Chapter 3 Parallel and Perpendicular Lines

Prerequisite Skills for the chapter "Parallel and Perpendicular Lines"

1. Adjacent angles have a common *vertex and side.*
2. Two angles are *supplementary* angles if the sum of their measures is 180°.
3. $m\angle 2 = 38^\circ$

   $m\angle 2 + m\angle 3 = 180^\circ$

   $38^\circ + m\angle 3 = 180^\circ$

   $m\angle 3 = 142^\circ$

   $m\angle 1 = m\angle 3 = 142^\circ$
4. $m\angle 2 = 90^\circ$

   $m\angle 2 + m\angle 3 = 180^\circ$

   $90^\circ + m\angle 3 = 180^\circ$

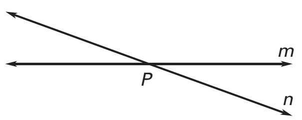
   $m\angle 3 = 90^\circ$

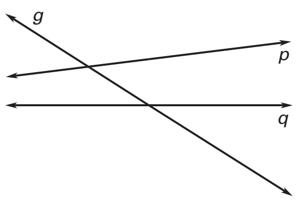
   $m\angle 1 = m\angle 3 = 90^\circ$
5. $m\angle 2 = 135^\circ$

   $m\angle 1 + m\angle 2 = 180^\circ$

   $m\angle 1 + 135^\circ = 180^\circ$

   $m\angle 1 = 45^\circ$

   $m\angle 3 = m\angle 1 = 45^\circ$
6. 

7. 

8. $\frac{36}{60} = \frac{3 \cdot 12}{5 \cdot 12} = \frac{3}{5}$
9. $\frac{54}{24} = \frac{9 \cdot 6}{4 \cdot 6} = \frac{9}{4}$
10. $\frac{16}{88} = \frac{2 \cdot 8}{11 \cdot 8} = \frac{2}{11}$
11. $\frac{21}{6} = \frac{7 \cdot 3}{2 \cdot 3} = \frac{7}{2}$

Lesson 3.1 Identify Pairs of Lines and Angles

Investigating Geometry Activity for the lesson "Identify Pairs of Lines and Angles"

1. $\overleftrightarrow{JM}$ and $\overleftrightarrow{LQ}$ will never intersect in space because they lie in different planes.
2. **a.** No; $\overleftrightarrow{JK}$ and $\overleftrightarrow{NR}$ lie in different planes.

   b. Yes; $\overleftrightarrow{QR}$ and $\overleftrightarrow{MR}$ intersect at point R.

   c. Yes; $\overleftrightarrow{LM}$ and $\overleftrightarrow{MR}$ intersect at point M.

   d. No; $\overleftrightarrow{KL}$ and $\overleftrightarrow{NQ}$ lie in different planes.
3. **a.** Yes; for example, plane JKR contains both lines.

   b. Yes; for example, plane QMR contains both lines.

   c. No; for example, points J, N, and L lie in one plane, and points J, N, and R lie in a different plane.

   d. Yes; for example, plane JLQ contains both lines.
4. Yes; *Sample answer:* When two lines intersect in space, they will lie in the same plane because three noncollinear points determine a unique plane.
5. Drawings will vary. No, the answers will remain the same because the drawing is labeled the same as Exercises 1–3.

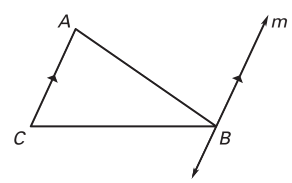
Guided Practice for the lesson "Identify Pairs of Lines and Angles"

1. Both $\overleftrightarrow{AH}$, $\overleftrightarrow{EH}$ appear skew to $\overleftrightarrow{CD}$ and contain point H.
2. Yes; $\overleftrightarrow{AC}$ is not perpendicular to $\overleftrightarrow{BF}$, because $\overleftrightarrow{MD}$ is perpendicular to $\overleftrightarrow{BF}$ and by the Perpendicular Postulate there is exactly one line perpendicular to $\overleftrightarrow{BF}$ through M.
3. Corresponding angles
4. Alternate exterior angles
5. Alternate interior angles

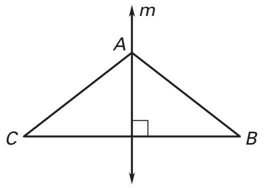
Exercises for the lesson "Identify Pairs of Lines and Angles"

Skill Practice

1. A line that intersects two other lines is a *transversal*.
2. The legs of the table and the top of the table cannot lie in parallel planes because the legs intersect the top of the table.
3. $\overleftrightarrow{AB}$ appears parallel to $\overleftrightarrow{CD}$.
4. $\overleftrightarrow{BC}$ appears perpendicular to $\overleftrightarrow{CD}$.
5. $\overleftrightarrow{BF}$ appears skew to $\overleftrightarrow{CD}$.
6. Plane ABE appears parallel to plane CDH.
7. $\overleftrightarrow{MK} \parallel \overleftrightarrow{LS}$
8. $\overleftrightarrow{NP} \perp \overleftrightarrow{PQ}$
9. $\overleftrightarrow{PN}$ is not parallel to $\overleftrightarrow{KM}$ because they intersect.
10. $\overleftrightarrow{PR}$ is not perpendicular to $\overleftrightarrow{NP}$. Because $\overleftrightarrow{PQ}$ is perpendicular to $\overleftrightarrow{NP}$ and by the Perpendicular Postulate there is exactly one line perpendicular to $\overleftrightarrow{NP}$ through P.
11. Corresponding angles are $\angle 1$ and $\angle 5$, $\angle 2$ and $\angle 6$, $\angle 3$ and $\angle 7$, and $\angle 4$ and $\angle 8$.
12. Alternate interior angles are $\angle 3$ and $\angle 6$, and $\angle 4$ and $\angle 5$.
13. Alternate exterior angles are $\angle 1$ and $\angle 8$, and $\angle 2$ and $\angle 7$.
14. Consecutive interior angles are $\angle 3$ and $\angle 5$, and $\angle 4$ and $\angle 6$.
15. $\angle 1$ and $\angle 8$ are not in corresponding positions in the diagram. $\angle 1$ and $\angle 8$ are alternate exterior angles.
16. One line can be drawn through B and parallel to $\overleftrightarrow{AC}$.



17. One line can be drawn through A and perpendicular to $\overleftrightarrow{BC}$.



18. $\angle 5$ and $\angle 1$ are corresponding angles.

19. $\angle 11$ and $\angle 13$ are consecutive interior angles.

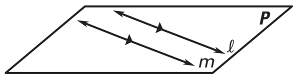
20. $\angle 6$ and $\angle 13$ are consecutive interior angles.

21. $\angle 10$ and $\angle 15$ are alternate exterior angles.

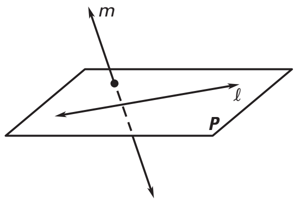
22. $\angle 2$ and $\angle 11$ are alternate interior angles.

23. $\angle 8$ and $\angle 4$ are corresponding angles.

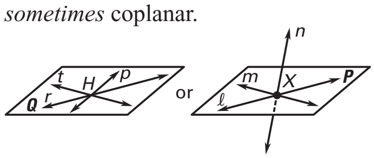
24. If two lines are parallel, then they are *always* coplanar.



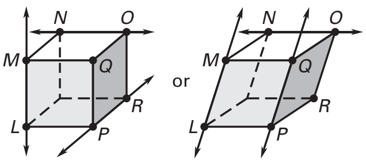
25. If two lines are not coplanar, then they *never* intersect.



26. If three lines intersect at one point, then they are *sometimes* coplanar.



27. If two lines are skew to a third line, then they are *sometimes* skew to each other.



28. B; $\angle RPQ$ and $\angle PRS$ are alternate interior angles.

29. $\angle BCG$, $\angle CFJ$, and $\angle GJH$ are corresponding angles.

30. $\angle BCG$ and $\angle HJC$ are consecutive interior angles.

31. $\angle FCJ$, $\angle HJC$, and $\angle DFC$ are alternate interior angles.

32. $\angle FCA$ and $\angle GJH$ are alternate exterior angles.

33. **a.** $m\angle 1 = 80°$; $m\angle 2 = 80°$

b. $m\angle 3 = 70°$; $m\angle 4 = 70°$

c. If parallel lines are cut by a transversal, then the alternate exterior angles are congruent.

Problem Solving

34. The platform is parallel to the ground.

35. The arm is skew to a telephone pole.

36. Answers will vary.

37. A; The horizontal bars are parallel.

38. **a.** $\angle 5$ and $\angle 9$, $\angle 6$ and $\angle 10$, $\angle 7$ and $\angle 11$, $\angle 8$ and $\angle 12$

b. $\angle 6$ and $\angle 11$, $\angle 8$ and $\angle 9$

c. $\angle 5$ and $\angle 12$, $\angle 7$ and $\angle 10$

d. $\angle 6$ and $\angle 9$, $\angle 8$ and $\angle 11$,

e. No; the rear leg is skew to the transversal.

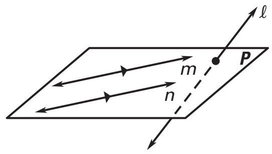
39. The adjacent interior angles are supplementary, so the measure of the other two interior angles must be 90°.

40. True; the plane containing the floor of the treehouse is parallel to the ground.

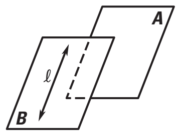
41. False; the lines containing the railings of the staircase intersect the ground, so they are not skew to the ground.

42. True; the lines containing the balusters are perpendicular to the plane containing the floor.

43. *Sample answer:*



44. *Sample answer:*



Lesson 3.2 Use Parallel Lines and Transversals

Investigating Geometry Activity for the lesson "Use Parallel Lines and Transversals"

1–2. Answers for the table will vary. So, $\angle AGE \cong \angle BGH \cong \angle CHG \cong \angle DHF$, $\angle EGB \cong \angle AGH \cong \angle GHD \cong \angle$CHF.

3. **a.** When two parallel lines are cut by a transversal, corresponding angles are congruent.

b. When two parallel lines are cut by a transversal, alternate interior angles are congruent.

4. When two parallel lines are cut by a transversal, the consecutive interior angles are supplementary.

When $m\angle AGH = 70°$, then $m\angle CHG = 110°$.

When $m\angle BGH = 130°$, then $m\angle GHD = 50°$.

Guided Practice for the lesson "Use Parallel Lines and Transversals"

1. $m\angle 4 = 105°$; Vertical Angles Congruence Theorem;

$m\angle 5 = 105°$; Corresponding Angles Postulate;

$m\angle 8 = 105°$; Alternate Exterior Angles Theorem

2. $m\angle 7 + m\angle 8 = 180°$

$m\angle 3 = m\angle 7$

$m\angle 3 + m\angle 8 = 180°$

$68° + (2x + 4)° = 180°$

$2x = 108$

$x = 54$

The value of x is 54.

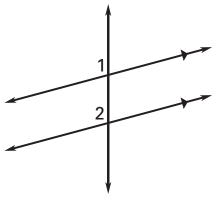
3. Yes. You could still prove the theorem because the congruence of $\angle 3$ and $\angle 2$ is not dependent on the congruence of $\angle 1$ and $\angle 3$.

4. Because the sun's rays are parallel, $\angle 1$ and $\angle 2$ are alternate interior angles. By the Alternate Interior Angles Theorem, $\angle 1 \cong \angle 2$. By the definition of congruent angles, $m\angle 1 = m\angle 2 = 41°$.

Exercises for the lesson "Use Parallel Lines and Transversals"

Skill Practice

1. *Sample answer:*



2. When two parallel lines are cut by a transversal, vertical angles, corresponding angles, alternate interior angles, and alternate exterior angles are congruent and adjacent angles, exterior angles on the same side of the transversal and consecutive interior angles are supplementary.

3. C; $\angle 4$ and $\angle 1$ are corresponding angles, so $m\angle 4 = m\angle 1$.

4. If $m\angle 4 = 65°$, then $m\angle 1 = 65°$, by the Vertical Angles Congruence Theorem.

5. If $m\angle 7 = 110°$, then $m\angle 2 = 110°$, by the Alternate Exterior Angles Theorem.

6. If $m\angle 5 = 71°$, then $m\angle 4 = 71°$, by the Alternate Interior Angles Theorem.

7. If $m\angle 3 = 117°$, then $m\angle 5 = 63°$, by the Consecutive Interior Angles Theorem.

8. If $m\angle 8 = 54°$, then $m\angle 1 = 54°$, by the Alternate Exterior Angles Theorem.

9. Corresponding Angles Postulate

10. Alternate Interior Angles Theorem

11. Alternate Interior Angles Theorem

12. Consecutive Interior Angles Theorem

13. Alternate Exterior Angles Theorem

14. Corresponding Angles Postulate

15. Alternate Exterior Angles Theorem

16. Consecutive Interior Angles Theorem

17. $m\angle 1 = 150°$, by the Corresponding Angles Postulate.

    $m\angle 2 = 150°$, by the Alternate Exterior Angles Theorem.

18. $m\angle 1 = 140°$, by the Alternate Interior Angles Theorem.

    $m\angle 2 = 40°$, by the Consecutive Interior Angles Theorem.

19. $m\angle 1 = 122°$, by the Alternate Interior Angles Theorem.

    $m\angle 2 = 58°$, by the Consecutive Interior Angles Theorem.

20. The student cannot conclude that $\angle 9 \cong \angle 10$ by the Corresponding Angles Postulate because the lines are not known to be parallel. There is not enough information given for the student to make any conclusion about the angles.

21. *Sample answer:* You can show that $\angle 1 \cong \angle 4$ by the Alternate Exterior Angles Theorem. You can show that $\angle 1 \cong \angle 3$ by the Corresponding Angles Postulate and $\angle 3 \cong \angle 4$ by the Vertical Angles Congruence Theorem. So, $\angle 1 \cong \angle 4$ by the Transitive Property of Angle Congruence.

22. $\angle 1$ and the 80° angle are supplementary by the Consecutive Interior Angles Theorem, so $m\angle 1 = 100°$. $\angle 1$ and $\angle 2$ are supplementary by the Consecutive Interior Angles Theorem, so $m\angle 2 = 80°$. $\angle 3$ and the 80° angle are supplementary by the Consecutive Interior Angles Theorem, so $m\angle 3 = 100°$.

23. $\angle 1$ and the 90° angle are supplementary by the Consecutive Interior Angles Theorem, so $m\angle 1 = 90°$. $\angle 2$ is congruent to the 115° angle by the Corresponding Angles Postulate, so $m\angle 2 = 115°$. $\angle 2$ and $\angle 3$ are supplementary by the Consecutive Interior Angles Theorem, so $m\angle 3 = 65°$.

24. $\angle 2$ is congruent to the 133° angle by the Alternate Interior Angles Theorem, so $m\angle 2 = 133°$. $\angle 3$ and the 133° angle are supplementary by the Consecutive Interior Angles Theorem, so $m\angle 3 = 47°$. $\angle 1 \cong \angle 3$ by the Alternate Interior Angles Theorem, so $m\angle 1 = 47°$.

25. *Sample answer:* If $\overleftrightarrow{AB} \parallel \overleftrightarrow{DC}$, then $\angle BAC \cong \angle DCA$ and $\angle CDB \cong \angle ABD$.

26. *Sample answer:* If $\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$, then $\angle BAD$ and $\angle ABC$ are supplementary and $\angle ADC$ and $\angle BCD$ are supplementary.

27. Using the Alternate Interior Angles Theorem, $x = 45$. Using the Corresponding Angles Postulate, $y = 85$.

28. Using the Consecutive Interior Angles Theorem:

$$3y° + 6y° = 180°$$
$$9y = 180$$
$$y = 20$$
$$2x° + 90° = 180°$$
$$2x = 90$$
$$x = 45$$

29. Using the Corresponding Angles Postulate, $x = 65$.

    Using the Angle Addition Postulate:

$$x° + y° + 55° = 180°$$
$$65 + y + 55 = 180$$
$$y = 60$$

30. Using the Corresponding Angles Postulate:

$$3x° = 60°$$
$$x = 20$$

    Using the Consecutive Interior Angles Theorem:

$$(5y - 5)° + 135° = 180°$$
$$5y = 50$$
$$y = 10$$

31. Using the Alternate Interior Angles Theorem:

$4x^\circ = 52^\circ$

$x = 13$

Using the Consecutive Interior Angles Theorem:

$[4x^\circ + (3y + 2)^\circ] + 90^\circ = 180^\circ$

$4(13) + 3y + 2 + 90 = 180$

$3y = 36$

$y = 12$

32. Using the Consecutive Interior Angles Theorem:

$5x^\circ + (14x - 10)^\circ = 180^\circ$

$19x = 190$

$x = 10$

$2y^\circ + (14x - 10)^\circ = 180^\circ$

$2y + 14(10) - 10 = 180$

$2y = 50$

$y = 25$

33. B; Using the Alternate Exterior Angles Theorem: $m\angle 1 = 110^\circ$

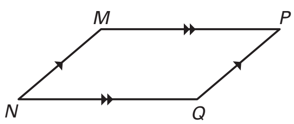
Using the Linear Pair Postulate:

$(y - 5)^\circ + m\angle 1 = 180^\circ$

$(y - 5)^\circ + 110^\circ = 180^\circ$

$y = 75$

34. *Sample answer:*



$\angle MNQ$ and $\angle PQN$, and $\angle PQN$ and $\angle QPM$ are both supplementary by the Consecutive Interior Angles Theorem. $\angle MNQ \cong \angle QPM$ by the Congruent Supplements Theorem. $\angle NMP$ and $\angle QPM$, and $\angle QPM$ and $\angle PQN$ are both supplementary by the Consecutive Interior Angles Theorem. $\angle NMP \cong \angle PQN$ by the Congruent Supplements Theorem. So, $\angle MNQ \cong \angle QPM$ and $\angle NMP \cong \angle PQN$.

35. Using the Consecutive Interior Angles Theorem:

$(2x - y)^\circ + 60^\circ = 180^\circ$

$2x - y = 120 \rightarrow y = 2x - 120$

$(2x + y)^\circ + 40^\circ = 180^\circ$

$2x + y = 140$

$2x^\circ + (2x - 120)^\circ = 140^\circ$

$4x = 260$

$x = 65$

$y = 2(65) - 120 = 10$

So, the value of x is 65 and the value of y is 10.

36. Using the Consecutive Interior Angles Theorem:

$150^\circ + (5x - y)^\circ = 180^\circ$

$5x - y = 30 \rightarrow y = 5x - 30$

$130^\circ + (5x + y)^\circ = 180^\circ$

$5x + y = 50$

$5x^\circ + (5x - 30)^\circ = 50^\circ$

$10x = 80$

$x = 8$

$y = 5(8) - 30 = 10$

The value of x is 8 and the value of y is 10.

Problem Solving

37.

Statements	Reasons
1. $p \parallel q$	**1.** Given
2. $\angle 1 \cong \angle 3$	**2.** Corresponding Angles Postulate
3. $\angle 3 \cong \angle 2$	**3.** Vertical Angles Congruence Postulate
4. $\angle 1 \cong \angle 2$	**4.** Transitive Property of Angle Congruence

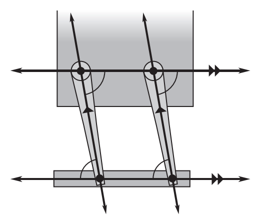
38. a. $\angle 4 \cong \angle 1$ by the Vertical Angles Congruence Theorem. $\angle 5 \cong \angle 1$ by the Corresponding Angles Postulate. $\angle 8 \cong \angle 1$ by the Alternate Exterior Angles Theorem.

b. $\angle 6$ and $\angle 8$ are a linear pair, so they are supplementary and $m\angle 6 + m\angle 8 = 180^\circ$. $m\angle 8 = m\angle 1 = 110^\circ$. So, $m\angle 6 + 110^\circ = 180^\circ$, or $m\angle 6 = 70^\circ$.

39. a. The following pairs of angles are always congruent: $\angle 1$ and $\angle 5$, and $\angle 2$ and $\angle 6$.

The following pairs of angles are always supplementary: $\angle 1$ and $\angle 2$, $\angle 1$ and $\angle 6$, $\angle 2$ and $\angle 5$, and $\angle 5$ and $\angle 6$.

b. Because the bars are parallel, the corresponding angles between the bars and the foot are congruent. Because the body and the foot are parallel, the bars act as transversals, and so the alternate interior angles are congruent. (See diagram.) This forces the foot to stay parallel with the floor.



40. a. Using the Alternate Interior Angles Theorem, $m\angle 2 = m\angle 1 = 70^\circ$. $\angle 2$ and $\angle 3$ are a linear pair, so $70^\circ + m\angle 3 = 180^\circ$, or $m\angle 3 = 110^\circ$.

b. $\angle ABC$ is a straight angle because $\angle 2$ and $\angle 3$ are supplementary, so the sum of the angle measures is 180°.

c. If $m\angle 1$ is 60°, $\angle ABC$ will still be a straight angle because $\angle 2$ and $\angle 3$ will still be supplementary. The opening of the box will be more steep because the measure of $\angle 1$ is smaller, so the slope of the line becomes greater.

41.

Statements	Reasons
1. $n \parallel p$	**1.** Given
2. $\angle 1 \cong \angle 3$	**2.** Alternate Interior Angles Theorem
3. $m\angle 1 = m\angle 3$	**3.** Def. of congruent angles
4. $\angle 2$ and $\angle 3$ are supplementary.	**4.** Linear Pair Postulate
5. $m\angle 2 + m\angle 3 = 180°$	**5.** Definition of supplementary angles
6. $m\angle 1 + m\angle 2 = 180°$	**6.** Substitution
7. $\angle 1$ and $\angle 2$ are supplementary.	**7.** Definition of supplementary angles

42.

Statements	Reasons
1. $t \perp r$, $r \parallel s$	**1.** Given
2. $\angle 1$ is a right angle.	**2.** Def. of perpendicular lines
3. $m\angle 1 = 90°$	**3.** Def. of right angle
4. $\angle 1 \cong \angle 2$	**4.** Corresponding Angles Post.
5. $m\angle 1 = m\angle 2$	**5.** Def. of congruent angles
6. $m\angle 2 = 90°$	**6.** Substitution
7. $\angle 2$ is a right angle.	**7.** Def. of right angle
8. $t \perp s$	**8.** Def. of perpendicular lines

43. $\angle 4 \cong \angle 2$ by the Alternate Interior Angles Theorem. $\angle 2 \cong \angle 3$ by the definition of angle bisector. $\angle 5 \cong \angle 1$ by the Corresponding Angles Postulate. $\angle 4 \cong \angle 5$ is given, so $\angle 1 \cong \angle 2 \cong \angle 3 \cong \angle 4 \cong \angle 5$. Because $m\angle 1 + m\angle 2 + m\angle 3 = 180°$, the measure of each angle is 60°. So, $m\angle 1 = 60°$.

Quiz for the lessons "Identify Pairs of Lines and Angles" and "Use Parallel Lines and Transversals"

1. $\angle 2$ and $\angle 6$ are corresponding angles.
2. $\angle 3$ and $\angle 5$ are consecutive interior angles.
3. $\angle 3$ and $\angle 6$ are alternate interior angles.
4. $\angle 2$ and $\angle 7$ are alternate exterior angles.
5. Using the Alternate Interior Angles Theorem:

   $2x° = 128°$

   $x = 64$
6. Using the Alternate Exterior Angles Theorem:

   $(2x + 1)° = 151°$

   $2x° = 150°$

   $x = 75$
7. Using the Consecutive Interior Angles Theorem:

   $(7x + 24)° + 72° = 180$

   $7x = 84$

   $x = 12$

Lesson 3.3 Prove Lines are Parallel

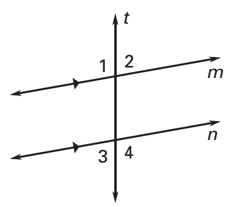
Guided Practice for the lesson "Prove Lines are Parallel"

1. Yes. The measure of the angle supplementary to the 105° angle is 75° by the Linear Pair Postulate. The corresponding angles are congruent by the definition of congruent angles. So, $m \parallel n$ by the Corresponding Angles Converse.
2. Postulate 16 exchanges the hypothesis and conclusion of Postulate 15, so Postulate 16 is the converse of Postulate 15.
3. You can prove the lines are parallel using the Alternate Exterior Angles Converse.
4. You can prove the lines are parallel using the Corresponding Angles Converse.
5. *Sample answer:* You cannot prove the lines are parallel because you do not know if $\angle 1 \cong \angle 2$, so you cannot use the Alternate Interior Angles Converse.
6. Given: $\angle 1 \cong \angle 8$

   Prove: $j \parallel k$
7. It is given that $\angle 4 \cong \angle 5$. By the *Vertical Angles Congruence Theorem,* $\angle 1 \cong \angle 4$. Then by the Transitive Property of Congruence, $\angle 1 \cong \angle 5$. So by the *Corresponding Angles Converse,* $g \parallel h$.
8. All of the steps are parallel. Since the bottom step is parallel to the ground, the Transitive Property of Parallel Lines applies and the top step is parallel to the ground.

Exercises for the lesson "Prove Lines are Parallel"

Skill Practice

1. 

   $\angle 1$ and $\angle 4$, and $\angle 2$ and $\angle 3$ are Alternate Exterior Angles.
2. Two lines cut by a transversal have congruent pairs of alternate interior angles if and only if the lines are parallel.

   Two lines cut by a transversal have congruent pairs of alternate exterior angles if and only if the lines are parallel.

   Two lines cut by a transversal have supplementary pairs of consecutive interior angles if and only if the lines are parallel.
3. Using the Corresponding Angles Converse:

   $3x° = 120°$

   $x = 40$

   The lines are parallel when $x = 40$.

4. Using the Corresponding Angles Converse:

$(2x + 15)° = 135°$

$2x = 120$

$x = 60$

The lines are parallel when $x = 60$.

5. Using the Consecutive Interior Angles Converse:

$(3x - 15)° + 150° = 180°$

$3x = 45$

$x = 15$

The lines are parallel when $x = 15$.

6. Using the Alternate Exterior Angles Converse:

$(180 - x)° = x°$

$180 = 2x$

$90 = x$

The lines are parallel when $x = 90$.

7. Using the Consecutive Interior Angles Converse:

$2x° + x° = 180°$

$3x = 180$

$x = 60$

The lines are parallel when $x = 60$.

8. Using the Alternate Interior Angles Theorem:

$(2x + 20)° = 3x°$

$20 = x$

The lines are parallel when $x = 20$.

9. The student cannot conclude that lines a and b are parallel because there is no indication that x equals y. There is not enough information given in order to make any conclusion.

10. Yes; Alternate Interior Angles Converse.

11. Yes; Alternate Exterior Angles Converse.

12. No, there is not enough information to prove $m \parallel n$.

13. Yes; Corresponding Angles Converse.

14. No, there is not enough information to prove $m \parallel n$.

15. Yes; Alternate Exterior Angles Converse.

16. Answers will vary.

17. a. Using the Angle Addition Postulate,

$m\angle DCG = m\angle DCE + m\angle ECG$

$= 38° + 77° = 115°$.

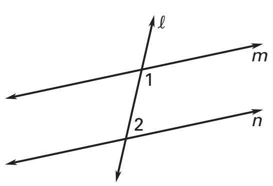
Using the Linear Pair Postulate,

$m\angle CGH + 115° = 180°$, so $m\angle CGH = 65°$.

b. $\angle DCG$ and $\angle CGH$ are consecutive interior angles and they are supplementary.

c. $\overleftrightarrow{DB} \parallel \overleftrightarrow{HF}$ by the Consecutive Interior Angles Converse.

18. a.



b. Given: $\angle 1$and $\angle 2$ are supplementary.

Prove: $m \parallel n$

19. Yes. You can prove $a \parallel b$ by using the Consecutive Interior Angles Converse.

20. Yes. You can prove $a \parallel b$ by the Alternate Exterior Angles Converse. Because the sum of 66° and 48° is 114°, the alternate exterior angles are congruent.

21. There is not enough information to prove $a \parallel b$.

22. The angles shown as congruent do not show that $\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$. By the Alternate Interior Angles Converse, $\overleftrightarrow{AB} \parallel \overleftrightarrow{DC}$.

23. D; There is not enough information given to make any of the listed conclusions.

24. One angle measure must be given in order to find the measure of every angle. *Sample answer:* Using the Vertical Angles Congruence Theorem, the Linear Pair Postulate, and the Corresponding Angles Postulate, the other angle measures can be found.

25. Given: $\angle 1$ and $\angle 7$ are supplementary.

Prove: $j \parallel k$

Show $\angle 1$ congruent to $\angle 4$ by the Vertical Angles Congruence Theorem, and show $\angle 4$ and $\angle 7$ to be supplementary by substitution. Then lines j and k are parallel by the Consecutive Interior Angles Converse.

26. $\overrightarrow{EA} \parallel \overrightarrow{HC}$ by the Corresponding Angles Converse because $\angle HEA \cong \angle GHC$.

$\overrightarrow{EB}$ is not parallel to $\overrightarrow{HD}$ because $\angle HEB$ is not congruent to $\angle GHD$.

27. a. There is only one line through R perpendicular to plane ABC.

b. There are infinitely many lines through R parallel to plane ABC.

c. There is only one plane through R parallel to plane ABC.

28. a. Using the Corresponding Angles Converse:

$(2x + 2)° = (x + 56)°$

$x + 2 = 56$

$x = 54$

Lines p and q are parallel when $x = 54$.

b. Using the Linear Pair Postulate and the Corresponding Angles Converse:

$180° - (y + 7)° = (3y - 17)°$

$173 - y = 3y - 17$

$173 = 4y - 17$

$190 = 4y$

$47.5 = y$

Lines r and s are parallel when $y = 47.5$.

c. Lines r and s cannot be parallel if lines p and q are parallel. If lines p and q are parallel, $x = 54$ and $y = 63$, but for lines r and s to be parallel, y must equal 47.5.

Problem Solving

29. Because the alternate interior angles are congruent, you know that the top of the picnic table is parallel to the ground by the Alternate Interior Angles Converse.

30. Because the corresponding angles are congruent, you know that line n is parallel to line m by the Corresponding Angles Converse.

31.

Statements	Reasons
1. $m\angle 1 = 115°$, $m\angle 2 = 65°$	**1.** Given
2. $115° + 65° = 180°$	**2.** Addition
3. $m\angle 1 + m\angle 2 = 180°$	**3.** Substitution Property of Equality
4. $\angle 1$ and $\angle 2$ are supplementary.	**4.** Definition of supplementary angles
5. $m \parallel n$	**5.** Consecutive Interior Angles Converse

32. Because the alternate exterior angles are congruent, you know that the bowling pins are set up in parallel lines by the Alternate Exterior Angles Converse.

33. E. 20th Ave., E. 19th Ave., and E. 17th Ave. are all parallel by the Corresponding Angles Converse. E. 18th Ave. and E. 17th Ave. are parallel by the Alternate Interior Angles Converse. All of the streets are parallel to each other by the Transitive Property of Parallel Lines.

34.

Statements	Reasons
1. $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$	**1.** Given
2. $\angle 2 \cong \angle 3$	**2.** Vertical Angles Congruence Theorem
3. $\angle 1 \cong \angle 4$	**3.** Substitution Property of Congruence
4. $\overline{AB} \parallel \overline{CD}$	**4.** Alternate Interior Angles Converse

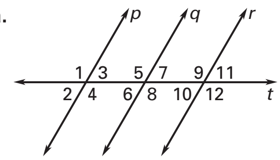
35.

Statements	Reasons
1. $a \parallel b$, $\angle 2 \cong \angle 3$	**1.** Given
2. $\angle 1 \cong \angle 3$	**2.** Alternate Interior Angles Theorem
3. $\angle 1 \cong \angle 2$	**3.** Substitution Property of Congruence
4. $c \parallel d$	**4.** Corresponding Angles Converse

36. It is given that $\angle 2 \cong \angle 7$. $\angle 7 \cong \angle 6$ by the Vertical Angles Congruence Theorem. $\angle 2 \cong \angle 6$ by the Transitive Property of Congruence. So, $m \parallel n$ by the Corresponding Angles Converse.

37. It is given that $\angle 3$ and $\angle 5$ are supplementary. $\angle 5$ and $\angle 7$ form a linear pair by the definition of a linear pair. $\angle 5$ and $\angle 7$ are supplementary by the Linear Pair Postulate. $\angle 3 \cong \angle 7$ by the Congruent Supplements Theorem. So, $m \parallel n$ by the Corresponding Angles Converse.

38. a.



b. Given: $p \parallel q$ and $q \parallel r$

Prove: $p \parallel r$

c.

Statements	Reasons
1. $p \parallel q$, $q \parallel r$	**1.** Given
2. $\angle 3 \cong \angle 6$	**2.** Alternate Interior Angles Theorem
3. $\angle 6 \cong \angle 10$	**3.** Corresponding Angles Postulate
4. $\angle 3 \cong \angle 10$	**4.** Substitution Property of Congruence
5. $p \parallel r$	**5.** Alternate Interior Angles Converse

39. a. Because the corresponding angles formed by the blue lines and the horizontal edge are congruent, you know that the blue lines are parallel by the Corresponding Angles Converse.

b. Slide the triangle along a fixed horizontal line using the edge that forms a 90° angle to draw vertical parallel lines.

40. *Sample answer:* Because the 114° angle and the 66° angle made by the transversal g are supplementary, you can show that $a \parallel b$ by the Consecutive Interior Angles Converse.

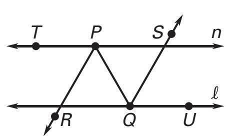
41. *Sample answer:* Because the two corresponding 66° angles made by the transversal g are congruent, you can show that $b \parallel c$ by the Corresponding Angles Converse.

42. *Sample answer:* Because the two corresponding 137° angles ($71° + 66° = 137°$) made by the transversal b are congruent, you can show that $d \parallel f$ by the Corresponding Angles Converse.

43. *Sample answer:* Because the two corresponding 66° angles ($180° - 114° = 66°$) made by the transversal b are congruent, you can show that $e \parallel g$ by the Corresponding Angles Converse.

44. *Sample answer:* Because the 114° angle and the 66° angle made by the transversal g are supplementary, you can show that $a \parallel c$ by the Consecutive Interior Angles Converse.

45. a. *Sample answer:*



b. Conjecture: The angle bisectors of a pair of alternate interior angles are parallel.

Given: $\ell \parallel n$, $\overrightarrow{PR}$ bisects $\angle QPT$, and $\overrightarrow{QS}$ bisects $\angle PQU$.

Prove: $\overrightarrow{PR} \parallel \overrightarrow{QS}$

Statements	Reasons
1. $\ell \parallel n$, $\overrightarrow{PR}$ bisects $\angle QPT$, and $\overrightarrow{QS}$ bisects $\angle PQU$	**1.** Given
2. $\angle QPT \cong \angle PQU$	**2.** Alternate Interior Angles Theorem
3. $\angle QPR \cong \angle RPT$ and $\angle PQS \cong \angle SQU$	**3.** Definition of angle bisector
4. $m\angle QPT = m\angle PQU$, $m\angle QPR = m\angle RPT$, and $m\angle PQS = m\angle SQU$	**4.** Definition of congruent angles
5. $m\angle QPT = m\angle QPR + m\angle RPT$; $m\angle PQU = m\angle PQS + m\angle SQU$	**5.** Angle Addition Postulate
6. $m\angle QPR + m\angle RPT = m\angle PQS + m\angle SQU$	**6.** Substitution Property of Equality
7. $m\angle QPR + m\angle QPR = m\angle PQS + m\angle PQS$	**7.** Substitution Property of Equality
8. $2 \bullet m\angle QPR = 2 \bullet m\angle PQS$	**8.** Distributive Property
9. $m\angle QPR = m\angle PQS$	**9.** Division Property of Equality
10. $\angle QPR \cong \angle PQS$	**10.** Definition of congruent angles
11. $\overrightarrow{PR} \parallel \overrightarrow{QS}$	**11.** Alternate Interior Angles Converse

Mixed Review of Problem Solving for the lessons "Identify Pairs of Lines and Angles", "Use Parallel Lines and Transversals", and "Prove Lines are Parallel"

1. a. Lines p and q, and lines k and m are parallel.

b. Lines q and m are skew lines.

c. Lines n and k, and lines n and m are perpendicular.

2. a. $\angle 1$ and $\angle 2$, and $\angle 1$ and $\angle 3$ form linear pairs. $\angle 1$ and $\angle 4$ are vertical angles. $\angle 1$ and $\angle 5$ are corresponding angles. $\angle 1$ and $\angle 6$, and $\angle 1$ and $\angle 8$ are supplementary angles. $\angle 1$ and $\angle 7$ are alternate exterior angles.

b. When $r \parallel s$, $\angle 3 \cong \angle 2$, $\angle 3 \cong \angle 6$, $\angle 3 \cong \angle 8$.

3. Because $\angle 1$ and $\angle 2$ are alternate exterior angles, you know that $\angle 1 \cong \angle 2$ by the Alternate Exterior Angles Theorem. $m\angle 1 = m\angle 2$ by the definition of congruent angles. So, $m\angle 2 = m\angle 1 = 53°$.

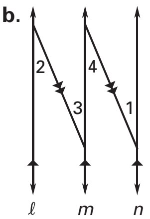
4. Because the alternate interior angles are congruent, you know that the top and bottom of the Z are parallel by the Alternate Interior Angles Converse.

5. a. Using the Alternate Interior Angles Converse:

$$(2x + 1)° = 23°$$
$$2x = 22$$
$$x = 11$$

Lines ℓ and m are parallel when $x = 11$.

b.



Because lines ℓ and m are parallel and the two transversals are parallel, you know that $\angle 2 \cong \angle 3$ and $\angle 3 \cong \angle 4$ by the Alternate Interior Angles Theorem. Because $\ell \parallel m$ and $\ell \parallel n$, you know that $m \parallel n$ by the Transitive Property of Parallel Lines. $\angle 4 \cong \angle 1$ by the Alternate Interior Angles Theorem. $\angle 1 \cong \angle 2$ by the Transitive Property of Congruence. So, $m\angle 1 = m\angle 2 = 23°$.

6. Using the Alternate Interior Angles Theorem and the Linear Pair Postulate, $m\angle 1 = 180° - 30° = 150°$.

7. Using the Linear Pair Postulate, $m\angle x = 180° - 88° = 92°$. Because alternate interior angles are congruent, you know that $c \parallel d$ by the Alternate Interior Angles Converse. Using the Consecutive Interior Angles Theorem, $m\angle y = 180° - 64° = 116°$. So, the value of x is 92 and the value of y is 116.

Lesson 3.4 Find and Use Slopes of Lines

Guided Practice for the lesson "Find and Use Slopes of Lines"

1. Slope of line b:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 0}{6 - 4} = 2$$

2. Slope of line c:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 4}{6 - 0} = 0$$

3. Slope of line m: $m_1 = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{1 - 3}{4 - (-1)} = -\dfrac{2}{5}$

Slope of line t: $m_2 = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-3 - (-1)}{3 - (-2)} = -\dfrac{2}{5}$

Because m and t have the same slope, they are parallel.

4. Slope of line n: $m_1 = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{5 - 2}{6 - 0} = \dfrac{3}{6} = \dfrac{1}{2}$

Slope of line m: $m_2 = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{0 - 4}{4 - 2} = \dfrac{-4}{2} = -2$

Because the product of m_1 and m_2 is -1, $n \perp m$.

5. The parachute in jump c was in the air approximately 1.25 seconds longer than the parachutes in jumps a and b. So, the parachute in jump c was in the air for the longest time.

6. The x-intercepts represent the time (in minutes) that it takes each parachute to reach the ground. You can eliminate choice B, because jumps b and c do not have the same x-intercept, so the parachutes were not open for the same amount of time.

7. Slope of line q: $m_1 = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{5 - 0}{-4 - 0} = -\dfrac{5}{4}$

   Slope of line t: $m_2 = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{7 - 0}{-10 - 0} = -\dfrac{7}{10}$

   Line q is steeper because the absolute value of the slope of its line is greater.

8. Slope $= \dfrac{\text{rise}}{\text{run}} = \dfrac{300}{350} = 0.857$

   The roller coaster is more steep than the Magnum because the slope of its line is greater ($0.875 > 0.5125$). The roller coaster is less steep than the Millennium Force because the slope of its line is less ($0.875 < 1$).

Exercises for the lesson "Find and Use Slopes of Lines"

Skill Practice

1. The slope of a non-vertical line is the ratio of vertical change to horizontal change between any two points on the line.

2. When you apply the slope formula to a horizontal line, the numerator of the resulting fraction is zero, so the slope is zero. When you apply the slope formula to a vertical line, the denominator of the resulting fraction is zero, so the slope is undefined.

3. D; m is positive.

4. A; m is negative.

5. B; m is zero.

6. C; m is undefined.

7. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{6 - 5}{5 - 3} = \dfrac{1}{2}$

   The slope is $\frac{1}{2}$.

8. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-6 - 2}{2 - (-2)} = \dfrac{-8}{4} = -2$

   The slope is -2.

9. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-1 - (-1)}{3 - (-5)} = \dfrac{0}{8} = 0$

   The slope is 0.

10. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{6 - 1}{0 - 2} = \dfrac{5}{-2} = -\dfrac{5}{2}$

    The slope is $-\frac{5}{2}$.

11. The slope of the line was computed using $\frac{\text{run}}{\text{rise}}$ instead of $\frac{\text{rise}}{\text{run}}$. The rise is 3 and the run is 4, so the slope is $\frac{3}{4}$.

12. The values of x_2 and x_1 were interchanged when computing the slope.

    $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{7 - 5}{2 - 4} = \dfrac{2}{-2} = -1$

13. Slope of line 1: $m_1 = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{4 - 0}{7 - 1} = \dfrac{2}{3}$

    Slope of line 2: $m_2 = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{6 - 0}{3 - 7} = \dfrac{6}{-4} = -\dfrac{3}{2}$

    Because $m_1 m_2 = \frac{2}{3} \cdot -\frac{3}{2} = -1$, the lines are perpendicular.

14. Slope of line 1: $m_1 = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-2 - 1}{-7 - (-3)} = \dfrac{3}{4}$

    Slope of line 2: $m_2 = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{4 - (-1)}{8 - 2} = \dfrac{5}{6}$

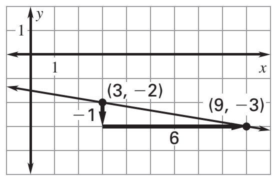
    Because $m_1 \neq m_2$ and $m_1 m_2 \neq -1$, the lines are neither parallel nor perpendicular.

15. Slope of line 1: $m_1 = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{7 - 3}{-5 - (-9)} = \dfrac{4}{4} = 1$

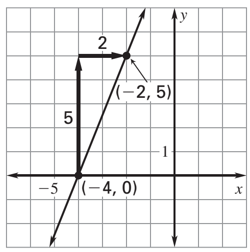
    Slope of line 2: $m_2 = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{2 - 6}{-7 - (-11)} = \dfrac{-4}{4} = -1$

    Because $m_1 m_2 = 1 \cdot (-1) = -1$, the lines are perpendicular.

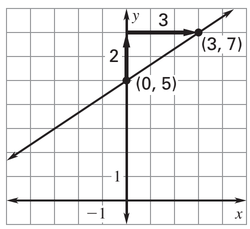
16. $P(3, -2)$, slope $-\frac{1}{6}$



17. $P(-4, 0)$, slope $\frac{5}{2}$



18. $P(0, 5)$, slope $\frac{2}{3}$



19. Slope of line 1: $m_1 = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{5 - 3}{3 - (-2)} = \dfrac{2}{5}$

    Slope of line 2: $m_2 = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{5 - 1}{6 - 3} = \dfrac{4}{3}$

    Line 2 is steeper because the slope of its line is greater $\left(\frac{4}{3} > \frac{2}{5}\right)$.

20. Slope of line 1: $m_1 = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-2 - (-1)}{1 - (-2)} = -\dfrac{1}{3}$

    Slope of line 2: $m_2 = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-4 - (-3)}{-1 - (-5)} = -\dfrac{1}{4}$

    Line 1 is steeper because the absolute value of the slope of its line is greater $\left(\left|-\frac{1}{3}\right| > \left|-\frac{1}{4}\right|\right)$.

21. Slope of line 1: $m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 2}{-3 - (-4)} = 4$

Slope of line 2: $m_2 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 6}{3 - 1} = \frac{2}{2} = 1$

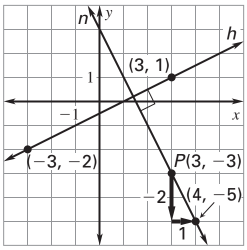
Line 1 is steeper because the slope of its line is greater ($4 > 1$).

22. You can determine which of two lines is steeper by comparing the slope of each line. The line that has the slope with the greater absolute value is steeper.

23. Slope of line h: $m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - (-2)}{3 - (-3)} = \frac{3}{6} = \frac{1}{2}$

Slope of line n: $\frac{1}{2} \cdot m_2 = -1$

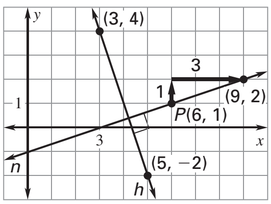
$m_2 = -2$



24. Slope of line h: $m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 4}{5 - 3} = \frac{-6}{2} = -3$

Slope of line n: $-3 \cdot m_2 = -1$

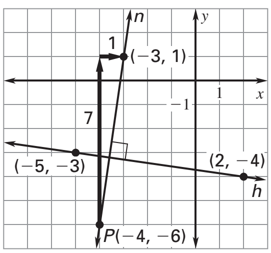
$m_2 = \frac{1}{3}$



25. Slope of line h: $m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - (-3)}{2 - (-5)} = -\frac{1}{7}$

Slope of line n: $-\frac{1}{7} \cdot m_2 = -1$

$m_2 = 7$



26. If the points all lie on the same line, then the slope of the line containing any two of the points will be the same.

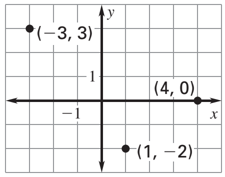
Use $(-3, 3)$ and $(1, -2)$:

$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 3}{1 - (-3)} = -\frac{5}{4}$

Use $(-3, 3)$ and $(4, 0)$:

$m_2 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 3}{4 - (-3)} = -\frac{3}{7}$

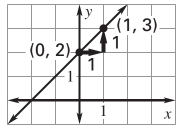
Because the slopes of lines are different, you know that the points do not all lie on the same line.



27. Parallel lines have the same slope.

Use $(-2, 4)$ and $(-5, 1)$:

$m_2 = m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 4}{-5 - (-2)} = 1$



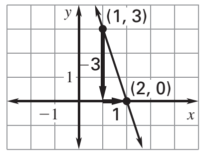
28. The product of the slopes of two perpendicular lines is -1.

Use $(-1, -1)$ and $(2, 0)$:

$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - (-1)}{2 - (-1)} = \frac{1}{3}$

$\frac{1}{3} \cdot m_2 = -1$

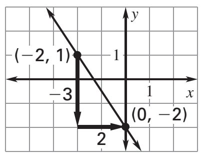
$m_2 = -3$



29. Parallel lines have the same slope.

Use $(3, 1)$ and $\left(4, -\frac{1}{2}\right)$:

$m_2 = m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-\frac{1}{2} - 1}{4 - 3} = -\frac{3}{2}$



30.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$-2 = \frac{y - 2}{0 - (-3)}$$

$$-2 = \frac{y - 2}{3}$$

$$-2(3) = y - 2$$

$$-6 = y - 2$$

$$-4 = y$$

31.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{1}{3} = \frac{0 - (-4)}{x - (-7)}$$

$$\frac{1}{3} = \frac{4}{x + 7}$$

$$x + 7 = 12$$

$$x = 5$$

32.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$-4 = \frac{1 - (-3)}{x - 4}$$

$$-4 = \frac{4}{x - 4}$$

$$-4(x - 4) = 4$$

$$-4x + 16 = 4$$

$$-4x = -12$$

$$x = 3$$

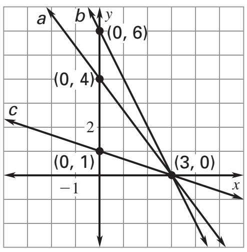
Problem Solving

33. slope $= \frac{\text{rise}}{\text{run}} = \frac{6}{9} = \frac{2}{3}$

The slope of the slide is $\frac{2}{3}$.

34. B; The slope of the line for car B is less steep than the slope of the line for car A, so the gas remaining in car B does not deplete as quickly as the distance driven increases. Car B has the better gas mileage.

35.



Line b is the most steep because the absolute value of its slope is the greatest. Line c is the least steep because the absolute value of its slope is the least.

36. a. $h = \frac{1}{4}v$

slope $= \frac{\text{rise}}{\text{run}} = \frac{v}{h} = \frac{v}{\frac{1}{4}v} = 4$

The recommended slope for a ladder is 4.

b. When $h = 6$:

$$\frac{v}{h} = 4$$

$$\frac{v}{6} = 4$$

$$v = 24$$

The ladder touches the building 24 feet above the ground.

c. When $v = 34$:

$$\frac{v}{h} = 4$$

$$\frac{34}{h} = 4$$

$$8.5 = h$$

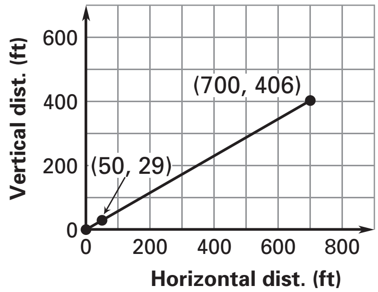
The base of the ladder is 8.5 feet away from the building.

37. a.

Horizontal, ft	0	50	100	150	200
Vertical, ft	0	29	58	87	116
Horizontal, ft	250	300	350	400	450
Vertical, ft	145	174	203	232	261
Horizontal, ft	500	550	600	650	700
Vertical, ft	290	319	348	377	406

At the top, the incline is 406 feet high.

b. slope $= \frac{\text{rise}}{\text{run}} = \frac{29}{50}$



c. slope $= \frac{\text{rise}}{\text{run}} = \frac{144}{271}$

The slope of the Burgenstock Incline is $\frac{144}{271}$.

The Duquesne Incline is steeper because the slope of its climb path is greater $\left(\frac{29}{50} > \frac{144}{271}\right)$.

38. It is given that $p \parallel q$, so by the Slopes of Parallel Lines Postulate, $m_p = m_q$. It is also given that $q \parallel r$, so by the Slopes of Parallel Lines Postulate, $m_q = m_r$. By the Transitive Property of Equality, $m_p = m_r$. Therefore, by the Slopes of Parallel Lines Postulate, $p \parallel r$.

39. average rate of change $= \frac{\text{change in profit}}{\text{change in time}}$

$$= \frac{15{,}400 - 8500}{2006 - 2000} = \frac{6900}{6} = \frac{1150}{1}$$

The average rate of change is \$1150 per year.

40. average rate of change $= \frac{\text{change in height}}{\text{change in time}}$

$$= \frac{706 - 400}{45} = \frac{306}{45} = \frac{6.8}{1}$$

The averate rate of change is 6.8 feet per minute.

41. a. Because the slope of the graph is steepest during that period, the NBA attendance increased the most from 1985 to 1990.

rate of change $= \frac{\text{change in attendance}}{\text{change in time}}$

$$= \frac{16 \text{ million} - 7 \text{ million}}{1990 - 1985}$$

$$= \frac{9 \text{ million}}{5} = 1.8$$

The rate of change from 1985–1990 is about 1.8 million people per year.

b. Because the slope of the graph is steepest during that period, the NHL attendance increased the most from 1995 to 2000.

$$\text{rate of change} = \frac{\text{change in attendance}}{\text{change in time}}$$
$$= \frac{18 \text{ million} - 6 \text{ million}}{5}$$
$$= \frac{12 \text{ million}}{5} = 2.4$$

The rate of change from 1995–2000 is about 2.4 million people per year.

c. The graph for the NFL attendance shows that there was a small but steady increase in attendance from 1985 to 2000.

42. The slope of the line using $(-3, 1)$ and $(0, k)$, $m_1 = \frac{k-1}{3}$, must be the same as the slope of the line using $(-3, 1)$ and $(k, 5)$, $m_2 = \frac{4}{k+3}$, in order for the points to be collinear.

$$m_1 = m_2$$
$$\frac{k-1}{3} = \frac{4}{k+3}$$

$(k - 1)(k + 3) = 12$

$k^2 + 2k - 15 = 0$

$(k + 5)(k - 3) = 0$

$k + 5 = 0$ or $k - 3 = 0$

$k = -5$ or $k = 3$

The two values of k are -5 and 3.

Quiz for the lessons "Prove Lines are Parallel" and "Find and Use Slopes of Lines"

1. Using the Consecutive Interior Angles Converse,

$2x + 54 = 180$

$2x = 126$

$x = 63.$

The lines are parallel when $x = 63$.

2. Using the Corresponding Angles Converse,

$3x - 5 = 145$

$3x = 150$

$x = 50.$

The lines are parallel when $x = 50$.

3. Using the Alternate Exterior Angles Converse,

$88 = 4x - 12$

$100 = 4x$

$25 = x.$

The lines are parallel when $x = 25$.

4. $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - (-1)}{3 - 1} = \frac{4}{2} = 2$

The slope of the line is 2.

5. $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 2}{4 - 1} = \frac{3}{3} = 1$

The slope of the line is 1.

6. $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6 - (-2)}{-7 - (-3)} = \frac{-4}{-4} = 1$

The slope of the line is 1.

Technology Activity for the lesson "Find and Use Slopes of Lines"

1. a–c. Answers will vary.

2. When one of the lines is vertical, its slope is undefined, so the product of the slopes is also undefined.

Lesson 3.5 Write and Graph Equations of Lines

Guided Practice for the lesson "Write and Graph Equations of Lines"

1. $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - (-1)}{3 - 0} = \frac{2}{3}$

y-intercept: -1

$y = mx + b$

$y = \frac{2}{3}x - 1$

2. $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 5}{1 - (-2)} = \frac{-3}{3} = -1$

$y = mx + b$

$2 = -1(1) + b$

$3 = b$

$y = -x + 3$

3. The line parallel to $y = 3x - 5$ has a slope of 3.

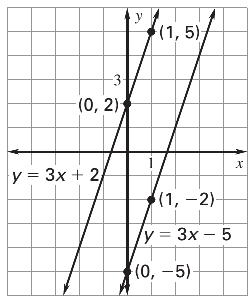
$m = 3, (x, y) = (1, 5)$

$y = mx + b$

$5 = 3(1) + b$

$2 = b$

$y = 3x + 2$

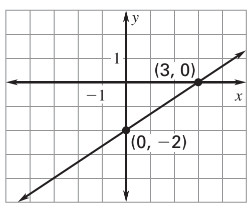


4. You know $x = 4$ and $y = 2$ are perpendicular because $x = 4$ is a vertical line and $y = 2$ is a horizontal line. Horizontal and vertical lines are always perpendicular.

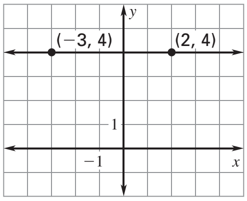
5. The slope is the monthly fee, \$50, and the y-intercept is the cost of joining the gym, \$125.

6. $2x - 3y = 6$

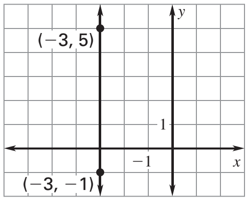
x-intercept:	y-intercept:
$2x - 3(0) = 6$	$2(0) - 3y = 6$
$2x = 6$	$-3y = 6$
$x = 3$	$y = -2$



7. $y = 4$

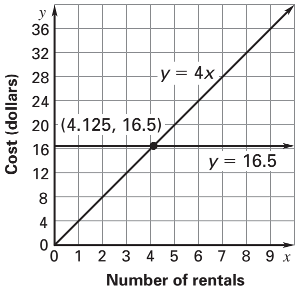


8. $x = -3$



9. Cost of one month's rental online: $y = 16.5$

Cost of one month's rental locally: $y = 4x$, where x represents the number of DVDs rented



The point of intersection is (4.125, 16.5). Using the graph, you can see that it is cheaper to rent online if you rent 5 or more DVDs per month.

10. With a 2-for-1 coupon, the equation for two or more local rentals is $y = 4(x - 1) = 4x - 4$. This graph intersects the graph of $y = 16.5$ at the point (5.125, 16.5). So, you would have to rent 6 or more DVDs to make the online rental the better buy.

Exercises for the lesson "Write and Graph Equations of Lines"

Skill Practice

1. In the expression *slope-intercept form*, the word *intercept* refers to the point where the line crosses the y-axis.

2. To find the x-intercept, let $y = 0$ and solve for x. To find the y-intercept, let $x = 0$ and solve for y.

3. $m = \dfrac{0 - (-4)}{3 - 0} = \dfrac{4}{3}$

y-intercept: -4

$y = mx + b$

$y = \frac{4}{3}x - 4$

4. $m = \dfrac{-2 - (-3)}{0 - (-5)} = \dfrac{1}{5}$

y-intercept: -2

$y = mx + b$

$y = \frac{1}{5}x - 2$

5. $m = \dfrac{-2 - 4}{1 - (-3)} = \dfrac{-6}{4} = -\dfrac{3}{2}$

$y = mx + b$

$4 = -\frac{3}{2}(-3) + b$

$4 = \frac{9}{2} + b$

$-\frac{1}{2} = b$

$y = -\frac{3}{2}x - \frac{1}{2}$

6. $m = \dfrac{-3 - 3}{2 - (-3)} = -\dfrac{6}{5}$

$y = mx + b$

$3 = -\frac{6}{5}(-3) + b$

$-\frac{3}{5} = b$

$y = -\frac{6}{5}x - \frac{3}{5}$

7. $m = \dfrac{6 - 0}{5 - 1} = \dfrac{6}{4} = \dfrac{3}{2}$

$y = mx + b$

$0 = \frac{3}{2}(1) + b$

$-\frac{3}{2} = b$

$y = \frac{3}{2}x - \frac{3}{2}$

8. $m = \dfrac{-3 - (-1)}{1 - (-5)} = \dfrac{-2}{6} = -\dfrac{1}{3}$

$y = mx + b$

$-3 = -\frac{1}{3}(1) + b$

$-\frac{8}{3} = b$

$y = -\frac{1}{3}x - \frac{8}{3}$

9. B; slope $= \dfrac{\text{rise}}{\text{run}} = \dfrac{-2}{4} = -\dfrac{1}{2}$

y-intercept: 1

$y = -\frac{1}{2}x + 1$

10. $m = -5, b = -12$

$y = mx + b$

$y = -5x - 12$

11. $m = 3, b = 2$

$y = mx + b$

$y = 3x + 2$

12. $m = 4, b = -6$

$y = mx + b$

$y = 4x - 6$

13. $m = -\frac{5}{2}, b = 0$

$y = mx + b$

$y = -\frac{5}{2}x$

14. $m = \frac{4}{9}, b = -\frac{2}{9}$

$y = mx + b$

$y = \frac{4}{9}x - \frac{2}{9}$

15. $m = -\frac{11}{5}, b = -12$

$y = mx + b$

$y = -\frac{11}{5}x - 12$

16. $P(-1, 0), m = -1$

$y = mx + b$

$0 = -1(-1) + b$

$-1 = b$

$y = -x - 1$

17. $P(5, 4), m = 4$

$y = mx + b$

$4 = 4(5) + b$

$-16 = b$

$y = 4x - 16$

18. $P(6, -2), m = 3$

$y = mx + b$

$-2 = 3(6) + b$

$-20 = b$

$y = 3x - 20$

19. $P(-8, -2), m = -\frac{2}{3}$

$y = mx + b$

$-2 = -\frac{2}{3}(-8) + b$

$-\frac{22}{3} = b$

$y = -\frac{2}{3}x - \frac{22}{3}$

20. $P(0, -3), m = -\frac{1}{6}$

$y = mx + b$

$-3 = -\frac{1}{6}(0) + b$

$-3 = b$

$y = -\frac{1}{6}x - 3$

21. $P(-13, 7), m = 0$

$y = mx + b$

$7 = 0(-13) + b$

$7 = b$

$y = 7$

22. If a line has an undefined slope, it is a vertical line. The equation of the vertical line that passes through the point $(3, -2)$ is $x = 3$.

23. A line parallel to $y = -2x + 3$ has a slope of -2.

$P(0, -1), m = -2$

$y = mx + b$

$-1 = -2(0) + b$

$-1 = b$

$y = -2x - 1$

24. A line parallel to $y = 16$ has a slope of 0.

$P(-7, -4), m = 0$

$y = mx + b$

$-4 = 0(-7) + b$

$-4 = b$

$y = -4$

25. A line parallel to $y - 1 = \frac{1}{5}(x + 4)$ has a slope of $\frac{1}{5}$.

$P(3, 8), m = \frac{1}{5}$

$y = mx + b$

$8 = \frac{1}{5}(3) + b$

$\frac{37}{5} = b$

$y = \frac{1}{5}x + \frac{37}{5}$

26. A line parallel to $x = -5$ is a vertical line with undefined slope.

$P(-2, 6)$, $m =$ undefined

$x = -2$

27. $10x + 4y = -8$

$4y = -10x - 8$

$y = -\frac{5}{2}x - 2$

A line parallel to $10x + 4y = -8$ has a slope of $-\frac{5}{2}$.

$P(-2, 1), m = -\frac{5}{2}$

$y = mx + b$

$1 = -\frac{5}{2}(-2) + b$

$-4 = b$

$y = -\frac{5}{2}x - 4$

28. $-x + 2y = 12$

$2y = x + 12$

$y = \frac{1}{2}x + 6$

A line parallel to $-x + 2y = 12$ has a slope of $\frac{1}{2}$.

$P(4, 0), m = \frac{1}{2}$

$y = mx + b$

$0 = \frac{1}{2}(4) + b$

$-2 = b$

$y = \frac{1}{2}x - 2$

29. D; $m = \frac{9 - 1}{2 - (-2)} = \frac{8}{4} = 2$

Choice D is the only equation with a slope of 2.

30. $P(0, 0), y = -9x - 1$

$-9 \cdot m = -1$

$m = \frac{1}{9}$

$y = mx + b$

$0 = \frac{1}{9}(0) + b$

$0 = b$

$y = \frac{1}{9}x$

31. $P(-1, 1), y = \frac{7}{3}x + 10$

$\frac{7}{3} \cdot m = -1$

$m = -\frac{3}{7}$

$y = mx + b$

$1 = -\frac{3}{7}(-1) + b$

$\frac{4}{7} = b$

$y = -\frac{3}{7}x + \frac{4}{7}$

32. $P(4, -6), y = -3$

The line $y = -3$ is horizontal, so a line perpendicular to it is vertical.

$x = 4$

33. $P(2, 3), y - 4 = -2(x + 3)$

$-2 \cdot m = -1$

$m = \frac{1}{2}$

$y = mx + b$

$3 = \frac{1}{2}(2) + b$

$2 = b$

$y = \frac{1}{2}x + 2$

34. $P(0, -5), x = 20$

The line $x = 20$ is vertical, so a line perpedicular to it is horizontal.

$y = -5$

35. $P(-8, 0)$

$3x - 5y = 6$

$-5y = -3x + 6$

$y = \frac{3}{5}x - \frac{6}{5}$

$\frac{3}{5} \cdot m = -1$

$m = -\frac{5}{3}$

$y = mx + b$

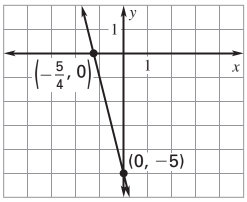
$0 = -\frac{5}{3}(-8) + b$

$-\frac{40}{3} = b$

$y = -\frac{5}{3}x - \frac{40}{3}$

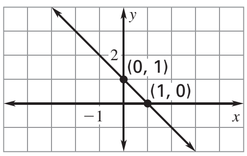
36. $8x + 2y = -10$

x-intercept:	y-intercept:
$8x + 2(0) = -10$	$8(0) + 2y = -10$
$8x = -10$	$2y = -10$
$x = -\frac{5}{4}$	$y = -5$



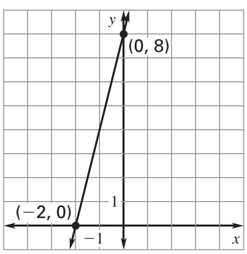
37. $x + y = 1$

x-intercept:	y-intercept:
$x + 0 = 1$	$0 + y = 1$
$x = 1$	$y = 1$



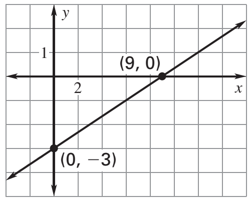
38. $4x - y = -8$

x-intercept:	y-intercept:
$4x - 0 = -8$	$4(0) - y = -8$
$4x = -8$	$y = 8$
$x = -2$	



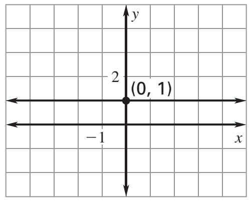
39. $-x + 3y = -9$

x-intercept:	y-intercept:
$-x + 3(0) = -9$	$-0 + 3y = -9$
$-x = -9$	$3y = -9$
$x = 9$	$y = -3$



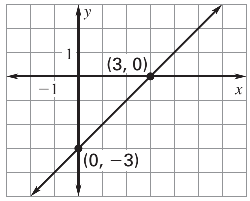
40. $y - 2 = -1$

$y = 1$



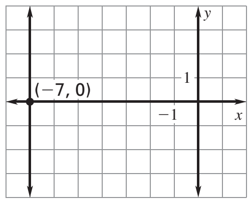
41. $y + 2 = x - 1$

$y = x - 3$



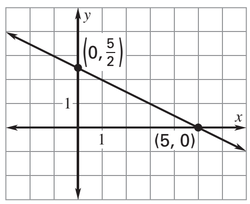
42. $x + 3 = -4$

$x = -7$



43. $2y - 4 = -x + 1$

x-intercept:	y-intercept:
$2(0) - 4 = -x + 1$	$2y - 4 = -0 + 1$
$-5 = -x$	$2y = 5$
$5 = x$	$y = \frac{5}{2}$

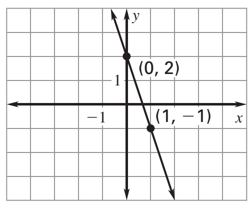


44. $3(x - 2) = -y - 4$

$3x - 6 = -y - 4$

$3x - 2 = -y$

$y = -3x + 2$



45. When finding the intercepts, the wrong variables were set equal to zero. To find the x-intercept, let $y = 0$:

$$5x - 3y = -15$$
$$5x - 3(0) = -15$$
$$x = -3$$

To find the y-intercept, let $x = 0$:

$$5x - 3y = -15$$
$$5(0) - 3y = -15$$
$$y = 5$$

46. midpoint of $\overleftrightarrow{PQ} = \left(\frac{-4+4}{2}, \frac{3+(-1)}{2}\right) = (0, 1)$

slope of $\overleftrightarrow{PQ} = \frac{-1-3}{4-(-4)} = \frac{-4}{8} = -\frac{1}{2}$

slope of perpendicular bisector $= 2$

The y-intercept of the perpendicular bisector is 1, because the bisector contains the midpoint (0, 1).

slope-intercept form of perp. bisector: $y = 2x + 1$

47. midpoint of $\overleftrightarrow{PQ} = \left(\frac{-5+3}{2}, \frac{-5+3}{2}\right) = (-1, -1)$

slope of $\overleftrightarrow{PQ} = \frac{3-(-5)}{3-(-5)} = \frac{2}{2} = 1$

slope of perpendicular bisector $= -1$

Find the y-intercept b of the perpendicular bisector by using $m = -1$ and $(x, y) = (-1, -1)$.

$y = mx + b$; $-1 = -1(-1) + b$; $-1 = 1 + b$; $b = -2$

slope-intercept form of perp. bisector: $y = -x - 2$

48. midpoint of $\overleftrightarrow{PQ} = \left(\frac{0+6}{2}, \frac{2+(-2)}{2}\right) = (3, 0)$

slope of $\overleftrightarrow{PQ} = \frac{-2-2}{6-0} = \frac{-4}{6} = -\frac{2}{3}$

slope of perpendicular bisector $= \frac{3}{2} = 1.5$

Find the y-intercept b of the perpendicular bisector by using $m = 1.5$ and $(x, y) = (3, 0)$.

$y = mx + b$; $0 = 1.5(3) + b$; $0 = 4.5 + b$; $b = -4.5$

slope-intercept form of perp. bisector: $y = 1.5x - 4.5$

49. x-intercept: 4

y-intercept: 4

$m = \frac{0-4}{4-0} = -\frac{4}{4} = -1$

$y = mx + b$

$y = -x + 4$

50. x-intercept: -2

y-intercept: -5

$m = \frac{-5-0}{0-(-2)} = -\frac{5}{2}$

$y = mx + b$

$y = -\frac{5}{2}x - 5$

51. x-intercept: -20

y-intercept: 10

$m = \frac{10-0}{0-(-20)} = \frac{10}{20} = \frac{1}{2}$

$y = mx + b$

$y = \frac{1}{2}x + 10$

52. $(-10, -3)$, $(6, 1)$

$m = \frac{1-(-3)}{6-(-10)} = \frac{4}{16} = \frac{1}{4}$

$y = mx + b$

$1 = \frac{1}{4}(6) + b$

$-\frac{1}{2} = b$

$y = \frac{1}{4}x - \frac{1}{2}$

$0 = \frac{1}{4}x - \frac{1}{2}$ $y = \frac{1}{4}(0) - \frac{1}{2}$

$\frac{1}{2} = \frac{1}{4}x$ $= -\frac{1}{2}$

$2 = x$

The x-intercept is 2. The y-intercept is $-\frac{1}{2}$.

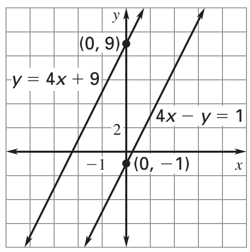
53. $y = 4x + 9$

$4x - y = 1$

$-y = -4x + 1$

$y = 4x - 1$

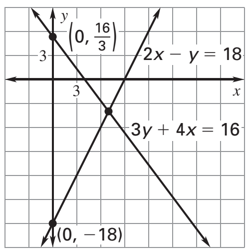
The lines are parallel, so they share no solutions.



54. $3y + 4x = 16$ $2x - y = 18$

$3y = -4x + 16$ $-y = -2x + 18$

$y = -\frac{4}{3}x + \frac{16}{3}$ $y = 2x - 18$

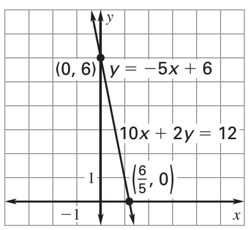


The equations share one solution.

55. $y = -5x + 6$ $10x + 2y = 12$

$2y = -10x + 12$

$y = -5x + 6$



The lines are the same, so they share infinitely many solutions.

56. $y = 4x + 9$

$4x - y = 1$

$4x - (4x + 9) = 1$

$4x - 4x - 9 = 1$

$-9 \neq 1$

The equations do not share any solutions.

$3y + 4x = 16$

$2x - y = 18$

$-y = -2x + 18$

$y = 2x - 18$

$3(2x - 18) + 4x = 16$

$6x - 54 + 4x = 16$

$10x = 70$

$x = 7$

$y = 2(7) - 18$

$y = -4$

The equations share one solution.

$y = -5x + 6$

$10x + 2y = 12$

$10x + 2(-5x + 6) = 12$

$10x - 10x + 12 = 12$

$12 = 12$

The equations share infinitely many solutions.

Sample answer: If a point is found, then the lines intersect at that point. If the variables drop out and the result is a false statement, then the lines are parallel. If the variables drop out and the result is a true statement, then the lines are the same line.

57. $y = x + 1$

$m = 1$

$(-1, k), (-7, -2)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$1 = \frac{-2 - k}{-7 - (-1)}$$

$-6 = -2 - k$

$-4 = -k$

$4 = k$

58. A line perpendicular to $y = x - \frac{28}{5}$ has a slope of $m = -1$. $(k, 2), (7, 0)$

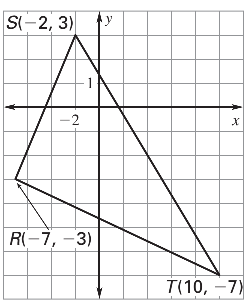
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$-1 = \frac{0 - 2}{7 - k}$$

$-7 + k = -2$

$k = 5$

59.



$$m_{RS} = \frac{3 - (-3)}{-2 - (-7)} = \frac{6}{5}$$

$y = mx + b$

$3 = \frac{6}{5}(-2) + b$

$\frac{27}{5} = b$

$\overleftrightarrow{RS}$: $y = \frac{6}{5}x + \frac{27}{5}$

$$m_{ST} = \frac{-7 - 3}{10 - (-2)} = \frac{-10}{12} = -\frac{5}{6}$$

$y = mx + b$

$3 = -\frac{5}{6}(-2) + b$

$\frac{4}{3} = b$

$\overleftrightarrow{ST}$: $y = -\frac{5}{6}x + \frac{4}{3}$

$$m_{RT} = \frac{-7 - (-3)}{10 - (-7)} = \frac{-4}{17} = -\frac{4}{17}$$

$y = mx + b$

$-7 = -\frac{4}{17}(10) + b$

$-\frac{79}{17} = b$

$\overleftrightarrow{RT}$: $y = -\frac{4}{17}x - \frac{79}{17}$

The slope of $\overleftrightarrow{RS}$ and $\overleftrightarrow{ST}$ are negative reciprocals, so the lines are perpendicular. So $\triangle RST$ has one right angle.

Problem Solving

60. $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{280 - 50}{10 - 0} = \frac{230}{10} = 23$

$y = mx + b$

$50 = 23(0) + b$

$50 = b$

An equation of the line is $y = 23x + 50$. The slope is the cost per month. The y-intercept is the initial fee.

$y = 23x + 50$

$y = 23(12) + 50$

$= 326$

The total cost of using the web hosting service for one year is \$326.

61.

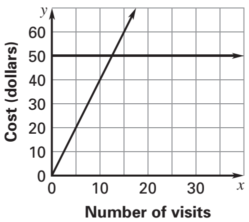
Total Weight	=	Rate of weight gain per day	•	Days since age 14	+	Weight at age 14
y	=	2.1	•	x	+	2000

The slope, 2.1, represents the rate of weight gain (in kilograms per day). The y-intercept, 2000, represents the weight (in kilograms) of the dinosaur at age 14.

62. **a.** Cost using a pass: $y = 50$

Cost paying per visit: $y = 4x$

b.



c. The point of intersection is (12.5, 50). You need to visit the park at least 13 times for the pass to be cheaper. The point of intersection represents the point at which the costs are equal. So, any number of visits beyond this point would be cheaper using the pass.

63. $Ax + By = C$

$2x + 3y = 24$

The value of A is the cost of a small slice, the value of B is the cost of a large slice, the value of C is the total cost.

64. $x =$ time (minutes)

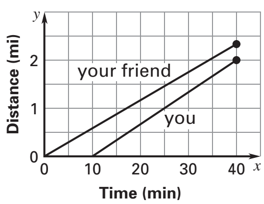
$y =$ distance run (miles)

Your friend: $\frac{3.5 \text{ mi}}{\text{h}} \cdot \frac{1 \text{ h}}{60 \text{ min}} = \frac{7}{120}$ mi/min

$y = \frac{7}{120}x$

You: $\frac{4 \text{ mi}}{\text{h}} \cdot \frac{1 \text{ h}}{60 \text{ min}} = \frac{1}{15}$ mi/min

$y = \frac{1}{15}(x - 10)$

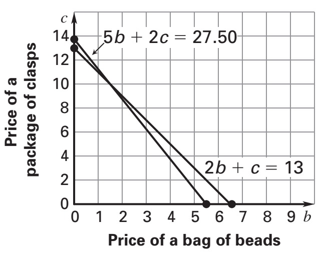


Because the lines on the graph do not intersect, you will not catch up to your friend.

65. **a.** Total cost for Audrey: $2b + c = 13$

Total cost for Sarah: $5b + 2c = 27.50$

b.



c. The intersection represents the prices of one bag of beads and one package of clasps.

66. $m_1 = \frac{174 - 112}{4 - 2} = 31$

$m_2 = \frac{102 - 62}{3 - 1} = 20$

Equation for first gym:

$y = mx + b$

$112 = 31(2) + b$

$50 = b$

$y = 31x + 50$

Equation for second gym:

$y = mx + b$

$62 = 20(1) + b$

$42 = b$

$y = 20x + 42$

Point of intersection:

$31x + 50 = 20x + 42$

$11x = -8$

$x = -\frac{8}{11}$

$y = 31\left(-\frac{8}{11}\right) + 50 = \frac{302}{11}$

The graphs intersect at $\left(-\frac{8}{11}, \frac{302}{11}\right)$.

The second gym is cheaper because it has a lower initial cost and a lower monthly cost.

Problem Solving Workshop for the lesson "Write and Graph Equations of Lines"

1. Cost to buy skates: $y = 130$

Cost to rent skates per hour: $y = 5x$, where x represents the number of hours

$y = 5x$

$130 = 5x$

$26 = x$

It is cheaper to buy skates if you skate more than 26 hours.

2. Cost to buy skates: $y = 130$

Cost to rent skates per day: $y = 12x$, where x represents the number of days

$y = 12x$

$130 = 12x$

$10.83 = x$

It is cheaper to buy skates if you skate 11 or more days.

3. Let $x =$ number of buttons sold

Charge per button • Number of buttons sold = Total spent

$2 \cdot x = 200 + 30$

$2x = 230$

$x = 115$

You need to sell 115 buttons to earn back what you spent.

4. Let $x =$ number of widgets

$15x = 1200 + 5x$

$10x = 1200$

$x = 120$

They need to sell 120 widgets to earn back the money spent on the machine.

5. Answers will vary.

6.

Years	Amount in savings	Amount in CD
0	\$1000	\$1000
1	\$1015	\$1030
2	\$1030.23	\$1060.90
3	\$1045.68	\$1092.73
4	\$1061.37	\$1125.51
5	\$1077.29	\$1159.28

It does not make sense to put your money in the savings account. Because the CD earns more interest, more money will always be earned there with the same initial amount invested.

Lesson 3.6 Prove Theorems about Perpendicular Lines

Activity in the lesson "Prove Theorems about Perpendicular Lines"

1. Right angles appear to be formed.
2. All four angles are congruent and they are right angles.

Guided Practice for the lesson "Prove Theorems about Perpendicular Lines"

1. By Theorem 3.8, $\angle ABD$ is a right angle because two lines intersect to form a linear pair of congruent angles. So, $\angle 3$ and $\angle 4$ are complementary by Theorem 3.10.
2. Each of the angles can be proven to be a right angle by the definition of perpendicular lines.
3. Yes; because $b \perp d$ and $a \perp d$, $b \parallel a$ by Theorem 3.12.
4. Yes, because $c \parallel d$ by Theorem 3.12 and $b \perp d$, then $b \perp c$ by Theorem 3.11.
5. Points on line c: $(-1, 0)$, $(0, 2)$

$m = \frac{2 - 0}{0 - (-1)} = 2$

$y = mx + b$

$2 = 2(0) + b$

$2 = b$

An equation for line c is $y = 2x + 2$.

Line perpendicular to line c passing through point A:

$A(-3, 2)$, $m = -\frac{1}{2}$

$y = mx + b$

$2 = -\frac{1}{2}(-3) + b$

$\frac{1}{2} = b$

$y = -\frac{1}{2}x + \frac{1}{2}$

Point of intersection:

$2x + 2 = -\frac{1}{2}x + \frac{1}{2}$ $\quad y = 2\left(-\frac{3}{5}\right) + 2$

$\frac{5}{2}x = -\frac{3}{2}$ $\quad y = \frac{4}{5}$

$x = -\frac{3}{5}$

Distance from point A to line c:

$d = \sqrt{\left(-3 - \left(-\frac{3}{5}\right)\right)^2 + \left(2 - \frac{4}{5}\right)^2} = \frac{6\sqrt{5}}{5} \approx 2.7$

6. Choose point $(0, -2)$ on line d.

Line perpendicular to line c passing through $(0, -2)$:

$m = -\frac{1}{2}$

$y = mx + b$

$-2 = -\frac{1}{2}(0) + b$

$-2 = b$

$y = -\frac{1}{2}x - 2$

Point of intersection:

$2x + 2 = -\frac{1}{2}x - 2$ $\quad y = 2\left(-\frac{8}{5}\right) + 2$

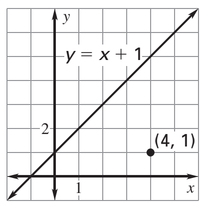
$\frac{5}{2}x = -4$ $\quad y = -\frac{6}{5}$

$x = -\frac{8}{5}$

Distance from line c to line d:

$d = \sqrt{\left(-\frac{8}{5} - 0\right)^2 + \left(-\frac{6}{5} - (-2)\right)^2} = \frac{4\sqrt{5}}{5} \approx 1.8$

7.



Line perpendicular to $y = x + 1$ passing through point $(4, 1)$:

$m = -1$, $y = mx + b$

$1 = -1(4) + b$

$5 = b$

$y = -x + 5$

Point of intersection:

$x + 1 = -x + 5$ $\quad y = 2 + 1$

$2x = 4$ $\quad y = 3$

$x = 2$

The point on the line that is the shortest distance from $(4, 1)$ is $(2, 3)$.

The distance between these two point is:

$d = \sqrt{(2 - 4)^2 + (3 - 1)^2} = \sqrt{4 + 4} \approx 2.8$

Exercises for the lesson "Prove Theorems about Perpendicular Lines"

Skill Practice

1. The length of $\overline{AB}$ is called the distance between the two parallel lines because it is the perpendicular segment joining the lines.
2. If two lines intersect to form a linear pair of congruent angles, then the lines are perpendicular.
3. If two sides of two adjacent acute angles are perpendicular, then the angles are complementary.
4. If two lines are perpendicular, then they intersect to form four right angles.
5. $m\angle 1 + 65° = 90°$

   $m\angle 1 = 25°$
6. $m\angle 1 = 90°$
7. $m\angle 1 + 38° = 90°$

   $m\angle 1 = 52°$
8. Lines m and n are both perpendicular to t, so by Theorem 3.12, $m \| n$.
9. Lines n and t intersect to form a linear pair of congruent angles, so by Theorem 3.8, $n \perp t$. Because m and n are both perpendicular to t, by Theorem 3.12, $m \| n$.
10. Because $x°$ is supplementary to a right angle, $x° = 90°$. So, $n \perp t$. Because m and n are both perpendicular to t, by Theorem 3.12, $m \| n$.
11. Draw a line using the straightedge. Draw a second line perpendicular to the first line using the protractor. Draw a third line perpendicular to the second line using the protractor. The first and third lines are parallel.
12. Fold the paper in thirds lengthwise, creating two parallel lines across the paper. Unfold the paper and then fold it in half widthwise, creating a line that is perpendicular to the first two parallel lines.
13. You would have to know that both y and z are perpendicular to x to know that $y \| z$. This information is not given.
14. $\overline{AC}$ is not perpendicular to $\overleftrightarrow{AB}$, so its length is not the distance from $\overleftrightarrow{AB}$ to point C.
15. $x + 14 + 63 = 90$

    $x + 77 = 90$

    $x = 13$
16. $x - 25 + 20 = 90$

    $x - 5 = 90$

    $x = 95$
17. $2x - 9 + x = 90$

    $3x = 99$

    $x = 33$
18. Lines n and p are parallel because they both are perpendicular to line k.
19. Lines f and g are parallel because they both are perpendicular to line d.
20. Lines v, w, and x are parallel because they all are perpendicular to line z or y. Lines z and y are parallel because they both are perpendicular to line w.
21. A; By Theorem 3.10, because $c \perp d$, $\angle 1$ and $\angle 2$ are complementary.
22. The distance between two lines is only defined for parallel lines because the distance between nonparallel lines is not constant.
23. Line through point (0, 0):

$$m = \frac{4-0}{1-0} = 4 \rightarrow y = 4x + b$$
$$0 = 4(0) + b$$
$$0 = b$$

$y = 4x$

Line perpendicular to $y = 4x$ and passing through (5, 3):

$$m = -\frac{1}{4} \rightarrow y = -\frac{1}{4}x + b$$
$$3 = -\frac{1}{4}(5) + b$$
$$\frac{17}{4} = b$$
$$y = -\frac{1}{4}x + \frac{17}{4}$$

Point of intersection:

$$4x = -\frac{1}{4}x + \frac{17}{4} \qquad y = 4(1)$$
$$\frac{17}{4}x = \frac{17}{4} \qquad y = 4$$
$$x = 1$$

The point of intersection is (1, 4).

Distance between (5, 3) and (1, 4):

$d = \sqrt{(1-5)^2 + (4-3)^2} = \sqrt{17} \approx 4.1$

So, the distance between the two parallel lines is about 4.1 units.

24. Line through point (2, 4):

$$m = \frac{0-4}{6-2} = -1 \rightarrow y = -1x + b$$
$$4 = -1(2) + b$$
$$6 = b$$

$y = -x + 6$

Line perpendicular to $y = -x + 6$ and passing through (1, 1):

$$m = 1 \rightarrow y = 1x + b$$
$$1 = 1(1) + b$$
$$0 = b$$

$y = x$

Point of intersection:

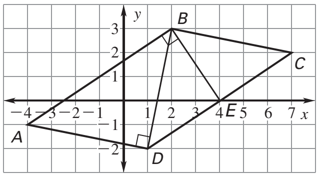
$$-x + 6 = x \qquad y = -(3) + 6$$
$$-2x = -6 \qquad y = 3$$
$$x = 3$$

The point of intersection is (3, 3).

Distance between (1, 1) and (3, 3):

$d = \sqrt{(3-1)^2 + (3-1)^2} = \sqrt{8} \approx 2.8$

So, the distance between the two parallel lines is about 2.8 units.

25. 

$\overline{AB}$ has slope $\frac{3-(-1)}{2-(-4)} = \frac{4}{6} = \frac{2}{3}$. The perpendicular to $\overline{AB}$ through B has slope $-\frac{3}{2}$ and intersects $\overline{DC}$ at $E(4, 0)$. The distance between $\overline{AB}$ and $\overline{DC}$ is

$BE = \sqrt{(4-2)^2 + (0-3)^2} = \sqrt{4+9} = \sqrt{13} \approx 3.6.$

$\overline{AD}$ has slope $\frac{-2-(-1)}{1-(-4)} = \frac{-1}{5}$. The perpendicular to $\overline{AD}$ through D has slope 5 and intersects $\overline{BC}$ at B.

The distance between $\overline{AD}$ and $\overline{BC}$ is

$BD = \sqrt{(1-2)^2 + (-2-3)^2} = \sqrt{1+25} = \sqrt{26} \approx 5.1.$

26. $m\angle 1 = 90°$ since it is a right angle.

$m\angle 3 = 30°$ by the Vertical Angles Congruence Theorem.

$m\angle 5 = 90°$ by the Vertical Angles Congruence Theorem.

$m\angle 2 + 30° = 90°$ because if the sides of two adjacent acute angles are perpendicular, then the angles are complementary. So, $m\angle 2 = 60°$. $m\angle 4 + 40° = m\angle 2$ by the Vertical Angles Congruence Theorem. So, $m\angle 4 = 20°$.

27. $y = \frac{3}{2}x + 4$

$$-3x + 2y = -1$$
$$2y = 3x - 1$$
$$y = \frac{3}{2}x - \frac{1}{2}$$

Both lines have the same slope, so they are parallel.

Line perpendicular to $y = \frac{3}{2}x + 4$ at point $(0, 4)$:

$$m = -\frac{2}{3} \rightarrow y = -\frac{2}{3}x + b$$
$$4 = -\frac{2}{3}(0) + b$$
$$4 = b$$
$$y = -\frac{2}{3}x + 4$$

Point of intersection of $y = \frac{3}{2}x - \frac{1}{2}$ and $y = -\frac{2}{3}x + 4$:

$$\frac{3}{2}x - \frac{1}{2} = -\frac{2}{3}x + 4 \qquad y = \frac{3}{2}\left(\frac{27}{13}\right) - \frac{1}{2}$$
$$\frac{13}{6}x = \frac{9}{2} \qquad y = \frac{34}{13}$$
$$x = \frac{27}{13}$$

The point of intersection is $\left(\frac{27}{13}, \frac{34}{13}\right)$.

Distance between $(0, 4)$ and $\left(\frac{27}{13}, \frac{34}{13}\right)$:

$$d = \sqrt{\left(\frac{27}{13} - 0\right)^2 + \left(\frac{37}{13} - 4\right)^2} = \frac{9\sqrt{13}}{13} \approx 2.5$$

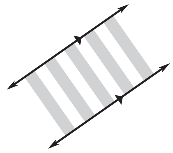
The distance between the lines is about 2.5 units.

28. Construct a perpendicular line to the plane passing through the point. You would find the distance from a point to a plane by finding the length of the perpendicular segment from the point to the point where the segment intersects the plane.

You can only find the distance from a line to a plane if they are parallel. If they are not parallel, the distance is not constant.

Problem Solving

29. You should jump to point C because the shortest distance between two parallel lines is the line perpendicular to them.

30. 

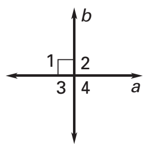
It would require less paint if the segments were perpendicular to the crosswalk because the shortest distance between the two sides of the crosswalk is a perpendicular segment.

31.

Statements	Reasons
1. $\angle 1$ and $\angle 2$ are a linear pair.	**1.** Given
2. $\angle 1$ and $\angle 2$ are supplementary.	**2.** Linear Pair Postulate
3. $m\angle 1 + m\angle 2 = 180°$	**3.** Definition of supplementary angles
4. $\angle 1 \cong \angle 2$	**4.** Given
5. $m\angle 1 = m\angle 2$	**5.** Definition of congruent angles
6. $m\angle 1 + m\angle 1 = 180°$	**6.** Substitution Property of Equality
7. $2(m\angle 1) = 180°$	**7.** Combine like terms.
8. $m\angle 1 = 90°$	**8.** Division Property of Equality
9. $\angle 1$ is a right angle.	**9.** Definition of a right angle
10. $g \perp h$	**10.** Definition of perpendicular lines

32. Given: $a \perp b$

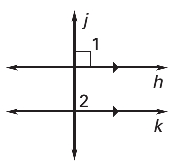
Prove: $\angle 1, \angle 2, \angle 3$, and $\angle 4$ are right angles.



Statements	Reasons
1. $a \perp b$	**1.** Given
2. $\angle 1$ is a right angle.	**2.** Definition of $\perp$ lines
3. $m\angle 1 = 90°$	**3.** Definition of right angle
4. $\angle 1$ and $\angle 4$ are vertical angles.	**4.** Definition of vertical angles
5. $m\angle 4 = 90°$	**5.** Vertical Angles Congruence Theorem
6. $\angle 4$ is a right angle.	**6.** Definition of right angle
7. $\angle 1$ and $\angle 2$ form a linear pair.	**7.** Definition of linear pair
8. $m\angle 1 + m\angle 2 = 180°$	**8.** Linear Pair Postulate
9. $90° + m\angle 2 = 180°$	**9.** Substitution Property of Equality
10. $m\angle 2 = 90°$	**10.** Subtraction Property of Equality
11. $\angle 2$ is a right angle.	**11.** Definition of right angle
12. $\angle 2$ and $\angle 3$ are vertical angles.	**12.** Definition of vertical angles
13. $m\angle 3 = 90°$	**13.** Vertical Angles Congruence Theorem
14. $\angle 3$ is a right angle.	**14.** Definition of right angle

33. Given: $h \parallel k, j \perp h$

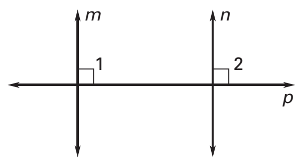
Prove: $j \perp k$



Statements	Reasons
1. $h \parallel k, j \perp h$	**1.** Given
2. $\angle 1 \cong \angle 2$	**2.** Corresponding Angles Postulate
3. $m\angle 1 = m\angle 2$	**3.** Definition of congruent angles
4. $\angle 1$ is a right angle.	**4.** Perpendicular lines intersect to form four right angles.
5. $m\angle 1 = 90°$	**5.** Definition of right angle
6. $m\angle 2 = 90°$	**6.** Substitution
7. $\angle 2$ is a right angle.	**7.** Definition of right angle
8. $j \perp k$	**8.** Definition of perpendicular lines

34. Given: $m \perp p, n \perp p$

Prove: $m \parallel n$



Statements	Reasons
1. $m \perp p, n \perp p$	**1.** Given
2. $\angle 1$ and $\angle 2$ are right angles.	**2.** Definition of perpendicular lines
3. $\angle 1 \cong \angle 2$	**3.** Right Angle Congruence Theorem
4. $m \parallel n$	**4.** Corresponding Angles Converse

35. $\overrightarrow{AB}$ is not necessarily perpendicular to $\overrightarrow{AC}$ because you do not know the relationship between $\angle 2$ and $\angle 3$.

36. $\overrightarrow{AB}$ is not necessarily perpendicular to $\overrightarrow{AC}$. You know $\angle 1$ and $\angle 2$ are complementary, but you do not know the relationship between $\angle 2$ and $\angle 3$.

37. If $m\angle 1 = m\angle 3$, $m\angle 2 = m\angle 4$, and $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 = 180°$, then $2m\angle 1 + 2m\angle 2 = 180°$. So, $m\angle 1 + m\angle 2 = 90°$. So, $\angle 1$ is complementary to $\angle 2$ and $\angle 3$ is complementary to $\angle 4$. By the Substitution Property, $\angle 2$ is complementary to $\angle 3$. So, $\overrightarrow{AB} \perp \overrightarrow{AC}$.

38. $m\angle 1 = 40°$ and $m\angle 4 = 50°$. The sum of the measures of all four angles is $180°$. $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 = 180°$. By substitution, $40° + m\angle 2 + m\angle 3 + 50° = 180°$. By the Subtraction Property of Equality, $m\angle 2 + m\angle 3 = 90°$. So, $\angle 2$ and $\angle 3$ form a right angle, and $\overrightarrow{AB} \perp \overrightarrow{AC}$.

Quiz for the lessons "Write and Graph Equations of Lines" and "Prove Theorems about Perpendicular Lines"

1. $P(0, 0), y = -3x + 1$

$m = -3$

$y = mx + b$

$0 = -3(0) + b$

$0 = b$

$y = -3x$

2. $P(-5, -6), y - 8 = 2x + 10$

$y = 2x + 18$

$m = 2$

$y = mx + b$

$-6 = 2(-5) + b$

$4 = b$

$y = 2x + 4$

3. $P(1, -2), x = 15$

The line parallel to $x = 15$ is vertical. The equation is $x = 1$.

4. $P(3, 4), y = 2x - 1$

$2m = -1 \rightarrow m = -\frac{1}{2}$

$y = mx + b$

$4 = -\frac{1}{2}(3) + b$

$\frac{11}{2} = b$

$y = -\frac{1}{2}x + \frac{11}{2}$

5. $P(2, 5), y = -6$

The line perpendicular to $y = -6$ is vertical. The equation is $x = 2$.

6. $(4, 0), 12x + 3y = 9$

$3y = -12x + 9$

$y = -4x + 3$

$-4m = -1 \rightarrow m = \frac{1}{4}$

$y = mx + b$

$0 = \frac{1}{4}(4) + b$

$-1 = b$

$y = \frac{1}{4}x - 1$

7. There is not enough information to determine whether any of the lines are parallel.

8. $b \parallel c$ because they are both perpendicular to the same line.

9. $m \parallel n$ because they are both perpendicular to the same line.

Investigating Geometry Construction for the lesson "Prove Theorems about Perpendicular Lines"

1. Postulate 16 (Corresponding Angles Converse)

2. *Sample answer:* In both types of constructions, the intersection of two arcs is used to perform the construction. In the construction of a segment bisector, the intersection points of the arcs determine two points on the segment bisector. In the construction of a perpendicular to a line, the two arcs determine a point that is used with a given point to draw the perpendicular.

3. Check constructions. The lines are perpendicular.

Theorem 3.11: If a transversal is perpendicular to one of two parallel lines, then it is perpendicular to the other.

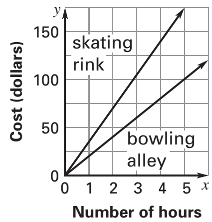
4. Check constructions. The lines are parallel.

Theorem 3.12: In a plane, if two lines are perpendicular to the same line, then they are parallel to each other.

Mixed Review of Problem Solving for the lessons "Find and Use Slopes of Lines", "Write and Graph Equations of Lines", and "Prove Theorems about Perpendicular Lines"

1. **a.** Roller skating rink: $y = 35x$

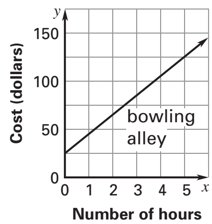
Bowling alley: $y = 20x$



b. The lines from part (a) are not parallel because they have differrent slopes.

c. The slope in each equation represents the cost per hour in dollars.

d. $y = 20x + 25$



This line is parallel to the line representing the bowling alley cost in part (a), because the lines have the same slope but different y-intercepts.

2. $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{275 - 200}{5 - 0} = \frac{75}{5} = 15$

The slope is 15.

3. *Sample answer:*

$2x + 3y = 6$

$3y = -2x + 6$

$y = -\frac{2}{3}x + 2$

A line parallel to this line is $y = -\frac{2}{3}x + 4$.
A line perpendicular to $y = -\frac{2}{3}x + 2$ is $y = \frac{3}{2}x$.

4. Equation of hiking path line:

$m = \frac{0 - 10}{50 - 25} = -\frac{10}{25} = -0.4$

$y = mx + b$

$0 = (-0.4)(50) + b$

$20 = b$

$y = -0.4x + 20$

Slope of the line perpendicular to the hiking path:

$m \cdot (-0.4) = -1$

$m = 2.5$

Equation of the line perpendicular to the hiking path, passing through point (60, 100):

$y = mx + b$

$100 = 2.5(60) + b$

$-50 = b$

$y = 2.5x - 50$

The point where the two lines intersect:

$$-0.4x + 20 = 2.5x - 50$$
$$70 = 2.9x$$
$$24.1 \approx x$$

$y = 2.5(24.1) - 50 = 10.25$

The distance between (60, 100) and (24.1, 10.25):

$$d = \sqrt{(24.1 - 60)^2 + (10.25 - 100)^2}$$
$$\approx \sqrt{1288.8 + 8055.1} \approx 96.7$$

The shortest distance you can walk to reach the hiking path is about 96.7 units. This distance is the shortest because it is a path perpendicular to the hiking path.

5. a. An equation for the height is $y = \frac{17}{25}x$, where x is the horizontal distance.

When $x = 500$: $y = \frac{17}{25}(500) = 340$

The car is 340 feet higher at the top than at its starting point.

b. The slope is $\frac{17}{25}$.

c. Because $\frac{7}{25} = 0.68$ and $0.7 > 0.68$, the Monongahela Incline is steeper.

Chapter Review for the chapter "Parallel and Perpendicular Lines"

1. Two lines that do not intersect and are not coplanar are called *skew lines*.

2. Alternate interior angle pairs lie between the two lines and on opposite sides of the transversal while consecutive interior angle pairs lie between the two lines and on the same side of the transversal.

3. $\angle 1$ and $\angle 5$ are corresponding angles.

4. $\angle 3$ and $\angle 6$ are alternate interior angles.

5. $\angle 4$ and $\angle 6$ are consecutive interior angles.

6. $\angle 7$ and $\angle 2$ are alternate interior angles.

7. The equation $14x - 2y = 26$ is in standard form.

8. The equation $y = 7x - 13$ is in slope-intercept form.

9. $\overline{NR} \perp \overleftrightarrow{QR}$

10. $\overline{NP} \parallel \overleftrightarrow{QR}$

11. $\overline{NJ}$ is skew to $\overleftrightarrow{QR}$.

12. Plane NJK is parallel to plane LMQ.

13. By the Vertical Angles Congruence Theorem, $m\angle 1 = 54°$. By the Alternate Interior Angles Theorem, $m\angle 2 = m\angle 1 = 54°$.

14. By the Consecutive Interior Angles Theorem, $m\angle 1 + 95° = 180°$. So, $m\angle 1 = 85°$. By the Alternate Interior Angles Theorem, $m\angle 2 = 95°$.

15. By the Corresponding Angles Postulate, $m\angle 1 = 135°$. $m\angle 1 + m\angle 2 = 180°$ because they are supplementary angles.

$$135° + m\angle 2 = 180°$$
$$m\angle 2 = 45°$$

16. By the Corresponding Angles Postulate, $y = 35$.

$x° + 35° = 180°$ because they are supplementary angles.

$$x + 35 = 180$$
$$x = 145$$

17. By the Alternate Interior Angles Theorem, $(5x - 17)° = 48°$.

$$5x - 17 = 48$$
$$5x = 65$$
$$x = 13$$

$48° + y° = 180°$ because they are supplementary angles.

$$48 + y = 180$$
$$y = 132$$

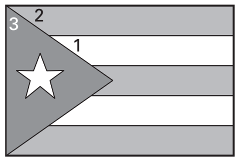
18. By the Corresponding Angles Postulate, $2y° = 58°$.

$$2y = 58$$
$$y = 29$$

$2x° + 58° = 180°$ because they are supplementary angles.

$$2x + 58 = 180$$
$$2x = 122$$
$$x = 61$$

19.



$\angle 2$ and $\angle 3$ are complementary. So, $m\angle 2 = 90° - m\angle 3 = 90° - 55° = 35°$. By the Corresponding Angles Postulate, $m\angle 1 = m\angle 2 = 35°$.

20. $x° + 73° = 180°$

$$x = 107$$

21. $(x + 14)° = 147°$

$$x = 133$$

22. $(2x + 20)° + 3x° = 180°$

$$5x + 20 = 180$$
$$5x = 160$$
$$x = 32$$

23. Line 1: (8, 12), (7, −5)

$$m_1 = \frac{-5 - 12}{7 - 8} = \frac{-17}{-1} = 17$$

Line 2: (−9, 3), (8, 2)

$$m_2 = \frac{2 - 3}{8 - (-9)} = \frac{-1}{17} = -\frac{1}{17}$$

The lines are perpendicular.

24. Line 1: (3, −4), (−1, 4)

$$m_1 = \frac{4 - (-4)}{-1 - 3} = \frac{8}{-4} = -2$$

Line 2: (2, 7), (5, 1)

$$m_2 = \frac{1 - 7}{5 - 2} = -\frac{6}{3} = -2$$

The lines are parallel.

25. a. $P(3, -1), y = 6x - 4$

$m = 6$

$y = mx + b$

$-1 = 6(3) + b$

$19 = b$

$y = 6x - 19$ is parallel to the given line.

b. $m = -\frac{1}{6}$

$y = mx + b$

$-1 = -\frac{1}{6}(3) + b$

$-\frac{1}{2} = b$

$y = -\frac{1}{6}x - \frac{1}{2}$ is perpendicular to the given line.

26. a. $P(-6, 5), 7y + 4x = 2$

$7y = -4x + 2$

$y = -\frac{4}{7}x + \frac{2}{7}$

$m = -\frac{4}{7}$

$y = mx + b$

$5 = -\frac{4}{7}(-6) + b$

$\frac{11}{7} = b$

$y = -\frac{4}{7}x + \frac{11}{7}$ is parallel to the given line.

b. $m = \frac{7}{4}$

$y = mx + b$

$5 = \frac{7}{4}(-6) + b$

$\frac{31}{2} = b$

$y = \frac{7}{4}x + \frac{31}{2}$ is perpendicular to the given line.

27. The lines have a slope of $\frac{3 - 6}{-1 - 0} = \frac{-3}{-1} = 3$, so a perpendicular line has a slope of $-\frac{1}{3}$.

The segment from $(-1, 3)$ to $(2, 2)$ has a slope of $\frac{2 - 3}{2 - (-1)} = -\frac{1}{3}$.

So, the distance between the lines is

$d = \sqrt{(2 - (-1))^2 + (2 - 3)^2} = \sqrt{10} \approx 3.2$ units.

28. The lines have a slope of $\frac{8 - 6}{3 - (-2)} = \frac{2}{5}$, so a perpendicular line has a slope of $-\frac{5}{2}$.

The segment from $(-2, 6)$ to $(0, 1)$ has a slope of $\frac{1 - 6}{0 - (-2)} = -\frac{5}{2}$.

So, the distance between the lines is

$d = \sqrt{(0 - (-2))^2 + (1 - 6)^2} = \sqrt{29} \approx 5.4$ units.

Chapter Test for the chapter "Parallel and Perpendicular Lines"

1. $\angle 1$ and $\angle 8$ are alternate exterior angles.

2. $\angle 2$ and $\angle 6$ are corresponding angles.

3. $\angle 3$ and $\angle 5$ are consecutive interior angles.

4. $\angle 4$ and $\angle 5$ are alternate interior angles.

5. $\angle 3$ and $\angle 7$ are corresponding angles.

6. $\angle 3$ and $\angle 6$ are alternate interior angles.

7. $x = 140$

8. $(18x - 22)° = 50°$

$18x = 72$

$x = 4$

9. $(4x + 11)° = 107°$

$4x = 96$

$x = 24$

10. $x° + 137° = 180°$

$x = 43$

11. $x° = (128 - x)°$

$2x = 128$

$x = 64$

12. $73° + (x + 17)° = 180°$

$x + 90 = 180$

$x = 90$

13. $(3, -1), (3, 4)$

$m = \frac{4 - (-1)}{3 - 3} = \frac{5}{0}$

The slope is undefined.

14. $(2, 7), (-1, -3)$

$m = \frac{-3 - 7}{-1 - 2} = \frac{-10}{-3} = \frac{10}{3}$

15. $(0, 5), (-6, 12)$

$m = \frac{12 - 5}{-6 - 0} = -\frac{7}{6}$

16. $P(-2, 4), m = 3$

$y = mx + b$

$4 = 3(-2) + b$

$10 = b$

$y = 3x + 10$

17. $P(7, 12), m = -\frac{1}{5}$

$y = mx + b$

$12 = -\frac{1}{5}(7) + b$

$\frac{67}{5} = b$

$y = -\frac{1}{5}x + \frac{67}{5}$

18. $P(3, 5), m = -8$

$y = mx + b$

$5 = -8(3) + b$

$29 = b$

$y = -8x + 29$

19. $P(1, 3), y = 2x - 1$

$2m = -1 \rightarrow m = -\frac{1}{2}$

$y = mx + b$

$3 = -\frac{1}{2}(1) + b$

$\frac{7}{2} = b$

$y = -\frac{1}{2}x + \frac{7}{2}$

20. $P(0, 2), y = -x + 3$

$-m = -1 \rightarrow m = 1$

$y = mx + b$

$2 = 1(0) + b$

$2 = b$

$y = x + 2$

21. $P(2, -3), x - y = 4$

$-y = -x + 4$

$y = x - 4$

$m = -1$

$y = mx + b$

$-3 = -1(2) + b$

$-1 = b$

$y = -x - 1$

22. $68° + x° = 90°$

$x = 22$

23. $51° + 3x° = 90°$

$3x = 39$

$x = 13$

24. $x° + (8x + 9)° = 90°$

$9x = 81$

$x = 9$

25. $(0, 30), (50, 60)$

$m = \frac{60 - 30}{50 - 0} = \frac{30}{50} = \frac{3}{5}$

$y = mx + b$

$30 = \frac{3}{5}(0) + b$

$30 = b$

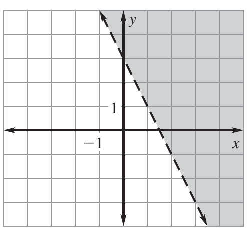
$y = \frac{3}{5}x + 30$

When $x = 100$: $y = \frac{3}{5}(100) + 30 = 90$

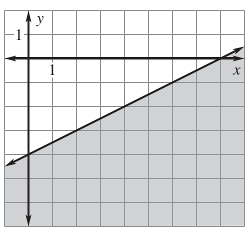
The cost of renting the van for a 100-mile trip is \$90.

Algebra Review for the chapter "Parallel and Perpendicular Lines"

1. $y > -2x + 3$

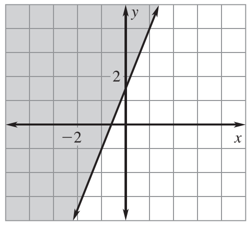


2. $y \leq 0.5x - 4$

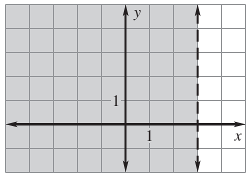


3. $-2.5x + y \geq 1.5$

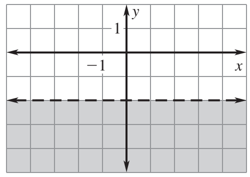
$y \geq 2.5x + 1.5$



4. $x < 3$



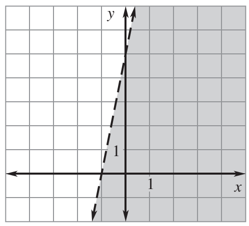
5. $y < -2$



6. $5x - y > -5$

$-y > -5x - 5$

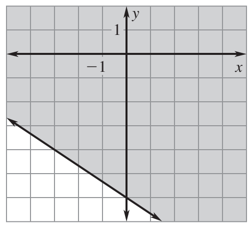
$y < 5x + 5$



7. $2x + 3y \geq -18$

$3y \geq -2x - 18$

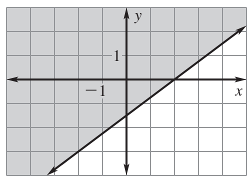
$y \geq -\frac{2}{3}x - 6$



8. $3x - 4y \leq 6$

$-4y \leq -3x + 6$

$y \geq \frac{3}{4}x - \frac{3}{2}$



9. Let x = number of months.

$8x \leq 46$

$x \leq 5.75$

It will take at most 6 months for Eric to repay his mother.

10. Let x = score on fourth quiz.

$\frac{76 + 81 + 77 + x}{4} \geq 80$

$\frac{234 + x}{4} \geq 80$

$234 + x \geq 320$

$x \geq 86$

He must get a score of at least 86 to have an average of at least 80.

11. $0.07x + 5 < 0.12x$

$5 < 0.05x$

$100 < x$

After 100 minutes of calls, the cost of using Company A is less than the cost of using Company B.

Extra Practice

For the chapter "Parallel and Perpendicular Lines"

1. $\angle 6$ and $\angle 2$ are corresponding angles.
2. $\angle 7$ and $\angle 2$ are alternate exterior angles.
3. $\angle 5$ and $\angle 3$ are consecutive interior angles.
4. $\angle 4$ and $\angle 5$ are alternate interior angles.
5. $\angle 1$ and $\angle 5$ are corresponding angles.
6. $\angle 3$ and $\angle 6$ are alternate interior angles.
7. $\angle AMB$ and $\angle HLM$ are corresponding angles.
   $\angle AMB$ and $\angle MJC$ are corresponding angles.
8. $\angle AML$ and $\angle MLK$ are alternate interior angles.
9. $\angle CJD$ and $\angle FKL$ are alternate exterior angles.
   $\angle CJD$ and $\angle AML$ are alternate exterior angles.
10. $\angle LMJ$ and $\angle MLK$ are consecutive interior angles.
    $\angle LMJ$ and $\angle MJK$ are consecutive interior angles.
11. $\overleftrightarrow{BG}$ is a transversal of $\overleftrightarrow{AD}$ and $\overleftrightarrow{HE}$.
    $\overleftrightarrow{CF}$ is a transversal of $\overleftrightarrow{AD}$ and $\overleftrightarrow{HE}$.
12. $m\angle 1 = 44°$, $m\angle 2 = 136°$; $m\angle 2 = 136°$ because when two parallel lines are cut by a transversal, the alternate exterior angles are congruent; $m\angle 1 = 44°$ because it is a linear pair with $\angle 2$ $(180° - 136° = 44°)$.
13. $m\angle 1 = 68°$, $m\angle 2 = 112°$; $m\angle 1 = 68°$ because when two parallel lines are cut by a transversal, the alternate interior angles are congruent; $m\angle 2 = 112°$ because it is a linear pair with $\angle 1$ $(180° - 68° = 112°)$.
14. $m\angle 1 = 106°$, $m\angle 2 = 106°$; $m\angle 1 = 106°$ because when two parallel lines are cut by a transversal, the corresponding angles are congruent; $m\angle 2 = 106°$ because vertical angles are congruent.
15. Because alternate interior angles are congruent,

$$9x° = 81°$$
$$x = 9$$

Because two angles that form a linear pair are supplementary,

$$(100 - y)° + 81° = 180°$$
$$181 - y = 180$$
$$181 = y + 180$$
$$1 = y$$

16. Because consecutive interior angles are supplementary,

$$3x° + 90° = 180° \qquad (13y + 5)° + (5y - 5)° = 180°$$
$$3x = 90 \qquad 18y = 180$$
$$x = 30 \qquad y = 10$$

17. Because alternate exterior angles are congruent,

$$(7y - 18)° = (6y + 1)°$$
$$y - 18 = 1$$
$$y = 19$$

Because two angles that form a linear pair are supplementary,

$$(6y + 1)° + (3x - 10)° = 180°$$
$$6(19) + 1 + 3x - 10 = 180$$
$$114 + 1 + 3x - 10 = 180$$
$$3x + 105 = 180$$
$$3x = 75$$
$$x = 25$$

18. No, there is not enough information to prove that $m \parallel n$.
19. Yes; You would use the Consecutive Interior Angles Converse Theorem. If two lines are cut by a transversal so that a pair of consecutive interior angles are supplementary, then the lines are parallel.
20. Yes; You would use the Consecutive Interior Angles Converse Theorem. If two lines are cut by a transversal so that a pair of consecutive interior angles are supplementary, then the lines are parallel.
21. Yes; If two lines are cut by a transversal so that alternate interior angles are congruent, then the lines are parallel.
22. Yes; *Sample answer:* If two lines are cut by a transversal so the alternate interior angles are congruent, then the lines are parallel.
23. Yes; If two lines are cut by a transversal so the consecutive interior angles are supplementary, then the lines are parallel.
24. Slope of Line 1: $m = \frac{5-4}{10-7} = \frac{1}{3}$

    Slope of Line 2: $m = \frac{5-3}{8-2} = \frac{2}{6} = \frac{1}{3}$

    Parallel; The lines are parallel because the slopes are equal.
25. Slope of Line 1: $m = \frac{5-1}{-2-(-3)} = \frac{4}{1} = 4$

    Slope of Line 2: $m = \frac{-2-(-3)}{5-(-1)} = \frac{1}{6}$

    Neither; The lines are not parallel because the slopes are not equal and the lines are not perpendicular because the product of the slopes is not -1.
26. Slope of Line 1: $m = \frac{7-0}{8-(-6)} = \frac{7}{14} = \frac{1}{2}$

    Slope of Line 2: $m = \frac{2-4}{2-1} = \frac{-2}{1} = -2$

    Perpendicular; The lines are perpendicular because the product of their slopes is -1.
27. Slope of Line 1: $m = \frac{-9-(-6)}{-4-0} = \frac{-3}{-4} = \frac{3}{4}$

    Slope of Line 2: $m = \frac{9-5}{1-(-2)} = \frac{4}{3}$

    Line 2 is steeper because $\left|\frac{4}{3}\right| > \left|\frac{3}{4}\right|$.
28. Slope of Line 1: $m = \frac{3-(-5)}{-1-(-1)} = \frac{8}{0}$ undefined

    Slope of Line 2: $m = \frac{4-4}{-5-(-3)} = \frac{0}{-2} = 0$

    Line 1 is steeper because Line 1 is a vertical line and Line 2 is a horizontal line.

29. Slope of Line 1: $m = \frac{6-1}{2-1} = \frac{5}{1} = 5$

Slope of Line 2: $m = \frac{10-1}{3-1} = \frac{9}{2} = 4\frac{1}{2}$

Line 1 is steeper because $|5| > \left|4\frac{1}{2}\right|$.

30. $y = mx + b$

When $m = 2$, $x = 4$, and $y = 7$:

$7 = 2(4) + b$

$7 = 8 + b$

$-1 = b$

So, $y = 2x - 1$.

31. $y = mx + b$

When $m = \frac{2}{3}$, $x = -3$, and $y = 0$:

$0 = \frac{2}{3}(-3) + b$

$0 = -2 + b$

$2 = b$

So, $y = \frac{2}{3}x + 2$.

32. $y = mx + b$

When $m = -\frac{1}{3}$, $x = 9$, and $y = 4$:

$4 = -\frac{1}{3}(9) + b$

$4 = -3 + b$

$7 = b$

So, $y = -\frac{1}{3}x + 7$.

33. $y = mx + b$

When $m = -2$, $x = 1$, and $y = -2$:

$-2 = -2(1) + b$

$-2 = -2 + b$

$0 = b$

So, $y = -2x$.

34. $y = mx + b$

When $m = -\frac{1}{3}$, $x = 6$, and $y = 3$:

$3 = -\frac{1}{3}(6) + b$

$3 = -2 + b$

$5 = b$

So, $y = -\frac{1}{3}x + 5$.

35. $y = mx + b$

When $m = 1$, $x = -7$, and $y = 3$:

$3 = 1(-7) + b$

$3 = -7 + b$

$10 = b$

So, $y = x + 10$.

36. $y = mx + b$

When $m = 4$, $x = 0$, and $y = 3$:

$3 = 4(0) + b$

$3 = 0 + b$

$3 = b$

So, $y = 4x + 3$.

37. $y = mx + b$

When $m = \frac{2}{5}$, $x = -9$, and $y = 4$:

$4 = \frac{2}{5}(-9) + b$

$4 = -\frac{18}{5} + b$

$\frac{38}{5} = b$

So, $y = \frac{2}{5}x + \frac{38}{5}$.

38. $y = mx + b$

When $m = 1$, $x = 8$, and $y = -3$:

$-3 = 1(8) + b$

$-3 = 8 + b$

$-11 = b$

So, $y = x - 11$.

39. $m\angle ADB + m\angle BDC = m\angle ADC$

$m\angle ADB + 21° = 90°$

$m\angle ADB = 69°$

40. $m\angle ADB = 90°$ because vertical angles are congruent.

41. $m\angle ADB + 17° = 90°$

$m\angle ADB = 73°$

42. $m\angle ADB + m\angle BDC = m\angle ADC$

$2x° + (x + 12)° = 90°$

$2x + x + 12 = 90$

$3x + 12 = 90$

$3x = 78$

$x = 26$

$m\angle ADB = 2x° = (2 \cdot 26)° = 52°$

43. $m\angle ADB + m\angle BDC = m\angle ADC$

$(3x + 32)° + 26x° = 90°$

$3x + 32 + 26x = 90$

$29x + 32 = 90$

$29x = 58$

$x = 2$

$m\angle ADB = (3x + 32)° = (3(2) + 32)°$

$= (6 + 32)° = 38°$

44. $m\angle ADB + m\angle BDC = m\angle ADC$

$(4x - 1)° + (2x + 1)° = 90°$

$4x - 1 + 2x + 1 = 90$

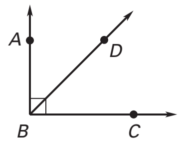
$6x = 90$

$x = 15$

$m\angle ADB = (4x - 1)° = (4(15) - 1)° = (60 - 1)° = 59°$

45. Given: $\overrightarrow{BA} \perp \overrightarrow{BC}$,
$\overrightarrow{BD}$ bisects $\angle ABC$.

Prove: $m\angle ABD = 45°$



Statements	Reasons
1. $\overrightarrow{BA} \perp \overrightarrow{BC}$	**1.** Given
2. $\angle ABC$ is a right angle.	**2.** Definition of perpendicular lines
3. $m\angle ABC = 90°$	**3.** Definition of right angle
4. $\overrightarrow{BD}$ bisects $\angle ABC$.	**4.** Given
5. $m\angle ABD = m\angle DBC$	**5.** Definition of angle bisector
6. $m\angle ABC = m\angle ABD + m\angle DBC$	**6.** Angle Addition Postulate
7. $m\angle ABD + m\angle DBC = 90°$	**7.** Transitive Property of Equality
8. $m\angle ABD + m\angle ABD = 90°$	**8.** Substitution Property of Equality
9. $2(m\angle ABD) = 90°$	**9.** Simplify.
10. $m\angle ABD = 45°$	**10.** Division Property of Equality