Chapter 4 Congruent Triangles

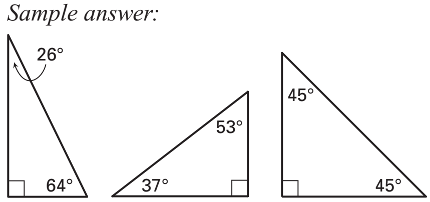
Prerequisite Skills for the chapter "Congruent Triangles"

1. $\angle A$ is an obtuse angle because $m\angle A > 90°$.
2. $\angle B$ is a right angle because $m\angle B = 90°$.
3. $\angle C$ is an acute angle because $m\angle C < 90°$.
4. $\angle D$ is an obtuse angle because $m\angle D > 90°$.
5. $70 + 2y = 180$
   $2y = 110$
   $y = 55$
6. $2x = 5x - 54$
   $-3x = -54$
   $x = 18$
7. $40 + x + 65 = 180$
   $x + 105 = 180$
   $x = 75$
8. $M\left(\frac{2 + (-1)}{2}, \frac{-5 + (-2)}{2}\right) = M\left(\frac{1}{2}, -\frac{7}{2}\right)$
   The midpoint of $\overline{PQ}$ is $\left(\frac{1}{2}, -\frac{7}{2}\right)$.
9. $M\left(\frac{-4 + 1}{2}, \frac{7 + (-5)}{2}\right) = M\left(-\frac{3}{2}, 1\right)$
   The midpoint of $\overline{PQ}$ is $\left(-\frac{3}{2}, 1\right)$.
10. $M\left(\frac{h + h}{2}, \frac{k + 0}{2}\right) = M\left(\frac{2h}{2}, \frac{k}{2}\right) = M\left(h, \frac{k}{2}\right)$
    The midpoint of $\overline{PQ}$ is $\left(h, \frac{k}{2}\right)$.
11. $\angle 2 \cong \angle 3$ by the Vertical Angles Congruence Theorem.
12. $\angle 1 \cong \angle 4$ by the Corresponding Angles Postulate.
13. $\angle 2 \cong \angle 6$ by the Alternate Interior Angles Theorem.
14. The angles are not congruent, unless they are right angles.

Lesson 4.1 Apply Triangle Sum Properties

Investigating Geometry Activity for the lesson "Apply Triangle Sum Properties"

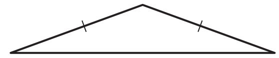
1. The measure of the third angle of a triangle can be found by subtracting the sum of the other two angles from 180°.
2. *Sample answer:*

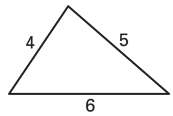


The two acute angles of each triangle sum to 90°, so the angles are complementary.

Guided Practice for the lesson "Apply Triangle Sum Properties"

1. *Sample answer:*





2. $AB = \sqrt{(3 - 0)^2 + (3 - 0)^2} = \sqrt{18} \approx 4.24$
   $AC = \sqrt{(-3 - 0)^2 + (3 - 0)^2} = \sqrt{18} \approx 4.24$
   $BC = \sqrt{(-3 - 3)^2 + (3 - 3)^2} = \sqrt{36} = 6$
   Because $AB = AC$, $\triangle ABC$ is an isosceles triangle.
   Slope $\overline{AB} = \frac{3 - 0}{3 - 0} = 1$
   Slope $\overline{AC} = \frac{3 - 0}{-3 - 0} = -1$
   The product of the slopes is $1(-1) = -1$, so $\overline{AB} \perp \overline{AC}$ and $\angle BAC$ is a right angle. Therefore, $\triangle ABC$ is a right isosceles triangle.
3. $40° + 3x° = (5x - 10)°$
   $50 = 2x$
   $25 = x$
   $m\angle 1 + 40° + 3(25)° = 180°$
   $m\angle 1 = 65°$
4. $m\angle A + m\angle B + m\angle C = 180°$
   $x° + 2x° + 3x° = 180°$
   $6x = 180$
   $x = 30$
   $m\angle A = x = 30°$
   $m\angle B = 2x = 60°$
   $m\angle C = 3x = 90°$
5. $2x° + (x - 6)° = 90°$
   $3x = 96$
   $x = 32$
   Therefore, the measures of the two acute angles are $2(32)° = 64°$ and $(32 - 6)° = 26°$.
6. $90° + 60° = 150°$
   By the Exterior Angle Theorem, the angle between the staircase and the extended segment is 150°.

Exercises for the lesson "Apply Triangle Sum Properties"

Skill Practice

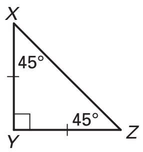
1. C; The triangle is right because it contains a 90° angle.
2. E; The triangle is equilateral because all sides are the same length.
3. F; The triangle is equiangular because all angles have the same measure.

4. A; The triangle is isosceles because two sides are the same length.

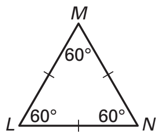
5. B; The triangle is scalene because each side has a different length.

6. D; The triangle is obtuse because it contains an angle with measure greater than 90°.

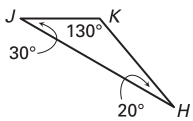
7. *Sample answer:* A right triangle cannot also be obtuse because the sum of the other two angles cannot be greater than 90°.

8. 

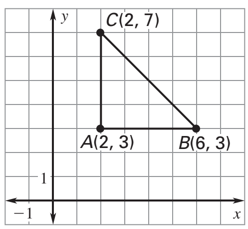
$\triangle XYZ$ is a right isosceles triangle.

9. 

$\triangle LMN$ is an equiangular equilateral triangle.

10. 

$\triangle JKH$ is an obtuse scalene triangle.

11. 

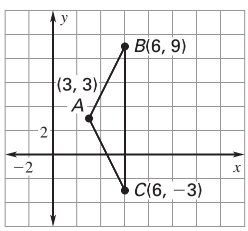
$AB = \sqrt{(6-2)^2 + (3-3)^2} = \sqrt{16} = 4$

$AC = \sqrt{(2-2)^2 + (7-3)^2} = \sqrt{16} = 4$

$BC = \sqrt{(2-6)^2 + (7-3)^2} = \sqrt{32} \approx 5.66$

So, $AB = AC$.

$\overline{CA}$ is vertical and $\overline{BA}$ is horizontal. So, $\overline{CA} \perp \overline{BA}$ and $\angle CAB$ is a right angle. Therefore, $\triangle ABC$ is a right isosceles triangle.

12. 

$AB = \sqrt{(6-3)^2 + (9-3)^2} = \sqrt{45} \approx 6.71$

$AC = \sqrt{(6-3)^2 + (-3-3)^2} = \sqrt{45} \approx 6.71$

$BC = \sqrt{(6-6)^2 + (-3-9)^2} = \sqrt{144} \approx 12$

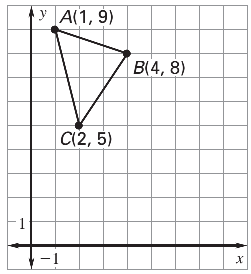
So $AB = AC$.

Slope $\overline{BA} = \dfrac{9-3}{6-3} = 2$

Slope $\overline{CA} = \dfrac{-3-3}{6-3} = -2$

Slope $\overline{BC} = \dfrac{-3-9}{6-6}$ is undefined.

So, there are no right angles. Therefore, $\triangle ABC$ is an isosceles triangle.

13. 

$AB = \sqrt{(4-1)^2 + (8-9)^2} = \sqrt{10} \approx 3.16$

$AC = \sqrt{(2-1)^2 + (5-9)^2} = \sqrt{17} \approx 4.12$

$BC = \sqrt{(4-2)^2 + (8-5)^2} = \sqrt{13} \approx 3.61$

So, there are no equal sides.

Slope $\overline{AB} = \dfrac{8-9}{4-1} = -\dfrac{1}{3}$

Slope $\overline{AC} = \dfrac{5-9}{2-1} = -4$

Slope $\overline{BC} = \dfrac{5-8}{2-4} = \dfrac{3}{2}$

So, there are no right angles. Therefore, $\triangle ABC$ is a scalene triangle.

14. $60° + 60° + x° = 180°$

$x = 60$

Because each angle measures 60°, the triangle is equiangular.

15. $x° + 3x° + 60° = 180°$

$4x = 120$

$x = 30$

The measures of the angles are 30°, 60°, and $3(30) = 90°$. So, the triangle is a right triangle.

16. $x° = 64° + 70°$

$x = 134$

The angles in the triangle measure 64°, 70°, and $180 - 134 = 46°$, so the triangle is acute.

17. $x° + 45° = (2x - 2)°$

$-x = -47$

$x = 47$

$2(47) - 2 = 92°$

18. $24° + (2x + 18)° = (3x + 6)°$

$36 = x$

$3(36) + 6 = 114°$

19. $90° + x° + (3x + 2)° = 180°$

$4x = 88$

$x = 22$

$m\angle 1 = 90 + 3(22) + 2 = 158°$

20. *Sample answer:* By the Corollary to the Triangle Sum Theorem, the acute angles must sum to 90°. So you would solve $3x + 2x = 90$ for x, then use substitution to find each angle measure.

21. $m\angle 1 + 40° = 90°$

$m\angle 1 = 50°$

22. $m\angle 2 = 90° + 40°$

$m\angle 2 = 130°$

23. $m\angle 3 = m\angle 1$

$m\angle 3 = 50°$

24. $m\angle 4 = m\angle 2$

$m\angle 4 = 130°$

25. $m\angle 5 + m\angle 3 = 90°$

$m\angle 5 + 50° = 90°$

$m\angle 5 = 40°$

26. $m\angle 6 + m\angle 4 + 20° = 180°$

$m\angle 6 + 130° + 20° = 180°$

$m\angle 6 = 30°$

27. Let $m\angle P = m\angle R = x°$. Then $m\angle Q = 2x°$.

$x° + x° + 2x° = 180°$

$4x = 180$

$x = 45$

So, $m\angle P = 45°$, $m\angle R = 45°$, and $m\angle Q = 2(45)° = 90°$.

28. Let $m\angle G = x°$. Then $m\angle F = 3x°$ and $m\angle E = (3x - 30)°$.

$x° + 3x° + (3x - 30)° = 180°$

$7x = 210$

$x = 30$

So, $m\angle G = 30°$, $m\angle F = 3(30)° = 90°$, and $m\angle E = (3(30) - 30)° = 60°$.

29. The second statement in incorrect. Being isosceles does not guarantee three congruent sides, only two. So, if $\triangle ABC$ is equilateral, then it is isosceles as well.

30. By the Exterior Angle Theorem, the measure of the exterior angle is equal to the sum of the measures of the two nonadjacent interior angles.
So, $m\angle 1 = 80° + 50° = 130°$.

31. B; If a triangle has two acute exterior angles, then it has two obtuse interior angles. This is not possible because the sum of the angles in the triangle must be 180°.

32. $x° = 43°$, so $x = 43$.

$y° = 180° - 43° - 105°$, so $y = 32$.

33. $x° = 118°$, so $x = 118$.

$y° = 180° - 62° - 22°$, so $y = 96$.

34. $x° = 180° - 70° - 25°$, so $x = 85$.

$y° = 180° - 95° - 20°$, so $y = 65$.

35. $x° = 180° - 90° - 64°$, so $x = 26$.

$y° = 180° - 90° - 26°$, so $y = 64$.

36. $x° = 47° + 15°$, so $x = 62$.

$y° = 180° - 90° - 62°$, so $y = 28$.

37. $y° = 180° - 90° - (35° + 18°)$, so $y = 37$.

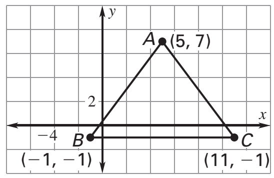
$x° = 180° - 90° - 18° - 37°$, so $x = 35$.

38. No. *Sample answer:* If an angle in a triangle is really close to zero, then the sum of the remaining two angles would be almost 180°. If these two angles were congruent, then the measure of each would be less than 90°, and the triangle would not be obtuse.

39. a. *Sample answer:* The three lines will always form a triangle as long as they do not all intersect at the same point and no two lines are parallel.

b. *Sample answer:* The three lines are $y = ax + b$, $y = x + 2$, and $y = 4x - 7$. If $a = 0$ and $b = 5$, then all three lines will intersect at only one point, (3, 5), so no triangle is formed.

c. $y = \frac{4}{3}x + \frac{1}{3}$, $y = -\frac{4}{3}x + \frac{41}{3}$, $y = -1$



$AB = \sqrt{(-1 - 5)^2 + (-1 - 7)^2} = \sqrt{100} = 10$

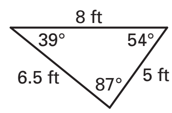
$AC = \sqrt{(11 - 5)^2 + (-1 - 7)^2} = \sqrt{100} = 10$

$BC = \sqrt{(11 - (-1))^2 + (-1 - (-1))^2} = \sqrt{144} = 12$

So, $AB = AC$, and the triangle is isosceles.

Problem Solving

40.



Because each side is a different length, the triangle is scalene. Also, because each angle measures less than 90°, the triangle is acute.

41. The side lengths are 2 inches because all sides must be equal in an equilateral triangle and $\frac{6}{3} = 2$. The angle measures will always be 60° in any equiangular triangle.

42. You could bend the strip again at 6 inches, so the sides would be 6, 6, and 8 inches. Or, you could bend the strip again at 7 inches, so the sides would be 6, 7, and 7 inches.

43. C; An angle cannot measure 0° or 180°, but it must measure between 0° and 180°.

44. $m\angle 6 + m\angle 3 = 180°$

$m\angle 6 + 65° = 180°$

$m\angle 6 = 115°$

45. $m\angle 5 = m\angle 2 + m\angle 3$

$m\angle 5 = 50° + 65° = 115°$

46. $m\angle 1 + m\angle 2 = 180°$

$m\angle 1 + 50° = 180°$

$m\angle 1 = 130°$

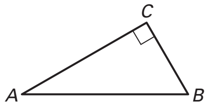
47. $m\angle 2 + m\angle 3 + m\angle 4 = 180°$

$50° + 65° + m\angle 4 = 180°$

$m\angle 4 = 65°$

48. Given: $\triangle ABC$ is a right triangle.

Prove: $\angle A$ and $\angle B$ are complementary.



Statements	Reasons
1. $\triangle ABC$ is a right triangle.	**1.** Given
2. $m\angle C = 90°$	**2.** Definition of right angle
3. $m\angle A + m\angle B + m\angle C = 180°$	**3.** Triangle Sum Theorem
4. $m\angle A + m\angle B + 90° = 180°$	**4.** Substitution Property of Equality
5. $m\angle A + m\angle B = 90°$	**5.** Subtraction Property of Equality
6. $\angle A$ and $\angle B$ are complementary.	**6.** Definition of complementary angles

49. a. $2\sqrt{2x} + 5\sqrt{2x} + 2\sqrt{2x} = 180$

b. $2\sqrt{2x} + 5\sqrt{2x} + 2\sqrt{2x} = 180$

$$9\sqrt{2x} = 180$$

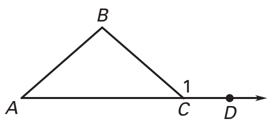
$$\sqrt{2x} = 20$$

The measures of the angles are $2(20)° = 40°$, $5(20)° = 100°$, and $2(20)° = 40°$.

c. The triangle is obtuse because it contains an angle greater than 90°.

50. Given: $\triangle ABC$, exterior angle $\angle BCD$

Prove: $m\angle BCD = m\angle CBA + m\angle BAC$



Statements	Reasons
1. $\triangle ABC$, exterior angle $\angle BCD$	**1.** Given
2. $m\angle ACD = 180°$	**2.** Definition of a straight angle
3. $m\angle ACB + m\angle BCD = m\angle ACD$	**3.** Angle Addition Postulate
4. $m\angle ACB + m\angle BCD = 180°$	**4.** Substitution Property of Equality
5. $m\angle BCD = 180° - m\angle ACB$	**5.** Subtraction Property of Equality
6. $m\angle ACB + m\angle CBA + m\angle BAC = 180°$	**6.** Triangle Sum Theorem
7. $m\angle CBA + m\angle BAC = 180° - m\angle ACB$	**7.** Subtraction Property of Equality
8. $m\angle BCD = m\angle CBA + m\angle BAC$	**8.** Transitive Property of Equality

51. *Sample answer:* Mary and Tom both reasoned correctly, but the initial plan is not correct. The measure of the exterior angle should be $100° + 50° = 150°$, not 145°.

52. a. If $AB = AC = x$ and $BC = 2x - 4$:

$$x + x + 2x - 4 = 32$$

$$4x = 36$$

$$x = 9$$

If $AB = x$ and $BC = AC = 2x - 4$:

$$x + (2x - 4) + (2x - 4) = 32$$

$$5x = 40$$

$$x = 8$$

The two possible values for x are 8 and 9.

b. If $AB = AC = x$ and $BC = 2x - 4$:

$$x + x + 2x - 4 = 12$$

$$4x = 16$$

$$x = 4$$

If $AB = x$ and $BC = AC = 2x - 4$:

$$x + (2x - 4) + (2x - 4) = 12$$

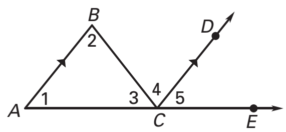
$$5x = 20$$

$$x = 4$$

There is only one possible value for x, which is 4.

53. Given: $\triangle ABC$, points D and E

Prove: $m\angle 1 + m\angle 2 + m\angle 3 = 180°$



Statements	Reasons
1. $\triangle ABC$, $\overline{AB} \parallel \overline{CD}$	**1.** Given
2. $m\angle ACE = 180°$	**2.** Definition of straight angle
3. $m\angle 3 + m\angle 4 + m\angle 5 = m\angle ACE$	**3.** Angle Addition Postulate
4. $m\angle 3 + m\angle 4 + m\angle 5 = 180°$	**4.** Substitution Property of Equality
5. $\angle 1 \cong \angle 5$	**5.** Corresponding Angles Postulate
6. $\angle 2 \cong \angle 4$	**6.** Alternate Interior Angles Theorem
7. $m\angle 1 = m\angle 5$, $m\angle 2 = m\angle 4$	**7.** Definition of congruent angles
8. $m\angle 3 + m\angle 2 + m\angle 1 = 180°$	**8.** Substitution Property of Equality

Lesson 4.2 Apply Congruence and Triangles

Guided Practice for the lesson "Apply Congruence and Triangles"

1. Corresponding angles: $\angle A \cong \angle C$, $\angle B \cong \angle D$, $\angle H \cong \angle F$, $\angle G \cong \angle E$

Corresponding sides: $\overline{AB} \cong \overline{CD}$, $\overline{BG} \cong \overline{DE}$, $\overline{GH} \cong \overline{EF}$, $\overline{HA} \cong \overline{FC}$

2.
$$\angle H \cong \angle F$$
$$m\angle H = m\angle F$$
$$(4x + 5)^\circ = 105^\circ$$
$$4x + 5 = 105$$
$$4x = 100$$
$$x = 25$$
$$m\angle H = 4(25) + 5 = 105^\circ$$

3. The sides of $\triangle PTS$ are congruent to the corresponding sides of $\triangle RTQ$ by the indicated markings. $\angle PTS \cong \angle RTQ$ by the Vertical Angles Theorem. Also, $\angle P \cong \angle R$ and $\angle S \cong \angle Q$ by the Alternate Interior Angles Theorem. Because all corresponding sides and angles are congruent, $\triangle PTS \cong \triangle RTQ$.

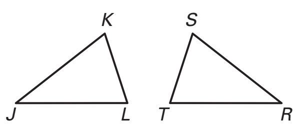
4. $m\angle DCN = 75^\circ$ by the Third Angles Theorem.

5. To show that $\triangle NDC \cong \triangle NSR$, you need to know that $\overline{DC} \cong \overline{SR}$ and $\overline{DN} \cong \overline{SN}$.

Exercises for the lesson "Apply Congruence and Triangles"

Skill Practice

1.



$\overline{JK} \cong \overline{RS}$, $\overline{KL} \cong \overline{ST}$, $\overline{JL} \cong \overline{RT}$,

$\angle J \cong \angle R$, $\angle K \cong \angle S$, $\angle L \cong \angle T$

2. *Sample answer:* To prove that two triangles are congruent, you need to show that all corresponding sides and angles are congruent.

3. $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, $\overline{AC} \cong \overline{DF}$,

$\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$

Sample answer: $\triangle CBA \cong \triangle FED$

4. $\overline{GH} \cong \overline{QR}$, $\overline{HJ} \cong \overline{RS}$, $\overline{JK} \cong \overline{ST}$, $\overline{KG} \cong \overline{TQ}$,

$\angle G \cong \angle Q$, $\angle H \cong \angle R$, $\angle J \cong \angle S$, $\angle K \cong \angle T$

Sample answer: $KJHG \cong TSRQ$

5. $m\angle Y = m\angle N = 124^\circ$

6. $m\angle M = m\angle X = 33^\circ$

7. $YX = NM = 8$

8. $\overline{YZ} \cong \overline{NL}$

9. $\triangle LNM \cong \triangle ZYX$

10. $\triangle YXZ \cong \triangle NML$

11. $\triangle XYZ \cong \triangle ZWX$ because all corresponding sides and angles are congruent.

12. The triangles cannot be proven congruent because $\overline{BC} \ncong \overline{DF}$ and only one pair of corresponding angles are shown congruent.

13. $\triangle BAG \cong \triangle CDF$ because all corresponding sides and angles are congruent.

14. $VWXYZ \cong KLMNJ$ because all corresponding sides and angles are congruent.

15. $m\angle M = 180^\circ - 70^\circ - 90^\circ = 20^\circ$

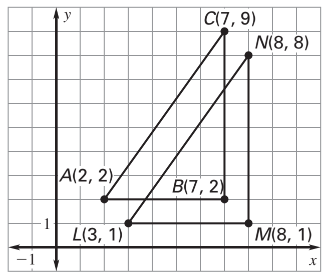
$m\angle M = m\angle X$, so $x = 20$.

16. $m\angle C = 180^\circ - 80^\circ - 45^\circ = 55^\circ$

$m\angle C = m\angle R$, so $55^\circ = 5x^\circ$, or $x = 11$.

17. The student has only shown that the corresponding angles are congruent. The student still needs to show that all corresponding sides are congruent, which they are not.

18. *Sample answer:*



$\triangle LMN \cong \triangle ABC$

19. $12x + 4y = 40$ → $12x + 4y = 40$

$17x - y = 50$ → (× 4) $68x - 4y = 200$

$$80x = 240$$
$$x = 3$$

$12(3) + 4y = 40 \rightarrow y = 1$

So, $x = 3$ and $y = 1$.

20.
$$4x + y = 22$$
$$6x - y = 28$$
$$10x = 50$$
$$x = 5$$

$4(5) + y = 22 \rightarrow y = 2$

So, $x = 5$ and $y = 2$.

21. B; $m\angle G = 90^\circ$ because $\triangle ABC \cong \triangle GIH$.

$m\angle I = 20^\circ$ because $\triangle EFD \cong \triangle GIH$.

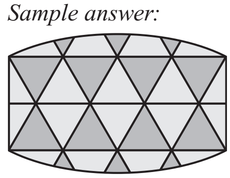
Therefore, for $\triangle GIH$, $m\angle H = 180^\circ - 90^\circ - 20^\circ = 70^\circ$.

22. *Sample answer:* The hexagon is regular because all angles are equal and all sides are congruent because they are corresponding parts of congruent triangles.

Problem Solving

23. The Transitive Property of Congruent Triangles guarantees that all triangles are congruent because each triangle in the rug is made from the same triangular shape.

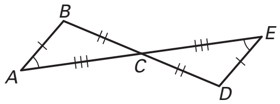
24. *Sample answer:*



25. The length, width, and depth of the new stereo must be congruent to the length, width, and depth of the old stereo in order to fit into the existing space.

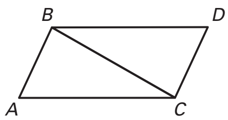
26. Given: $\overline{AB} \cong \overline{ED}$, $\overline{BC} \cong \overline{DC}$, $\overline{CA} \cong \overline{CE}$, $\angle BAC \cong \angle DEC$

Prove: $\triangle ABC \cong \triangle EDC$



Statements	Reasons
1. $\overline{AB} \cong \overline{ED}$, $\overline{BC} \cong \overline{DC}$, $\overline{CA} \cong \overline{CE}$, $\angle BAC \cong \angle DEC$	**1.** Given
2. $\angle BCA \cong \angle DCE$	**2.** Vertical Angles Congruence Theorem
3. $\angle ABC \cong \angle EDC$	**3.** Third Angles Theorem
4. $\triangle ABC \cong \triangle EDC$	**4.** Definition of congruent triangles

27. *Sample answer:*



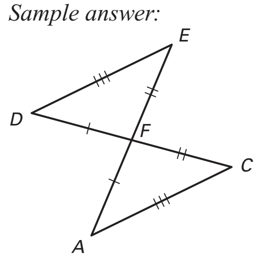
Yes, $\overline{AC} \parallel \overline{BD}$. Because $\triangle ABC \cong \triangle DCB$, the alternate interior angles are congruent.

28. Given: $\angle A \cong \angle D$, $\angle B \cong \angle E$

Prove: $\angle C \cong \angle F$

Statements	Reasons
1. $\angle A \cong \angle D$, $\angle B \cong \angle E$	**1.** Given
2. $m\angle A + m\angle B + m\angle C = 180°$, $m\angle D + m\angle E + m\angle F = 180°$	**2.** Triangle Sum Theorem
3. $m\angle A + m\angle B + m\angle C = m\angle D + m\angle E + m\angle F$	**3.** Transitive Property of Equality
4. $m\angle A = m\angle D$, $m\angle B = m\angle E$	**4.** Definition of congruent angles
5. $m\angle D + m\angle E + m\angle C = m\angle D + m\angle E + m\angle F$	**5.** Substitution Property of Equality
6. $m\angle C = m\angle F$	**6.** Subtraction Property of Equality
7. $\angle C \cong \angle F$	**7.** Definition of congruent figures

29. *Sample answer:*



No; $\triangle AFC \cong \triangle DFE$, but F is not the midpoint of $\overline{AD}$ and $\overline{EC}$.

30. *Sample answer:* You can measure two angles of the triangle and use the Triangle Sum Theorem to find the third angle. The angles in the quadrilateral can be found using the angle measures of the triangle.

31. a. $ABEF \cong CDEF$, so $\overline{BE} \cong \overline{DE}$ and $\angle ABE \cong \angle CDE$ because corresponding parts of congruent figures are congruent.

b. $\angle GBE$ and $\angle GDE$ are both supplementary to congruent angles $\angle ABE$ and $\angle CDE$ respectively, so $\angle GBE \cong \angle GDE$.

c. $\angle GEB \cong \angle GED$ because $\angle GED$ is a right angle and $\angle GEB$ and $\angle GED$ are supplementary.

d. Yes; You found that $\angle GBE \cong \angle GDE$ and $\angle GEB \cong \angle GED$. By the Third Angles Theorem, $\angle BGE \cong \angle DGE$. From the diagram, $\overline{BG} \cong \overline{DG}$, and $\overline{BE} \cong \overline{DE}$ because $ABEF \cong GDEF$. By the Reflexive Property, $\overline{GE} \cong \overline{GE}$. So, $\triangle BEG \cong \triangle DEG$.

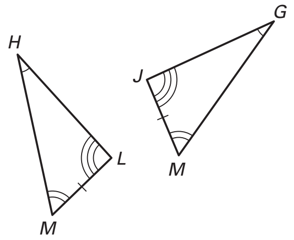
32. Given: $\overline{WX} \perp \overrightarrow{VZ}$ at Y, Y is the midpoint of $\overline{WX}$, $\overline{VW} \cong \overline{VX}$, and $\overrightarrow{VZ}$ bisects $\angle WVX$.

Prove: $\triangle VWY \cong \triangle VXY$

Statements	Reasons
1. $\overline{WX} \perp \overrightarrow{VZ}$ at Y, Y is the midpoint of $\overline{WX}$, $\overline{VW} \cong \overline{VX}$, and $\overrightarrow{VZ}$ bisects $\angle WVX$.	**1.** Given
2. $\angle WYV$ and $\angle XYV$ are right angles.	**2.** Definition of perpendicular lines
3. $\angle WYV \cong \angle XYV$	**3.** Right Angle Congruence Theorem
4. $\overline{WY} \cong \overline{XY}$	**4.** Definition of midpoint
5. $\angle WVY \cong \angle XVY$	**5.** Definition of angle bisector
6. $\angle VWY \cong \angle VXY$	**6.** Third Angles Theorem
7. $\overline{VY} \cong \overline{VY}$	**7.** Reflexive Property of Congruence
8. $\triangle VWY \cong \triangle VXY$	**8.** Definition of $\cong$ ▲s

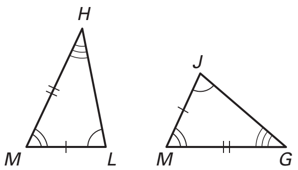
Problem Solving Workshop for the lesson "Apply Congruence and Triangles"

1. a.



$\angle L \cong \angle J$ by the Third Angles Theorem.

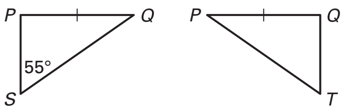
b.



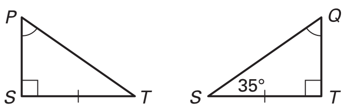
$\angle H \cong \angle G$ by the Third Angles Theorem.

$\overline{HM} \cong \overline{GM}$ because $\overline{HJ} \cong \overline{GL}$ and $\overline{JM} \cong \overline{LM}$.

2.



To find $m\angle PTS$:



By the Third Angle Theorem, $\angle QST \cong \angle PTS$. Because $m\angle QST$ is given as 35°, then $m\angle PTS = 35°$.

Lesson 4.3 Relate Transformations and Congruence

Investigating Geometry for the lesson "Relate Transformations and Congruence"

Explore 1

Step 4. a. *Sample answer:* The transformation is a flip, or reflection, in the y-axis. The image is congruent to the preimage.

b. *Sample answer:* The transformation is an enlargement, or dilation. The image is not congruent to the preimage.

c. *Sample answer:* The transformation is a 90° turn, or rotation, counterclockwise about the origin. The image is congruent to the preimage.

d. *Sample answer:* The transformation is a stretch, or shear, in a vertical direction. The image is not congruent to the preimage.

Explore 2

Step 4. a. yes **b.** yes **c.** no

Draw Conclusions

1. $(x, y) \rightarrow (x, -y)$; yes

2. $(x, y) \rightarrow (y, -x)$; yes

3. $(x, y) \rightarrow (-x, -y)$; yes

4. $(x, y) \rightarrow (-2x, 3y)$

$(-1, 2) \rightarrow (-2 \cdot (-1), 3 \cdot 2) = (2, 6)$,

$(1, 3) \rightarrow (-2 \cdot 1, 3 \cdot 3) = (-2, 9)$,

$(2, 0) \rightarrow (-2 \cdot 2, 3 \cdot 0) = (-4, 0)$;

No; neither lengths nor angles are preserved, so it is not a rigid motion.

5. translation, reflection, rotation

6. translation, reflection, rotation, dilation

7. translation, reflection, rotation

8. No; a transformation such as $(x, y) \rightarrow (2x, 0.5y)$ preserves area but does not preserve length or angle measure, so it is not a rigid motion.

Guided Practice for the lesson "Relate Transformations and Congruence"

1. translation and then rotation

2. translation and then reflection

3. not congruent

4. congruent; reflection

Exercises for the lesson "Relate Transformations and Congruence"

Skill Practice

1. Examples of transformations that are rigid motions are *translations*, *reflections*, and *rotations*.

2. A transformation that maps one figure onto a congruent figure preserves lengths and angle measures, so it is a rigid motion.

3. rotation **4.** translation **5.** reflection

6–8. Check students' drawings.

9. In choice C, the lengths are not preserved. The correct answer is C.

10. No; a reflection maps one side to a congruent side, but other sides are not congruent.

11. yes; reflection in the line $y = x$

12. yes; translation 3 units right and 2 units down

13. yes; rotation 90° counterclockwise about the origin

14. yes; translation 3 units right and 5 units up

15. No; a rotation does not map one figure onto the other, because corresponding side lengths are not congruent.

16. *Sample answer:* The function rule describes a translation 3 units to the right and 1 unit down. A translation is a rigid motion.

17. *Sample answer:* The function rule moves points 1 unit to the left and then stretches points vertically away from the x-axis. The transformation is not a rigid motion, because lengths and angles are not preserved. The triangle with vertices (0, 0), (1, 0), and (1, 1) is transformed to a taller triangle with vertices (−1, 0), (0, 0), and (0, 2).

18. No; check students' drawings.

19. Yes; check students' drawings; a translation followed by a reflection, or a rotation followed by a reflection.

Problem Solving

20. 90° rotation (either way), followed by translation across and down

21. 180° rotation around the midpoint of $\overline{BC}$, reflection across $\overline{BC}$ followed by reflection across the perpendicular bisector of $\overline{BC}$

22. 180° rotation

23. 90° rotation counterclockwise

24. 120° rotation clockwise, followed by translation

25. Check students' designs.

26. a. Check students' rotations.

b. *Sample answer:* Yellow tiles that share an edge can be rotated either 72° around the vertex of the smaller angle or 108° around the vertex of the larger angle. Red tiles that share an edge can be rotated 36° around the vertex of the smaller angle or 144° around the vertex of the larger angle.

c. *Sample answer:* You can calculate the angles of the tiles by observing how many of each tile type meet at various vertices in the design.

27. Cutting both pieces together will make all corresponding sides and angles of the pattern congruent.

28. a. The rigid motion of reflection across a vertical line maps $\triangle RTX$ onto $\triangle VTX$ so $\triangle RTX \cong \triangle VTX$.

b. The same rigid motion that maps $\triangle RTX$ onto $\triangle VTX$ also maps $\triangle STW$ onto $\triangle UTW$, so $\triangle STW \cong \triangle UTW$. Because the triangles are congruent, the corresponding sides are congruent, so $SW = UW$.

c. $\frac{TS}{SW} = \frac{TR}{RX}$

$\frac{16}{SW} = \frac{16 + 14}{15}$

$30SW = 240$

$SW = 8$

$(SW)^2 + (TW)^2 = (TS)^2$

$8^2 + (TW)^2 = 16^2$

$64 + (TW)^2 = 256$

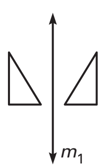
$(TW)^2 = 192$

$TW = \sqrt{192}$

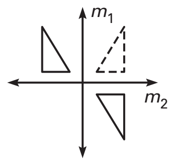
≈ 13.9

Therefore, $SW = 8$ ft and $TW = 13.9$ ft.

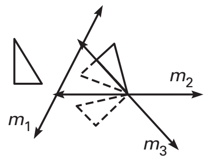
29. a.



b.



c.



Lesson 4.4 Prove Triangles Congruent by SSS

Investigating Geometry Activity for the lesson "Prove Triangles Congruent by SSS"

1. No. *Sample answer:* In the activity, once the triangle lengths were established it was impossible to create two different triangles.

2. Yes. *Sample answer:* The straw activity indicates that two triangles with three pairs of congruent sides are congruent.

3. Yes. *Sample answer:* In the activity, it was possible to change the angles in the quadrilateral, thus changing the shape.

4. No. *Sample answer:* The activity established that the angles could change, thus two quadrilaterals with pairs of congruent sides are not necessarily congruent.

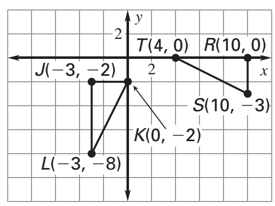
Guided Practice for the lesson "Prove Triangles Congruent by SSS"

1. Yes; $\triangle DFG \cong \triangle HJK$ by SSS.

2. No; $\triangle ACB \ncong \triangle CAD$ because $\overline{AB} \ncong \overline{CD}$.

3. Yes; $\triangle QPT \cong \triangle RST$ by SSS.

4.



By counting, $KJ = SR = 3$ and $JL = RT = 6$.

$LK = \sqrt{(-3 - 0)^2 + (-8 - (-2))^2} = \sqrt{45} = 3\sqrt{5}$

$TS = \sqrt{(10 - 4)^2 + (-3 - 0)^2} = \sqrt{45} = 3\sqrt{5}$

So $LK = TS$.

Because the three sides of $\triangle JKL$ are congruent to the three sides of $\triangle RST$, $\triangle JKL \cong \triangle RST$ by SSS.

5. The square is not stable because it has no diagonal support so the shape could change.

6. The figure is stable because it has diagonal support and fixed sides.

7. The figure is not stable because the bottom half does not have diagonal support and could change shape.

Exercises for the lesson "Prove Triangles Congruent by SSS"

Skill Practice

1. corresponding angles
2. neither
3. corresponding sides
4. neither
5. not true; $\overline{RS} \ncong \overline{TQ}$
6. true; $\overline{AB} \cong \overline{CD}$, $\overline{AD} \cong \overline{CB}$, and $\overline{BD} \cong \overline{DB}$, so $\triangle ABD \cong \triangle CDB$ by SSS.
7. true; $\overline{DE} \cong \overline{DG}$, $\overline{DF} \cong \overline{DF}$, and $\overline{EF} \cong \overline{GF}$, so $\triangle DEF \cong \triangle DGF$ by SSS.
8. The triangle vertices do not correspond.

   Sample answer: $\overline{WX} \cong \overline{YZ}$, $\overline{WZ} \cong \overline{YX}$, and $\overline{XZ} \cong \overline{ZX}$, so $\triangle WXZ \cong \triangle YZX$ by SSS.
9. $\triangle ABC$:

   $AB = \sqrt{(4 - (-2))^2 + (-2 - (-2))^2} = \sqrt{36} = 6$

   $BC = \sqrt{(4 - 4)^2 + (6 - (-2))^2} = \sqrt{64} = 8$

   $AC = \sqrt{(4 - (-2))^2 + (6 - (-2))^2} = \sqrt{100} = 10$

   $\triangle DEF$:

   $DE = \sqrt{(5 - 5)^2 + (1 - 7)^2} = \sqrt{36} = 6$

   $EF = \sqrt{(13 - 5)^2 + (1 - 1)^2} = \sqrt{64} = 8$

   $DF = \sqrt{(13 - 5)^2 + (1 - 7)^2} = \sqrt{100} = 10$

   $AB = DE$, $BC = EF$, and $AC = DF$, so $\triangle ABC \cong \triangle DEF$ by SSS.
10. $\triangle ABC$:

    $AB = \sqrt{(3 - (-2))^2 + (-3 - 1)^2} = \sqrt{41}$

    $BC = \sqrt{(7 - 3)^2 + (5 - (-3))^2} = \sqrt{80} = 4\sqrt{5}$

    $AC = \sqrt{(7 - (-2))^2 + (5 - 1)^2} = \sqrt{97}$

    $\triangle DEF$:

    $DE = \sqrt{(8 - 3)^2 + (2 - 6)^2} = \sqrt{41}$

    $EF = \sqrt{(10 - 8)^2 + (11 - 2)^2} = \sqrt{85}$

    $DF = \sqrt{(10 - 3)^2 + (11 - 6)^2} = \sqrt{74}$

    The sides of $\triangle ABC$ are not congruent to the sides of $\triangle DEF$, so the triangles are not congruent.
11. $\triangle ABC$:

    $AB = \sqrt{(6 - 0)^2 + (5 - 0)^2} = \sqrt{61}$

    $BC = \sqrt{(9 - 6)^2 + (0 - 5)^2} = \sqrt{34}$

    $AC = \sqrt{(9 - 0)^2 + (0 - 0)^2} = \sqrt{81} = 9$

    $\triangle DEF$:

    $DE = \sqrt{(6 - 0)^2 + (-6 - (-1))^2} = \sqrt{61}$

    $EF = \sqrt{(9 - 6)^2 + (-1 - (-6))^2} = \sqrt{34}$

    $DF = \sqrt{(9 - 0)^2 + (-1 - (-1))^2} = \sqrt{81} = 9$

    $AB = DE$, $BC = EF$, and $AC = DF$, so $\triangle ABC \cong \triangle DEF$ by SSS.
12. $\triangle ABC$:

    $AB = \sqrt{(-5 - (-5))^2 + (2 - 7)^2} = \sqrt{25} = 5$

    $BC = \sqrt{(0 - (-5))^2 + (2 - 2)^2} = \sqrt{25} = 5$

    $AC = \sqrt{(0 - (-5))^2 + (2 - 7)^2} = \sqrt{50} = 5\sqrt{2}$

    $\triangle DEF$:

    $DE = \sqrt{(0 - 0)^2 + (1 - 6)^2} = \sqrt{25} = 5$

    $EF = \sqrt{(4 - 0)^2 + (1 - 1)^2} = \sqrt{16} = 4$

    $DF = \sqrt{(4 - 0)^2 + (1 - 6)^2} = \sqrt{41}$

    The sides of $\triangle ABC$ are not congruent to the sides of $\triangle DEF$, so the triangles are not congruent.
13. Stable; The figure has diagonal supports that form triangles with fixed side lengths, so it is stable.
14. Not stable; The figure does not have diagonal support, so it is not stable.
15. Stable; The figure has diagonal supports that form triangles with fixed side lengths, so it is stable.
16. B; $\overline{FJ} \ncong \overline{FH}$ because $2(FJ) = FH$.
17. B; $\overline{AB} \ncong \overline{AD}$ because two adjacent sides of a rectangle are not necessarily congruent.
18. $\triangle ABC \ncong \triangle DEF$ because $AC = FC + 2$ and $DF = FC + 3$, so $AC \neq DF$.
19. $\triangle ABC \ncong \triangle DEF$ because the vertices do not correspond.
20. $\overline{JP} \cong \overline{JP}$ by the Reflexive Property of Congruence. So, $\triangle JPK \cong \triangle JPL$ by SSS.
21. If $\overline{AB} \cong \overline{CD}$ and $\overline{AC} \cong \overline{BD}$:

    $5x = 3x + 10$ and $5x - 2 = 4x + 3$

    $2x = 10$ $\quad x = 5$ ✓

    $x = 5$ ✓

    If $\overline{AB} \cong \overline{BD}$ and $\overline{AC} \cong \overline{CD}$:

    $5x = 4x + 3$ $\quad 5x - 2 = 3x + 10$

    $x = 3$ $\quad 2x = 12$

    $x = 6$

    5 is the only value for x that will make the triangles congruent.

Problem Solving

22. Yes; Use the string to measure each side of one triangle and then measure the sides of the second triangle to see if they are congruent to the corresponding sides of the first triangle.
23. Gate 1; *Sample answer:* Gate 1 has a diagonal support that forms two triangles with fixed side lengths, so these triangles cannot change shape. Gate 2 is not stable because the gate is a quadrilateral that can be many shapes.

24. Given: $\overline{GH} \cong \overline{JK}$, $\overline{HJ} \cong \overline{KG}$

Prove: $\triangle GHJ \cong \triangle JKG$

Statements	Reasons
1. $\overline{GH} \cong \overline{JK}$ and $\overline{HJ} \cong \overline{KG}$	**1.** Given
2. $\overline{GJ} \cong \overline{JG}$	**2.** Reflexive Property of Congruence
3. $\triangle GHJ \cong \triangle JKG$	**3.** SSS Congruence Postulate

25. Given: $\overline{WX} \cong \overline{VZ}$, $\overline{WY} \cong \overline{VY}$, $\overline{YZ} \cong \overline{YX}$

Prove: $\triangle VWX \cong \triangle WVZ$

Statements	Reasons
1. $\overline{WX} \cong \overline{VZ}$, $\overline{WY} \cong \overline{VY}$, $\overline{YZ} \cong \overline{YX}$	**1.** Given
2. $\overline{WV} \cong \overline{VW}$	**2.** Reflexive Property of Congruence
3. $WY = VY$, $YZ = YX$	**3.** Definition of congruent segments
4. $WY + YZ = VY + YZ$	**4.** Addition Property of Equality
5. $WY + YZ = VY + YX$	**5.** Substitution Property of Equality
6. $WZ = VX$	**6.** Segment Addition Postulate
7. $\overline{WZ} \cong \overline{VX}$	**7.** Definition of congruent segments
8. $\triangle VWX \cong \triangle WVZ$	**8.** SSS Congruence Postulate

26. Given: $\overline{AE} \cong \overline{CE}$, $\overline{AB} \cong \overline{CD}$, E is the midpoint of $\overline{BD}$.

Prove: $\triangle EAB \cong \triangle ECD$

Statements	Reasons
1. $\overline{AE} \cong \overline{CE}$ and $\overline{AB} \cong \overline{CD}$	**1.** Given
2. E is the midpoint of $\overline{BD}$.	**2.** Given
3. $\overline{EB} \cong \overline{ED}$	**3.** Definition of midpoint
4. $\triangle EAB \cong \triangle ECD$	**4.** SSS Congruence Postulate

27. Given: $\overline{FM} \cong \overline{FN}$, $\overline{DM} \cong \overline{HN}$, $\overline{EF} \cong \overline{GF}$, $\overline{DE} \cong \overline{HG}$

Prove: $\triangle DEN \cong \triangle HGM$

Statements	Reasons
1. $\overline{FM} \cong \overline{FN}$, $\overline{DM} \cong \overline{HN}$, $\overline{EF} \cong \overline{GF}$, $\overline{DE} \cong \overline{HG}$	**1.** Given
2. $\overline{MN} \cong \overline{NM}$	**2.** Reflexive Property of Congruence
3. $FM = FN$, $DM = HN$, $EF = FG$, $MN = NM$	**3.** Definition of congruent segments
4. $EF + FN = GF + FN$, $DM + MN = HN + MN$	**4.** Addition Property of Equality
5. $EF + FN = GF + FM$, $DM + MN = HN + NM$	**5.** Substitution Property of Equality
6. $EN = GM$, $DN = HM$	**6.** Segment Addition Postulate
7. $\overline{EN} \cong \overline{GM}$, $\overline{DN} \cong \overline{HM}$	**7.** Definition of congruent segments
8. $\triangle DEN \cong \triangle HGM$	**8.** SSS Congruence Postulate

28. **a.** Door 1 has diagonal support and fixed side lengths, so now it is stable.

b. No, this would not be a good choice because it would be hard for rescuers to pass through the door.

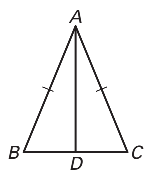
c. Yes, rescuers would be able to pass easily through the door.

d. Door 2 is more stable because it contains a diagonal support.

29. You can find third base because only one triangle can be formed from three fixed sides.

30. Given: $\triangle ABC$ is isosceles; D is the midpoint of $\overline{BC}$.

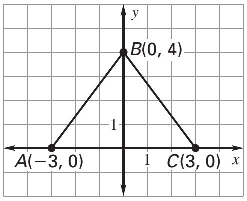
Prove: $\triangle ABD \cong \triangle ACD$



Statements	Reasons
1. $\triangle ABC$ is isosceles; D is the midpoint of $\overline{BC}$.	**1.** Given
2. $\overline{AB} \cong \overline{AC}$	**2.** Definition of isosceles triangle
3. $\overline{BD} \cong \overline{CD}$	**3.** Definition of midpoint
4. $\overline{AD} \cong \overline{AD}$	**4.** Reflexive Property of Congruence
5. $\triangle ABD \cong \triangle ACD$	**5.** SSS Congruence Postulate

Quiz for the lessons "Apply Triangle Sum Properties", "Apply Congruence and Triangles", "Relate Transformations and Congruence" and "Prove Triangles Congruent by SSS"

1.



$AB = \sqrt{(-3 - 0)^2 + (0 - 4)^2} = \sqrt{25} = 5$

$AC = \sqrt{(-3 - 3)^2 + (0 - 0)^2} = \sqrt{36} = 6$

$BC = \sqrt{(3 - 0)^2 + (0 - 4)^2} = \sqrt{25} = 5$

So $AB = BC$.

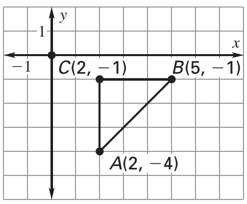
slope $\overline{AB} = \frac{4 - 0}{0 - (-3)} = \frac{4}{3}$

slope $\overline{AC} = \frac{0 - 0}{3 - (-3)} = 0$

slope $\overline{BC} = \frac{0 - 4}{3 - 0} = -\frac{4}{3}$

So there are no right angles. Therefore, $\triangle ABC$ is an isosceles triangle.

2.



$AB = \sqrt{(2 - 5)^2 + (-4 - (-1))^2} = \sqrt{18}$

$AC = \sqrt{(2 - 2)^2 + (-4 - (-1))^2} = \sqrt{9} = 3$

$BC = \sqrt{(5 - 2)^2 + (-1 - (-1))^2} = \sqrt{9} = 3$

So $AC = BC$.

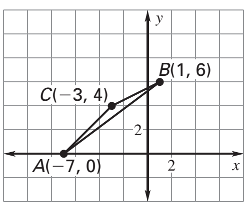
slope $\overline{AB} = \frac{-1 - (-4)}{5 - 2} = \frac{3}{3} = 1$

slope $\overline{AC} = \frac{-4 - (-1)}{2 - 2}$ is undefined.

slope $\overline{BC} = \frac{-1 - (-1)}{5 - 2} = 0$

So $AC \perp BC$. Therefore, $\triangle ABC$ is a right isosceles triangle.

3.



$AB = \sqrt{(-7 - 1)^2 + (0 - 6)^2} = \sqrt{100} = 10$

$BC = \sqrt{(1 - (-3))^2 + (6 - 4)^2} = \sqrt{20} = 2\sqrt{5}$

$AC = \sqrt{(-7 - (-3))^2 + (0 - 4)^2} = \sqrt{32} = 4\sqrt{2}$

So there are no equal sides.

slope $\overline{AB} = \frac{6 - 0}{1 - (-7)} = \frac{6}{8} = \frac{3}{4}$

slope $\overline{BC} = \frac{6 - 4}{1 - (-3)} = \frac{2}{4} = \frac{1}{2}$

slope $\overline{AC} = \frac{4 - 0}{-3 - (-7)} = \frac{4}{4} = 1$

So there are no right angles. Therefore, $\triangle ABC$ is a scalene triangle.

4. $5x - 11 = 3x + 7$

$2x = 18$

$x = 9$

5. $(5y + 36)° = 61°$

$5y = 25$

$y = 5$

6. The function rule describes a translation 3 units to the left and 4 units down. A translation is a rigid motion.

7. The function rule moves points 1 unit to the right and then stretches points vertically away from the x-axis. The transformation is not a rigid motion, because lengths and angles are not preserved.

8. Given: $\overline{AB} \cong \overline{AC}$, $\overline{AD}$ bisects $\overline{BC}$.

Prove: $\triangle ABD \cong \triangle ACD$

Statements	Reasons
1. $\overline{AB} \cong \overline{AC}$	**1.** Given
2. $\overline{AD}$ bisects $\overline{BC}$.	**2.** Given
3. $\overline{BD} \cong \overline{CD}$	**3.** Definition of bisector
4. $\overline{AD} \cong \overline{AD}$	**4.** Reflexive Property of Congruence
5. $\triangle ABD \cong \triangle ACD$	**5.** SSS Congruence Postulate

Lesson 4.5 Prove Triangles Congruent by SAS and HL

Guided Practice for the lesson "Prove Triangles Congruent by SAS and HL"

1. Given: $\overline{SV} \cong \overline{VU}$, $\overline{RT} \perp \overline{SU}$

Prove: $\triangle SVR \cong \triangle UVR$

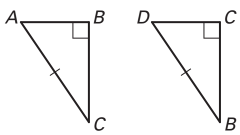
Statements	Reasons
1. $\overline{SV} \cong \overline{VU}$	**1.** Given
2. $\overline{RT} \perp \overline{SU}$	**2.** Given
3. $\angle SVR$ and $\angle UVR$ are right angles.	**3.** Definition of perpendicular lines
4. $\angle SVR \cong \angle UVR$	**4.** Right Angle Congruence Theorem
5. $\overline{VR} \cong \overline{VR}$	**5.** Reflexive Property for Congruence
6. $\triangle SVR \cong \triangle UVR$	**6.** SAS Congruence Postulate

2. Given: *ABCD* is a square; *R*, *S*, *T*, and *U* are midpoints of *ABCD*; $\angle B$ and $\angle D$ are right angles.

Prove: $\triangle BSR \cong \triangle DUT$

Statements	Reasons
1. *ABCD* is a square; *R*, *S*, *T*, and *U* are midpoints of *ABCD*; $\angle B$ and $\angle D$ are right angles.	**1.** Given
2. $\overline{BC} \cong \overline{DA}$, $\overline{BA} \cong \overline{DC}$	**2.** Definition of a square
3. $BC = DA$, $BA = DC$	**3.** Definition of congruent segments
4. $BS = SC$, $DU = UA$, $BR = RA$, $DT = TC$	**4.** Definition of midpoint
5. $BS + SC = BC$, $DU + UA = DA$, $BR + RA = BA$, $DT + TC = DC$	**5.** Segment Addition Postulate
6. $2BS = BC$, $2DU = DA$, $2BR = BA$, $2DT = DC$	**6.** Substitution Property of Equality
7. $2BS = 2DU$, $2BR = 2DT$	**7.** Substitution Property of Equality
8. $BS = DU$, $BR = DT$	**8.** Division Property of Equality
9. $\overline{BS} \cong \overline{DU}$, $\overline{BR} \cong \overline{DT}$	**9.** Definition of congruent segments
10. $\angle B \cong \angle D$	**10.** Right Angle Congruence Theorem
11. $\triangle BSR \cong \triangle DUT$	**11.** SAS Congruence Postulate

3.



4. Given: $\angle ABC$ and $\angle DCB$ are right angles; $\overline{AC} \cong \overline{DB}$

Prove: $\triangle ACB \cong \triangle DBC$

Statements	Reasons
1. $\angle ABC$ and $\angle DCB$ are right angles.	**1.** Given
2. $\overline{AC} \cong \overline{DB}$	**2.** Given
3. $\triangle ACB$ and $\triangle DBC$ are right triangles.	**3.** Definition of a right triangle
4. $\overline{CB} \cong \overline{BC}$	**4.** Reflexive Property of Congruence
5. $\triangle ACB \cong \triangle DBC$	**5.** HL Congruence Theorem

Exercises for the lesson "Prove Triangles Congruent by SAS and HL"

Skill Practice

1. The angle between two sides of a triangle is called the included angle.

2. *Sample answer:* SAS requires two sides and the included angle of one triangle to be congruent to the corresponding two sides and included angle of a second triangle. SSS requires that the three sides of one triangle be congruent to the corresponding sides of a second triangle.

3. $\angle XYW$ is between $\overline{XY}$ and $\overline{YW}$.

4. $\angle WZY$ is between $\overline{WZ}$ and $\overline{ZY}$.

5. $\angle ZWY$ is between $\overline{ZW}$ and $\overline{YW}$.

6. $\angle WXY$ is between $\overline{WX}$ and $\overline{YX}$.

7. $\angle XYZ$ is between $\overline{XY}$ and $\overline{YZ}$.

8. $\angle XWZ$ is between $\overline{WX}$ and $\overline{WZ}$.

9. not enough information; The congruent angles are not between the congruent sides.

10. enough information;
sides: $\overline{LM} \cong \overline{NQ}$, $\overline{MN} \cong \overline{QP}$
included angle: $\angle LMN \cong \angle NQP$

11. not enough information; There are no congruent pairs of sides.

12. not enough information; The congruent angles are not between the congruent sides.

13. enough information;
sides: $\overline{EF} \cong \overline{GH}$, $\overline{FH} \cong \overline{HF}$
included angle: $\angle EFH \cong \angle GHF$

14. not enough information; the congruent angles are not between the congruent sides.

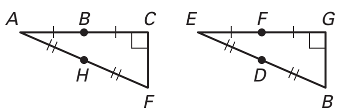
15. B; $\triangle ABC \ncong \triangle DEF$ because the corresponding congruent angle is not between the corresponding congruent sides.

16. $\triangle BAD \cong \triangle BCD$ by SAS because $\overline{BA} \cong \overline{BC}$, $\overline{AD} \cong \overline{CD}$ and $\angle BAD \cong \angle BCD$ because *ABCD* is a square.

17. $\triangle STU \cong \triangle UVR$ by SAS because $\overline{ST} \cong \overline{UV}$, $\overline{TU} \cong \overline{VR}$, and $\angle STU \cong \angle UVR$ because *RSTUV* is a regular pentagon.

18. $\triangle KMN \cong \triangle KLN$ by SAS because $\overline{KM}$, $\overline{MN}$, $\overline{KL}$, and $\overline{LN}$ are all congruent radii, and $\angle KMN \cong \angle KLN$ by definition of $\perp$ lines.

19.



Because $\overline{BD} \cong \overline{DE} \cong \overline{AH} \cong \overline{HF}$, then $BD + DE = AH + HF$, so $\overline{FA} \cong \overline{BE}$. By the same reasoning, $\overline{AC} \cong \overline{EG}$. Because $\triangle ACF$ and $\triangle EGB$ are right triangles, then $\triangle ACF \cong \triangle EGB$ by the HL Congruence Theorem.

20. not enough information; The corresponding congruent angle is not between the corresponding congruent sides.

21. enough information; SAS;
sides: $\overline{XZ} \cong \overline{QZ}$, $\overline{YZ} \cong \overline{PZ}$
included angle: $\angle XZY \cong \angle QZP$

22. enough information; HL Congruence Theorem;
hypotenuse: $\overline{NL} \cong \overline{LS}$
leg: $\overline{MN} \cong \overline{RL}$
right angle: $\angle LMN \cong \angle SRL$

23. Yes; Because the right angles are between the pairs of corresponding congruent legs, the triangles are congruent by the SAS Congruence Postulate.

24. Because $\triangle XZY \cong \triangle WZY$ by SAS, $\overline{YX}$ and $\overline{YW}$ have the same length.

$$5x - 1 = 4x + 6$$
$$x = 7$$

25. $\overline{AB} \cong \overline{DE}$, $\overline{CB} \cong \overline{FE}$, $\overline{AC} \cong \overline{DF}$

26. $\angle A \cong \angle D$, $\overline{CA} \cong \overline{FD}$, $\overline{BA} \cong \overline{ED}$

27. $\angle B \cong \angle E$, $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$

28. Yes; Because $\overline{LN}$ bisects $\angle KLM$, $\angle KLN \cong \angle MLN$. Also, because the triangles are isosceles, $\overline{LN} \cong \overline{LK} \cong \overline{LM}$. Therefore, $\triangle KLN \cong \triangle MLN$ by the SAS Congruence Postulate.

29. Because M is the midpoint of $\overline{PQ}$, $\overline{PM} \cong \overline{QM}$. Also, because $\overline{RM} \perp \overline{PQ}$, $\angle PMR$ and $\angle QMR$ are both right angles, so $\angle PMR \cong \angle QMR$. Finally, by the Reflexive Property of Congruence, $\overline{RM} \cong \overline{RM}$. Therefore, $\triangle RMP \cong \triangle RMQ$ by the SAS Congruence Postulate.

30. First, $\angle DAC = \angle DAB + \angle BAC$ and $\angle FAB = \angle FAC + \angle BAC$. Because $\angle DAB$ and $\angle FAC$ are both right angles, they are also congruent. By substitution, $\angle DAC \cong \angle FAB$. Therefore, $\triangle ACD \cong \triangle ABF$ by the SAS Congruence Postulate.

Problem Solving

31. You would use the SAS Congruence Postulate.

32. You would use the SAS Congruence Postulate.

33. SAS: The two sides and the included angle of one sail need to be congruent to two sides and the included angle of the second sail.

HL: The two sails need to be right triangles with congruent hypotenuses and one pair of congruent legs.

34. Given: Point M is the midpoint of $\overline{LN}$.
$\triangle PMQ$ is an isosceles triangle with $\overline{MP} \cong \overline{MQ}$
$\angle L$ and $\angle N$ are right angles.
Prove: $\triangle LMP \cong \triangle NMQ$

Statements	Reasons
1. $\angle L$ and $\angle N$ are right angles.	**1.** Given
2. $\triangle LMP$ and $\triangle NMQ$ are right triangles.	**2.** Definition of a right triangle
3. Point M is the midpoint of $\overline{LN}$.	**3.** Given
4. $\overline{LM} \cong \overline{NM}$	**4.** Definition of midpoint
5. $\overline{MP} \cong \overline{MQ}$	**5.** Given
6. $\triangle LMP \cong \triangle NMQ$	**6.** HL Congruence Theorem

35. Given: $\overline{PQ}$ bisects $\angle SPT$; $\overline{SP} \cong \overline{TP}$
Prove: $\triangle SPQ \cong \triangle TPQ$

Statements	Reasons
1. $\overline{PQ}$ bisects $\angle SPT$.	**1.** Given
2. $\angle SPQ \cong \angle TPQ$	**2.** Definition of angle bisector
3. $\overline{SP} \cong \overline{TP}$	**3.** Given
4. $\overline{PQ} \cong \overline{PQ}$	**4.** Reflexive Property of Congruence
5. $\triangle SPQ \cong \triangle TPQ$	**5.** SAS Congruence Postulate

36. Given: $\overline{VX} \cong \overline{XY}$, $\overline{XW} \cong \overline{YZ}$, $\overline{XW} \parallel \overline{YZ}$
Prove: $\triangle VXW \cong \triangle XYZ$

Statements	Reasons
1. $\overline{VX} \cong \overline{XY}$, $\overline{XW} \cong \overline{YZ}$	**1.** Given
2. $\overline{XW} \parallel \overline{YZ}$	**2.** Given
3. $\angle VXW \cong \angle XYZ$	**3.** Corresponding Angles Postulate
4. $\triangle VXW \cong \triangle XYZ$	**4.** SAS Congruence Postulate

37. Given: $\overline{JM} \cong \overline{LM}$
Prove: $\triangle JKM \cong \triangle LKM$

Statements	Reasons
1. $\angle J$ and $\angle L$ are right angles.	**1.** Given in diagram
2. $\triangle JKM$ and $\triangle LKM$ are right triangles.	**2.** Definition of a right triangle
3. $\overline{JM} \cong \overline{LM}$	**3.** Given
4. $\overline{KM} \cong \overline{KM}$	**4.** Reflexive Property of Congruence
5. $\triangle JKM \cong \triangle XYZ$	**5.** HL Congruence Theorem

38. Given: D is the midpoint of $\overline{AC}$.

Prove: $\triangle ABD \cong \triangle CBD$

Statements	Reasons
1. D is the midpoint of $\overline{AC}$.	**1.** Given
2. $\overline{AD} \cong \overline{CD}$	**2.** Definition of midpoint
3. $\overline{BD} \cong \overline{BD}$	**3.** Reflexive Property of Congruence
4. $\overline{BD} \perp \overline{AC}$	**4.** Given in diagram
5. $\angle BDA$ and $\angle BDC$ are right angles.	**5.** Definition of perpendicular lines
6. $\angle BDA \cong \angle BDC$	**6.** Right Angle Congruence Theorem
7. $\triangle ABD \cong \triangle CBD$	**7.** SAS Congruence Postulate

39. D; Because $m\angle ADC + m\angle ADB + m\angle BDE = 180$, $m\angle ADB = 180° - 70° - 40° = 70°$. So $\angle ADB \cong \angle ACE$. Therefore, $\triangle AEC \cong \triangle ABD$ by the SAS Congruence Postulate. Because $\angle FED$ and $\angle ABF$ are corresponding angles of congruent triangles, $\angle FED \cong \angle ABF$.

40. Given: $\overline{CR} \cong \overline{CS}$, $\overline{QC} \perp \overline{CR}$, $\overline{QC} \perp \overline{CS}$

Prove: $\triangle QCR \cong \triangle QCS$

Statements	Reasons
1. $\overline{QC} \perp \overline{CR}$ and $\overline{QC} \perp \overline{CS}$	**1.** Given
2. $\angle QCR$ and $\angle QCS$ are right angles.	**2.** Definition of perpendicular lines
3. $\angle QCR \cong \angle QCS$	**3.** Right Angle Congruence Theorem
4. $\overline{CR} \cong \overline{CS}$	**4.** Given
5. $\overline{QC} \cong \overline{QC}$	**5.** Reflexive Property of Congruence
6. $\triangle QCR \cong \triangle QCS$	**6.** SAS Congruence Postulate

41. To use the SSS Congruence Postulate, you need to find the length of each side of the two triangles and show that pairs of corresponding sides have the same length and therefore are congruent.

To use the SAS Congruence Postulate:

slope $\overline{ON} = \frac{8-0}{8-0} = 1$

slope $\overline{MP} = \frac{0-4}{8-4} = \frac{-4}{4} = -1$

Because $1(-1) = -1$, $\overline{MP} \perp \overline{ON}$. So $\angle PMO$ and $\angle PMN$ are right angles and $\angle PMO \cong \angle PMN$.

$MO = \sqrt{(4-0)^2 + (4-0)^2} = \sqrt{32}$

$MN = \sqrt{(8-4)^2 + (8-4)^2} = \sqrt{32}$

So $\overline{MO} \cong \overline{MN}$.

By the Reflexive Property of Congruence, $\overline{PM} \cong \overline{PM}$. Therefore, by SAS, $\triangle PMO \cong \triangle PMN$.

In this case, either method will work to prove congruence.

Technology Activity for the lesson "Prove Triangles Congruent by SAS and HL"

Step 3.

$\overline{BD} \cong \overline{BE}$ because both segments are radii of the same circle. In $\triangle ABD$ and $\triangle ABE$, $\overline{AB} \cong \overline{AB}$ and $\angle BAD \cong \angle BAE$.

1. $\triangle ABD \ncong \triangle ABE$ because $\overline{DA} \ncong \overline{EA}$. $\overline{EA}$ is obviously much longer than $\overline{DA}$.

2. $\angle BDA$ is a right angle. Because $\triangle ABD$ is a right triangle, the Hypotenuse-Leg Congruence Theorem guarantees that any triangle with these dimensions will also be congruent to $\triangle ABD$.

3. In this activity, SSA can yield two non-congruent triangles as in Exercise 1. However, HL results in only one triangle.

Mixed Review of Problem Solving for the lessons "Apply Triangle Sum Properties", "Apply Congruence and Triangles", "Relate Transformations and Congruence", "Prove Triangles Congruent by SSS", and "Prove Triangles Congruent by SAS and HL"

1. a. acute triangles: $\triangle EBC$, $\triangle FCG$, $\triangle DCG$, $\triangle ACB$

obtuse triangles: $\triangle AEC$, $\triangle DFC$

b. All triangles in the figure are scalene.

2. *Sample answer:* You know that $\triangle PQR \cong \triangle STR$ by the SSS Congruence Postulate.

$PQ = TS = 18$

$PR = QR = TR = SR = 3\sqrt{34}$

3. $m\angle 1 + 90° = 160°$

$m\angle 1 = 70°$

4. Yes; $\overline{CE} \cong \overline{AG}$ and $\overline{AH} \cong \overline{DE}$, so $\overline{CE} - \overline{DE} \cong \overline{AG} - \overline{AH}$, or $\overline{CD} \cong \overline{GH}$. Likewise, $\overline{BC} \cong \overline{FG}$. Because $\angle G$ and $\angle C$ are both right angles, $\angle G \cong \angle C$. Therefore, $\triangle BCD \cong \triangle FGH$ by the SAS Congruence Postulate.

5. a. Given: $\overline{FG} \cong \overline{HG}$, $\overline{BG} \perp \overline{FH}$

Prove: $\triangle FGB \cong \triangle HGB$

Statements	Reasons
1. $\overline{FG} \cong \overline{HG}$	**1.** Given
2. $\overline{BG} \perp \overline{FH}$	**2.** Given
3. $\angle BGF$ and $\angle BGH$ are right angles.	**3.** Definition of perpendicular lines
4. $\angle BGF \cong \angle BGH$	**4.** Right Angle Congruence Theorem
5. $\overline{GB} \cong \overline{GB}$	**5.** Reflexive Property of Congruence
6. $\triangle FGB \cong \triangle HGB$	**6.** SAS Congruence Postulate

b. Yes;
Given: $\overline{DF} \cong \overline{EH}$, $m\angle EHB = 25°$, $m\angle BFG = 65°$, $\overline{DF} \perp \overline{AG}$ at point F.
Prove: $\triangle BDF \cong \triangle BEH$

Statements	Reasons
1. $\overline{DF} \cong \overline{EH}$, $m\angle EHB = 25°$, $m\angle BFG = 65°$, $\overline{DF} \perp \overline{AG}$ at point F.	**1.** Given
2. $\triangle FGB \cong \triangle HGB$	**2.** Proven in Ex. 5(a)
3. $\overline{FB} \cong \overline{HB}$	**3.** Corr. parts of $\cong$ △ are $\cong$.
4. $\angle DFG$ is a right angle.	**4.** Definition of perpendicular lines
5. $m\angle DFG = 90°$	**5.** Definition of a right angle
6. $m\angle DFB + m\angle BFG = m\angle DFG$	**6.** Angle Addition Postulate
7. $m\angle DFB + 65° = 90°$	**7.** Substitution Property of Equality
8. $m\angle DFB = 25°$	**8.** Subtraction Property of Equality
9. $m\angle DFB = m\angle EHB$	**9.** Transitive Property of Equality
10. $\angle DFB \cong \angle EHB$	**10.** Definiton of congruent angles
11. $\triangle BDF \cong \triangle BEH$	**11.** SAS Congruence Postulate

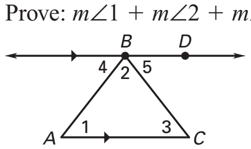
6. $(4x + 18)° = 110°$
$4x = 92$
$x = 23$

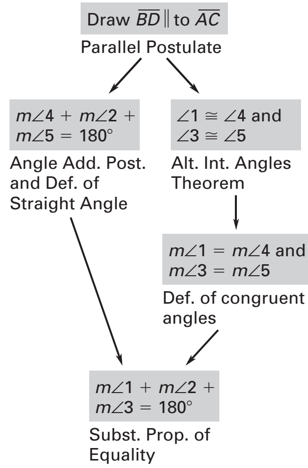
Lesson 4.6 Prove Triangles Congruent by ASA and AAS

Guided Practice for the lesson "Prove Triangles Congruent by ASA and AAS"

1. You would use AAS to prove $\triangle RST \cong \triangle VUT$. $\overline{RS} \cong \overline{VU}$ and $\angle RST \cong \angle VUT$ are given. The second pair of angles, $\angle RTS$ and $\angle VTU$, are congruent by the Vertical Angles Theorem.

2. Given: $\triangle ABC$
Prove: $m\angle 1 + m\angle 2 + m\angle 3 = 180°$





3. The AAS Congruence Theorem could be used to prove $\triangle ABE \cong \triangle ADE$ because $\angle ABE \cong \angle ADE$, $\angle AEB \cong \angle AED$, and $\overline{AE} \cong \overline{AE}$.

4. No, towers B and C could not be used to locate the fire. No triangle is formed by the towers and the fire, so the fire could be anywhere between towers B and C.

Exercises for the lesson "Prove Triangles Congruent by ASA and AAS"

Skill Practice

1. *Sample answer:* A flow proof shows the flow of a logical argument.

2. You need to show that any pair of corresponding sides are also congruent.

3. Yes; $\triangle ABC \cong \triangle QRS$ by the AAS Congruence Theorem.

4. No; it cannot be proven that $\triangle XYZ \cong \triangle JKL$.

5. Yes; $\triangle PQR \cong \triangle RSP$ by the ASA Congruence Postulate.

6. There is no AAA congruence postulate or theorem.

7. B;
two angles: $\angle CBA \cong \angle KJH$ and $\angle CAB \cong \angle KHJ$
non-included side: $\overline{BC} \cong \overline{JK}$
So $\triangle ABC \cong \triangle HJK$ by AAS.

8. $\overline{GH} \cong \overline{MN}$, $\angle G \cong \angle M$, $\angle F \cong \angle L$

9. $\overline{FG} \cong \overline{LM}$, $\angle G \cong \angle M$, $\angle F \cong \angle L$

10. $\overline{FH} \cong \overline{LN}$, $\angle H \cong \angle N$, $\overline{HG} \cong \overline{NM}$

11. Given: $\overline{AF} \cong \overline{DF}$ and $\overline{FE} \cong \overline{FB}$; $\angle AFE \cong \angle DFB$ by the Vertical Angles Congruence Theorem.

12. Given: $\angle EAD \cong \angle DBE$ and $\angle AED \cong \angle BDE$; $\overline{ED} \cong \overline{DE}$ by the Reflexive Property of Segment Congruence.

13. Given: $\angle AED \cong \angle BDC$ and $\overline{ED} \cong \overline{DC}$; $\angle EDA \cong \angle DCB$ by the Corresponding Angles Postulate.

14. Yes; $\triangle ABC \cong \triangle DEF$ by the SAS Congruence Postulate.

15. No; you cannot determine if triangles are congruent because there is no AAA congruence postulate or theorem.

16. No; you cannot determine if triangles are congruent because $\overline{AC}$ and $\overline{DE}$ are not corresponding sides.

17. No; you cannot determine if triangles are congruent because none of the congruent sides are corresponding.

18. No; you cannot prove that $\triangle ABC \cong \triangle DEC$.

19. Yes; $\triangle TUV \cong \triangle TWV$ can be proved by the SAS Congruence Postulate.

20. No; you cannot prove that $\triangle QML \cong \triangle LPN$.

21. a. slope $\overline{BC} = \frac{6-5}{6-2} = \frac{1}{4}$

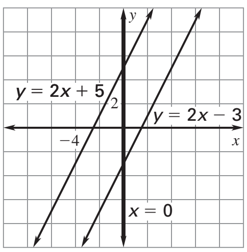
slope $\overline{AD} = \frac{2-1}{4-0} = \frac{1}{4}$

Because slope $\overline{BC}$ = slope $\overline{AD}$, $\overline{BC}$ and $\overline{AD}$ are parallel. So $\angle CAD \cong \angle ACB$ by the Alternate Interior Angles Theorem.

b. From part (a), $\overline{BC}$ and $\overline{AD}$ are parallel. So $\angle ACD \cong \angle CAB$ by the Alternate Interior Angles Theorem.

c. From parts (a) and (b), $\angle CAD \cong \angle ACB$ and $\angle ACD \cong \angle CAB$. Also, $\overline{AC} \cong \overline{CA}$ by the Reflexive Property. So $\triangle ABC \cong \triangle CDA$ by the ASA Congruence Postulate.

22. a.



b. Any real value except $m = 2$ will form two triangles in the graph. The triangles will be congruent right triangles when $m = -\frac{1}{2}$ or 0. The resulting triangles are congruent by the ASA Congruence Postulate or the AAS Congruence Theorem.

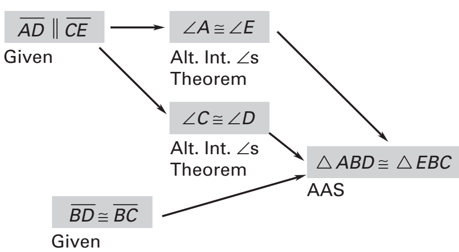
Problem Solving

23. In the picture, two pairs of angles and the included pair of sides are shown to be congruent, so the triangles are congruent by ASA.

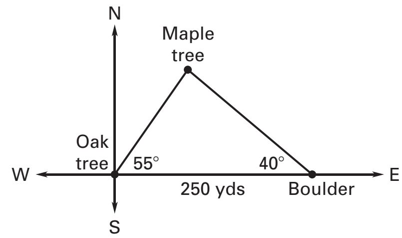
24. In the picture, two pairs of angles and a nonincluded pair of sides are shown to be congruent, so the triangles are congruent by AAS.

25. Given: $\overline{AD} \parallel \overline{CE}$, $\overline{BD} \cong \overline{BC}$

Prove: $\triangle ABD \cong \triangle EBC$



26.



Yes, you will be able to locate the maple tree. Because two angles and the included side are given, there is only one possible triangle that can be formed.

27. $\triangle ABC \cong \triangle DEF$ by the AAS Congruence Theorem.

28. All right angles are congruent and the right angles are included between the pairs of congruent legs in the triangle. So the triangles are congruent by SAS.

29. All right angles are congruent and another pair of angles are given to be congruent. If the congruent legs are between the congruent pairs of angles, then the triangles are congruent by ASA. If the congruent legs are not included between the congruent pairs of angles, then the triangles are congruent by AAS.

30. All right angles are congruent. Because another pair of angles and the nonincluded sides are given to be congruent, the triangles are congruent by AAS.

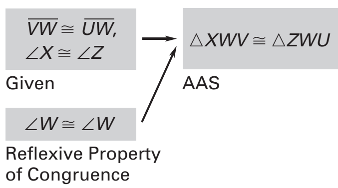
31. Given: $\overline{AK} \cong \overline{CJ}$, $\angle BJK \cong \angle BKJ$, $\angle A \cong \angle C$

Prove: $\triangle ABK \cong \triangle CBJ$

Statements	Reasons
1. $\overline{AK} \cong \overline{CJ}$, $\angle A \cong \angle C$, $\angle BJK \cong \angle BKJ$	**1.** Given
2. $\triangle ABK \cong \triangle CBJ$	**2.** ASA Congruence Postulate

32. Given: $\overline{VW} \cong \overline{UW}$, $\angle X \cong \angle Z$

Prove: $\triangle XWV \cong \triangle ZWU$



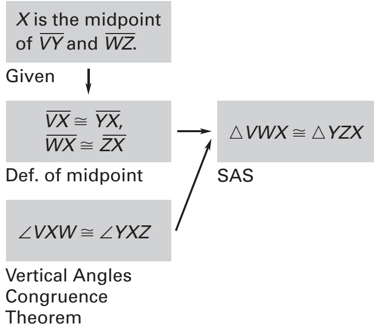
33. Given: $\angle NKM \cong \angle LMK$, $\angle L \cong \angle N$

Prove: $\triangle NMK \cong \triangle LKM$

Statements	Reasons
1. $\angle NKM \cong \angle LMK$, $\angle L \cong \angle N$	**1.** Given
2. $\overline{MK} \cong \overline{KM}$	**2.** Reflexive Property of Congruence
3. $\triangle NMK \cong \triangle LKM$	**3.** AAS Congruence Theorem

34. Given: X is the midpoint of $\overline{VY}$ and $\overline{WZ}$.

Prove: $\triangle VWX \cong \triangle YZX$



35. Given: $\triangle ABF \cong \triangle DFB$, F is the midpoint of $\overline{AE}$, B is the midpoint of $\overline{AC}$.

Prove: $\triangle FDE \cong \triangle BCD \cong \triangle ABF$

Statements	Reasons
1. F is the midpoint of $\overline{AE}$. B is the midpoint of $\overline{AC}$.	**1.** Given
2. $\overline{FE} \cong \overline{AF}$, $\overline{BC} \cong \overline{AB}$	**2.** Definition of midpoint
3. $\triangle ABF \cong \triangle DFB$	**3.** Given
4. $\overline{AF} \cong \overline{DB}$, $\overline{AB} \cong \overline{DF}$	**4.** Corr. parts of ≅ ⚠ are ≅.
5. $\overline{FE} \cong \overline{DB}$, $\overline{BC} \cong \overline{DF}$	**5.** Transitive Property of Congruence
6. $\angle AFB \cong \angle DBF$, $\angle ABF \cong \angle DFB$ $\angle FAB \cong \angle BDF$	**6.** Corr. parts of ≅ ⚠ are ≅.
7. $m\angle AFB = m\angle DBF$, $m\angle ABF = m\angle DFB$, $m\angle FAB = m\angle BDF$	**7.** Definition of congruent angles
8. $m\angle AFB + m\angle DFB + m\angle EFD = 180°$, $m\angle ABF + m\angle DBF + m\angle CBD = 180°$	**8.** Definition of a straight angle
9. $m\angle AFB + m\angle ABF + m\angle EFD = 180°$, $m\angle ABF + m\angle AFB + m\angle CBD = 180°$	**9.** Substitution Property of Equality
10. $m\angle EFD = 180° - m\angle AFB - m\angle ABF$, $m\angle CBD = 180° - m\angle AFB - m\angle ABF$	**10.** Subtraction Property of Equality
11. $m\angle AFB + m\angle ABF + m\angle FAB = 180°$	**11.** Triangle Sum Theorem
12. $m\angle FAB = 180° - m\angle AFB - m\angle ABF$	**12.** Subtraction Property of Equality
13. $m\angle FAB = m\angle EFD = m\angle CBD$	**13.** Substitution Property of Equality
14. $\angle FAB \cong \angle EFD \cong \angle CBD$	**14.** Definition of congruent angles
15. $\triangle FDE \cong \triangle BCD \cong \triangle ABF$	**15.** SAS Congruence Postulate

Investigating Geometry Construction for the lesson "Prove Triangles Congruent by ASA and AAS"

1. reflection in the line that contains $\overline{AB}$
2. reflection in the perpendicular bisector of $\overline{AB}$
3. yes; rotation of 180° around the midpoint of $\overline{AB}$
4. Rigid motions can be used to transform the triangles onto each other, so the triangles are all congruent.
5. Yes; the same rigid motions of reflection can be used to show that the triangles are all congruent.

Lesson 4.7 Use Congruent Triangles

Guided Practice for the lesson "Use Congruent Triangles"

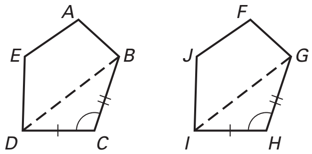
1. $\overline{BD} \cong \overline{BD}$ by the Reflexive Property, so $\triangle ABD \cong \triangle CBD$ by SSS. Corresponding parts of congruent triangles are congruent, so $\angle A \cong \angle C$.
2. No; Because M is the midpoint of $\overline{NK}$, $\overline{NM} \cong \overline{MK}$. No matter how far apart the stakes at K and M are placed, the triangles will always be congruent by ASA.
3. You are given that $\overline{TU} \cong \overline{QP}$ and you can deduce that $\overline{PU} = \overline{UP}$ by the Reflexive Property. Now you only need to show that $\overline{PT} \cong \overline{UQ}$ to prove congruence by SSS. To do this, you can show that triangles QSP and TRU are right and congruent by HL. This leads to right triangles USQ and PRT being congruent by HL, which gives $\overline{PT} \cong \overline{UQ}$.
4. $\overline{AC}$ and $\overline{AB}$

Exercises for the lesson "Use Congruent Triangles"

Skill Practice

1. Corresponding parts of congruent triangles are congruent.
2. *Sample answer:* You might choose to use congruent triangles to measure the distance across a river if you are unable to cross it. You could also use congruent triangles to measure the distance across a lake.
3. $\triangle CBA \cong \triangle CBD$ by SSS.
4. $\triangle QPR \cong \triangle TPS$ by SAS.
5. $\triangle JKM \cong \triangle LKM$ by HL.
6. $\triangle CAD \cong \triangle BDA$ by AAS.
7. $\triangle JNH \cong \triangle KLG$ by AAS.
8. $\triangle VRT \cong \triangle QVW$ by AAS.
9. $\angle ABC \cong \angle CDA$ is given, but this angle is not included between the pairs of congruent sides $\overline{BC}$ and $\overline{DA}$, and $\overline{CA}$ and $\overline{AC}$. So the triangles cannot be proven to be congruent.
10. Show $\overline{VT} \cong \overline{TV}$ by the Reflexive Property. So $\triangle VST \cong \triangle TUV$ by SSS. Because corresponding parts of congruent triangles are congruent, $\angle S \cong \angle U$.
11. Show $\angle NLM \cong \angle PLQ$ by the Vertical Angles Congruence Theorem. So $\triangle NLM \cong \triangle PLQ$ by AAS. Because corresponding parts of congruent triangles are congruent, $\overline{LM} \cong \overline{LQ}$.

12. *Sample answer:*



$ABCDE \cong FGHIJ$

When connecting any pair of corresponding vertices of congruent pentagons, a pair of congruent triangles will be formed by SAS. Because these diagonals are now corresponding parts of congruent triangles, the segments must be congruent.

13.

$m\angle D = m\angle A$	$m\angle E = m\angle B$	$m\angle F = m\angle C$
$(3x + 10)° = 70°$	$\left(\frac{y}{3} + 20\right)° = 60°$	$(z^2 + 14)° = 50°$
$3x = 60$	$\frac{y}{3} = 40$	$z^2 = 36$
$x = 20$	$y = 120$	$z = \pm 6$

14. B; $\overline{BA} \cong \overline{BC}$ and $\angle BDC \cong \angle BDA$ are given. Also, $\overline{BD} \cong \overline{BD}$ by the Reflexive Property. However, two pairs of congruent sides and a pair of non-included congruent angles are not enough to prove triangle congruence because there is no SSA congruence property.

15. Show $\overline{FG} \cong \overline{GF}$ by the Reflexive Property, so $\triangle KFG \cong \triangle HGF$ by AAS. Because $\overline{FK}$ and $\overline{GH}$ are corresponding parts of congruent triangles, $\overline{FK} \cong \overline{GH}$. Also, $\angle FJK \cong \angle GJH$ by the Vertical Angles Congruence Theorem, so $\triangle FJK \cong \triangle GJH$ by AAS. Because corresponding parts of congruent triangles are congruent, $\angle 1 \cong \angle 2$.

16. $\triangle EAB \cong \triangle EDC$ by AAS. Because corresponding parts of congruent triangles are congruent, $\angle EBA \cong \angle ECD$. Because angles 1 and 2 are supplementary to congruent angles, $\angle 1 \cong \angle 2$.

17. $\angle STR \cong \angle QTP$ by the Vertical Angles Congruence Theorem. So $\triangle STR \cong \triangle QTP$ by ASA. $\overline{TR}$ and $\overline{TP}$ are corresponding sides of congruent triangles, so $\overline{TR} \cong \overline{TP}$. Because $\angle PTS$ and $\angle RTQ$ are vertical angles, $\triangle PTS \cong \triangle RTQ$ by SAS. Corresponding parts of congruent triangles are congruent, so $\angle 1 \cong \angle 2$.

18. Show $\triangle ABE \cong \triangle CBE$ by ASA, which gives you $\overline{AE} \cong \overline{CE}$. Use the Angle Addition Postulate and congruent angles to show $\angle FAE \cong \angle DCE$. Then $\triangle AEF \cong \triangle CED$ by SAS, and $\angle 1 \cong \angle 2$.

19. Let P be the point where $\overleftrightarrow{NL}$ intersects $\overline{KM}$. Then $\triangle PKN \cong \triangle PMN$ by SSS. Corresponding parts of congruent triangles are congruent, so $\angle KPN \cong \angle MPN$. $\overline{PL} \cong \overline{PL}$ by the Reflexive Property, so $\triangle MPL \cong \triangle KPL$ by SAS. Because $\angle 1$ and $\angle 2$ are corresponding parts of congruent triangles, $\angle 1 \cong \angle 2$.

20. First, $\triangle TVY \cong \triangle UXZ$ by SAS. $\overline{TY}$ and $\overline{UZ}$ are corresponding parts of congruent triangles, so $\overline{TY} \cong \overline{UZ}$. Then since $\overline{TY} \parallel \overline{UZ}$, $\angle YTW \cong \angle UZW$ and $\angle TYW \cong \angle ZUW$ by the Alternate Interior Angles Theorem. So $\triangle TWY \cong \triangle ZWU$ by ASA. Then because corresponding parts of congruent triangles are congruent, $\overline{TW} \cong \overline{ZW}$ and $\overline{YW} \cong \overline{UW}$. $\angle TWU \cong \angle ZWY$ by the Vertical Angles Theorem, so $\triangle TWU \cong \triangle ZWY$ by SAS. $\angle 1 \cong \angle 2$ because corresponding parts of congruent triangles are congruent.

21. $AB = \sqrt{(6 - 3)^2 + (11 - 7)^2} = \sqrt{25} = 5$

$DE = \sqrt{(5 - 2)^2 + (-8 - (-4))^2} = \sqrt{25} = 5$

So $\overline{AB} \cong \overline{DE}$.

$BC = \sqrt{(11 - 6)^2 + (13 - 11)^2} = \sqrt{29}$

$EF = \sqrt{(10 - 5)^2 + (-10 - (-8))^2} = \sqrt{29}$

So $\overline{BC} \cong \overline{EF}$.

$AC = \sqrt{(11 - 3)^2 + (13 - 7)^2} = \sqrt{100} = 10$

$DF = \sqrt{(10 - 2)^2 + (-10 - (-4))^2} = \sqrt{100} = 10$

So $\overline{AC} \cong \overline{DF}$.

Because all pairs of corresponding sides are congruent, $\triangle ABC \cong \triangle DEF$ by SSS. Therefore, since corresponding parts of congruent triangles are congruent, $\angle A \cong \angle D$.

22. $AB = \sqrt{(3 - 3)^2 + (2 - 8)^2} = \sqrt{36} = 6$

$DE = \sqrt{(5 - (-1))^2 + (5 - 5)^2} = \sqrt{36} = 6$

So $\overline{AB} \cong \overline{DE}$.

$BC = \sqrt{(11 - 3)^2 + (2 - 2)^2} = \sqrt{64} = 8$

$EF = \sqrt{(5 - 5)^2 + (13 - 5)^2} = \sqrt{64} = 8$

So $\overline{BC} \cong \overline{EF}$.

$AC = \sqrt{(11 - 3)^2 + (2 - 8)^2} = \sqrt{100} = 10$

$DF = \sqrt{(5 - (-1))^2 + (13 - 5)^2} = \sqrt{100} = 10$

So $\overline{AC} \cong \overline{DF}$.

Because all pairs of corresponding sides are congruent, $\triangle ABC \cong \triangle DEF$ by SSS. Therefore, because corresponding parts of congruent triangles are congruent, $\angle A \cong \angle D$.

23. Given: $\angle T \cong \angle U$, $\angle Z \cong \angle X$, $\overline{YZ} \cong \overline{YX}$

Prove: $\angle VYX \cong \angle WYZ$

Statements	Reasons
1. $\angle T \cong \angle U$, $\angle Z \cong \angle X$, $\overline{YZ} \cong \overline{YX}$	**1.** Given
2. $\triangle TYZ \cong \triangle UYX$	**2.** AAS
3. $\angle TYZ \cong \angle UYX$	**3.** Corr. parts of $\cong$ $\triangle$s are $\cong$.
4. $m\angle TYZ = m\angle UYX$	**4.** Definiton of congruent angles
5. $m\angle TYW + m\angle WYZ = m\angle TYZ$, $m\angle TYW + m\angle VYX = m\angle UYX$	**5.** Angle Addition Postulate
6. $m\angle TYW + m\angle WYZ = m\angle TYW + m\angle VYX$	**6.** Substitution
7. $m\angle WYZ = m\angle VYX$	**7.** Subtraction Property of Equality
8. $\angle WYZ \cong \angle VYX$	**8.** Definition of congruent angles

24. Given: $\overline{FG} \cong \overline{HG} \cong \overline{JG} \cong \overline{KG}$, $\overline{JM} \cong \overline{KM} \cong \overline{LM} \cong \overline{NM}$

Prove: $\overline{FL} \cong \overline{HN}$

Statements	Reasons
1. $\overline{FG} \cong \overline{HG} \cong \overline{JG} \cong \overline{KG}$, $\overline{JM} \cong \overline{KM} \cong \overline{LM} \cong \overline{NM}$	**1.** Given
2. $\angle FGJ \cong \angle HGK$, $\angle JML \cong \angle KMN$	**2.** Vertical Angles Congruence Theorem
3. $\triangle FGJ \cong \triangle HGK$, $\triangle JML \cong \triangle KMN$	**3.** SAS
4. $\overline{FJ} \cong \overline{HK}$, $\overline{JL} \cong \overline{KN}$	**4.** Corr. parts of ≅ ⊿ are ≅.
5. $FJ = HK$, $JL = KN$	**5.** Definition of congruent segments
6. $FJ + JL = HK + KN$	**6.** Addition Property of Equaltiy
7. $FL = HN$	**7.** Segment Addition Postulate
8. $\overline{FL} \cong \overline{HN}$	**8.** Definition of congruent segments

25. Given: $\angle PRU \cong \angle QVS$, $\overline{RS} \cong \overline{UV}$, $\angle TSU \cong \angle USW \cong \angle TUS \cong \angle SUW$

Prove: $\triangle PUX \cong \triangle QSY$

Statements	Reasons
1. $\angle PRU \cong \angle QVS$, $\overline{RS} \cong \overline{UV}$, $\angle TSU \cong \angle USW \cong \angle TUS \cong \angle SUW$	**1.** Given
2. $\overline{SU} \cong \overline{SU}$	**2.** Reflexive Property of Congruence
3. $SU = SU$, $RS = UV$	**3.** Def. of congruent angles
4. $RS + SU = SU + UV$	**4.** Addition Prop. of Equality
5. $RU = SV$	**5.** Segment Add. Postulate
6. $\overline{RU} \cong \overline{SV}$	**6.** Def. of congruent segments
7. $\triangle QSV \cong \triangle PUR$	**7.** ASA
8. $\overline{PU} \cong \overline{QS}$, $\angle RPU \cong \angle VQS$	**8.** Corr. parts of ≅ ⊿ are ≅.
9. $m\angle TSU + m\angle USW = m\angle TSW$, $m\angle TUS + m\angle SUW = m\angle TUW$	**9.** Angle Addition Postulate
10. $m\angle TSU = m\angle USW = m\angle TUS = m\angle SUW$	**10.** Definition of congruent angles
11. $m\angle TSU = m\angle TSU = m\angle TSW$, $m\angle TSU + m\angle TSU = m\angle TUW$	**11.** Substitution
12. $m\angle TSW = m\angle TUW$	**12.** Transitive Property of Equality
13. $\angle TSW \cong m\angle TUW$	**13.** Definition of congruent angles
14. $\triangle PUX \cong \triangle QSY$	**14.** ASA

26. Given: $\overline{AD} \cong \overline{BD} \cong \overline{FD} \cong \overline{GD}$

Prove: $\overline{AC} \cong \overline{GE}$

Statements	Reasons
1. $\overline{AD} \cong \overline{BD} \cong \overline{FD} \cong \overline{GD}$	**1.** Given
2. $\angle ADF \cong \angle BDG$	**2.** Vertical Angles Congruence Theorem
3. $\triangle ADF \cong \triangle GDB$	**3.** SAS
4. $\angle CAD \cong \angle EGD$	**4.** Corr. parts of ≅ ⊿ are ≅.
5. $\angle ADC \cong \angle GDE$	**5.** Vertical Angles Congruence Theorem
6. $\triangle ADC \cong \triangle GDE$	**6.** ASA
7. $\overline{AC} \cong \overline{GE}$	**7.** Corr. parts of ≅ ⊿ are ≅.

27. $\triangle ABC \cong \triangle NPQ$ by ASA. Then $\overline{BC} \cong \overline{PQ} \cong \overline{EF} \cong \overline{HJ}$. So, $\triangle ABC \cong \triangle DEF$ by HL and $\triangle ABC \cong \triangle GHJ$ by SSS. $\triangle ABC$, $\triangle DEF$, $\triangle GHJ$, and $\triangle NPQ$ are all congruent.

Problem Solving

28. Because $\overline{CD} \perp \overline{DE}$ and $\overline{CD} \perp \overline{AC}$, $\angle D$ and $\angle C$ are congruent right angles. $\angle DBE$ and $\angle CBA$ are vertical angles, so they are congruent. Because $\overline{DB} \cong \overline{CB}$, $\triangle DBE \cong \triangle CBA$ by ASA. Then because corresponding parts of congruent triangles are congruent, $\overline{AC} \cong \overline{DE}$. So you can find AC, the distance across the canyon, by measuring DE.

29. Given: $\overline{PQ} \parallel \overline{VS}$, $\overline{QU} \parallel \overline{ST}$, $\overline{PQ} \cong \overline{VS}$

Prove: $\angle Q \cong \angle S$

Statements	Reasons
1. $\overline{PQ} \parallel \overline{VS}$ and $\overline{QU} \parallel \overline{ST}$.	**1.** Given
2. $\angle QPU \cong \angle SVT$, $\angle QUP \cong \angle STV$	**2.** Corresponding Angles Postulate
3. $\overline{PQ} \cong \overline{VS}$	**3.** Given
4. $\triangle QPU \cong \triangle SVT$	**4.** AAS
5. $\angle Q \cong \angle S$	**5.** Corr. parts of ≅ ⊿ are ≅.

30. By ASA, $\triangle ABC \cong \triangle EDC$ so $\overline{ED} \cong \overline{AB}$. Because $EC \approx 11.5$ m and $CD \approx 2.5$ m, $ED \approx \sqrt{11.5^2 - 2.5^2} \approx 11.2$ by the Pythagorean Theorem. Because $\overline{ED} \cong \overline{AB}$, then AB, the distance across the half pipe, is approximately 11.2 meters.

31. A; The SAS Congruence Postulate would not appear in the proof because you only have one pair of congruent sides, $\overline{WZ} \cong \overline{ZW}$.

32. Given: $\overline{AB} \cong \overline{AC}$, $\overline{BG} \cong \overline{CG}$

Prove: $\overrightarrow{AG}$ bisects $\angle A$.

Statements	Reasons
1. $\overline{AB} \cong \overline{AC}$, $\overline{BG} \cong \overline{CG}$	1. Given
2. $\overline{AG} \cong \overline{AG}$	2. Reflexive Property for Congruence
3. $\triangle CAG \cong \triangle BAG$	3. SSS
4. $\angle CAG \cong \angle BAG$	4. Corr. parts of ≅ $\triangle$s are ≅.
5. $\overrightarrow{AG}$ bisects $\angle A$.	5. Definition of angle bisector

33. No; the given congruent angles are not the included angles, so you cannot prove that $\overline{AB} \cong \overline{BC}$.

34. Yes; $\triangle ADE \cong \triangle CDE$, so $\overline{AE} \cong \overline{CE}$ because corresponding parts of congruent triangles are congruent. Also, $\angle CEB \cong \angle AEB$ by the Right Angle Congruence Theorem. $\overline{BE} \cong \overline{BE}$ by the Reflexive Property, so $\triangle BAE \cong \triangle BCE$ by SAS. Because corresponding parts of congruent triangles are congruent, $\overline{AB} \cong \overline{BC}$.

35. Yes; By definition of a bisector, $\overline{AD} \cong \overline{CD}$. $\overline{DB} \cong \overline{DB}$ by the Reflexive Property and $\angle ADB \cong \angle CDB$ by the Right Angles Congruence Theorem. So, $\triangle ADB \cong \triangle CDB$ by SAS. Corresponding parts of congruent triangles are congruent, so $\overline{AB} \cong \overline{BC}$.

36. a.

Statements	Reasons
1. $\overline{AP} \cong \overline{BP}$ $\overline{AQ} \cong \overline{BQ}$	1. Given by construction
2. $\overline{PQ} \cong \overline{PQ}$	2. Reflexive Property of Congruence
3. $\triangle APQ \cong \triangle BPQ$	3. SSS Congruence Postulate
4. $\angle APM \cong \angle BPM$	4. Corr. parts of ≅ $\triangle$s are ≅.
5. $\overline{PM} \cong \overline{PM}$	5. Reflexive Property of Congruence
6. $\triangle APM \cong \triangle BPM$	6. SAS Congruence Postulate
7. $\angle AMP \cong \angle BMP$	7. Corr. parts of ≅ $\triangle$s are ≅.
8. $\overleftrightarrow{PM} \perp \overleftrightarrow{AB}$	8. If 2 lines intersect to form a linear pair of ≅ $\angle$s, then the lines are ⊥.
9. $\angle AMP$ and $\angle BMP$ are right angles.	9. If 2 lines are ⊥, then they intersect to form 4 rt. $\angle$s.

b.

Statements	Reasons
1. $\overline{AP} \cong \overline{BP}$, $\overline{AQ} \cong \overline{BQ}$	1. Given by construction
2. $\overline{PQ} \cong \overline{PQ}$	2. Reflexive Property of Congruence
3. $\triangle APQ \cong \triangle BPQ$	3. SSS Congruence Postulate
4. $\angle QPA \cong \angle QPB$	4. Corr. parts of ≅ $\triangle$s are ≅.
5. $\overleftrightarrow{PQ} \perp \overleftrightarrow{AB}$	5. If 2 lines intersect to form a linear pair of ≅ $\angle$s, then the lines are ⊥.
6. $\angle QPA$ and $\angle QPB$ are right angles.	6. If 2 lines are ⊥, then they intersect to form 4 rt. $\angle$s.

37. Given: $\overline{MN} \cong \overline{KN}$, $\angle PMN \cong \angle NKL$, $\angle MJN$ and $\angle KQN$ are right angles.
Prove: $\angle 1 \cong \angle 2$

Statements	Reasons
1. $\angle PMN \cong \angle NKL$	1. Given
2. $\overline{MN} \cong \overline{KN}$	2. Given
3. $\angle PNM \cong \angle LNK$	3. Vertical Angles Congruence Theorem
4. $\triangle PNM \cong \triangle LNK$	4. ASA
5. $\overline{PM} \cong \overline{LK}$, $\angle MPJ \cong \angle KLQ$	5. Corr. parts of ≅ $\triangle$s are ≅.
6. $\angle PJM$ and $\angle LQK$ are right angles.	6. Theorem 3.9
7. $\angle PJM \cong \angle LQK$	7. Right Angles Congruence Theorem
8. $\triangle PJM \cong \triangle LQK$	8. AAS
9. $\angle 1 \cong \angle 2$	9. Corr. parts of ≅ $\triangle$s are ≅.

38. Given: $\overline{TS} \cong \overline{TV}$, $\overline{SR} \cong \overline{VW}$
Prove: $\angle 1 \cong \angle 2$

Statements	Reasons
1. $\overline{TS} \cong \overline{TV}$, $\overline{SR} \cong \overline{VW}$	1. Given
2. $TS = TV$, $SR = VW$	2. Definition of congruent segments
3. $TS + SR = TR$, $TV + VW = TW$	3. Segment Addition Postulate
4. $TV + SR = TR$, $TV + SR = TW$	4. Substitution Property of Equality
5. $TR = TW$	5. Transitive Property of Equality
6. $\overline{TR} \cong \overline{TW}$	6. Definition of congruent segments

Statements	Reasons
7. $\angle RTV \cong \angle WTS$	**7.** Reflexive Property of Congruence
8. $\triangle RTV \cong \triangle WTS$	**8.** SAS
9. $\overline{RV} \cong \overline{WS}$	**9.** Corr. parts of $\cong$ $\triangle$s are $\cong$.
10. $\overline{SV} \cong \overline{VS}$	**10.** Reflexive Property of Congruence
11. $\triangle RSV \cong \triangle WVS$	**11.** SSS
12. $\angle RSV \cong \angle WVS$	**12.** Corr. parts of $\cong$ $\triangle$s are $\cong$.
13. $\angle RSV$ and $\angle 1$ are supplementary. $\angle WVS$ and $\angle 2$ are supplementary.	**13.** Linear Pair Postulate
14. $\angle 1 \cong \angle 2$	**14.** Congruent Supplements Theorem

39. Given: $\overline{BA} \cong \overline{BC}$, D and E are midpoints, $\angle A \cong \angle C$, $\overline{DF} \cong \overline{EF}$

Prove: $\overline{FG} \cong \overline{FH}$

Statements	Reasons
1. $\overline{BA} \cong \overline{BC}$, D and E are midpoints, $\angle A \cong \angle C$, $\overline{DF} \cong \overline{EF}$.	**1.** Given
2. $\overline{BD} \cong \overline{DA}$, $\overline{BE} \cong \overline{EC}$	**2.** Definition of midpoint
3. $BD = DA$, $BE = EC$	**3.** Definition of congruent segments
4. $BD + DA = BA$, $BE + EC = BC$	**4.** Segment Addition Postulate
5. $BD + DA = BE + EC$	**5.** Substitution Property
6. $BD + BD = BE + BE$, $DA + DA = EC + EC$	**6.** Substitution Property of Equality
7. $2BD = 2BE$, $2DA = 2EC$	**7.** Simplify.
8. $BD = BE$, $DA = EC$	**8.** Division Property of Equality
9. $\overline{BD} \cong \overline{BE}$, $\overline{DA} \cong \overline{EC}$	**9.** Definition of congruent segments
10. $\overline{BJ}$ containing point F	**10.** Construction
11. $\overline{BF} \cong \overline{BF}$	**11.** Reflexive Property of Congruence
12. $\triangle BFD \cong \triangle BFE$	**12.** SSS
13. $\angle BFE \cong \angle BFD$, $\angle BEF \cong \angle BDF$	**13.** Corr. parts of $\cong$ $\triangle$s are $\cong$.
14. $\angle BFE \cong \angle GFJ$, $\angle BFD \cong \angle HFJ$	**14.** Vertical angles Congruence Theorem
15. $\angle GFJ \cong \angle HFJ$	**15.** Substitution
16. $\overline{FJ} \cong \overline{FJ}$	**16.** Reflexive Property of Segment Congruence
17. $\angle BEF$ and $\angle CEG$, $\angle BDF$ and $\angle ADH$ form linear pairs.	**17.** Definition of linear pair
18. $\angle BEF$ and $\angle CEG$, $\angle BDF$ and $\angle ADH$ are supplementary.	**18.** Linear Pair Postulate
19. $\angle CEG \cong \angle ADH$	**19.** Congruent Supplements Theorem
20. $\triangle CEG \cong \triangle ADH$	**20.** ASA
21. $\angle EGJ \cong \angle DHJ$	**21.** Corr. parts of $\cong$ $\triangle$s are $\cong$.
22. $\triangle GFJ \cong \triangle HFJ$	**22.** AAS
23. $\overline{FG} \cong \overline{FH}$	**23.** Corr. parts of $\cong$ $\triangle$s are $\cong$.

40. Given: $\overline{AB} \parallel \overline{EC}$, $\overline{AC} \parallel \overline{ED}$, $\overline{AB} \cong \overline{ED}$, $\overline{AC} \cong \overline{EC}$

Prove: $\overline{AD} \cong \overline{EB}$

Statements	Reasons
1. $\overline{AB} \parallel \overline{EC}$, $\overline{AC} \parallel \overline{ED}$, $\overline{AB} \cong \overline{ED}$, $\overline{AC} \cong \overline{EC}$	**1.** Given
2. $\angle DEC \cong \angle ECA$, $\angle ECA \cong \angle BAC$	**2.** Alternate Interior Angles Theorem
3. $\angle DEC \cong \angle BAC$	**3.** Transitive Property of Angle Congruence
4. $\triangle DEC \cong \triangle BAC$	**4.** SAS
5. $\overline{BC} \cong \overline{CD}$, $\angle BCA \cong \angle DCE$	**5.** Corr. parts of $\cong$ $\triangle$s are $\cong$.
6. $m\angle BCA = m\angle DCE$	**6.** Definition of congruent angles
7. $m\angle BCA + m\angle ACE = m\angle DCE + m\angle ACE$	**7.** Addition Property of Equality
8. $m\angle BCE = m\angle DCA$	**8.** Angle Addition Postulate
9. $\angle BCE \cong \angle DCA$	**9.** Definition of congruent angles
10. $\triangle BCE \cong \triangle DCA$	**10.** SAS
11. $\overline{AD} \cong \overline{EB}$	**11.** Corr. parts of $\cong$ $\triangle$s are $\cong$.

Quiz for the lessons "Prove Triangles Congruent by SAS and HL", "Prove Triangles Congruent by ASA and AAS", and "Use Congruent Triangles"

1. SAS can be used to prove that the triangles are congruent.
2. HL can be used to prove that the right triangles are congruent.
3. AAS can be used to prove that the triangles are congruent.
4. Given: $\angle BAC \cong \angle DCA$, $\overline{AB} \cong \overline{CD}$

   Prove: $\triangle ABC \cong \triangle CDA$

Statements	Reasons
1. $\overline{AB} \cong \overline{CD}$	1. Given
2. $\angle BAC \cong \angle DCA$	2. Given
3. $\overline{AC} \cong \overline{CA}$	3. Reflexive Property of Segment Congruence
4. $\triangle ABC \cong \triangle CDA$	4. SAS

5. Given: $\angle W \cong \angle Z$, $\overline{VW} \cong \overline{YZ}$

   Prove: $\triangle VWX \cong \triangle YZX$

Statements	Reasons
1. $\angle W \cong \angle Z$	1. Given
2. $\angle WXV \cong \angle YXZ$	2. Vertical Angles Congruence Theorem
3. $\overline{VW} \cong \overline{YZ}$	3. Given
4. $\triangle VWX \cong \triangle YZX$	4. AAS

6. Show that $\angle P \cong \angle M$ by the Right Angles Congruence Theorem and $\angle PLQ \cong \angle NLM$ by the Vertical Angles Theorem. $\overline{PQ} \cong \overline{MN}$ is given, so $\triangle PQL \cong \triangle MNL$ by AAS. Then because corresponding parts of congruent triangles are congruent, $\overline{QL} \cong \overline{NL}$.

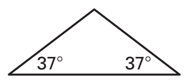
Lesson 4.8 Use Isosceles and Equilateral Triangles

Guided Practice for the lesson "Use Isosceles and Equilateral Triangles"

1. If $\overline{HG} \cong \overline{HK}$, then $\angle HGK \cong \angle HKG$.
2. If $\angle KHJ \cong \angle KJH$, then $\overline{HK} \cong \overline{KJ}$.
3. $ST = TU$, so $ST = 5$.
4. No; the Triangle Sum Theorem guarantees that the sum of the angles in a triangle will be 180°. Because an equilateral triangle contains three equal angles, each angle must measure exactly $\frac{180}{3}$, or 60°.
5. The large triangle is equilateral, so $x = 60$. The smaller triangle is isosceles, with base angles $90° - 60°$, or 30° each. So $y = 180 - 30 - 30$, or 120.
6. From part (b) in Example 4, $\triangle PQT$ is isosceles, so $\overline{PT} \cong \overline{QT}$. Then from part (c), $\triangle PTS \cong \triangle QTR$, so $\overline{TR} \cong \overline{TS}$ because corresponding parts of congruent triangles are congruent. So $\overline{QS} \cong \overline{PR}$ by substitution and the Segment Addition Postulate. $\overline{QP} \cong \overline{PQ}$ by the Reflexive Property and $\overline{PS} \cong \overline{QR}$ is given, so $\triangle QPS \cong \triangle PQR$ by SSS.

Exercises for the lesson "Use Isosceles and Equilateral Triangles"

Skill Practice

1. The vertex angle of an isosceles triangle is the angle formed by the congruent legs of the triangle.
2. In an isosceles triangle, the base angles are congruent.
3. If $\overline{AE} \cong \overline{DE}$, then $\angle A \cong \angle D$ by the Base Angles Theorem.
4. If $\overline{AB} \cong \overline{EB}$, then $\angle A \cong \angle BEA$ by the Base Angles Theorem.
5. If $\angle D \cong \angle CED$, then $\overline{EC} \cong \overline{CD}$ by the Converse of Base Angles Theorem.
6. If $\angle EBC \cong \angle ECB$, then $\overline{EB} \cong \overline{EC}$ by the Converse of Base Angles Theorem.
7. $\triangle ABC$ is equilateral, so $AB = 12$.
8. $\triangle MNL$ is equilateral, so $ML = 16$.
9. $\triangle RST$ is equilateral, so $\angle STR$ is 60°.
10. 

The vertex angle measures $180 - 37 - 37$, or 106°.

11. $3x° = 60°$

    $x = 20$

12. $5x + 5 = 35$

    $5x = 30$

    $x = 6$

13. $9x° = 27°$

    $x = 8$

14. $\overline{AC}$ is not congruent to $\overline{BC}$. By the Converse of Base Angles Theorem, $\overline{AB} \cong \overline{BC}$, so $BC = 5$.
15. $180° - 102° = 78°$

    $x° = y° = \frac{1}{2}(78) = 39°$

    So $x = y = 39$.
16. $(x + 7)° = 55°$

    $x = 48$

    $y = 180° - 2(55)°$

    $y = 70$
17. $180° - 90° = 90°$

    $x° = 9y° = \frac{1}{2}(90) = 45°$

    $x = 45$

    $9y = 45$

    $y = 5$
18. No; Isosceles triangles could have a right or obtuse vertex angle, which would make the triangle right or obtuse.

19. B;

$3x + 4 = 22$

$3x = 18$

$x = 6$

20. First, you find y by the Triangle Sum Theorem and the Base Angles Theorem.

$2(2y + 64)° + 50° = 180°$

$4y + 128 + 50 = 180$

$4y = 2$

$y = \frac{1}{2}$

Then by the definition of a linear pair, the left triangle has a vertex measuring $180° - \left(2\left(\frac{1}{2}\right) + 64\right)°$, or 115°.

Then to find x, use the Base Angles Theorem.

$2\left(45 - \frac{x}{4}\right)° + 115° = 180°$

$-\frac{x}{2} + 205 = 180$

$-\frac{x}{2} = -25$

$x = 50$

21. There is not enough information to find x or y. One of the vertex angles must be given.

22. By the Transitive Property of Congruence, all sides involving x and y are congruent. So first, solve the two expressions involving y, then use one of those expressions to solve for x.

$5y - 4 = y + 12$

$4y = 16$

$y = 4$

$3x^2 - 32 = y + 12$

$3x^2 - 32 = 4 + 12$

$3x^2 = 48$

$x^2 = 16$

$x = \pm 4$

23. $2x + 1 = x + 3$

$x = 2$

Perimeter $= (x + 3) + (2x + 1) + 6$

$= 2 + 3 + 2(2) + 1 + 6 = 16$ feet

24. $4x + 1 = x + 4$

$3x = 3$

$x = 1$

Perimeter $= (4x + 1) + (x + 4) + 7$

$= 4(1) + 1 + 1 + 4 + 7 = 17$ inches

25. $x + 5 = 21 - x$

$2x = 16$

$x = 8$

Perimeter $= 3(21 - x) = 3(21 - 8) = 39$ inches

26. Not possible; The left triangle is isosceles with legs of length 7. If $x = 90$, then the triangle would have base angles greater than 90°, which is not possible.

27. The x, y, and z values given are possible.

28. Not possible; If $x = 25$ and $y = 25$, then the supplementary angle to the left of y would be 155°. So the top left triangle would have an angle sum exceeding 180°, which is not possible.

29. The x, y, and z values given are possible.

30. $m\angle D + m\angle E + m\angle F = 180°$

$(4x + 2)° + (6x - 30)° + 3x° = 180°$

$13x = 208$

$x = 16$

$m\angle D = 4x + 2 = 4(16) + 2 = 66°$

$m\angle E = 6x - 30 = 6(16) - 30 = 66°$

$m\angle F = 3x = 3(16) = 48°$

$\triangle DEF$ is isosceles. Because two angles have the same measure, two sides are the same length by the Converse of the Base Angles Theorem.

31. D is the midpoint of $\overline{AC}$, so $\overline{AD} \cong \overline{DC}$. Also, $\overline{BD} \perp \overline{AC}$, so $\angle ADB \cong \angle CDB$ by the Right Angles Congruence Theorem. Then by the Reflexive Property, $\overline{BD} \cong \overline{BD}$, so $\triangle BDA \cong \triangle BDC$ by SAS. Because corresponding parts of congruent triangles are congruent, $\overline{BA} \cong \overline{BC}$, so $\triangle ABC$ is isosceles.

32. One triangle is equiangular, so all angles are 60°. The other two triangles are congruent, so the vertices will be equal.

$x° + x° + 60° = 360°$

$2x = 300$

$x = 150$

33. The small top triangle is isosceles, with $x°$ being a base angle. Because x and y are supplementary, $x + y = 180°$. Next consider the angles in the overall triangle. One angle measures $x°$ and one is 90°. The other will be $\frac{180 - y}{2}$, because the small bottom triangle is isosceles. The sum of the angles will be $x° + 90° + \frac{180 - y}{2} = 180°$, which simplifies to $y = 2x$. Now you solve the system $x + y = 180$ and $y = 2x$.

$x + 2x = 180$ by substitution

$3x = 180$

$x = 60$

$60 + y = 10$

$y = 120$

34. The top triangle is equiangular, and thus equilateral. So all angles in this triangle measure 60° and all sides have length 40. Then the bottom triangle would have base angles of 30°, so the third angle is 120°. So $x° = 60° + 30°$, or $x = 90$. Now the overall triangle is a right triangle with one leg of length 40 and the hypotenuse of length 80. Use the Pythagorean Theorem to find y.

$8y = \sqrt{80^2 - 40^2}$

$8y \approx 69.28$

$y \approx 8.66$

35. There are two possible cases.

1. If the exterior angle is formed by extending a leg through the vertex angle, then the base angles must each be 65° and the vertex angle would be 50°.
2. If the exterior angle is formed by extending the base, then the base angles would each be 50° and the vertex angle would be 80°.

36. Because $\angle A$ is the vertex angle of isosceles $\triangle ABC$, $\angle B$ must be congruent to $\angle C$. Because 2 times any integer angle measure will always be an even integer, an even integer will be subtracted from 180 to find $m\angle A$. 180 minus an even integer will always be an even integer, therefore $m\angle A$ must be even.

37. If the exterior angle is formed by extending the base, the three angle measures will be: $180 - x$, $180 - x$, and $180 - (180 - x) - (180 - x)$, or $2x - 180$. If the exterior angle is formed by extending a leg, the three angle measures will be $180 - x$, $\frac{x}{2}$, and $\frac{x}{2}$.

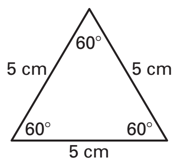
Problem Solving

38. $x = 79$

$y° = 180° - 2(79)°$

$y = 22$

39.



Each side is 5 centimeters and each angle measures 60°.

40. $180° - 2(85)° = 10°$

The vertex angle of the triangle is about 10°.

41. a. $\angle BAC \cong \angle CBD$ and $\angle BCA \cong \angle CDB$ by the Alternate Interior Angles Theorem. $\overline{BC} \cong \overline{BC}$ by the Reflexive Property. So $\triangle ABC \cong \triangle BCD$ by AAS.

b. The isosceles triangles are: $\triangle ABC$, $\triangle BCD$, $\triangle CDE$, $\triangle DEF$, $\triangle EFG$

c. Angles congruent to $\angle ABC$ are: $\angle BCD$, $\angle CDE$, $\angle DEF$, $\angle EFG$

42. a. The sides of each new triangle all contain the same number of congruent segments, so the triangles will be equilateral.

b. The areas of the first four triangles are 1, 4, 9, and 16 square units.

c. The area pattern is $1^2, 2^2, 3^2, 4^2 \ldots$ This sequence is the sequence of perfect squares. The seventh triangle will have an area of 7^2, or 49 square units.

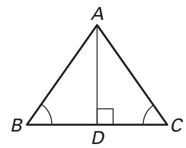
43. $\overline{PQ} \cong \overline{PR}$, so $\angle Q \cong \angle R$.

$m\angle P = 90°$, so $m\angle Q = m\angle R = \frac{180° - 90°}{2}$, or 45°.

44. If a triangle is equilateral, it is also isosceles. Using this fact, it can be shown that all angles in an equilateral triangle must be the same.

45. Given: $\angle B \cong \angle C$

Prove: $\overline{AB} \cong \overline{AC}$



Statements	Reasons
1. $\angle B \cong \angle C$	**1.** Given
2. Draw $\overline{AD}$, so $\overline{AD} \perp \overline{BC}$.	**2.** Perpendicular Postulate
3. $\angle ADC$ and $\angle ADB$ are right angles.	**3.** Definition of perpendicular lines
4. $\angle ADC \cong \angle ADB$	**4.** Right Angles Congruence Theorem
5. $\overline{AD} \cong \overline{AD}$	**5.** Reflexive Property of Congruence
6. $\triangle ADC \cong \triangle ADB$	**6.** AAS
7. $\overline{AB} \cong \overline{AC}$	**7.** Corr. parts of $\cong$ △ are $\cong$.

46. a. $\overline{AE} \cong \overline{DE}$, $\angle BAE \cong \angle CDE$, and $\overline{BA} \cong \overline{CD}$ are all given, so $\triangle ABE \cong \triangle DCE$ by SAS.

b. $\triangle AED$ and $\triangle BEC$ are isosceles triangles.

c. $\angle EDA$, $\angle EBC$, and $\angle ECB$ are all congruent to $\angle EAD$.

d. No; $\triangle AED$ and $\triangle BEC$ will still be isosceles triangles with $\angle BEC \cong \angle AED$. So the angle congruencies in part (c) will remain the same.

47. No; $m\angle 1 = 50$, so $m\angle 2 = 50°$. If $p \parallel q$, then the 45° angle would be the angle corresponding to $\angle 2$. Since $50° \neq 45°$, p is not parallel to q.

48. Yes; $m\angle ABC = 50°$ by the Vertical Angles Congruence Theorem and $m\angle CAB = 50°$ by the Linear Pair Postulate, so $\angle ABC \cong \angle CAB$. By the Converse of the Base Angles Theorem, $\overline{AC} \cong \overline{BC}$, so $\triangle ABC$ is isosceles.

49. Given: $\triangle ABC$ is equilateral, $\angle CAD \cong \angle ABE \cong \angle BCF$.

Prove: $\triangle DEF$ is equilateral.

Statements	Reasons
1. $\triangle ABC$ is equilateral, $\angle CAD \cong \angle ABE \cong \angle BCF$.	**1.** Given
2. $m\angle CAD = m\angle ABE = m\angle BCF$	**2.** Definition of congruent angles
3. $m\angle CAD + m\angle DAB = m\angle CAB$, $m\angle ABE + m\angle EBC = m\angle ABC$, $m\angle BCF + m\angle FCA = m\angle BCA$	**3.** Angle Addition Postulate
4. $m\angle CAB = m\angle ABC = m\angle BCA$	**4.** Corollary to the Base Angles Theorem
5. $m\angle CAD + m\angle DAB = m\angle ABE + m\angle EBC = m\angle BCF + m\angle FCA$	**5.** Substitution Property of Equality

Statements	Reasons
6. $m\angle CAD + m\angle DAB = m\angle CAD + m\angle EBC = m\angle CAD + m\angle FCA$	**6.** Substitution Property of Equality
7. $m\angle DAB = m\angle EBC = m\angle FCA$	**7.** Subtraction Property of Equality
8. $\angle DAB \cong \angle EBC \cong \angle FCA$	**8.** Definition of angle congruence
9. $\triangle ACF \cong \triangle CBE \cong \triangle BAD$	**9.** ASA
10. $\angle BEC \cong \angle ADB \cong \angle CFA$	**10.** Corr. parts of $\cong$ ▲s are $\cong$.
11. $\angle BEC$ and $\angle DEF$, $\angle ADB$ and $\angle EDF$, $\angle CFA$ and $\angle DFE$ are supplementary.	**11.** Linear Pair Postulate
12. $\angle DEF \cong \angle EDF \cong \angle DFE$	**12.** Congruent Supplements Theorem
13. $\triangle DEF$ is equiangular.	**13.** Definition of equiangular triangle
14. $\triangle DEF$ is equilateral.	**14.** Corollary to the Converse of Base Angles Theorem

50. If V is at (2, 2), then the points T, U, and V will form a line, not a triangle. If V is anywhere else on the line $y = x$, $\triangle TUV$ will be formed and $\overline{TV} \cong \overline{UV}$ because T and U will always be the same distance from point V. So $\triangle TUV$ will be isosceles.

51. 3 possibilities:

$5t - 12 = 3t$	$5t - 12 = t + 20$	$3t = t + 20$
$2t = 12$	$4t = 32$	$2t = 20$
$t = 6$	$t = 8$	$t = 10$

If $t = 6$, 8, or 10, then the triangle will be isosceles.

Lesson 4.9 Perform Congruence Transformations

Investigating Geometry Activity for the lesson "Perform Congruence Transformations"

1. In a slide, the x-coordinates are changed by the amount the triangle was shifted up or down. The y-coordinates are changed by the amount the triangle was shifted left or right.
In a flip, only one coordinate of the triangle's vertices changes. The x-coordinate changes sign if the triangle is flipped over the y-axis or the y-coordinate changes sign if the triangle is flipped over the x-axis.

2. Yes; yes; When sliding or flipping a triangle, the size and shape do not change, only the position changes. So the original triangle is congruent to the new triangle.

Guided Practice for the lesson "Perform Congruence Transformations"

1. The transformation shown is a reflection.

2. The new coordinates are found by adding 1 to each x-coordinate and subtracting 1 from each y-coordinate.
$(x, y) \to (x + 1, y - 1)$

3. The y-coordinates are multiplied by -1, so $\overline{RS}$ was reflected in the x-axis. $(x, y) \to (x, -y)$

4. $\triangle PQR$ is a 180° rotation of $\triangle STR$.

5. $PQ = ST = 2$, so $\overline{PQ} \cong \overline{ST}$. $PR = SR = 3$, so $\overline{PR} \cong \overline{SR}$. $\triangle PQR$ and $\triangle STR$ are right triangles, so $\triangle PQR \cong \triangle STR$ by HL. Therefore the transformation is a congruence transformation.

Exercises for the lesson "Perform Congruence Transformations"

Skill Practice

1. The new coordinates are formed by subtracting 1 from each x-coordinate and adding 4 to each y-coordinate.

2. The term congruence transformation is used because when an object is translated, reflected, or rotated, the new image is congruent to the original figure.

3. The transformation is a translation.

4. The transformation is a rotation.

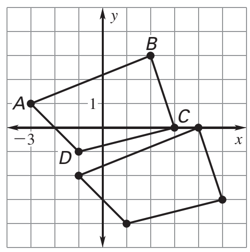
5. The transformation is a reflection.

6. Yes; The moving part of the window is a translation.

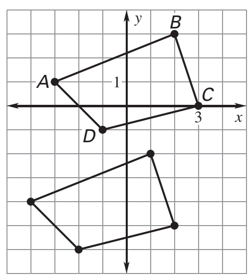
7. No; The moving part of the window is not a translation.

8. Yes; The moving part of the window is a translation.

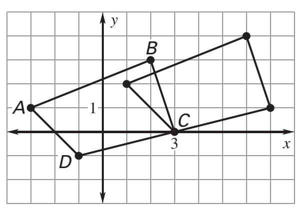
9. $(x, y) \to (x + 2, y - 3)$
$A(-3, 1) \to (-1, -2)$
$B(2, 3) \to (4, 0)$
$C(3, 0) \to (5, -3)$
$D(-1, -1) \to (1, -4)$



10. $(x, y) \to (x - 1, y - 5)$
$A(-3, 1) \to (-4, -4)$
$B(2, 3) \to (1, -2)$
$C(3, 0) \to (2, -5)$
$D(-1, -1) \to (-2, -6)$



11. $(x, y) \to (x + 4, y + 1)$
$A(-3, 1) \to (1, 2)$
$B(2, 3) \to (6, 4)$
$C(3, 0) \to (7, 1)$
$D(-1, -1) \to (3, 0)$



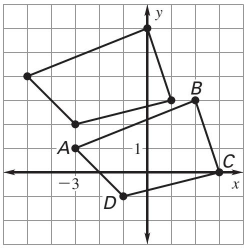
12. $(x, y) \to (x - 2, y + 3)$

$A(-3, 1) \to (-5, 4)$

$B(2, 3) \to (0, 6)$

$C(3, 0) \to (1, 3)$

$D(-1, -1) \to (-3, 2)$



13. $(x, y) \to (x - 4, y - 2)$ **14.** $(x, y) \to (x + 6, y + 3)$

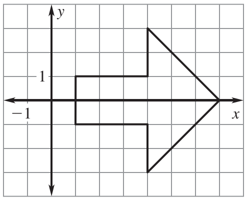
15. $(x, y) \to (x + 2, y - 1)$ **16.** $(x, y) \to (x - 7, y + 9)$

17. $(x, y) \to (x, -y)$

$(1, 1) \to (1, -1)$

$(4, 1) \to (4, -1)$

$(4, 3) \to (4, -3)$



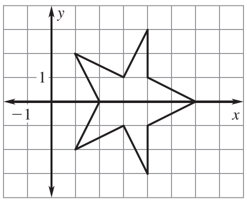
18. $(x, y) \to (x, -y)$

$(1, 2) \to (1, -2)$

$(3, 1) \to (3, -1)$

$(4, 3) \to (4, -3)$

$(4, 1) \to (4, -1)$

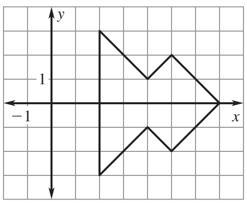


19. $(x, y) \to (x, -y)$

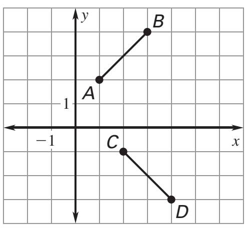
$(2, 3) \to (2, -3)$

$(4, 1) \to (4, -1)$

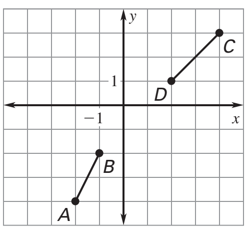
$(5, 2) \to (5, -2)$



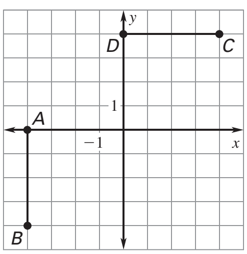
20. $\overline{CD}$ is a 90° clockwise rotation of $\overline{AB}$.



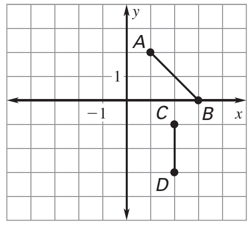
21. $\overline{CD}$ is not a rotation of $\overline{AB}$ because $m\angle AOC > m\angle BOD$.



22. $\overline{CD}$ is not a rotation of $\overline{AB}$ because the points are rotated in different directions.

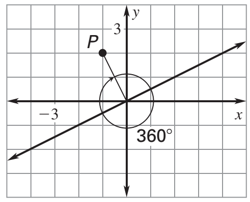


23. $\overline{CD}$ is not a rotation of $\overline{AB}$ because $m\angle AOC > m\angle BOD$.



24. To find the angle of rotation, corresponding angles of the triangles should be used to find the rotation angle. The red triangle is rotated 90° clockwise.

25. Yes; Any point or line segment can be rotated 360° and be its own image.



26. If (0, 3) translates to (0, 0), then (2, 5) translates to (2, 2).

27. If (0, 3) translates to (1, 2), then (2, 5) translates to (3, 4).

28. If (0, 3) translates to $(-3, -2)$, then (2, 5) translates to $(-1, 0)$.

29. $(x, y) \to (x + 2, y - 3)$

$(x, y) \to (4, 0)$

$x + 2 = 4 \qquad y - 3 = 0$

$x = 2 \qquad y = 3$

The point on the original figure is (2, 3).

30. $(x, y) \to (-x, y)$

$(x, y) \to (-3, 5)$

$-x = -3 \qquad y = 5$

$x = 3$

The point on the original figure is (3, 5).

31. $(x, y) \to (x - 7, y - 4)$

$(x, y) \to (6, -9)$

$x - 7 = 6 \qquad y - 4 = -9$

$x = 13 \qquad y = -5$

The point on the original figure is $(13, -5)$.

32. Length of sides of upper triangle:

$\sqrt{(2 - 1)^2 + (3 - 2)^2} = \sqrt{2}$

$\sqrt{(5 - 2)^2 + (1 - 3)^2} = \sqrt{13}$

$\sqrt{(5 - 1)^2 + (1 - 2)^2} = \sqrt{17}$

Length of sides of lower triangle:

$\sqrt{(0 - (-1))^2 + (1 - 0)^2} = \sqrt{2}$

$\sqrt{(3 - 0)^2 + (-1 - 1)^2} = \sqrt{13}$

$\sqrt{(3 - (-1))^2 + (-1 - 0)^2} = \sqrt{17}$

Both triangles have congruent side lengths, so the triangles are congruent by SSS.

33. $\overline{UV}$ is a 90° clockwise rotation of $\overline{ST}$ about E.

34. $\overline{AV}$ is a 90° counterclockwise rotation of $\overline{BX}$ about E.

35. $\triangle DST$ is a 180° rotation of $\triangle BWX$ about E.

36. $\triangle XYC$ is a 180° rotation of $\triangle TUA$ about E.

37. $(x, y) \rightarrow (x - 2, y + 1)$

$A(2, 3) \rightarrow (0, 4) = C(m - 3, 4)$

$B(4, 2a) \rightarrow (2, 2a + 1) = D(n - 9, 5)$

$m - 3 = 0 \qquad n - 9 = 2 \qquad 2a + 1 = 5$

$m = 3 \qquad n = 11 \qquad 2a = 4$

$a = 2$

$(x, y) \rightarrow (x, -y)$

$C(0, 4) \rightarrow (0, -4) = E(0, g - 6)$

$D(2, 5) \rightarrow (2, -5) = F(8h, -5)$

$g - 6 = -4 \qquad 8h = 2$

$g = 2 \qquad h = \frac{1}{4}$

So the variables are $m = 3$, $n = 11$, $a = 2$, $g = 2$, $h = \frac{1}{4}$.

Problem Solving

38. **a.** The designer can reflect the kite layout in the horizontal line.

b. The width of the top half of the kite is 2 feet, so the maximum width of the entire kite is 4 feet.

39. Starting at A, you will move the stencil 90° clockwise to get the design at B. To go from A to C, you move the stencil 90° counterclockwise.

40. **a.** *Sample answer:*

b. *Sample answer:*

41. **a.** The Black Knight moves up 2 spaces and to the left 1 space, so the translation is $(x, y) \rightarrow (x - 1, y + 2)$.

b. The White Knight moves down 1 space and to the right 2 spaces, so the translation is $(x, y) \rightarrow (x + 2, y - 1)$.

c. No; the White Knight is directly below the Black Knight, so the Black Knight would miss the White Knight because it would have to move at least one square horizontally according to the rules.

42. slope $\overline{FE} = \frac{2 - 3}{-1 - (-2)} = \frac{-1}{1} = -1$

slope $\overline{DE} = \frac{0 - 2}{-3 - (-1)} = \frac{-2}{-2} = 1$

So $\overline{FE} \perp \overline{DE}$ and $\angle FED$ is a right angle.

slope $CB = \frac{1 - 2}{4 - 3} = \frac{-1}{1} = -1$

slope $AB = \frac{-1 - 1}{2 - 4} = \frac{-2}{-2} = 1$

So $\overline{CB} \perp \overline{AB}$ and $\angle CBA$ is a right angle.

Length of hypotenuses:

$FD = \sqrt{(-2 - (-3))^2 + (3 - 0)^2} = \sqrt{10}$

$CA = \sqrt{(3 - 2)^2 + (2 - (-1))^2} = \sqrt{10}$

So $\overline{FD} \cong \overline{CA}$.

Length of corresponding legs:

$FE = \sqrt{(-1 - (-2))^2 + (2 - 3)^2} = \sqrt{2}$

$CB = \sqrt{(4 - 3)^2 + (1 - 2)^2} = \sqrt{2}$

So $\overline{FE} \cong \overline{CB}$.

By the HL Congruence Theorem, $\triangle ABC \cong \triangle DEF$, so $\triangle DEF$ is a congruence transformation of $\triangle ABC$.

43. B; Figure B represents the reflection of the folded paper over the folded line, or the unfolded paper.

44. Undo translation and rotation:

$(x, y) \rightarrow (x, y - 3) \rightarrow$ rotate 90° clockwise

$A(-4, 4) \rightarrow (-4, 1) \rightarrow (1, 4)$

$B(-1, 6) \rightarrow (-1, 3) \rightarrow (3, 1)$

$C(-1, 4) \rightarrow (-1, 1) \rightarrow (1, 1)$

The vertices of the original triangle are (1, 4), (3, 1), and (1, 1). The final image would be different if the original triangle was translated up 3 units and then rotated counterclockwise 90°. The final triangle would have vertices (−7, 1), (−4, 3), and (−4, 1).

Quiz for the lessons "Use Isosceles and Equilateral Triangles" and "Perform Congruence Transformations"

1. $6x + 12 = 24$

$6x = 12$

$x = 2$

2. $(3x + 48)° = 60°$

$3x = 12$

$x = 4$

3. $4x + 30 = 50$

$4x = 20$

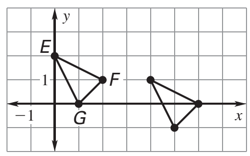
$x = 5$

4. $(x, y) \rightarrow (x + 4, y - 1)$

$E(0, 2) \rightarrow (4, 1)$

$F(2, 1) \rightarrow (6, 0)$

$G(1, 0) \rightarrow (5, -1)$



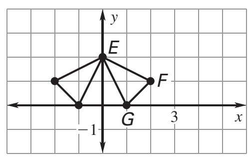
The transformation is a translation.

5. $(x, y) \rightarrow (-x, y)$

$E(0, 2) \rightarrow (0, 2)$

$F(2, 1) \rightarrow (-2, 1)$

$G(1, 0) \rightarrow (-1, 0)$



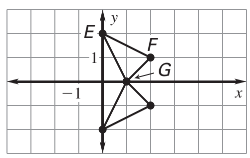
The transformation is a reflection in the y-axis.

6. $(x, y) \rightarrow (x, -y)$

$E(0, 2) \rightarrow (0, -2)$

$F(2, 1) \rightarrow (2, -1)$

$G(1, 0) \rightarrow (1, 0)$



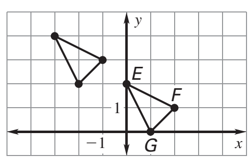
The transformation is a reflection in the x-axis.

7. $(x, y) \rightarrow (x - 3, y + 2)$

$E(0, 2) \rightarrow (-3, 4)$

$F(2, 1) \rightarrow (-1, 3)$

$G(1, 0) \rightarrow (-2, 2)$



The transformation is a translation.

8. No, Figure B is not a rotation of Figure A about the origin because not all the angles formed by connecting corresponding vertices are the same.

Mixed Review of Problem Solving for the lessons "Prove Triangles Congruent by ASA and AAS", "Use Congruent Triangles", "Use Isosceles and Equilateral Triangles", and "Perform Congruence Transformations"

1. a. Figure B is a reflection of Figure A in the *y*-axis.

b. Figure C is a $90°$ counterclockwise rotation of Figure A.

c. Figure D is a reflection of Figure A in the *x*-axis.

d. The pattern will be completed by rotating Figure A $90°$ clockwise and $180°$.

2. No; because the given angle is not included between the given sides, more than one triangle could have these dimensions.

3. You friend could tell you either of the remaining two angles or the length of the other side that forms the $34°$ angle.

4. Yes; yes; $\angle ACD \cong \angle BCE$ by the Vertical Angles Congruence Theorem. $\overline{AC} \cong \overline{BC}$ and $\overline{DC} \cong \overline{EC}$ are given, so $\triangle ACD \cong \triangle BCE$ by SAS. Because corresponding parts of congruent triangles are congruent, $\overline{AD} \cong \overline{BE}$.

5. a. In $\triangle ABC$ it is given that $\overline{BA} \cong \overline{BC}$, so $\angle BCE \cong \angle BAE$ by the Base Angles Theorem.

b. Given: $\overline{AB} \cong \overline{CB}$, $\overline{BE} \perp \overline{AC}$, $\angle AEF \cong \angle CED$, $\angle EAF \cong \angle ECD$

Prove: $\overline{AF} \cong \overline{CD}$

Statements	Reasons
1. $\overline{AB} \cong \overline{CB}$, $\overline{BE} \perp \overline{AC}$, $\angle AEF \cong \angle CED$, $\angle EAF \cong \angle ECD$	**1.** Given
2. $\angle AEB$ and $\angle CEB$ are right angles.	**2.** Definition of perpendicular lines
3. $\angle AEB \cong \angle CEB$	**3.** Right Angles Congruence Theorem
4. $\angle BCE \cong \angle BAE$	**4.** Proven in Ex. 5(a).
5. $\triangle AEB \cong \triangle CEB$	**5.** AAS
6. $\overline{AE} \cong \overline{CE}$	**6.** Corr. parts of $\cong$ $\triangle$s are $\cong$.
7. $\triangle AEF \cong \triangle CED$	**7.** ASA
8. $\overline{AF} \cong \overline{CD}$	**8.** Corr. parts of $\cong$ $\triangle$s are $\cong$.

6. The triangles are congruent by AAS.

$(4x + 17)$ in. $= 45$ in.

$4x = 28$

$x = 7$

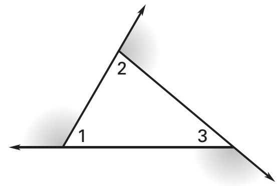
Chapter Review for the chapter "Congruent Triangles"

1. A triangle with three congruent angles is called equiangular.

2. In an isosceles triangle, base angles are opposite the congruent sides while the congruent sides form the vertex angle.

3. An isosceles triangle has at least two congruent sides while a scalene triangle has no congruent sides.

4. *Sample answer:*



5. Corresponding angles: $\angle P$ and $\angle L$, $\angle Q$ and $\angle M$, $\angle R$ and $\angle N$

Corresponding sides: $\overline{PQ}$ and $\overline{LM}$, $\overline{PR}$ and $\overline{LN}$, $\overline{QR}$ and $\overline{MN}$

6. $(2x - 25)° = x° + 20°$

$x = 45$

$(2(45) - 25)° = 65°$

7. $8x° = 2x° + 90°$

$6x = 90$

$x = 15$

$8(15)° = 120°$

8. $(9x + 9)° = 5x° + 45°$

$4x = 36$

$x = 9$

$(9(9) + 9)° = 90°$

9. $m\angle B = 180° - 50° - 70° = 60°$

10. $\overline{AB} \cong \overline{UT}$, so $AB = 15$ m.

11. $\angle T \cong \angle B$, so $m\angle T = 60°$.

12. $\angle V \cong \angle A$, so $m\angle V = 50°$.

13. $(2x + 4)° = 180° - 120° - 20°$

$2x + 4 = 40$

$2x = 36$

$x = 18$

14. $5x° = 180° - 35° - 90°$

$5x = 55$

$x = 11$

15. The figure has been slid, so the transformation is a translation up and right.

16. The figure has been turned, so the transformation is a rotation.

17. The figure has been flipped, so the transformation is a reflection.

18. Check students' drawings.

19. True; $\overline{XY} \cong \overline{RS}$, $\overline{YZ} \cong \overline{ST}$, and $\overline{XZ} \cong \overline{RT}$, so $\triangle XYZ \cong \triangle RST$ by SSS.

20. Not true; $AC = 5$ and $DB = 4$, so $\overline{AC} \ncong \overline{DB}$. Therefore $\triangle ABC \ncong \triangle DCB$.

21. True; $\angle QSR \cong \angle TSU$ by the Vertical Angles Theorem. Because $\overline{QS} \cong \overline{TS}$ and $\overline{RS} \cong \overline{US}$, $\triangle QRS \cong \triangle TUS$ by SAS.

22. Not true; The triangle vertices are in the incorrect order. $\overline{DE} \cong \overline{HG}$ and $\overline{EF} \cong \overline{GF}$, so $\triangle DEF \cong \triangle HGF$ by HL.

23. $\overline{DE} \cong \overline{GH}$, $\angle D \cong \angle G$, $\angle F \cong \angle J$

24. $\overline{DF} \cong \overline{GJ}$, $\angle F \cong \angle J$, $\angle D \cong \angle G$

25. Show $\triangle ACD$ and $\triangle BED$ are congruent by AAS, which makes $\overline{AD}$ congruent to $\overline{BD}$. $\triangle ABD$ is then an isosceles triangle, which makes $\angle 1$ and $\angle 2$ congruent.

26. Show $\triangle FKH \cong \triangle FGH$ by HL. So $\angle 1 \cong \angle 2$ because corresponding parts of congruent triangles are congruent.

27. Show $\triangle QSV \cong \triangle QTV$ by SSS. So $\angle QSV \cong \angle QTV$ because corresponding parts of congruent triangles are congruent. Using vertical angles and the Transitive Property, you get $\angle 1 \cong \angle 2$.

28. $\angle L \cong \angle N$, so $x = 65$.

29. $\triangle WXY$ is equilateral;

$\left(\frac{3}{2}x + 30\right)^\circ = 60^\circ$

$\frac{3}{2}x = 30$

$x = 20$

30. $\overline{TU} \cong \overline{VU}$;

$7x + 5 = 13 - x$

$8x = 8$

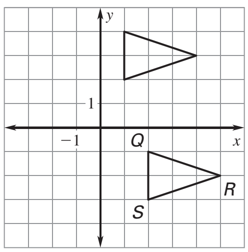
$x = 1$

31. $(x, y) \rightarrow (x - 1, y + 5)$

$Q(2, -1) \rightarrow (1, 4)$

$R(5, -2) \rightarrow (4, 3)$

$S(2, -3) \rightarrow (1, 2)$

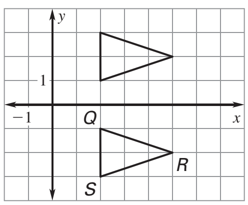


32. $(x, y) \rightarrow (x, -y)$

$Q(2, -1) \rightarrow (2, 1)$

$R(5, -2) \rightarrow (5, 2)$

$S(2, -3) \rightarrow (2, 3)$

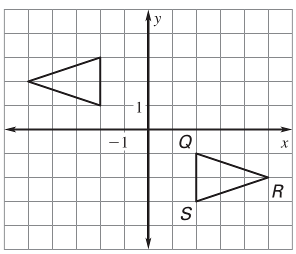


33. $(x, y) \rightarrow (-x, -y)$

$Q(2, -1) \rightarrow (-2, 1)$

$R(5, -2) \rightarrow (-5, 2)$

$S(2, -3) \rightarrow (-2, 3)$



Chapter Test for the chapter "Congruent Triangles"

1. The triangle is equilateral and acute (or equiangular).

2. The triangle is scalene and right.

3. The triangle is isosceles and obtuse.

4. $x^\circ = 180^\circ - 30^\circ - 80^\circ$

$x = 70$

5. $2x^\circ + x^\circ = 180^\circ - 90^\circ$

$3x = 90$

$x = 30$

6. $x^\circ = 180^\circ - 55^\circ - 50^\circ$

$x = 75$

7. $3x - 5 = 10$ $\quad$ $(15x + y)^\circ = 90^\circ$

$3x = 15$ $\quad$ $15(5) + y = 90$

$x = 5$ $\quad$ $y = 15$

8. *Sample answer:* Rotation about point M and then a translation right and up.

9. $\triangle ABC \cong \triangle EDC$ can be proven by SAS because $\overline{AC} \cong \overline{EC}$, $\overline{BC} \cong \overline{DC}$, and $\angle ACB \cong \angle ECD$ by the Vertical Angles Congruence Theorem.

10. $\triangle FGH \cong \triangle JKL$ can be proven by ASA because both triangles are equilateral, so all angles are congruent by the Corollary to the Base Angles Theorem.

11. $\triangle MNP \cong \triangle PQM$ can be proven by SSS because $\overline{MN} \cong \overline{PQ}$, $\overline{NP} \cong \overline{QM}$, and $\overline{MP} \cong \overline{PM}$ by the Reflexive Property.

12. Given: $\triangle ABC$ is isosceles. $\overline{BD}$ bisects $\angle B$.

Prove: $\triangle ABD \cong \triangle CBD$

Statements	Reasons
1. $\triangle ABC$ is isosceles with base $\overline{AC}$.	1. Given
2. $\angle BAD \cong \angle BCD$	2. Base Angles Theorem
3. $\overline{BD} \cong \overline{BD}$	3. Reflexive Property of Congruence
4. $\overline{BD}$ bisects $\angle B$.	4. Given
5. $\angle ABD \cong \angle CBD$	5. Definition of angle bisector
6. $\triangle ABD \cong \triangle CBD$	6. AAS Congruence Theorem

13. a. $\triangle PQR \cong \triangle STU$ by HL if $\overline{PQ} \cong \overline{ST}$ or if $\overline{QR} \cong \overline{TU}$.

b. $\triangle PQR \cong \triangle STU$ by AAS if $\angle P \cong \angle S$ or if $\angle R \cong \angle U$.

14. The figure transformation is a reflection.

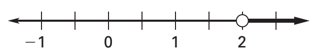
15. The triangle transformation is a reflection.

16. The triangle transformation is a translation.

Algebra Review for the chapter "Congruent Triangles"

1. $x - 6 > -4$

$x > 2$



2. $7 - c \leq -1$

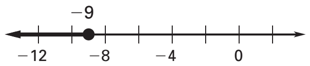
$-c \leq -8$

$c \geq 8$

3. $-54 \geq 6x$

$-9 \geq x$

$x \leq -9$



4. $\frac{5}{2}t + 8 \leq 33$

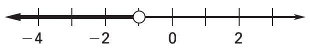
$\frac{5}{2}t \leq 25$

$t \leq 10$

5. $3(y + 2) < 3$

$y + 2 < 1$

$y < -1$



6. $\frac{1}{4}z < 2$

$z < 8$

7. $5k + 1 \geq -11$

$5k \geq -12$

$k \geq -\frac{12}{5}$

8. $13.6 > -0.8 - 7.2r$

$14.4 > -7.2r$

$-2 < r$

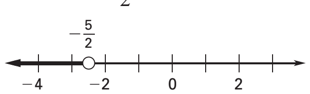
$r > -2$

9. $6x + 7 < 2x - 3$

$4x < -10$

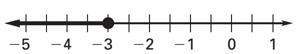
$x < -\frac{10}{4}$

$x < -\frac{5}{2}$



10. $-v + 12 \leq 9 - 2v$

$v \leq -3$

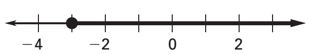


11. $4(n + 5) \geq 5 - n$

$4n + 20 \geq 5 - n$

$5n \geq -15$

$n \geq -3$



12. $5y + 3 \geq 2(y - 9)$

$5y + 3 \geq 2y - 18$

$3y \geq -21$

$y \geq -7$

13. $|x - 5| = 3$

$x - 5 = 3$ or $x - 5 = -3$

$x = 8$ $\quad$ $x = 2$

The solutions are 2 and 8.

14. $|x + 6| = 2$

$x + 6 = 2$ or $x + 6 = -2$

$x = -4$ $\quad$ $x = -8$

The solutions are -8 and -4.

15. $|4 - x| = 4$

$4 - x = 4$ or $4 - x = -4$

$0 = x$ $\quad$ $8 = x$

The solutions are 0 and 8.

16. $|2 - x| = 0.5$

$2 - x = 0.5$ or $2 - x = -0.5$

$1.5 = x$ $\quad$ $2.5 = x$

The solutions are 1.5 and 2.5.

17. $|3x - 1| = 8$

$3x - 1 = 8$ or $3x - 1 = -8$

$3x = 9$ $\quad$ $3x = -7$

$x = 3$ $\quad$ $x = -\frac{7}{3}$

The solutions are $-\frac{7}{3}$ and 3.

18. $|4x + 5| = 7$

$4x + 5 = 7$ or $4x + 5 = -7$

$4x = 2$ $\quad$ $4x = -12$

$x = \frac{1}{2}$ $\quad$ $x = -3$

The solutions are -3 and $\frac{1}{2}$.

19. $|x - 1.3| = 2.1$

$x - 1.3 = 2.1$ or $x - 1.3 = -2.1$

$x = 3.4$ $\quad$ $x = -0.8$

The solutions are -0.8 and 3.4.

20. $|3x - 15| = 0$

$3x - 15 = 0$

$3x = 15$

$x = 5$

The solution is 5.

21. $|6x - 2| = 4$

$6x - 2 = 4$ or $6x - 2 = -4$

$6x = 6$ $\quad$ $6x = -2$

$x = 1$ $\quad$ $x = -\frac{1}{3}$

The solutions are $-\frac{1}{3}$ and 1.

22. $|8x + 1| = 17$

$8x + 1 = 17$ or $8x + 1 = -17$

$8x = 16$ $\quad$ $8x = -18$

$x = 2$ $\quad$ $x = -\frac{9}{4}$

The solutions are $-\frac{9}{4}$ and 2.

23. $|9 - 2x| = 19$

$9 - 2x = 19$ or $9 - 2x = -19$

$-2x = 10$ $\quad$ $-2x = -28$

$x = -5$ $\quad$ $x = 14$

The solutions are -5 and 14.

24. $|0.5x - 4| = 2$

$0.5x - 4 = 2$ or $0.5x - 4 = -2$

$0.5x = 6$ $\quad$ $0.5x = 2$

$x = 12$ $\quad$ $x = 4$

The solutions are 4 and 12.

25. $|5x - 2| = 8$

$5x - 2 = 8$ or $5x - 2 = -8$

$5x = 10$ $\quad$ $5x = -6$

$x = 2$ $\quad$ $x = -\frac{6}{5}$

The solutions are $-\frac{6}{5}$ and 2.

26. $|7x + 4| = 11$

$7x + 4 = 11$ or $7x + 4 = -11$

$7x = 7$ $\quad$ $7x = -15$

$x = 1$ $\quad$ $x = -\frac{15}{7}$

The solutions are $-\frac{15}{7}$ and 1.

27. $|3x - 11| = 4$

$3x - 11 = 4$ or $3x - 11 = -4$

$3x = 15$ $\quad$ $3x = 7$

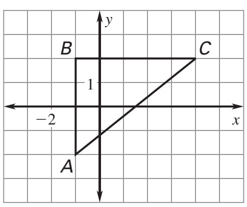
$x = 5$ $\quad$ $x = \frac{7}{3}$

The solutions are $\frac{7}{3}$ and 5.

Extra Practice

For the chapter "Congruent Triangles"

1.



$AB = \sqrt{(-1 - (-1))^2 + (2 - (-2))^2}$

$= \sqrt{0 + 16} = \sqrt{16} = 4$

$AC = \sqrt{(4 - (-1))^2 + (2 - (-2))^2} = \sqrt{25 + 16} = \sqrt{41}$

$BC = \sqrt{(4 - (-1))^2 + (2 - 2)^2} = \sqrt{25 + 0} = \sqrt{25} = 5$

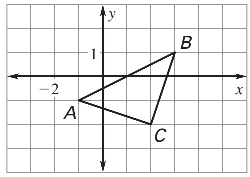
Because no sides are congruent, the triangle is scalene.

Slope of $\overline{AB}$: $m = \frac{2 - (-2)}{-1 - (-1)} = \frac{4}{0}$ undefined

Slope of $\overline{BC}$: $m = \frac{2 - 2}{4 - (-1)} = \frac{0}{5} = 0$

Because $\overline{AB}$ is vertical and $\overline{BC}$ is horizontal, $\overline{AB} \perp \overline{BC}$. So, the triangle is a right triangle.

2.



$AB = \sqrt{(3 - (-1))^2 + (1 - (-1))^2} = \sqrt{16 + 4} = \sqrt{20}$

$AC = \sqrt{(2 - (-1))^2 + (-2 - (-1))^2} = \sqrt{9 + 1} = \sqrt{10}$

$BC = \sqrt{(2 - 3)^2 + (-2 - 1)^2} = \sqrt{1 + 9} = \sqrt{10}$

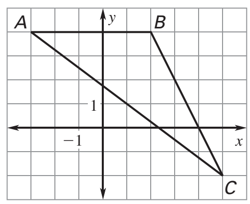
Because two sides are congruent, the triangle is isosceles.

Slope of $\overline{AC}$: $m = \frac{-2 - (-1)}{2 - (-1)} = \frac{-1}{3} = -\frac{1}{3}$

Slope of $\overline{BC}$: $m = \frac{-2 - 1}{2 - 3} = \frac{-3}{-1} = 3$

Because $-\frac{1}{3} \cdot 3 = -1$, $\overline{AC} \perp \overline{BC}$. So, the triangle is a right triangle.

3.



$AB = \sqrt{(2 - (-3))^2 + (4 - 4)^2} = \sqrt{25 + 0} = \sqrt{25} = 5$

$AC = \sqrt{(5 - (-3))^2 + (-2 - 4)^2}$

$= \sqrt{64 + 36} = \sqrt{100} = 10$

$BC = \sqrt{(5 - 2)^2 + (-2 - 4)^2} = \sqrt{9 + 36} = \sqrt{45}$

Because no sides are congruent, the triangle is scalene.

Slope of $\overline{AB}$: $m = \frac{4 - 4}{2 - (-3)} = \frac{0}{5} = 0$

Slope of $\overline{AC}$: $m = \frac{-2 - 4}{5 - (-3)} = \frac{-6}{8} = -\frac{3}{4}$

Slope of $\overline{BC}$: $m = \frac{-2 - 4}{5 - 2} = \frac{-6}{3} = -2$

Because there are not any negative reciprocals, there are no perpendicular lines. So, the triangle is not a right triangle.

4. $x° + 3x° + 56° = 180°$

$4x + 56 = 180$

$4x = 124$

$x = 31$

The angles of the triangle are 31°, $(3 \times 31)° = 93°$, and 56°. So, the triangle is an obtuse triangle.

5. $x° + (x + 1)° + (x + 5)° = 180°$

$$3x + 6 = 180$$
$$3x = 174$$
$$x = 58$$

The angles of the triangle are 58°, $(58 + 1)° = 59°$, and $(58 + 5)° = 63°$. So, the triangle is an acute triangle.

6. Because the angles form a linear pair, $60° + x° = 180°$. So, $x = 120$. The angles of the triangle are 90°, 60°, and $(180 - 90 - 60)° = 30°$. So, the triangle is a right triangle.

7. $\triangle DFG \cong \triangle FDE$; it is given that $\angle DGF \cong \angle FED$ and $\angle GFD \cong \angle EDF$; $\angle FDG \cong \angle DFE$ by the Third Angles Theorem; it is given that $\overline{DG} \cong \overline{FE}$ and $\overline{GF} \cong \overline{ED}$; $\overline{FD} \cong \overline{FD}$ by the Reflexive Property of Congruence; $\triangle DFG \cong \triangle FDE$ by the definition of congruence.

8. $\triangle JNM \cong \triangle KML$; it is given that all pairs of corresponding sides are congruent and that $\angle J \cong \angle K$; $\angle N \cong \angle KML$ by the Corresponding Angles Postulate; $\angle JMN \cong \angle L$ by the Third Angles Theorem; $\triangle JNM \cong \triangle KML$ by the definition of congruence.

9. $STWX \cong UTWV$; all pairs of corresponding angles and sides are congruent.

10. $5x° + 36° + 49° = 180°$

$$5x + 85 = 180$$
$$5x = 95$$
$$x = 19$$

11. $(7x - 5)° = 44°$

$$7x = 49$$
$$x = 7$$

12. No; the labels are not in the appropriate order to match the sides that are congruent. A true congruence statement would be $\triangle PQR \cong \triangle TVU$.

13. No; the labels are not in the appropriate order to match the sides that are congruent. A true congruence statement would be $\triangle JKM \cong \triangle LKM$.

14. No; a true congruence statement would be $\triangle PQR \cong \triangle TVU$.

15. No; a true congruence statement would be $\triangle JKM \cong \triangle LKM$.

16. Yes; use the Sedment Addition Postulate to get $\overline{AC} \cong \overline{BD}$. Also, $\overline{CD} \cong \overline{CD}$, so use the SSS Congruence Postulate.

17. $\triangle XUV \cong \triangle VWX$; because $\overline{XV} \cong \overline{XV}$, the triangles are congruent by the HL Congruence Theorem.

18. $\triangle NRM \cong \triangle PRQ$; because $\angle NRM \cong \angle PRQ$ by the Vertical Angles Congruence Theorem, $\triangle NRM \cong \triangle PRQ$ by the SAS Congruence Postulate.

19. $\triangle HJL \cong \triangle KLJ$; $\angle HJL \cong \angle JLK$ by the Alternate Interior Angles Theorem. Because $\overline{JL} \cong \overline{LJ}$, $\triangle HJL \cong \triangle KLJ$ by the SAS Congruence Postulate.

20. Yes; because $\angle HLG \cong \angle KLJ$ by the Vertical Angles Congruence Theorem, $\triangle GHL \cong \triangle JKL$ by the ASA Congruence Postulate.

21. Yes; because $\overline{QN} \cong \overline{QN}$, $\triangle MNQ \cong \triangle PNQ$ by the AAS Congruence Theorem.

22. No; you can only show that all 3 angles are congruent.

23. Yes; $\triangle ABC \cong \triangle DEF$ by the ASA Congruence Postulate.

24. No; there is no SSA Congruence Postulate.

25. State the given information from the diagram, and state that $\overline{AC} \cong \overline{AC}$ by the Reflexive Property of Congruence. Then use the SAS Congruence Postulate to prove $\triangle ABC \cong \triangle CDA$, and state $\angle 1 \cong \angle 2$ because corresponding parts of congruent triangles are congruent.

26. State the given information from the diagram. Prove $\triangle DEF \cong \triangle GHJ$ by the HL Congruence Theorem, and state $\angle 1 \cong \angle 2$ because corresponding parts of congruent triangles are congruent.

27. State the given information from the diagram, and state that $\overline{SR} \cong \overline{SR}$ by the Reflexive Property of Congruence. Then use the Segment Addition Postulate to show that $\overline{PR} \cong \overline{US}$. Use the SAS Congruence Postulate to prove $\triangle QPR \cong \triangle TUS$, and state $\angle 1 \cong \angle 2$ because corresponding parts of congruent triangles are congruent.

28. $AB = \sqrt{(6 - 0)^2 + (0 - 8)^2} = \sqrt{36 + 64} = \sqrt{100} = 10$

$AC = \sqrt{(0 - 0)^2 + (0 - 8)^2} = \sqrt{0 + 64} = \sqrt{64} = 8$

$BC = \sqrt{(0 - 6)^2 + (0 - 0)^2} = \sqrt{36 + 0} = \sqrt{36} = 6$

$DE = \sqrt{(9 - 3)^2 + (2 - 10)^2}$

$= \sqrt{36 + 64} = \sqrt{100} = 10$

$DF = \sqrt{(3 - 3)^2 + (2 - 10)^2} = \sqrt{0 + 64} = \sqrt{64} = 8$

$EF = \sqrt{(3 - 9)^2 + (2 - 2)^2} = \sqrt{36 + 0} = \sqrt{36} = 6$

Because all 3 pairs of corresponding sides are congruent, $\triangle ABC \cong \triangle DEF$ by the SSS Congruence Postulate. Then, $\angle A \cong \angle D$ because corresponding parts of congruent triangles are congruent.

29. $AB = \sqrt{(-2 - (-3))^2 + (3 - (-2))^2}$

$= \sqrt{1 + 25} = \sqrt{26}$

$AC = \sqrt{(2 - (-3))^2 + (2 - (-2))^2}$

$= \sqrt{25 + 16} = \sqrt{41}$

$BC = \sqrt{(2 - (-2))^2 + (2 - 3)^2} = \sqrt{16 + 1} = \sqrt{17}$

$DE = \sqrt{(6 - 5)^2 + (6 - 1)^2} = \sqrt{1 + 25} = \sqrt{26}$

$DF = \sqrt{(10 - 5)^2 + (5 - 1)^2} = \sqrt{25 + 16} = \sqrt{41}$

$EF = \sqrt{(10 - 6)^2 + (5 - 6)^2} = \sqrt{16 + 1} = \sqrt{17}$

Because all 3 pairs of corresponding sides are congruent, $\triangle ABC \cong \triangle DEF$ by the SSS Congruence Postulate. Then, $\angle A \cong \angle D$ because corresponding parts of congruent triangles are congruent.

30. $x° + y° + 132° = 180°$

$$x° + x° + 132° = 180°$$
$$2x° = 48°$$
$$x = 24$$
$$y = x = 24$$

31. $9x + 12 = 12x - 6$

$-3x = -18$

$x = 6$

$(9x - 12)° + (12x - 6)° + y° = 180°$

$9x + 12 + 12x - 6 + y = 180$

$21x + 6 + y = 180$

$y = 174 - 21x$

$y = 174 - 21(6) = 48$

32. $2x - 3 = 11$ $\quad$ $y + 4 = 11$

$2x = 14$ $\quad$ $y = 7$

$x = 7$

33. $6x - 5 = x + 5$

$5x = 10$

$x = 2$

34. $2(x + 1)° = 62°$ $\quad$ $2(x + 1)° + 62° + y° = 180°$

$2x + 2 = 62$ $\quad$ $2x + 2 + 62 + y = 180$

$2x = 60$ $\quad$ $2x + 64 + y = 180$

$x = 30$ $\quad$ $y = 116 - 2x$

$y = 116 - 2(30)$

$= 56$

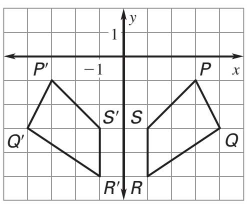
35. Because the triangle is a right triangle with two congruent sides, the 2 remaining angles must measure $\frac{180° - 90°}{2} = \frac{90°}{2} = 45°$.

$(2x - 11)° = 45°$ $\quad$ $(y + 16)° = 45°$

$2x = 56$ $\quad$ $y = 29$

$x = 28$

36.



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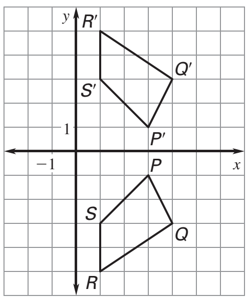

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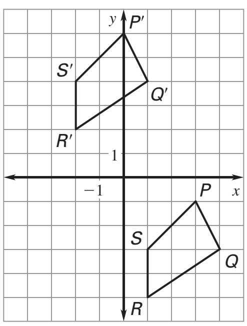

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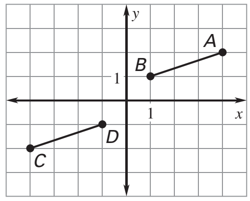
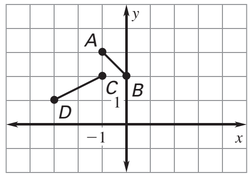


Yes, $\overline{CD}$ is a rotation of $\overline{AB}$; the rotation is 180°.

40.



No, $\overline{CD}$ is not a rotation of $\overline{AB}$.