°$ by the Alternate Interior Angles Theorem.

20. By the Triangle Sum Theorem, $53° + 45° + m\angle PNQ = 180°$, so $m\angle PNQ = 82°$.

21. $\triangle LSN \sim \triangle QRN$ by the AA Similarity Postulate, so

$\frac{QR}{LS} = \frac{RN}{SN}$

$\frac{18}{12} = \frac{RN}{16}$

$24 = RN.$

22. $\triangle LMN \sim \triangle QPN$ by the AA Similarity Postulate, so

$\frac{PQ}{ML} = \frac{RN}{SN}$

$\frac{PQ}{28} = \frac{24}{16}$

$PQ = 42.$

23. $LM = LS + SM$ $\qquad PQ = PR + RQ$

$28 = 12 + SM \qquad 42 = PR + 18$

$16 = SM \qquad 24 = PR$

$\triangle MSN \sim \triangle PRN$ by the AA Similarity Postulate, so

$$\frac{SM}{RP} = \frac{NM}{NP}$$

$$\frac{16}{24} = \frac{NM}{24\sqrt{2}}$$

$$16\sqrt{2} = NM.$$

24. $\triangle LSN \sim \triangle QRN$, $\triangle MSN \sim \triangle PRN$, and $\triangle LMN \sim \triangle QPN$, by the AA Similarity Postulate.

25. Scale factor of $\triangle VWX \sim \triangle ABC$: $\frac{VX}{AC} = \frac{51}{34} = \frac{3}{2}$

26. Find bases: $\frac{YX}{DC} = \frac{3}{2} \qquad \frac{WY}{BD} = \frac{3}{2}$

$\frac{45}{DC} = \frac{3}{2} \qquad \frac{WY}{12} = \frac{3}{2}$

$DC = 30 \qquad WY = 18$

So, $BC = 12 + 30 = 42$ and $XW = 45 + 18 = 63$.

Use the Pythagorean Theorem to find the height of each triangle.

$(AD)^2 + (DC)^2 = (AC)^2 \qquad (VY)^2 + (YX)^2 = (VX)^2$

$(AD)^2 + 30^2 = 34^2 \qquad (VY)^2 + 45^2 = 51^2$

$(AD)^2 = 256 \qquad (VY)^2 = 576$

$AD = 16 \qquad VY = 24$

Ratios of areas: $\frac{\text{Area of } \triangle VWX}{\text{Area of } \triangle ABC} = \frac{\frac{1}{2}(63)(24)}{\frac{1}{2}(42)(16)} = \frac{756}{336} = \frac{9}{4}$

27. Conjecture: In similar triangles, the ratio of the areas is the square of the scale factor.

Sample answer: Let the base and height of $\triangle VWX$ be $3a$ and $3b$. Let the base and height of $\triangle ABC$ be $2a$ and $2b$. The ratio of their areas is

$\frac{\frac{3a(3b)}{2}}{\frac{2a(2b)}{2}} = \frac{\frac{9ab}{2}}{\frac{4ab}{2}} = \frac{9ab}{4ab} = \frac{9}{4}$. Notice that $\frac{9}{4} = \left(\frac{3}{2}\right)^2$.

Problem Solving

28. AA Similarity Postulate; You know $\angle A \cong \angle A$ by the Reflexive Property. Because $\overline{BG} \parallel \overline{CF}$ and $\overline{CF} \parallel \overline{DE}$, you know that $\angle ABG \cong \angle ACF \cong \angle ADE$ and $\angle AGB \cong \angle AFC \cong \angle AED$ by the Corresponding Angles Postulate. So, $\triangle ABG \sim \triangle ACF$, $\triangle ABG \sim \triangle ADE$, $\triangle ACF \sim \triangle ADE$ by the AA Similarity Postulate.

29. Compare the first piece to the second piece:

Shortest sides	Longest sides	Remaining sides
$\frac{3}{4}$	$\frac{5}{7}$	$\frac{3}{4}$

The ratios are not all equal, so the first and second pieces are not similar.

Compare the second piece to the third piece:

Shortest sides	Longest sides	Remaining sides
$\frac{4}{3}$	$\frac{7}{5.25} = \frac{4}{3}$	$\frac{4}{3}$

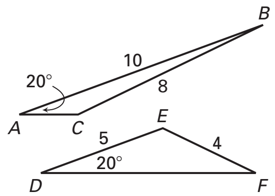
All of the ratios are equal, so the second and third pieces are similar.

The second and third pieces are similar, but the first and second pieces are not similar, so the first and third pieces are not similar.

30. You need to know $\frac{DC}{EC}$ is also equal to the other two ratios of corresponding side lengths.

31. You need to know that the included angles are congruent, $\angle CBD \cong \angle CAE$.

32.



You can see that $\frac{AB}{DE} = \frac{BC}{EF}$ and $\angle A \cong \angle D$, but it is obvious that $\frac{AC}{DF} \neq \frac{AB}{DE}$. So, there is no SSA Similarity Postulate.

33. a. The triangles are similar by the AA Similarity Postulate.

b. Let x be the height of the tree.

$$\frac{x \text{ in.}}{66 \text{ in.}} = \frac{(95 + 7) \text{ ft}}{7 \text{ ft}}$$

$7x = 6732$

$x \approx 962$

The height of the tree is about 962 inches, or about 80 feet.

c. Let x be the distance from Curtis to the tree.

$$\frac{962 \text{ in.}}{75 \text{ in.}} = \frac{(6 + x) \text{ ft}}{6 \text{ ft}}$$

$5772 = 450 + 75x$

$70.96 = x$

Curtis is about 71 feet from the tree.

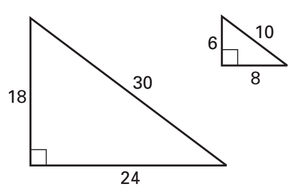
34. a. Using the Pythagorean Theorem:

$a^2 + b^2 = c^2 \qquad a^2 + b^2 = c^2$

$6^2 + b^2 = 10^2 \qquad 18^2 + b^2 = 30^2$

$b^2 = 64 \qquad b^2 = 576$

$b = 8 \qquad b = 24$



b. $\frac{8}{24} = \frac{1}{3}$

c. Ratios of corresponding side lengths:

Shortest sides	Longest sides	Remaining sides
$\frac{6}{18} = \frac{1}{3}$	$\frac{10}{30} = \frac{1}{3}$	$\frac{8}{24} = \frac{1}{3}$

All of the ratios are equal, so the two triangles are similar. This suggests a Hypotenuse-Leg Similarity Theorem for right triangles.

35. *Sample answer:* Because D, E, and F are midpoints, $\overline{DE} \parallel \overline{AC}$ and $\overline{EF} \parallel \overline{AB}$ by the Midsegment Theorem. So, $\angle A \cong \angle BDE$ by the Corresponding Angles Postulate and $\angle BDE \cong \angle DEF$ by the Alternate Interior Angles Theorem. Therefore, $m\angle DEF = 90°$.

36. Yes; All pairs of corresponding angles are in proportion when the two triangles are similar.

37. *Sample answer:* Locate G on $\overline{AB}$ so that $GB = DE$. Draw $\overline{GH}$ so that $\overline{GH} \parallel \overline{AC}$. Because $\angle A \cong \angle BGH$ and $\angle C \cong \angle BHG$ by the Corresponding Angles Postulate, $\triangle ABC \sim \triangle GBH$ by the AA Similarity Postulate. So, $\frac{AC}{GH} = \frac{AB}{GB}$. But $\frac{AB}{DE} = \frac{AC}{DF}$ and $GB = DE$, so $\frac{AC}{GH} = \frac{AC}{DF}$.

Therefore, $GH = DF$. Because $\angle BGH \cong \angle EDF$, $\triangle GBH \cong \triangle DEF$ by the SAS Congruence Postulate. So, $\angle B \cong \angle E$, and $\triangle ABC \sim \triangle DEF$ by the AA Similarity Postulate.

38. Because $\angle ADC$ and $\angle BCD$ are right angles, $\overline{AD} \parallel \overline{BC}$ by the Consecutive Interior Angles Converse. So, $\angle ADE \cong \angle B$ and $\angle A \cong \angle ACB$ by the Alternate Interior Angles Theorem. Therefore, $\triangle AED \sim \triangle CEB$ by the AA Similarity Postulate.

The scale factor of $\triangle AED$ to $\triangle CEB$ is $\frac{AD}{CB} = \frac{40}{25} = \frac{8}{5}$.

Let $AE = 8y$, $EC = 5y$, $DE = 8x$, and $BE = 5x$.
$\left(\text{Notice that the ratios of corresponding side lengths are } \frac{8}{5}.\right)$

Also notice that $\triangle ADC \sim \triangle EFC$ by the AA Similarity Postulate because $\angle A \cong \angle A$ and $\angle ADC \cong \angle EFC$. So, you can write the proportion:

$$\frac{AC}{EC} = \frac{AD}{EF}$$

$$\frac{8y + 5y}{5y} = \frac{40}{EF}$$

$$\frac{13}{5} = \frac{40}{EF}$$

$$\frac{200}{13} = EF$$

So, the length of $\overline{EF}$ is $\frac{200}{13}$ feet, or about 15.4 feet.

Quiz for the lessons "Use Similar Polygons", "Transformation and Similarity" , "Prove Triangles Similar by AA", and "Prove Triangles Similar by SSS and SAS"

1. $\frac{AD}{KN} = \frac{60}{36} = \frac{5}{3}$

The scale factor of $ABCD$ to $KLMN$ is $\frac{5}{3}$.

2. Find x: $\frac{DC}{NM} = \frac{5}{3}$ Find y: $\frac{AB}{KL} = \frac{5}{3}$

$\frac{70}{x} = \frac{5}{3}$ $\frac{45}{y} = \frac{5}{3}$

$210 = 5x$ $135 = y \cdot 5$

$42 = x$ $27 = y$

Find z: $m\angle A = m\angle K$

$85° = z°$

$85 = z$

3. Perimeter of $KLMN = KL + LM + MN + NK$

$= 27 + 10 + 42 + 36 = 115$

Perimeter of $ABCD$:
Use Theorem 6.1 to find the perimeter x.

$$\frac{x}{115} = \frac{5}{3}$$

$$3x = 575$$

$$x = 191\frac{2}{3}$$

The perimeter of $ABCD$ is $191\frac{2}{3}$ and the perimeter of $KLMN$ is 115.

4-6. Check students' drawings.

7. Both $\angle P$ and $\angle D$ are right angles, so $\angle P \cong \angle D$. Ratios of the lengths of the sides that include $\angle P$ and $\angle D$:

Shorter sides	Longer sides
$\frac{WP}{ZD} = \frac{30}{9} = \frac{10}{3}$	$\frac{YP}{ND} = \frac{36}{12} = \frac{3}{1}$

Because the lengths of the sides that include $\angle P$ and $\angle D$ are not proportional, $\triangle WPY$ and $\triangle ZDN$ are not similar.

8. Ratios of corresponding side lengths:

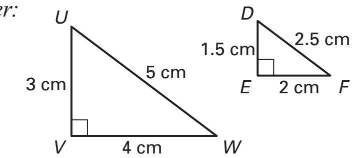
Shortest sides	Longest sides	Remaining sides
$\frac{AC}{XR} = \frac{20}{25} = \frac{4}{5}$	$\frac{CF}{RS} = \frac{32}{40} = \frac{4}{5}$	$\frac{FA}{SX} = \frac{28}{35} = \frac{4}{5}$

All of the ratios are equal, so $\triangle ACF \sim \triangle XRS$ by the SSS Similarity Theorem.

9. Both $m\angle M$ and $m\angle J$ equal 42°, so $\angle M \cong \angle J$. $\angle LGM \cong \angle HGJ$ by the Vertical Angles Congruence Theorem. So, $\triangle LGM \cong \triangle HGJ$ by the AA Similarity Theorem.

Mixed Review of Problem Solving for the lessons"Use Similar Polygons", "Transformation and Similarity", "Prove Triangles Similar by AA", and "Prove Triangles Similar by SSS and SAS"

1. *Sample answer:*



Scale factor of $\triangle UVW$ to $\triangle XYZ$: $\frac{1}{2}$

2. $\frac{AC}{DF} = \frac{3}{5}$

$\frac{AC}{12} = \frac{3}{5}$

$AC = 7.2$

3. Dilation with scale factor 3 and then a translation right.

4. Yes; triangle SRQ is a dilation and reflection of triangle LMN.

5. No; After a dilation of triangle LMN, side MN would match with RS, but side LM would not line up with side QS.

6. *Sample answer:* You would need to know $\frac{XW}{XV} = \frac{XY}{XZ}$.

7. a. Because $\angle B$ and $\angle D$ are right angles, $\angle B \cong \angle D$ by the Right Angles Congruence Theorem. $\angle BCA \cong \angle DCE$ by the Vertical Angles Congruence Theorem. So, $\triangle ABC \sim \triangle EDC$ by the AA Similarity Theorem.

b. $\frac{DE}{BA} = \frac{CD}{CB}$

$\frac{2}{8} = \frac{CD}{7.5 - CD}$

$15 - 2CD = 8CD$

$15 = 10CD$

$1.5 = CD$

CD is 1.5 miles.

c. Use the Pythagorean Theorem to find the lengths AC and EC.

$8^2 + 6^2 = AC^2$ $\qquad$ $1.5^2 + 2^2 = EC^2$

$100 = AC^2$ $\qquad$ $6.25 = EC^2$

$10 = AC$ $\qquad$ $2.5 = EC$

The distance between your house and the mall is 10 miles + 2.5 miles = 12.5 miles.

Lesson 6.5 Use Proportionality Theorems

Guided Practice for the lesson "Use Proportionality Theorems"

1. $\frac{XW}{WV} = \frac{XY}{YZ}$

$\frac{44}{35} = \frac{36}{YZ}$

$\frac{1260}{44} = YZ$

$\frac{315}{11} = YZ$

2. $\frac{NP}{PQ} = \frac{90}{50} = \frac{9}{5}$

$\frac{NS}{SR} = \frac{72}{40} = \frac{9}{5}$

$\overline{PS}$ is parallel to $\overline{QR}$ by the Converse of the Triangle Proportionality Theorem.

3. $\frac{AB}{16} = \frac{18}{15}$

$15 \cdot AB = 288$

$AB = 19.2$

4. $DB = DC = \frac{AB}{AC}$

$\frac{4}{4} = \frac{AB}{4\sqrt{2}}$

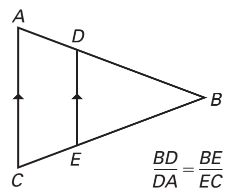
$4\sqrt{2} = AB$

Exercises for the lesson "Use Proportionality Theorems"

Skill Practice

1. The Triangle Proportionality Theorem: If a line parallel to one side of a triangle intersects the other two sides, then it divides the two sides proportionally.

If $\overline{DE} \parallel \overline{AC}$, then $\frac{BD}{DA} = \frac{BE}{EC}$.



2. *Sample answer:* In the Midsegment Theorem, the segment connecting the midpoints of two sides of a triangle is parallel to the third side. This is a special case of the Converse of the Triangle Proportionality Theorem.

3. $\frac{BA}{CB} = \frac{DE}{CD}$

$\frac{BA}{3} = \frac{12}{4}$

$BA = 9$

The length of $\overline{AB}$ is 9.

4. $\frac{AE}{ED} = \frac{AB}{BC}$

$\frac{14}{12} = \frac{AB}{18}$

$21 = AB$

The length of $\overline{AB}$ is 21.

5. $\frac{CK}{KS} = \frac{8}{5}$ $\qquad$ $\frac{LM}{MN} = \frac{12}{7.5} = \frac{8}{5}$

Because $\frac{LK}{KJ} = \frac{LM}{MN}$, $\overline{KM} \parallel \overline{JN}$ by the Converse of the Triangle Proportionality Theorem.

6. $\frac{LK}{KS} = \frac{18}{10} = \frac{9}{5}$ $\qquad$ $\frac{LM}{MN} = \frac{24}{15} = \frac{8}{5}$

Because $\frac{LK}{KJ} \neq \frac{LM}{MN}$, $\overline{KM}$ is not parallel to $\overline{JN}$.

7. $\frac{LK}{KJ} = \frac{25}{22.5} = \frac{10}{9}$ $\qquad$ $\frac{LM}{MN} = \frac{20}{18} = \frac{10}{9}$

Because $\frac{LK}{KJ} = \frac{LM}{MN}$, $\overline{KM} \parallel \overline{JN}$ by the Converse of the Triangle Proportionality Theorem.

8. C; If $\frac{QR}{RS} = \frac{TS}{RS}$, then $QR = TS$, which may not be true. The correct answer is C.

9. $\frac{x}{15} = \frac{14}{21}$

$21x = 210$

$x = 10$

10. $\frac{4}{6} = \frac{8}{y}$

$48 = 4y$

$12 = y$

11. $\frac{2}{1.5} = \frac{3}{4.5}$

$4.5z = 4.5$

$z = 1$

12. The length of $\overline{CD}$ is not 20. The length of $\overline{AC}$ is 20.

Let $CD = x$.

$\frac{AB}{CB} = \frac{AD}{C} \rightarrow \frac{10}{16} = \frac{20 - x}{x}$

13. C; $\frac{6x}{2x + 1 + 2x} = \frac{18}{7.5 + 6}$

$\frac{6x}{4x + 1} = \frac{18}{13.5}$

$81x = 72x + 18$

$x = 2$

The correct answer is C.

14. $\frac{11}{29 - 11} = \frac{16.5}{p}$

$11p = 297$

$p = 27$

15. $\frac{q}{16 - q} = \frac{36}{28}$

$28q = 576 - 36q$

$64q = 576$

$q = 9$

16. Find f:

$\frac{f}{12} = \frac{10}{15}$

$15f = 120$

$f = 8$

Find d:

$\frac{d}{12.5} = \frac{10}{10 + 15}$

$25d = 125$

$d = 5$

Find e:

$\frac{12 + 8}{e} = \frac{10 + 15}{5}$

$100 = 25e$

$4 = e$

Find c:

$\frac{c}{d} = \frac{5 + 15 + 10}{10}$

$10c = 30d$

$10c = 30(5)$

$c = 15$

Find b:

$\frac{b}{5 + 15 + 10} = \frac{12.5}{e + 12 + f}$

$b(e + 12 + f) = 375$

$24b = 375$

$b = 15.625$

Find a:

$\frac{a}{12.5} = \frac{6 + 5 + 15 + 10}{15 + 10}$

$25a = 12.5(b + 30)$

$25a = 570.3125$

$a = 22.8125$

17. Find d:

$\frac{3}{d} = \frac{9}{6}$

$18 = 9d$

$2 = d$

Find a:

$\frac{9 + a}{6 + 6} = \frac{6}{6}$

$b(9 + a) = 72$

$4(9 + a) = 72$

$a = 9$

Find b:

$\frac{6}{b} = \frac{9}{6}$

$36 = 9b$

$4 = b$

Find c:

$\frac{9 + a + 4.5}{6 + 6 + c} = \frac{7.5}{5}$

$67.5 + 5a = 90 + 7.5c$

$67.5 + 5(9) = 90 + 7.5c$

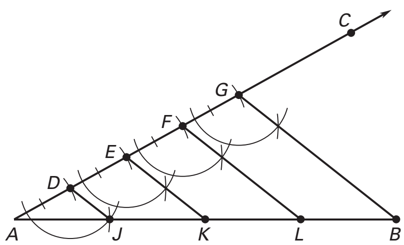
$22.5 = 7.5c$

$3 = c$

18. $\overline{AD}$ must bisect $\angle BAC$ to use Theorem 6.7. This information is not given, so the student cannot conclude that $AB = AC$.

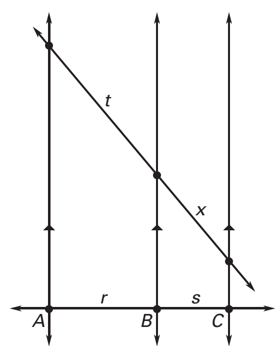
19. (a)–(b) see figure in part (c).

c. *Sample answer:*



Theorem 6.6 guarantees that parallel lines divide transversals proportionally. Because $\frac{AD}{DE} = \frac{DE}{EF} = \frac{EF}{FG} = 1$, you know $\frac{AJ}{JK} = \frac{JK}{KL} = \frac{KL}{LB} = 1$, which means $AJ = JK = KL = LB$.

20. *Sample answer:*



$\frac{r}{s} = \frac{t}{x}$ by Theorem 6.6.

Problem Solving

21. $\frac{400 \text{ yd}}{200 \text{ yd}} = \frac{700 \text{ yd}}{x \text{ yd}}$

$400x = 140{,}000$

$x = 350$

The distance along University Avenue from 12th Street to Washington Street is 350 yards.

22. Because $\overline{QS} \parallel \overline{TU}$, $\angle Q \cong \angle RTU$ and $\angle S \cong \angle RUT$ by the Corresponding Angles Postulate. By the AA Similarity Postulate, $\triangle SRQ \sim \triangle URT$. So, $\frac{QR}{TR} = \frac{SR}{UR}$ by the definition of similar triangles. Because $QR = QT + TR$ and $SR = SU + UR$, $\frac{QT + TR}{TR} = \frac{SU + UR}{UR}$, which simplifies to $\frac{QT}{TR} = \frac{SU}{UR}$ as shown:

$\frac{QT + TR}{TR} = \frac{SU + UR}{UR}$

$\frac{QT}{TR} + \frac{TR}{TR} = \frac{SU}{UR} + \frac{UR}{UR}$

$\frac{QT}{TR} + 1 = \frac{SU}{UR} + 1$

$\frac{QT}{TR} = \frac{SU}{UR}$

23. Label the point where the auxiliary line and $\overline{BE}$ intersect as point G. Because $k_1 \parallel k_3$ and $k_2 \parallel k_3$, $\frac{CB}{BA} = \frac{DG}{GA}$ and $\frac{DG}{GA} = \frac{DE}{EF}$ by the Triangle Proportionality Theorem. So, by the Transitive Property of Equality, $\frac{CB}{BA} = \frac{DE}{EF}$.

24. a. Lot A: $\frac{55 + 61}{48} = \frac{174 - x}{x}$

$116x = 8352 - 48x$

$164x = 8352$

$x \approx 50.9$

Lot B: $\frac{61}{55} = \frac{174 - 50.9 - y}{y}$

$61y = 6770.5 - 55y$

$116y = 6770.5$

$y \approx 58.4$

Lot C: $174 - 50.9 - 58.4 \approx 64.7$

Lot A has about 50.9 yards, Lot B has about 58.4 yards, and Lot C has about 64.7 yards of lake frontage.

b. Lot C should be listed for the highest price because it has the most lake frontage.

c. Because lot prices are in the same ratio as lake frontages, write and solve proportions to find the prices.

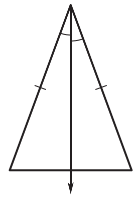
Lot B: $\frac{100{,}000}{x} = \frac{50.9}{58.4}$ Lot C: $\frac{100{,}000}{y} = \frac{50.9}{64.7}$

$5{,}840{,}000 = 50.9x$ $6{,}470{,}000 = 50.9y$

$114{,}735 \approx x$ $127{,}112 \approx y$

Lot B is about \$114,735 and Lot C is about \$127,112.

25.



Sample answer: In an isosceles triangle, the legs are congruent, so the ratio of their lengths is 1 : 1. By Theorem 6.7, this ratio is equal to the ratio of the lengths of the segments created by the ray, so it is also 1 : 1.

26. *Sample answer:* Given $\frac{RT}{TQ} = \frac{RU}{US}$, obtain $\frac{RT + TQ}{TQ} = \frac{RU + US}{US}$ and simplify to $\frac{RQ}{TQ} = \frac{RS}{US}$. Use proportions to solve for $\frac{TQ}{US}$. Show that $\frac{RQ}{RT} = \frac{RS}{US}$ and use the SAS Similarity Theorem to show $\triangle RTU \sim \triangle RQS$. Then show $\angle RTU \cong \angle RQS$ by the definition of similar triangles and that $\overline{QS} \parallel \overline{TU}$ by the Corresponding Angles Converse.

27. *Sample answer:* Because $\overline{AZ} \parallel \overline{XW}$, $\angle A \cong \angle YXW$ by the Corresponding Angles Postulate and $\angle XZA \cong \angle WXZ$ by the Alternate Interior Angles Theorem. So, $\triangle AXZ$ is isosceles by the converse of the Base Angles Theorem because $\angle A \cong \angle XZA$. Therefore, $AX = XZ$. Because $\overline{AZ} \parallel \overline{XW}$, $\frac{YW}{WZ} = \frac{XY}{AX}$ by the Triangle Proportionality Theorem. Substituting XZ for AX gives $\frac{YW}{WZ} = \frac{XY}{XZ}$.

28. a. $\frac{5.4}{x} = \frac{19 - 8.4}{8.4}$

$10.6x = 45.36$

$x \approx 4.3$

The length of the bottom edge of the drawing of Car 2 is about 4.3 centimeters.

b. *Sample answer:* The vertical edges of each car are parallel to each other; the triangle with vertices consisting of the vanishing point, the top left of Car 1, and the bottom left of Car 1 is similar to the triangle with vertices consisting of the vanishing point, the top left of Car 2, and the bottom left of Car 2.

c. $\frac{5.4}{4.3} = \frac{19 - 8.4 - x}{x}$

$5.4x = 45.58 - 4.3x$

$9.7x = 45.58$

$x \approx 4.7$

The length of the top edge of the drawing of Car 2 is about 4.7 centimeters.

29. Draw $\overline{AN}$ and $\overline{CM}$ so they are both parallel to $\overline{BY}$. Because $\overline{AN} \parallel \overline{CM}$, $\angle PAN \cong \angle PMC$ and $\angle PNA \cong \angle PCM$ by the Alternate Interior Angles Theorem. So, $\triangle PAN \sim \triangle PMC$ by the AA Similarity Postulate. Similarly, $\triangle CXM \sim \triangle BXP$ and $\triangle BZP \sim \triangle AZN$. From $\triangle PAN \sim \triangle PMC$, you know

$\frac{PA}{PM} = \frac{AN}{MC}$ using the definition of similarity. Similarly from $\triangle CXM \sim \triangle BXP$ and $\triangle BZP \sim \triangle AZN$, you know $\frac{CX}{BX} = \frac{MC}{PB}$ and $\frac{BZ}{AZ} = \frac{BP}{AN}$, respectively. Also, $\frac{AY}{YC} = \frac{PA}{PM}$ by the Triangle Proportionality Theorem. So,
$\frac{AY}{YC} \cdot \frac{CX}{BX} \cdot \frac{BZ}{AZ} = \frac{AN}{MC} \cdot \frac{MC}{PB} \cdot \frac{BP}{AN} = 1.$

Problem Solving Workshop for the lesson "Use Proportionality Theorems"

1. a. $DE = 3 \cdot FE = 3 \cdot 90 = 270$

DE is 270 yards.

b. The alley is one fourth of the way from E to D.

$\frac{1}{4}(270) = 67.5$

The distance from E to the alley along $\overleftrightarrow{FD}$ is 67.5 yards.

2. Using the Corresponding Angles Postulate, you know that the triangles with bases of lengths d, e, and f are similar by the AA Similarity Postulate. So,

$\frac{a}{a+b} = \frac{d}{e}, \frac{a}{a+b+c} = \frac{d}{f}$, and $\frac{a+b}{a+b+c} = \frac{e}{f}$

by the definition of similar triangles.

3. The distance when leaving from Point B is $\frac{0.9}{0.6} = 1.5$ times as far as leaving from Point A. If the person leaving Point A walks at a speed of 3 miles per hour, then the person leaving Point B must walk 1.5 times as fast, or $1.5(3) = 4.5$ miles per hour.

4. The actual distance walked is not needed. Only the ratio of the distances is needed to find the desired walking speed.

5. Seven is 3.5 times as large as 2, so x is 3.5 times as large as 1.5.

$x = 3.5(1.5) = 5.25$

Ten is $\frac{10}{7}$ times as large as 7, so y is $\frac{10}{7}$ times as large as 5.25

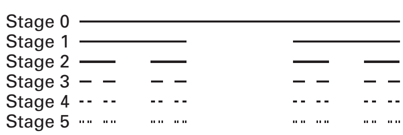
$y = \frac{10}{7}(5.25) = 7.5$

Extension for the lesson "Use Proportionality Theorems"

1. $\frac{\text{edge length of triangle in Stage 0}}{\text{edge length of triangle in Stage 1}} = \frac{1}{\frac{1}{3}} = \frac{3}{1}$;

Sample answer: The perimeter in Stage 1 is one unit longer. The three edges that were one unit each become twelve edges that are $\frac{1}{3}$ unit each.

2. a.



b.

Stage number	Number of segments	Segment length	Total length
0	1	1	1
1	2	$\frac{1}{3}$	$\frac{2}{3}$
2	4	$\frac{1}{9}$	$\frac{4}{9}$
3	8	$\frac{1}{27}$	$\frac{8}{27}$
4	16	$\frac{1}{81}$	$\frac{16}{81}$
5	32	$\frac{1}{243}$	$\frac{32}{243}$

c. Stage 10: Number of segments $= 2^{10} = 1024$;

segment length $= \frac{1}{3^{10}} = \frac{1}{59{,}049}$;

total length $= \frac{1024}{59{,}049}$, or about 0.01734 unit.

Stage 20: Number of segments $= 2^{20} = 1{,}048{,}576$;

segment length $= \frac{1}{3^{20}} = \frac{1}{3{,}486{,}784{,}401}$

total length $= \frac{1{,}048{,}576}{3{,}486{,}784{,}401}$, or

about 0.0003007 unit.

Stage n: Number of segments $= 2^n$;

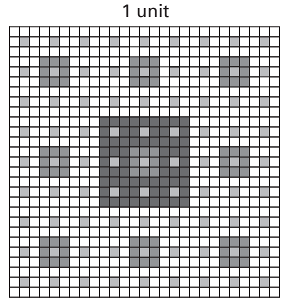
segment length $= \frac{1}{3^n}$

total length $= \frac{2^n}{3^n} = \left(\frac{2}{3}\right)^n$

3. a.

1 unit

1 unit



b. *Sample answer:* The upper lefthand square is a smaller version of the whole square.

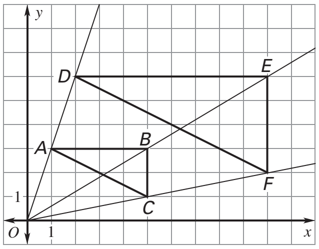
c.

Stage	Number of new colored squares	Area of 1 colored square	Total area
0	0	0	0
1	1	$\left(\frac{1}{3}\right)^2 = \frac{1}{9}$	$\frac{1}{9}$
2	8	$\left(\frac{1}{9}\right)^2 = \frac{1}{81}$	$\frac{1}{9} + \frac{8}{81} = \frac{17}{81}$
3	64	$\left(\frac{1}{27}\right)^2 = \frac{1}{729}$	$\frac{17}{81} + \frac{64}{729} = \frac{217}{729}$

Lesson 6.6 Perform Similarity Transformations

Investigating Geometry Activity for the lesson "Perform Similarity Transformations"

Explore: *Sample drawing:*

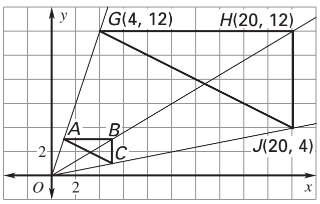


1. $AB = 4, BC = 2, DE = 8, EF = 4$

$\frac{DE}{AB} = \frac{EF}{BC}$ or $\frac{8}{4} = \frac{4}{2}$

$\angle B$ and $\angle E$ are right angles, so $\angle B \cong \angle E$ by the Right Angles Congruence Theorem. The ratios are equal, so the two sides including the congruent angles are proportional. So, $\triangle ABC \sim \triangle DEF$ by the SAS Similarity Theorem.

2.



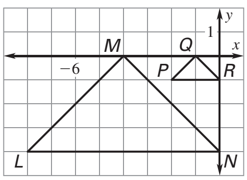
Guided Practice for the lesson "Perform Similarity Transformations"

1. $(x, y) \rightarrow (4x, 4y)$

$P(-2, -1) \rightarrow L(-8, -4)$

$Q(-1, 0) \rightarrow M(-4, 0)$

$R(0, -1) \rightarrow N(0, -4)$

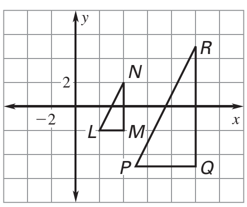


2. $(x, y) \rightarrow (0.4x, 0.4y)$

$P(5, -5) \rightarrow L(2, -2)$

$Q(10, -5) \rightarrow M(4, -2)$

$R(10, 5) \rightarrow N(4, 2)$



3. The scale factor is the ratio of a side length of the sticker image to a side length of the original photo, or $\frac{1.1}{5.5}$. In simplest form, the scale factor is $\frac{1}{5}$.

4. A dilation with respect to the origin and scale factor K can be described as $(x, y) \rightarrow (kx, ky)$. If $(x, y) = (0, 0)$, then $(kx, ky) = (k \cdot 0, k \cdot 0) = (0, 0)$.

Exercises for the lesson "Perform Similarity Transformations"

Skill Practice

1. In dilation, the image is *similar* to the original figure.

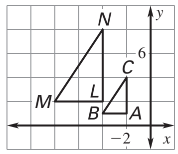
2. You find the scale factor of a dilation by setting up the ratio of a side length of the new figure to a side length of the original figure. A dilation is an enlargement if the scale factor is greater than 1 and a reduction if the scale factor is between 0 and 1.

3. $(x, y) \rightarrow (2x, 2y)$

$A(-2, 1) \rightarrow L(-4, 2)$

$B(-4, 1) \rightarrow M(-8, 2)$

$C(-2, 4) \rightarrow N(-4, 8)$

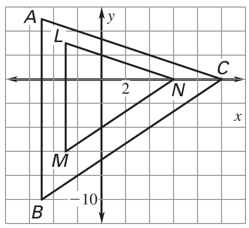


4. $(x, y) \rightarrow \left(\frac{3}{5}x, \frac{3}{5}y\right)$

$A(-5, 5) \rightarrow L(-3, 3)$

$B(-5, -10) \rightarrow M(-3, -6)$

$C(10, 0) \rightarrow N(6, 0)$

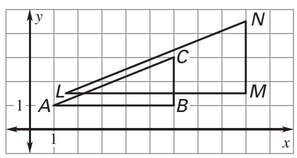


5. $(x, y) \rightarrow (1.5x, 1.5y)$

$A(1, 1) \rightarrow L(1.5, 1.5)$

$B(6, 1) \rightarrow M(9, 1.5)$

$C(6, 3) \rightarrow N(9, 4.5)$

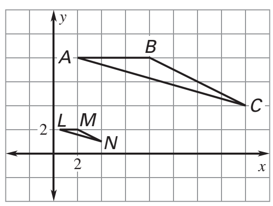


6. $(x, y) \rightarrow (0.25x, 0.25y)$

$A(2, 8) \rightarrow L(0.5, 2)$

$B(8, 8) \rightarrow M(2, 2)$

$C(16, 4) \rightarrow N(4, 1)$



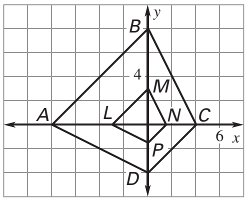
7. $(x, y) \rightarrow \left(\frac{3}{8}x, \frac{3}{8}y\right)$

$A(-8, 0) \rightarrow L(-3, 0)$

$B(0, 8) \rightarrow M(0, 3)$

$C(4, 0) \rightarrow N\left(\frac{3}{2}, 0\right)$

$D(0, -4) \rightarrow P\left(0, -\frac{3}{2}\right)$



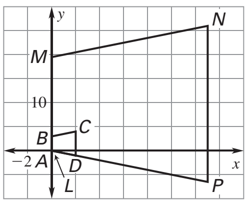
8. $(x, y) \rightarrow \left(\frac{13}{2}x, \frac{13}{2}y\right)$

$A(0, 0) \rightarrow L(0, 0)$

$B(0, 3) \rightarrow M\left(0, \frac{39}{2}\right)$

$C(2, 4) \rightarrow N(13, 26)$

$D(2, -1) \rightarrow P\left(13, -\frac{13}{2}\right)$



9. The dilation from Figure A to Figure B is a reduction. The scale factor is the length of B to the length of A, or $\frac{3}{6}$. In simplest form, the scale factor is $\frac{1}{2}$.

10. The dilation from Figure A to Figure B is an enlargement. The scale factor is the length of B to the length of A, or $\frac{3}{2}$.

11. The dilation from Figure A to Figure B is an enlargement. The scale factor is the length of one of the sides of B to the length of one of the sides of A, or $\frac{6}{2}$. In simplest form, the scale factor is 3.

12. The dilation from Figure A to Figure B is a reduction. The scale factor is the length of one of the sides of B to the length of one of the sides of A, or $\frac{1}{3}$.

13. C; $k(-4, 0) \rightarrow Q(-8, 0)$

The scale factor of $JKLM$ to $PQRS$ is 2.

$(x, y) \rightarrow (2x, 2y)$

$M(-1, -2) \rightarrow S(-2, -4)$

The coordinates of S are $(-2, -4)$, which is answer choice C.

14. The student found the scale factor of $\overline{AB}$ to $\overline{CD}$ by taking the ratio $\frac{AB}{CD}$, not $\frac{CD}{AB}$ like he/she should have. $\frac{CD}{AB} = \frac{5}{2}$

The scale factor of the dilation from $\overline{AB}$ to $\overline{CD}$ is $\frac{5}{2}$.

15. The left sides of △A and △B have a scale factor from A to B of $\frac{2}{4} = \frac{1}{2}$, but the tops of △A and △B have a scale factor from A to B of $\frac{4}{6} = \frac{1}{3}$. Because the scale factors of two lengths of the triangles are not equal, the triangles are not similar and therefore this figure is not a dilation.

16. The transformation shown is a rotation.

17. The transformation shown is a reflection.

18. The transformation shown is a dilation.

19. Scale factor of figure A to Figure B: $\frac{6}{3} = 2$

$2m = 8$ $\quad 2n = 10$

$m = 4$ $\quad n = 5$

20. Scale factor of Figure A to Figure B: $\frac{9}{12} = \frac{3}{4}$

$\frac{3}{4}p = 3$ $\quad \frac{3}{4}q = 9$ $\quad \frac{3}{4}r = 3$

$p = 4$ $\quad q = 12$ $\quad r = 4$

21. C; The ratios of the medians will be the same as the scale factor.

Scale factor of $\triangle DEO$ to $\triangle ABO$

$\frac{\text{length of bottom of } \triangle ABO}{\text{length of bottom of } \triangle DEO} = \frac{8}{6} = \frac{4}{3}$

$\frac{4}{3}$ is $133\frac{1}{3}\%$, so the answer choice is C.

22. When you dilate a figure using a scale factor of 2, you multiply both the x and y coordinates by 2: $(x, y) \rightarrow (2x, 2y)$. If you take that image and then dilate it by a scale factor of $\frac{1}{2}$, you multiply both the x and y coordinates by $\frac{1}{2}$: $(2x, 2y) \rightarrow (x, y)$. The new image is the same size and shape as the original figure.

23. *Sample answer:* Use a scale factor of 2 from $\triangle ABC$ to $\triangle DEF$. Then reflect through the y-axis.

24. *Sample answer:* Use a scale factor of $\frac{1}{3}$ from $\triangle ABC$ to $\triangle DEF$. Then translate 2 units left and 3 units up.

Problem Solving

25. If they use a scale factor of 24, multiply both dimensions by 24.

$12(24) = 288$ inches $\quad 6(24) = 144$ inches

$288 \text{ inches} \cdot \frac{1 \text{ foot}}{12 \text{ inches}} = 24 \text{ feet}$

$144 \text{ inches} \cdot \frac{1 \text{ foot}}{12 \text{ inches}} = 12 \text{ feet}$

The dimensions of the billboard are 24 feet by 12 feet.

26. The scale factor is the ratio of the width of the postcard to the width of the poster, or $\frac{5}{8}$. You should use a scale of $\frac{5}{8}$ for the image on the postcard.

27. The scale factor of the enlargement is the ratio of the height of the shadow to the height of your friend, or $\frac{15}{6}$. In simplest form, the scale factor of the enlargement is $\frac{5}{2}$.

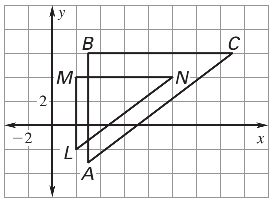
28. *Sample answer:* Multiply the coordinates of the smallest quadrilateral by 2, 3, and 4 to create each of the larger quadrilaterals.

29. a. $(x, y) \to \left(\frac{2}{3}x, \frac{2}{3}y\right)$

$A(3, -3) \to L(2, -2)$

$B(3, 6) \to M(2, 4)$

$C(15, 6) \to N(10, 4)$



b. Lengths of $\triangle ABC$:

$AB = 6 - (-3) = 9$, $BC = 15 - 3 = 12$, and

$CA = \sqrt{(6 - (-3))^2 + (15 - 3)^2} = 15$

Lengths of $\triangle LMN$:

$LM = 4 - (-2) = 6$, $MN = 10 - 2 = 8$, and

$LN = \sqrt{(4 - (-2))^2 + (10 - 2)^2} = 10$

Scale factor of $\triangle ABC$ to $\triangle LMN$: $\frac{LM}{AB} = \frac{6}{9} = \frac{2}{3}$

$$\frac{\text{perimeter of } \triangle LMN}{\text{perimeter of } \triangle ABC} = \frac{LM + MN + LN}{AB + BC + CA} = \frac{6 + 8 + 10}{9 + 12 + 15} = \frac{2}{3}$$

The ratio of the perimeters is the same as the scale factor.

c. $$\frac{\text{area of } \triangle LMN}{\text{area of } \triangle ABC} = \frac{\frac{1}{2} \cdot 6 \cdot 8}{\frac{1}{2} \cdot 9 \cdot 12} = \frac{24}{54} = \frac{4}{9}$$

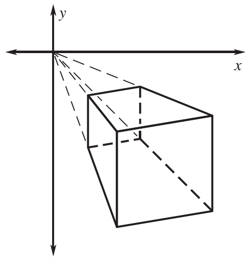
The ratio of the areas is the square of the scale factor.

30. a. A dilation with $-1 < k < 0$ would be a reduction.

b. A dilation with $k < -1$ would be an enlargement.

c. A dilation with $k = -1$ would be a figure with scale factor 1 (meaning it would stay the same size), but with a rotation of 180°.

31. *Sample answer:* First draw the x- and y-axis. The origin is your center of dilation (or vanishing point). Next draw a polygon. Then perform a dilation of the polygon by drawing rays and using a compass to measure equal lengths. Erase all hidden lines, and you just made a perspective drawing using dilations.



32. Let $P(a, b)$ and $Q(c, d)$ be the coordinates of the endpoints of $\overline{PQ}$ with midpoint $\left(\frac{a + c}{2}, \frac{b + d}{2}\right)$. Since $\overline{XY}$ is a dilation of $\overline{PQ}$ with scale factor k, you have $X(ka, kb)$ and $Y(kc, kd)$ with midpoint $\left(\frac{ka + kc}{2}, \frac{kb + kd}{2}\right) = \left(k\left(\frac{a + c}{2}\right)\right), \left(k\left(\frac{b + d}{2}\right)\right)$. Thus, the image of the midpoint of $\overline{PQ}$ is the midpoint of $\overline{XY}$.

33. The slope of $\overline{PQ}$ is $\frac{d - b}{c - a}$ and the slope of $\overline{XY}$ is

$$\frac{kd - kb}{kc - ka} = \frac{k(d - b)}{k(c - a)} = \frac{d - b}{c - a}.$$

Since the slopes are the same, the lines are parallel.

34. The length of the rectangle is $9 - 0 = 9$ units long and the width is $6 - 0 = 6$ units long. The area of the rectangle is $9 \cdot 6 = 54$ square units. Dilation involves a scale factor that is multiplied by all lengths. To get a rectangle with double area $2(54) = 108$, multiply both 9 and 6 by the same value x.

$9x \cdot 6x = 108$

$54x^2 = 108$

$x^2 = 2$

$x = \sqrt{2}$

Dilate the rectangle by a scale factor of $\sqrt{2}$ to get an area twice that of the original area. To produce an image whose area is n times the area of the original polygon, multiply by a scale factor of $\sqrt{n}$.

Quiz for the lessons "Use Proportionality Theorems" and "Perform Similarity Transformations"

1. $\frac{x}{4} = \frac{14}{7}$

$7x = 56$

$x = 8$

2. $\frac{7}{x} = \frac{6}{18}$

$126 = 6x$

$21 = x$

3. $\frac{2}{7} = \frac{3}{x}$

$2x = 21$

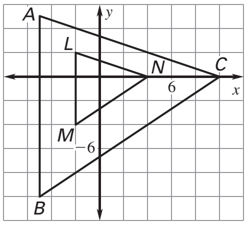
$x = 10.5$

4. $(x, y) \rightarrow (0.4x, 0.4y)$

$A(-5, 5) \rightarrow L(-2, 2)$

$B(-5, -10) \rightarrow M(-2, -4)$

$C(10, 0) \rightarrow N(4, 0)$

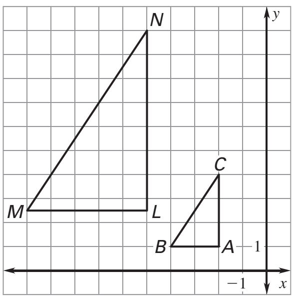


5. $(x, y) \rightarrow (2.5x, 2.5y)$

$A(-2, 1) \rightarrow L(-5, 2.5)$

$B(-4, 1) \rightarrow M(-10, 2.5)$

$C(-2, 4) \rightarrow N(-5, 10)$



Extension for the lesson "Perform Similarity Transformations"

1. slope of $\overline{AB} = \frac{4-3}{8-1} = \frac{1}{7}$

run: $\frac{4}{5}$ of $7 = \frac{28}{5} = 5.6$

rise: $\frac{4}{5}$ of $1 = \frac{4}{5} = 0.8$

The coordinates of P are $(1 + 5.6, 3 + 0.8) = (6.6, 3.8)$. The ratio of AP to PB is 4 to 1.

2. slope of $\overline{AB} = \frac{5-1}{4-(-2)} = \frac{4}{6}$

run: $\frac{3}{10}$ of $6 = \frac{18}{10} = 1.8$

rise: $\frac{3}{10}$ of $4 = \frac{12}{10} = 1.2$

The coordinates of P are $(-2 + 1.8, 1 + 1.2) = (-0.2, 2.2)$. The ratio of AP to PB is 3 to 7.

3. slope of $\overline{AB} = \frac{-2-0}{3-8} = \frac{-2}{-5}$

run: $\frac{1}{5}$ of $-5 = \frac{-5}{5} = -1$

rise: $\frac{1}{5}$ of $-2 = \frac{-2}{5} = -0.4$

The coordinates of P are $(8 - 1, 0 - 0.4) = (7, -0.4)$. The ratio of AP to PB is 1 to 4.

4. slope of $\overline{AB} = \frac{1-(-4)}{6-(-2)} = \frac{5}{8}$

run: $\frac{3}{5}$ of $8 = \frac{24}{5} = 4.8$

rise: $\frac{3}{5}$ of $5 = \frac{15}{5} = 3$

The coordinates of P are $(-2 + 4.8, -4 + 3) = (2.8, -1)$. The ratio of AP to PB is 3 to 2.

5–8. Check students' constructions.

9. *Sample answer*: If parallel lines intersect two transversals, they divide the transversals proportionally. Since $AD = DE = EF = FG$, any two segments have a ratio of 1. Therefore any two segments of $\overline{AB}$ have a ratio of 1, which means $AJ = JK = KL = LB$.

10. It is point P.

11. *Sample answer*: To find a point that lies beyond point B, use a fraction that is greater than 1 along with the rise and run from A to B to find the required coordinates.

Mixed Review of Problem Solving for the lessons "Use Proportionality Theorems" and "Perform Similarity Transformations"

1. The triangle formed by you and your shadow is similar to the triangle formed by the cactus and its shadow.

Set up the proportion $\frac{6}{8} = \frac{x}{84}$ and solve to get $x = 63$.

The cactus is 63 feet tall.

2. Sometimes. *Sample answer:* One possibility is for line ℓ_2 to be vertical with ℓ_1 and ℓ_3 being non-vertical so that the ratios of lengths $2x : x$ and $2y : y$ are achieved. In this case, ℓ_1, ℓ_2, and ℓ_3 are not parallel. It is also possible for lines ℓ_1, ℓ_2, and ℓ_3 to be parallel with the line labeled with $2x$ and $2y$ drawn at enough of an angle to the three parallel lines so that the ratios of lengths $2x : x$ and $2y : y$ are achieved.

3. $\frac{LM}{LJ} = \frac{MK}{KH}$

$\frac{LM}{4.7} = \frac{8}{7}$

$7LM = 37.6$

$LM \approx 5.4$

LM is about 5.4 meters.

4. The scale factor is the ratio of the length of the photograph to the length of the greeting card which is $\frac{2}{5}$, or 0.4.

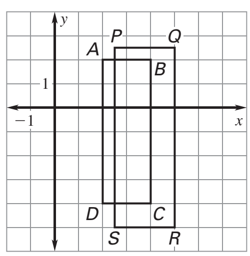
5. a. $(x, y) \rightarrow \left(\frac{5}{4}x, \frac{5}{4}y\right)$

$A(2, 2) \rightarrow P\left(\frac{5}{2}, \frac{5}{2}\right)$

$B(4, 2) \rightarrow Q\left(5, \frac{5}{2}\right)$

$C(4, -4) \rightarrow R(5, -5)$

$D(2, -4) \rightarrow S\left(\frac{5}{2}, -5\right)$



b. $\dfrac{\text{perimeter of } PQRS}{\text{perimeter of } ABCD} = \dfrac{\frac{5}{2} + \frac{15}{2} + \frac{5}{2} + \frac{15}{2}}{2 + 6 + 2 + 6} = \dfrac{5}{4}$

Scale factor $= \dfrac{PQ}{AB} = \dfrac{\frac{5}{2}}{2} = \dfrac{5}{4}$

The ratio of the perimeters is equal to the scale factor.

c. $\dfrac{\text{area of } PQRS}{\text{area of } ABCD} = \dfrac{bh}{bh} = \dfrac{\frac{5}{2} \cdot \frac{15}{2}}{2 \cdot 6} = \dfrac{25}{16}$

The ratio of the areas is equal to the square of the scale factor.

Chapter Review for the chapter "Similarity"

1. A *dilation* is a transformation in which the original figure and its image are similar.

2. If a dilation in a figure that is smaller than the original, it is a *reduction*.

3. The ratio of the side lengths of two similar figures is the *scale factor*.

4. All angles are right angles, so corresponding angles are congruent. Corresponding side lengths are proportional:

$\dfrac{AB}{EF} = \dfrac{12}{9} = \dfrac{4}{3}$ $\quad \dfrac{BC}{FG} = \dfrac{8}{6} = \dfrac{4}{3}$

$\dfrac{CD}{GH} = \dfrac{12}{9} = \dfrac{4}{3}$ $\quad \dfrac{AD}{EH} = \dfrac{8}{6} = \dfrac{4}{3}$

So, $ABCD \sim EFGH$.

The scale factor of $ABCD$ to $EFGH$ is $\frac{4}{3}$.

5. Corresponding side lengths are proportional:

$\dfrac{XY}{PQ} = \dfrac{25}{10} = \dfrac{5}{2}$ $\quad \dfrac{YZ}{QR} = \dfrac{15}{6} = \dfrac{5}{2}$ $\quad \dfrac{XZ}{PR} = \dfrac{20}{8} = \dfrac{5}{2}$

Angles R and Z are right angles, so they are congruent. Assuming $\angle P \cong \angle X$ and $\angle Q \cong \angle Y$, all angles are congruent. So, $\triangle XYZ \sim \triangle PQR$. The scale factor of $\triangle XYZ$ to $\triangle PQR$ is $\frac{5}{2}$.

6. $\dfrac{\text{small poster's perimeter}}{\text{large poster's perimeter}} = \dfrac{4}{5}$

$$\frac{x \text{ in.}}{85 \text{ in.}} = \frac{4}{5}$$

$$5x = 340$$

$$x = 68$$

The small poster's perimeter is 68 inches.

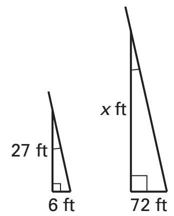
7. *Sample answer*: dilation with scale factor $\frac{2}{3}$ followed by a reflection through a vertical line.

8. *Sample answer*: translation down and left followed by a dilation with scale factor 2.

9. Because $m\angle Q$ and $m\angle T$ both equal 35°, $\angle Q \cong \angle T$. You know $\angle QSR \cong \angle TSU$ by the Vertical Angles Congruence Theorem. So, $\triangle QRS \sim \triangle TUS$ by the AA Similarity Postulate.

10. Because they are right angles, $\angle C \cong \angle F$. By the Triangle Sum Theorem, $60° + 90° + m\angle B = 180°$, so $m\angle B = 30°$ and $\angle B \cong \angle E$. So, $\triangle ABC \sim \triangle DEF$ by the AA Similarity Postulate.

11.



Not drawn to scale

$\dfrac{27 \text{ ft}}{x \text{ ft}} = \dfrac{6 \text{ ft}}{72 \text{ ft}}$

$4944 = 6x$

$324 = x$

The tower is 324 feet tall.

12. By the Reflexive Property, $\angle C \cong \angle C$.

Ratios of the lengths of the sides that include $\angle C$:

Shorter sides: $\dfrac{CD}{CE} = \dfrac{3.5}{10.5} = \dfrac{1}{3}$ $\quad$ Larger sides: $\dfrac{CB}{CA} = \dfrac{4}{12} = \dfrac{1}{3}$

The lengths of the sides that include $\angle C$ are proportional. So, $\triangle CBD \sim \triangle CAE$ by the SAS Similarity Theorem.

13. Ratios of the lengths of corresponding sides:

Shortest sides: $\dfrac{QU}{QT} = \dfrac{9}{13.5} = \dfrac{2}{3}$ $\quad$ Longest sides: $\dfrac{QR}{QS} = \dfrac{14}{21} = \dfrac{2}{3}$ $\quad$ Remaining sides: $\dfrac{UR}{TS} = \dfrac{10}{15} = \dfrac{2}{3}$

All the ratios are equal, so $\triangle RUQ \sim \triangle STQ$ by the SSS Similarity Theorem.

14. $\dfrac{EB}{BD} = \dfrac{16}{10} = \dfrac{8}{5}$ $\quad \dfrac{EA}{AC} = \dfrac{28}{20} = \dfrac{7}{5}$

Because $\dfrac{EB}{BD} \neq \dfrac{EA}{AC}$, $\overline{AB}$ is not parallel to $\overline{CD}$.

15. $\dfrac{EB}{BD} = \dfrac{20}{12} = \dfrac{5}{3}$ $\quad \dfrac{EA}{AC} = \dfrac{22.5}{13.5} = \dfrac{5}{3}$

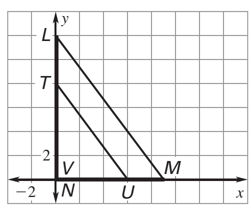
Because $\dfrac{EB}{BD} = \dfrac{EA}{AC}$, $\overline{AB} \parallel \overline{CD}$ by the Converse of the Triangle Proportionality Theorem.

16. $(x, y) \rightarrow \left(\frac{3}{2}x, \frac{3}{2}y\right)$

$T(0, 8) \rightarrow L(0, 12)$

$U(6, 0) \rightarrow M(9, 0)$

$V(0, 0) \rightarrow N(0, 0)$



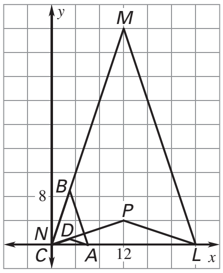
17. $(x, y) \to (4x, 4y)$

$A(6, 0) \to L(24, 0)$

$B(3, 9) \to M(12, 36)$

$C(0, 0) \to N(0, 0)$

$D(3, 1) \to P(12, 4)$



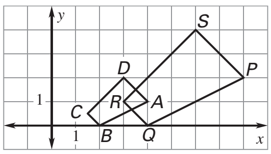
18. $(x, y) \to (0.5x, 0.5y)$

$P(8, 2) \to A(4, 1)$

$Q(4, 0) \to B(2, 0)$

$R(3, 1) \to C(1.5, 0.5)$

$S(6, 4) \to D(3, 2)$



Chapter Test for the chapter "Similarity"

1. $\angle R \cong \angle C, \angle Q \cong \angle B, \angle P \cong \angle A$

2. Because $\triangle PQR \sim \triangle ABC$

$\frac{PR}{AC} = \frac{PQ}{AB} = \frac{QR}{BC}$. So, $\frac{12}{10} = \frac{21}{x} = \frac{24}{20}$.

3. $\frac{21}{x} = \frac{12}{10}$

$12x = 210$

$x = 17.5$

4. Ratios of the lengths of the corresponding sides:

Shortest sides	Longest sides	Remaining sides
$\frac{MN}{YZ} = \frac{20}{11}$	$\frac{LN}{XZ} = \frac{30}{18} = \frac{5}{3}$	$\frac{LM}{XY} = \frac{25}{15} = \frac{5}{3}$

The ratios are not all equal, so $\triangle LMN$ and $\triangle XYZ$ are not similar.

5. By the Reflexive Property, $\angle D \cong \angle D$. By the Triangle Sum Theorem, $62° + 33° + m\angle B = 180°$, so $m\angle B = 85°$. Because $m\angle B$ and $m\angle ECD$ both equal $85°$, $\angle B \cong \angle ECD$. So, $\triangle ABD \sim \triangle ECD$ by the AA Similarity Postulate.

6. By the Vertical Angles Congruence Theorem, $\angle LNM \cong \angle JNK$. Ratios of the lengths of the sides that include $\angle LNM$ and $\angle JNK$:

Shorter sides	Longer sides
$\frac{LN}{JN} = \frac{6}{18} = \frac{1}{3}$	$\frac{MN}{KN} = \frac{9}{27} = \frac{1}{3}$

The lengths of the sides that include $\angle LNM$ and $\angle JNK$ are proportional. So, $\triangle LNM \sim \triangle JNK$ by the SAS Similarity Theorem.

7. $\frac{DA}{DE} = \frac{AB}{EC}$

$\frac{10 + 8}{10} = \frac{x}{9}$

$162 = 10x$

$16.2 = x$

The length of $\overline{AB}$ is 16.2.

8. $\frac{FE}{ED} = \frac{AB}{BC}$

$\frac{21}{35} = \frac{AB}{40}$

$24 = AB$

The length of $\overline{AB}$ is 24.

9. $\frac{DA}{CD} = \frac{BA}{BC}$

$\frac{52}{20} = \frac{BA}{30}$

$78 = BA$

The length of $\overline{AB}$ is 78.

10. The dilation from Figure A to Figure B is an enlargement.

Scale factor: $\frac{\text{length of left side of B}}{\text{length of left side of A}} = \frac{5}{2}$

11. The dilation from Figure A to Figure B is a reduction.

Scale factor: $\frac{\text{length of left side of B}}{\text{length of left side of A}} = \frac{2}{4} = \frac{1}{2}$

12. The distance around the bases is the perimeter of a square.

$\frac{\text{model's perimeter}}{\text{actual perimeter}} = \frac{1}{180}$

$\frac{x \text{ ft}}{360 \text{ ft}} = \frac{1}{180}$

$x = 2$

The distance around the bases in your model is 2 feet.

Algebra Review for the chapter "Similarity"

1. $x^2 + 8 = 108$

$x^2 = 100$

$x = \pm 10$

2. $2x^2 - 1 = 49$

$2x^2 = 50$

$x^2 = 25$

$x = \pm 5$

3. $x^2 - 9 = 8$
$x^2 = 17$
$x = \pm\sqrt{17}$

4. $5x^2 + 11 = 1$
$5x^2 = -10$
$x^2 = -2$
no solution

5. $2(x^2 - 7) = 6$
$x^2 - 7 = 3$
$x^2 = 10$
$x = \pm\sqrt{10}$

6. $9 = 21 + 3x^2$
$-12 = 3x^2$
$-4 = x^2$
no solution

7. $3x^2 - 17 = 43$
$3x^2 = 60$
$x^2 = 20$
$x = \pm\sqrt{20}$
$x = \pm 2\sqrt{5}$

8. $56 - x^2 = 20$
$36 = x^2$
$\pm 6 = x$

9. $-3(-x^2 + 5) = 39$
$-x^2 + 5 = -13$
$18 = x^2$
$\pm\sqrt{18} = x$
$\pm 3\sqrt{2} = x$

10. $\sqrt{\frac{7}{81}} = \frac{\sqrt{7}}{\sqrt{81}} = \frac{\sqrt{7}}{9}$

11. $\sqrt{\frac{3}{5}} = \frac{\sqrt{3}}{\sqrt{5}} = \frac{\sqrt{3}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{15}}{5}$

12. $\sqrt{\frac{24}{27}} = \sqrt{\frac{8}{9}} = \frac{\sqrt{8}}{\sqrt{9}} = \frac{2\sqrt{2}}{3}$

13. $\frac{3\sqrt{7}}{\sqrt{12}} = \frac{3\sqrt{7}}{\sqrt{12}} \cdot \frac{\sqrt{12}}{\sqrt{12}} = \frac{3\sqrt{84}}{12} = \frac{\sqrt{84}}{4} = \frac{\sqrt{21}}{2}$

14. $\sqrt{\frac{75}{64}} = \frac{\sqrt{75}}{\sqrt{64}} = \frac{5\sqrt{3}}{8}$

15. $\frac{\sqrt{2}}{\sqrt{200}} = \frac{\sqrt{2}}{10\sqrt{2}} = \frac{1}{10}$

16. $\frac{9}{\sqrt{27}} = \frac{9}{3\sqrt{3}} = \frac{3}{\sqrt{3}} = \frac{3}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{3}}{3} = \sqrt{3}$

17. $\sqrt{\frac{21}{42}} = \sqrt{\frac{1}{2}} = \frac{\sqrt{1}}{\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

Extra Practice

For the chapter "Similarity"

1. $x + 3x + 5x = 180°$
$9x = 180°$
$x = 20°$
The angle measures are $20°$, $3(20°) = 60°$, and $5(20°) = 100°$.

2. $x + 5x + 6x = 180°$
$12x = 180°$
$x = 15°$
The angle measures are $15°$, $5(15°) = 75°$, and $6(15°) = 90°$.

3. $2x + 3x + 5x = 180°$
$10x = 180°$
$x = 18°$
The angle measures are $2(18°) = 36°$, $3(18°) = 54°$, and $5(18°) = 90°$.

4. $5x + 6x + 9x = 180°$
$20x = 180°$
$x = 9°$
The angle measures are $5(9°) = 45°$, $6(9°) = 54°$, and $9(9°) = 81°$.

5. $\frac{x}{14} = \frac{6}{21}$
$x \cdot 21 = 14 \cdot 6$
$21x = 84$
$x = 4$

6. $\frac{15}{y} = \frac{20}{4}$
$15 \cdot 4 = y \cdot 20$
$60 = 20y$
$3 = y$

7. $\frac{3}{2z + 1} = \frac{1}{7}$
$3 \cdot 7 = (2z + 1) \cdot 1$
$21 = 2z + 1$
$20 = 2z$
$10 = z$

8. $\frac{a - 3}{2} = \frac{2a - 1}{6}$
$(a - 3)6 = 2(2a - 1)$
$6a - 18 = 4a - 2$
$2a = 16$
$a = 8$

9. $\frac{6}{3} = \frac{x + 8}{-1}$
$6(-1) = 3(x + 8)$
$-6 = 3x + 24$
$-30 = 3x$
$-10 = x$

10. $\frac{x + 6}{3} = \frac{x - 5}{2}$
$(x + 6)2 = 3(x - 5)$
$2x + 12 = 3x - 15$
$27 = x$

11. $\frac{x - 2}{4} = \frac{x + 10}{10}$
$(x - 2)10 = 4(x + 10)$
$10x - 20 = 4x + 40$
$6x = 60$
$x = 10$

12. $\frac{12}{8} = \frac{5 + t}{t - 3}$
$12(t - 3) = 8(5 + t)$
$12t - 36 = 40 + 8t$
$4t = 76$
$t = 19$

13. $x = \sqrt{4 \cdot 9} = \sqrt{36} = 6$
The geometric mean of 4 and 9 is 6.

14. $x = \sqrt{3 \cdot 48} = \sqrt{144} = 12$
The geometric mean of 3 and 48 is 12.

15. $x = \sqrt{9 \cdot 16} = \sqrt{144} = 12$

The geometric mean of 9 and 16 is 12.

16. $x = \sqrt{7 \cdot 11} = \sqrt{77}$

The geometric mean of 7 and 11 is $\sqrt{77} \approx 8.8$.

17. If $\frac{7}{x} = \frac{9}{y}$, then $\frac{x}{7} = \frac{y}{9}$ by the Reciprocal Property of Proportions.

18. If $\frac{2}{8} = \frac{1}{x}$, then $\frac{8+2}{2} = \frac{x+1}{1}$, because you can apply the Reciprocal Property of Proportions and then add the value of each ratio's denominator to its numerator.

19. $\frac{NJ}{NK} = \frac{NL}{NM}$

$\frac{6}{NK} = \frac{6+15}{14}$

$6 \cdot 14 = NK(6+15)$

$84 = 21 \cdot NK$

$4 = NK$

20. $\frac{CB}{DE} = \frac{BA}{EF}$

$\frac{CB}{BA} = \frac{DE}{EF}$

$\frac{BA}{CB} = \frac{EF}{DE}$

$\frac{BA+CB}{CB} = \frac{EF+DE}{DE}$

$\frac{CA}{10} = \frac{12+8}{8}$

$CA(8) = 10(12+8)$

$8CA = 200$

$CA = 25$

21. The diagram shows $\angle R \cong \angle S$, $\angle Q \cong \angle T$, $\angle P \cong \angle U$, and $\angle N \cong \angle V$.

$\frac{RQ}{ST} = \frac{11}{20}$, $\frac{QP}{TU} = \frac{8.8}{16} = \frac{88}{160} = \frac{11}{20}$, $\frac{PN}{UV} = \frac{11}{20}$ and

$\frac{RN}{SV} = \frac{8.8}{16} = \frac{88}{160} = \frac{11}{20}$

Because corresponding angles are congruent and corresponding side lengths are proportional, $RQPN \sim STUV$. The scale factor of $RQPN$ to $STUV$ is equal to the ratio of any two corresponding lengths, or 11 : 20.

22. The diagram shows $\angle D \cong \angle J$, $\angle E \cong \angle L$, and $\angle F \cong \angle K$.

$\frac{DE}{JL} = \frac{6}{3} = 2$, $\frac{DF}{JK} = \frac{8}{4} = 2$, $\frac{EF}{LK} = \frac{3}{1.5} = 2$

Because corresponding angles are congruent and corresponding side lengths are proportional, $\triangle DEF \sim \triangle JLK$. The scale factor of $\triangle DEF$ to $\triangle JLK$ is equal to the ratio of any two corresponding lengths, or 2 : 1.

23. The scale factor of $\triangle PQR$ to $\triangle LMN$: $\frac{QR}{MN} = \frac{36}{12} = \frac{3}{1}$

24. $m\angle P + m\angle Q + m\angle R = 180°$

$x° + 90° + 22.6° = 180°$

$x = 67.4$

$\frac{y}{13} = \frac{3}{1}$ $\qquad$ $\frac{15}{z} = \frac{3}{1}$

$y = 13 \cdot 3 = 39$ $\qquad$ $15 = z \cdot 3$

$5 = z$

.25. Perimeter of $\triangle PQR$: $15 + 36 + 39 = 90$

Perimeter of $\triangle LMN$: $5 + 12 + 13 = 30$

26. The blue special segments are altitudes of triangles.

$\frac{y+8}{y} = \frac{27}{18}$

$(y+8)18 = y(27)$

$18y + 144 = 27y$

$144 = 9y$

$16 = y$

27. The blue special segments are angle bisectors at corresponding vertices.

$\frac{4y+2}{3y+4} = \frac{36}{30}$

$(4y+2)30 = (3y+4)36$

$120y + 60 = 108y + 144$

$12y = 84$

$y = 7$

28. In $\triangle PQR$, $63° + 78° + m\angle R = 180°$

$m\angle R = 39°$

So, $\angle R \cong \angle V$ and $\angle P \cong \angle W$.

Therefore, $\triangle PQR \sim \triangle WUV$ by AA Similarity Postulate.

29. In $\triangle BFG$, $33° + 110° + m\angle G = 180°$

$m\angle G = 37°$

So, $\triangle ABC$ is not similar to $\triangle FBG$, because $\angle C \ncong \angle G$.

30. Because $\overline{VW} \perp \overline{WX}$ and $\overline{XY} \perp \overline{WX}$, $\overline{VW} \parallel \overline{XY}$ by the Lines Perpendicular to a Transversal Theorem. So, $\angle 1 \cong \angle 3$ by the Corresponding Angles Postulate. Also $\angle W \cong \angle Z$ by the Right Angles Congruence Theorem. So, $\triangle VWX \sim \triangle XZY$ by the AA Similarity Postulate.

31. Because $\overline{JK} \parallel \overline{NP}$ and $\overline{KL} \parallel \overline{PM}$, $\angle J \cong \angle PNM$ and $\angle L \cong \angle PMN$ by the Corresponding Angles Postulate. Therefore, $\triangle JKL \sim \triangle NPM$ by the AA Similarity Postulate.

32. In $\triangle VXW$ and $\triangle ZXY$, $\angle VXW$ and $\angle ZXY$ are vertical angles, so $\angle VXW \cong \angle ZXY$.

$\frac{VX}{ZX} = \frac{3}{6} = \frac{1}{2}$ and $\frac{WX}{YX} = \frac{4}{8} = \frac{1}{2}$

Because an angle of $\triangle VXW$ is congruent to an angle of $\triangle ZXY$, and the lengths of the sides including these angles are proportional, $\triangle VXW \sim \triangle ZXY$ by the SAS Similarity Theorem.

33. In $\triangle HJK$ and $\triangle SRT$, by comparing the corresponding sides in order from smallest to largest you have

$$\frac{HJ}{SR} = \frac{18}{30} = \frac{3}{5}, \frac{JK}{RT} = \frac{24}{40} = \frac{3}{5}, \text{ and } \frac{HK}{ST} = \frac{27}{45} = \frac{3}{5}.$$

Because the corresponding side lengths are proportional, $\triangle HJK \sim \triangle SRT$ by the SSS Similarity Theorem.

34. An angle of the triangle is bisected, so Theorem 6.7 applies.

$$\frac{a}{17} = \frac{21}{34}$$

$$34a = 357$$

$$a = 10.5$$

35. Apply the Triangle Proportionality Thoerem.

$$\frac{5}{2} = \frac{7.5}{x}$$

$$5x = 15$$

$$x = 3$$

Because $\frac{7.5}{3 + 7.5} = \frac{7.5}{10.5} = \frac{5}{7}$ and $\frac{5}{5 + 2} = \frac{5}{7}$, the two triangles are similar by the SAS Similarity Theorem.

So, $\frac{5}{7} = \frac{6}{y}$.

$$5y = 42$$

$$y = \frac{42}{5} = 8.4$$

36. Because three parallel lines intersect two transversals, apply Theorem 6.6.

$$\frac{x}{5} = \frac{24}{6}$$

$$6x = 120$$

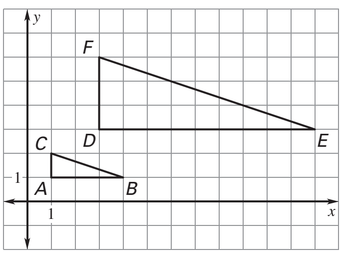
$$x = 20$$

37. $(x, y) \rightarrow (3x, 3y)$

$A(1, 1) \rightarrow D(3, 3)$

$B(4, 1) \rightarrow E(12, 3)$

$C(1, 2) \rightarrow F(3, 6)$



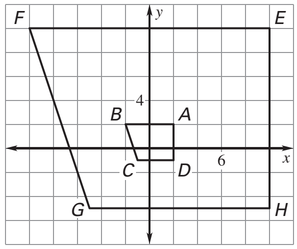
38. $(x, y) \rightarrow (5x, 5y)$

$A(2, 2) \rightarrow E(10, 10)$

$B(-2, 2) \rightarrow F(-10, 10)$

$C(-1, -1) \rightarrow G(-5, -5)$

$D(2, -1) \rightarrow H(10, -5)$

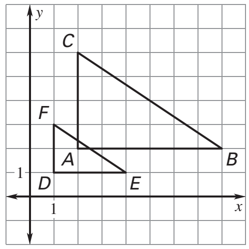


39. $(x, y) \rightarrow \left(\frac{1}{2}x, \frac{1}{2}y\right)$

$A(2, 2) \rightarrow D(1, 1)$

$B(8, 2) \rightarrow E(4, 1)$

$C(2, 6) \rightarrow F(1, 3)$



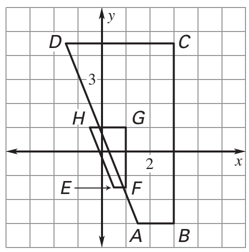
40. $(x, y) \rightarrow \left(\frac{1}{3}x, \frac{1}{3}y\right)$

$A(3, -6) \rightarrow E(1, -2)$

$B(6, -6) \rightarrow F(2, -2)$

$C(6, 9) \rightarrow G(2, 3)$

$D(-3, 9) \rightarrow H(-1, 3)$



41. Enlargement; Scale factor: $\frac{\text{length of B}}{\text{length of A}} = \frac{6}{2} = 3$ or $3:1$

42. Reduction; Scale factor: $\frac{\text{length of B}}{\text{length of A}} = \frac{5}{10} = \frac{1}{2}$ or $1:2$