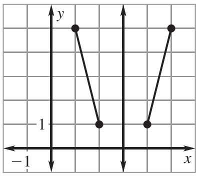
# Chapter 9 Quadratic Equations and Functions

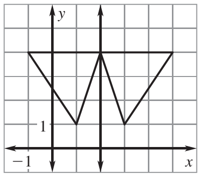
***Prerequisite Skills for the chapter "Quadratic Equations and Functions"***

1. The $x$-coordinate of a point where a graph crosses the $x$-axis is an *$x$-intercept*.
2. An *exponential function* is a function of the form $y = a \cdot b^x$ where $a \neq 0$, $b > 0$, and $b \neq 1$.

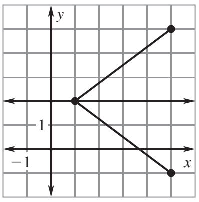
**3.**



**4.**



**5.**



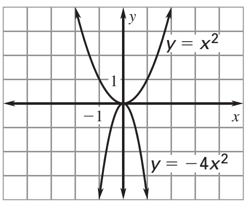
6. $\sqrt{81} = \sqrt{9 \cdot 9} = 9$
7. $-\sqrt{25} = -\sqrt{5 \cdot 5} = -5$
8. $\sqrt{1} = \sqrt{1 \cdot 1} = 1$
9. $\pm\sqrt{64} = \pm\sqrt{8 \cdot 8} = \pm 8$

## Lesson 9.1 Graph $y = ax^2 + c$

***Guided Practice for the lesson "Graph $y = ax^2 + c$"***

**1.**

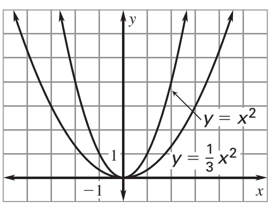
| **x** | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| **y** | −16 | −4 | 0 | −4 | −16 |




Both graphs have the same vertex, (0, 0), and axis of symmetry, $x = 0$, the graph of $y = -4x^2$ is narrower and opens down, because it is a vertical stretch (by a factor of 4) with a reflection in the $x$-axis.

**2.**

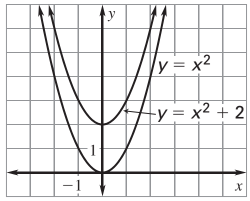
| **x** | −3 | −1 | 0 | 1 | 3 |
|---|---|---|---|---|---|
| **y** | 3 | $\frac{1}{3}$ | 0 | $\frac{1}{3}$ | 3 |




Both graphs open up, have the same vertex, (0, 0), and have the same axis of symmetry, $x = 0$. The graph of $y = \frac{1}{3}x^2$ is wider because it is a vertical shrink $\left(\text{by a factor of } \frac{1}{3}\right)$.

**3.**

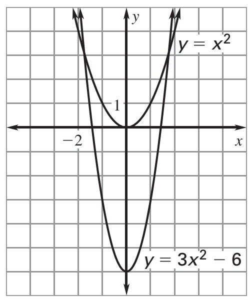
| **x** | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| **y** | 6 | 3 | 2 | 3 | 6 |




Both graphs open up and have the same axis of symmetry, $x = 0$. The vertex of the graph of $y = x^2 + 2$, (0, 2), is different than the vertex of the graph of $y = x^2$, (0, 0), because it is a vertical translation (of 2 units up).

**4.**

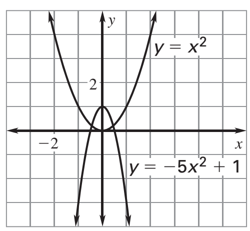
| **x** | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| **y** | 6 | −3 | −6 | −3 | 6 |




Both graphs open up and have the same axis of symmetry, $x = 0$. The graph of $y = 3x^2 - 6$ is narrower and has a lower vertex than the graph of $y = x^2$ because it is a vertical stretch and a vertical translation.

**5.**

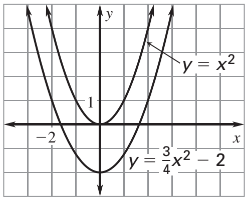
| **x** | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| **y** | −19 | −4 | 1 | −4 | −19 |




Both graphs have the same axis of symmetry. The graph of $y = -5x^2 + 1$ is narrower, opens down, and has a higher vertex because of a vertical stretch (by a factor of 5), a reflection through the $x$-axis; and a vertical translation.

6.

| x | −4 | −2 | 0 | 2 | 4 |
|---|---|---|---|---|---|
| y | 10 | 1 | −2 | 1 | 10 |




Both graphs open up and have the same axis of symmetry, $x = 0$. However, the graph of $y = \frac{3}{4}x^2 - 2$ is wider and has a lower vertex, because of a vertical shrink $\left(\text{by a factor of } \frac{3}{4}\right)$ and a vertical translation.

7. The graph of $y = x^2 + 2$ has vertex (0, 2) and the graph of $y = x^2 - 2$ has vertex (0, −2). Moving the vertex from (0, 2) to (0, −2) translates the graph 4 units down.

8. If the reflective surface extends 4 meters on either side of the origin, the domain is $-4 \le x \le 4$.

   Lowest point: $y = 0$

   Highest point: $y = 0.09(4)^2 = 1.44$

   The range is $0 \le y \le 1.44$.

### *Exercises for the lesson "Graph $y = ax^2 + c$"*

### *Skill Practice*

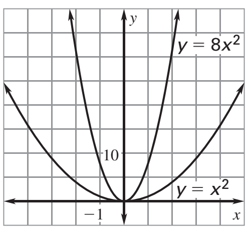
1. Every quadratic function has a U-shaped graph called a *parabola*.

2. The graph of the quadratic function $y = ax^2 + bx + c$ opens up if $a > 0$ and opens down if $a < 0$.

3. C   4. A   5. B

6.

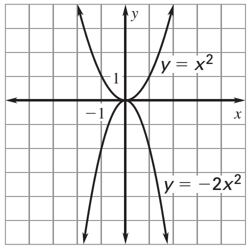
| x | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| y | 32 | 8 | 0 | 8 | 32 |




Both graphs open up, and have the same vertex, (0, 0), and have the same axis of symmetry, $x = 0$. The graph of $y = 8x^2$ is narrower because of a vertical stretch (by a factor of 8).

7.

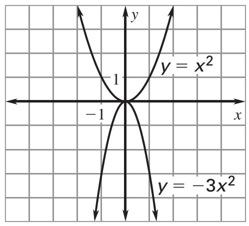
| x | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| y | −8 | −2 | 0 | −2 | −8 |




Both graphs have the same vertex, (0, 0), and the same axis of symmetry, $x = 0$. The graph of $y = -2x^2$ opens down and is narrower because of a reflection in the $x$-axis and a vertical stretch (by a factor of 2).

8.

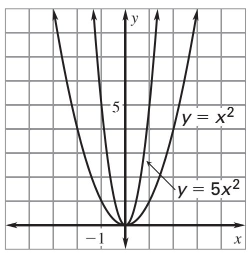
| x | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| y | −12 | −3 | 0 | −3 | −12 |




Both graphs have the same vertex, (0, 0), and the same axis of symmetry, $x = 0$. The graph of $y = -3x^2$ opens down and is narrower because of a reflection in the $x$-axis and a vertical stretch (by a factor of 3).

9.

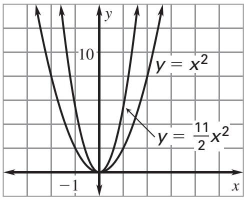
| x | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| y | 20 | 5 | 0 | 5 | 20 |




Both graphs open up, have the same vertex, (0, 0), and have the same axis of symmetry, $x = 0$. The graph of $y = 5x^2$ is narrower because of a vertical stretch (by a factor of 5).

**10.**

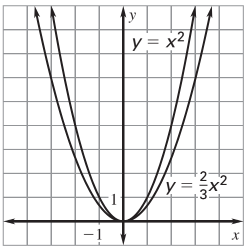
| $x$ | $-2$ | $-1$ | $0$ | $1$ | $2$ |
|---|---|---|---|---|---|
| $y$ | $22$ | $\frac{11}{2}$ | $0$ | $\frac{11}{2}$ | $22$ |




Both graphs open up, have the same vertex, (0, 0), and have the same axis of symmetry. The graph of $y = \frac{11}{2}x^2$ is narrower because of a vertical stretch $\left(\text{by a factor of } \frac{11}{2}\right)$.

**11.**

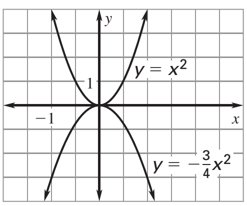
| $x$ | $-3$ | $-1$ | $0$ | $1$ | $3$ |
|---|---|---|---|---|---|
| $y$ | $6$ | $\frac{2}{3}$ | $0$ | $\frac{2}{3}$ | $6$ |




Both graphs open up, have the same vertex, (0, 0), and have the same axis of symmetry, $x = 0$. The graph of $y = \frac{2}{3}x^2$ is wider because of a vertical shrink $\left(\text{by a factor of } \frac{2}{3}\right)$.

**12.**

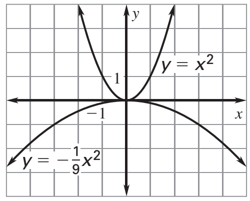
| $x$ | $-2$ | $-1$ | $0$ | $1$ | $2$ |
|---|---|---|---|---|---|
| $y$ | $-3$ | $-\frac{3}{4}$ | $0$ | $-\frac{3}{4}$ | $-3$ |




Both graphs have the same vertex, (0, 0), and have the same axis of symmetry, $x = 0$. The graph of $y = -\frac{3}{4}x^2$ opens down and is wider because of a reflection in the $x$-axis and a vertical shrink $\left(\text{by a factor of } \frac{3}{4}\right)$.

**13.**

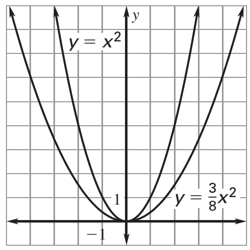
| $x$ | $-6$ | $-3$ | $0$ | $3$ | $6$ |
|---|---|---|---|---|---|
| $y$ | $-4$ | $-1$ | $0$ | $-1$ | $-4$ |




Both graphs have the same vertex, (0, 0), and the same axis of symmetry, $x = 0$. The graph of $y = -\frac{1}{9}x^2$ opens down and is wider because of a reflection in the $x$-axis and a vertical shrink $\left(\text{by a factor of } \frac{1}{9}\right)$.

**14.**

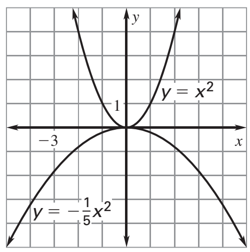
| $x$ | $-4$ | $-2$ | $0$ | $2$ | $4$ |
|---|---|---|---|---|---|
| $y$ | $6$ | $\frac{3}{2}$ | $0$ | $\frac{3}{2}$ | $6$ |




Both graphs open up, have the same vertex, (0, 0), and have the same axis of symmetry, $x = 0$. The graph of $y = \frac{3}{8}x^2$ is wider because of a vertical shrink $\left(\text{by a factor of } \frac{3}{8}\right)$.

**15.**

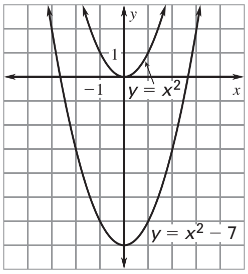
| $x$ | $-5$ | $-1$ | $0$ | $1$ | $5$ |
|---|---|---|---|---|---|
| $y$ | $-5$ | $-\frac{1}{5}$ | $0$ | $-\frac{1}{5}$ | $-5$ |




Both graphs have the same vertex, (0, 0), and the same axis of symmetry, $x = 0$. The graph of $y = -\frac{1}{5}x^2$ opens down and is wider because of a reflection in the $x$-axis and a vertical shrink $\left(\text{by a factor of } \frac{1}{5}\right)$.

16.

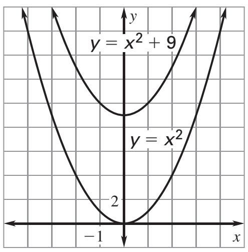
| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **x** | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
| **y** | 2 | −3 | −6 | −7 | −6 | −3 | 2 |




Both graphs open up and have the same axis of symmetry, $x = 0$. The graph of $y = x^2 - 7$ has a lower vertex because of a vertical translation (of 7 units down).

17.

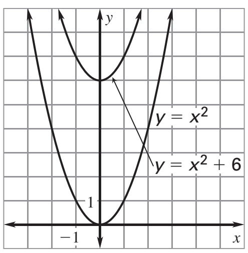
| | | | | | |
|---|---|---|---|---|---|
| **x** | −2 | −1 | 0 | 1 | 2 |
| **y** | 13 | 10 | 9 | 10 | 13 |




Both graphs open up and have the same axis of symmetry, $x = 0$. The vertex of the graph of $y = x^2 + 9$, (0, 9), is higher because of a vertical translation (of 9 units up).

18.

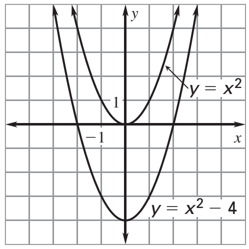
| | | | | | |
|---|---|---|---|---|---|
| **x** | −2 | −1 | 0 | 1 | 2 |
| **y** | 10 | 7 | 6 | 7 | 10 |




Both graphs open up and have the same axis of symmetry, $x = 0$. The vertex of the graph of $y = x^2 + 6$, (0, 6), is higher because of a vertical translation (of 6 units up).

19.

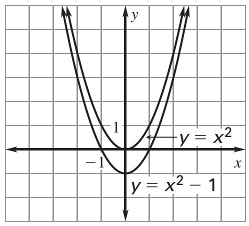
| | | | | | |
|---|---|---|---|---|---|
| **x** | −2 | −1 | 0 | 1 | 2 |
| **y** | 0 | −3 | −4 | −3 | 0 |




Both graphs open up and have the same axis of symmetry, $x = 0$. The vertex of the graph of $y = x^2 - 4$, (0, −4), is lower because of a vertical translation (of 4 units down).

20.

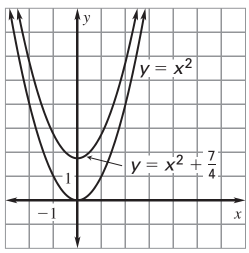
| | | | | | |
|---|---|---|---|---|---|
| **x** | −2 | −1 | 0 | 1 | 2 |
| **y** | 3 | 0 | −1 | 0 | 3 |




Both graphs open up and have the same axis of symmetry, $x = 0$. The vertex of the graph of $y = x^2 - 1$, (0, −1), is lower because of a vertical translation (1 unit down).

21.

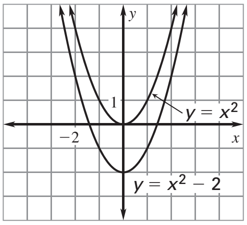
| | | | | | |
|---|---|---|---|---|---|
| **x** | −2 | −1 | 0 | 1 | 2 |
| **y** | $\frac{23}{4}$ | $\frac{11}{4}$ | $\frac{7}{4}$ | $\frac{11}{4}$ | $\frac{23}{4}$ |




Both graphs open up and have the same axis of symmetry, $x = 0$. The vertex of the graph of $y = x^2 + \frac{7}{4}$, $\left(0, \frac{7}{4}\right)$, is higher because of a vertical translation $\left(\frac{7}{4} \text{ units up}\right)$.

22. C; Because the graph of $y = -\frac{3}{4}x^2 + 7$ is a vertical translation of 7 units up, its vertex is (0, 7).

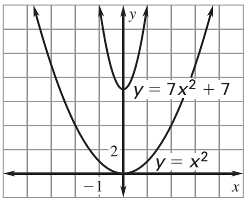
**23.** The graph of $y = x^2 - 2$ should be shifted 2 units down, not 2 units up. The vertex should be at $(0, -2)$.




Both graphs open up and have the same axis of symmetry. However, the vertex of the graph of $y = x^2 - 2$, $(0, -2)$ is 2 units below the vertex of the graph of $y = x^2$, $(0, 0)$.

**24.**

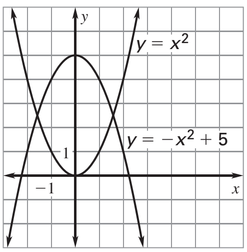
| **x** | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| **y** | 35 | 14 | 7 | 14 | 35 |




Both graphs open up and have the same axis of symmetry, $x = 0$. The graph of $y = 7x^2 + 7$ is narrower and has a higher vertex because of a vertical stretch and a vertical translation.

**25.**

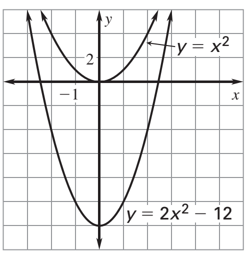
| **x** | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| **y** | 1 | 4 | 5 | 4 | 1 |




Both graphs have the same axis of symmetry, $x = 0$. The graph of $y = -x^2 + 5$ opens down and has a higher vertex because of a reflection in the $x$-axis and a vertical translation.

**26.**

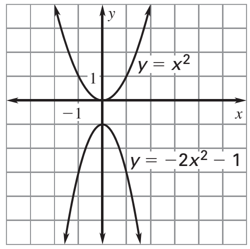
| **x** | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| **y** | −4 | −10 | −12 | −10 | −4 |




Both graphs open up and have the same axis of symmetry, $x = 0$. The graph of $y = 2x^2 - 12$ is narrower and has a lower vertex because of a vertical stretch and a vertical translation.

**27.**

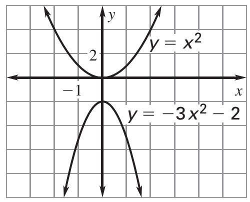
| **x** | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| **y** | −9 | −3 | −1 | −3 | −9 |




Both graphs open up and have the same axis of symmetry, $x = 0$. The graph of $y = -2x^2 - 1$ opens down, is narrower, and has a lower vertex because of a reflection in the $x$-axis, a vertical stretch, and a vertical translation.

**28.**

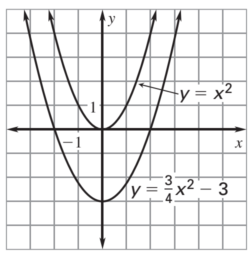
| **x** | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| **y** | −14 | −5 | −2 | −5 | −14 |




Both graphs open up and have the same axis of symmetry, $x = 0$. The graph of $y = -3x^2 - 2$ opens down, is narrower, and has a lower vertex because of a reflection in the $x$-axis, a vertical stretch, and a vertical translation.

**29.**

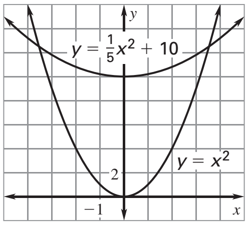
| **x** | −4 | −2 | 0 | 2 | 4 |
|---|---|---|---|---|---|
| **y** | 9 | 0 | −3 | 0 | 9 |




Both graphs open up and have the same axis of symmetry, $x = 0$. The graph of $y = \frac{3}{4}x^2 - 3$ opens down, is wider, and has a lower vertex than the graph of $y = x^2$ because of a vertical shrink and a vertical translation.

**30.**

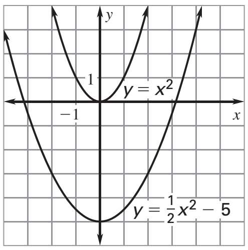
| $x$ | $-10$ | $-5$ | $0$ | $5$ | $10$ |
|---|---|---|---|---|---|
| $y$ | $30$ | $15$ | $10$ | $15$ | $30$ |




Both graphs open up and have the same axis of symmetry, $x = 0$. The graph of $y = \frac{1}{5}x^2 + 10$ is wider and has a higher vertex because of a vertical shrink and a vertical translation.

**31.**

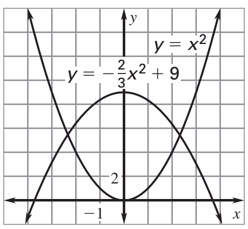
| $x$ | $-4$ | $-2$ | $0$ | $2$ | $4$ |
|---|---|---|---|---|---|
| $y$ | $3$ | $-3$ | $-5$ | $-3$ | $3$ |




Both graphs open up and have the same axis of symmetry, $x = 0$. The graph of $y = \frac{1}{2}x^2 + 5$ is wider and has a lower vertex because of a vertical shrink and a vertical translation.

**32.**

| $x$ | $-6$ | $-3$ | $0$ | $3$ | $6$ |
|---|---|---|---|---|---|
| $y$ | $-15$ | $3$ | $9$ | $3$ | $-15$ |




Both graphs open up and have the same axis of symmetry, $x = 0$. The graph of $y = -\frac{2}{3}x^2 + 9$ opens down is wider, and has a higher vertex because of a reflection in the $x$–axis, a vertical shrink, and a vertical translation.

**33.** B; The vertex of the graph of $y = x^2 + 3$ is (0, 3). The vertex of the graph of $y = x^2 + 9$ is (0, 9). Moving the vertex from (0, 3) to (0, 9) translates the graph 6 units up.

**34.** You can obtain the graph of $g(x) = x^2 + 8$ with vertex $(0, 8)$ from the graph of $f(x) = x^2 - 5$ with vertex $(0, -5)$ by translating 13 units up.

**35.** You can obtain the graph of $g(x) = 3x^2 - 16$ with vertex $(0, -16)$ from the graph of $f(x) = 3x^2 - 11$ with vertex $(0, -11)$ by translating 5 units down.

**36.** Both graphs have the same vertex, but the graph of $f(x) = 4x^2$ is vertically stretched by a factor of 4 and the graph of $g(x) = 2x^2$ is vertically stretched only by a factor of 2. You can obtain the graph of $g$ from the graph of $f$ by vertically stretching it by another factor of 2 or multiplying it by 2.

**37.** Use the form $y = ax^2 + c$.

| When $x = -1$ and $y = 9$: | When $x = 0$ and $y = 3$: |
|---|---|
| $9 = a(-1)^2 + c$ | $3 = a(0)^2 + c$ |
| $9 = a + c$ | $3 = c$ |

When $c = 3$: $9 = a + 3 \rightarrow 6 = a$

So, an equation is $y = 6x^2 + 3$.

**38.** Use the form $y = ax^2 + c$.

| When $x = 2$ and $y = 1$: | When $x = 5$ and $y = -20$: |
|---|---|
| $1 = a(2)^2 + c$ | $-20 = a(5)^2 + c$ |
| $1 = 4a + c$ | $-20 = 25a + c$ |
| $1 - 4a = c$ | |
| When $c = 1 - 4a$: | When $a = -1$: |
| $-20 = 25a + (1 - 4a)$ | $c = 1 - 4(-1)$ |
| $-21 = 21a$ | $c = 5$ |
| $-1 = a$ | |

So, an equation is $y = -x^2 + 5$.

**39.** Use the form $y = ax^2 + c$.

| When $x = -2$ and $y = -16.5$: | When $x = 1$ and $y = 4.5$: |
|---|---|
| $-16.5 = a(-2)^2 + c$ | $4.5 = a(1)^2 + c$ |
| $-16.5 = 4a + c$ | $4.5 = a + c$ |
| | $4.5 - a = c$ |
| When $c = 4.5 - a$: | When $a = -7$: |
| $-16.5 = 4a + (4.5 - a)$ | $4.5 - (-7) = c$ |
| $-21 = 3a$ | $11.5 = c$ |
| $-7 = a$ | |

So, an equation is $y = -7x^2 + 11.5$.

### *Problem Solving*

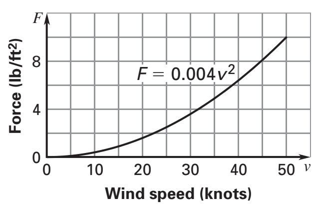
**40. a.** In the graph, the reflective surface extends 32 meters on either side of the origin. So, the domain is $-32 \le x \le 32$.

**b.** Lowest point: $y = 0$

Highest point: $y = 0.012(32)^2 = 12.288$

The range is $0 \le y \le 12.288$.

**41. a.**



**b.** A wind speed of about 16 knots will produce a force of 1 pound per square foot on a sail.

**c.** A wind speed of about 35 knots will produce a force of 5 pounds per square foot on a sail.

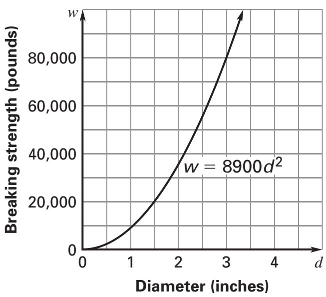
**42. a.** Vertical motion model: $h = -16t^2 + s$

Initial height of 45 feet: $h = -16t^2 + 45$

Initial height of 32 feet: $h = -16t^2 + 32$

**b.** The graph of $h = -16t^2 + 45$ is a vertical translation (of 13 units up) of the graph of $h = -16t^2 + 32$.

**43. a.**




**b.** Rope 1: $w_1 = 8900d_1^2$ Rope 2: $w_2 = 8900d_2^2$

$w_2 = 4w_1$

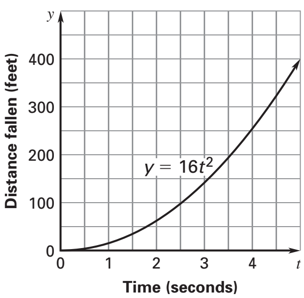
$w_2 = 4(8900)d_1^2$

$w_2 = 8900(4)d_1^2$

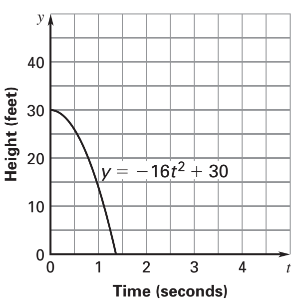
$w_2 = 8900(2d_1)^2$

So, $d_2 = 2d_1$. Rope 2 does not have 4 times the diameter of rope 1.

**44. a.**




**b.**




**c.** The graph from part (b) is a reflection in the $x$-axis of the graph from part (a), with a translation of 30 units up. To find the numbers of seconds after which the container has fallen 10 feet, in graph (a) find the $t$-value when $y = 10$, and in graph (b) find the $t$-value when $y = 20$.

**45.** $E = \frac{1}{2}mv^2$

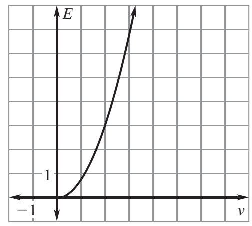
$918.75 = \frac{1}{2}m(35)^2$

$918.75 = 612.5m$

$1.5 = m$

$E = \frac{1}{2}(1.5)v^2$

$E = 0.75v^2$




## Lesson 9.2 Graph $y = ax^2 + bx + c$

### *Guided Practice for the lesson "Graph $y = ax^2 + bx + c$"*

**1.** $y = x^2 - 2x - 3$

Axis of symmetry: $x = -\frac{b}{2a} = -\frac{-2}{2(1)} = 1$

When $x = -\frac{b}{2a} = 1$: $y = (1)^2 - 2(1) - 3 = -4$

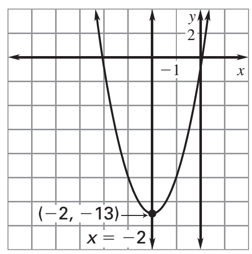
Vertex: $(-1, -4)$

**2.** $y = 3x^2 + 12x - 1$

Axis of symmetry: $x = -\frac{b}{2a} = -\frac{12}{2(3)} = -2$

When $x = -2$: $y = 3(-2)^2 + 12(-2) - 1 = -13$

Vertex: $(-2, -13)$




**3.** $f(x) = 6x^2 + 18x + 13$

Because $a = 6$ and $6 > 0$, the function has a minimum value.

$x = -\frac{b}{2a} = -\frac{18}{2(6)} = -\frac{3}{2}$

$f\left(-\frac{3}{2}\right) = 6\left(-\frac{3}{2}\right)^2 + 18\left(-\frac{3}{2}\right) + 13 = -\frac{1}{2}$

The minimum value is $-\frac{1}{2}$.

**4.** $y = 0.00014x^2 - 0.4x + 507$

$x = -\frac{b}{2a} = \frac{(-0.4)}{2(0.00014)} \approx 1429$

$y = 0.00014(1429)^2 - 0.4(1429) + 507 \approx 221$

The vertex is (1429, 221), so the cable is about 221 feet above the water at its lowest point.

### *Exercises for the lesson "Graph $y = ax^2 + bx + c$"*

### *Skill Practice*

**1.** The function $y = ax^2 + bx + c$ has a minimum value if $a > 0$ and a maximum value if $a < 0$.

**2.** 1. Determine whether the parabola opens up or down.

2. Find and draw the axis of symmetry.

3. Find and plot the vertex.

4. Plot two points (two $x$-values less than the $x$-coordinate of the vertex).

5. Reflect the points (in the step above) in the axis of symmetry.

6. Draw a parabola through the plotted points.

**3.** $y = 2x^2 - 8x + 6$

Axis of symmetry: $x = -\frac{b}{2a} = -\frac{8}{2(2)} = 2$

When $x = 2$: $y = 2(2)^2 - 8(2) + 6 = -2$

The vertex is $(2, -2)$.

**4.** $y = x^2 - 6x + 11$

Axis of symmetry: $x = -\frac{b}{2a} = -\frac{-6}{2(1)} = 3$

When $x = 3$: $y = (3)^2 - 6(3) + 11 = 2$

The vertex is $(3, 2)$.

**5.** $y = -3x^2 + 24x - 22$

Axis of symmetry: $x = -\frac{b}{2a} = -\frac{24}{2(-3)} = 4$

When $x = 4$: $y = -3(4)^2 + 24(4) - 22 = 26$

The vertex is $(4, 26)$.

**6.** $y = -x^2 - 10x$

Axis of symmetry: $x = -\frac{b}{2a} = \frac{-10}{2(-1)} = -5$

When $x = -5$: $y = -(-5)^2 - 10(-5) = 25$

The vertex is $(-5, 25)$.

**7.** $y = 6x^2 + 6x$

Axis of symmetry: $x = -\frac{b}{2a} = -\frac{6}{2(6)} = -\frac{1}{2}$

When $x = -\frac{1}{2}$: $y = 6\left(-\frac{1}{2}\right)^2 + 6\left(-\frac{1}{2}\right) = -\frac{3}{2}$

The vertex is $\left(-\frac{1}{2}, -\frac{3}{2}\right)$.

**8.** $y = 4x^2 + 7$

Axis of symmetry: $x = -\frac{b}{2a} = -\frac{0}{2(4)} = 0$

When $x = 0$: $y = 4(0)^2 + 7 = 7$

The vertex is $(0, 7)$.

**9.** $y = -\frac{2}{3}x^2 - 1$

Axis of symmetry: $x = -\frac{b}{2a} = -\frac{0}{2\left(-\frac{2}{3}\right)} = 0$

When $x = 0$: $y = -\frac{2}{3}(0)^2 - 1 = -1$

The vertex is $(0, -1)$.

**10.** $y = \frac{1}{2}x^2 + 8x - 9$

Axis of symmetry: $x = -\frac{b}{2a} = -\frac{8}{2\left(\frac{1}{2}\right)} = -8$

When $x = -8$: $y = \frac{1}{2}(-8)^2 + 8(-8) - 9 = -41$

The vertex is $(-8, -41)$.

**11.** $y = -\frac{1}{4}x^2 + 3x - 2$

Axis of symmetry: $x = -\frac{b}{2a} = -\frac{3}{2\left(-\frac{1}{4}\right)} = 6$

When $x = 6$: $y = -\frac{1}{4}(6)^2 + 3(6) - 2 = 7$

The vertex is $(6, 7)$.

**12.** D; $y = -3x^2 + 18x - 13$

$x = -\frac{b}{2a} = -\frac{18}{2(-3)} = 3$

$y = -3(3)^2 + 18(3) - 13 = 14$

The vertex is $(3, 14)$.

**13.** The equation for the axis of symmetry is $x = -\frac{b}{2a}$, not $x = \frac{b}{2a}$.

For the function $y = 2x^2 + 16x - 1$, the axis of symmetry is

$x = -\frac{b}{2a} = -\frac{16}{2(2)} = -4$.

**14.** $-\frac{3}{2}$ should be substituted for $a$, not $\frac{3}{2}$.

For the function $y = -\frac{3}{2}x^2 + 18x - 5$, the axis of symmetry is

$x = -\frac{b}{2a} = -\frac{18}{2\left(-\frac{3}{2}\right)} = 6$.

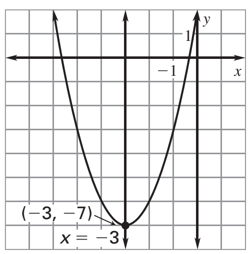
**15.** $y = x^2 + 6x + 2$

Because $a > 0$, the parabola opens up.

Axis of symmetry: $x = \frac{b}{2a} = -\frac{6}{2(1)} = -3$

When $x = -3$: $y = (-3)^2 + 6(-3) + 2 = -7$

The vertex is $(-3, -7)$.



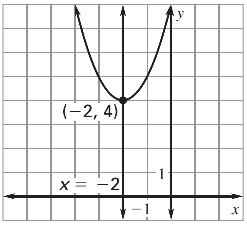

**16.** $y = x^2 + 4x + 8$

Because $a > 0$, the parabola opens up.

Axis of symmetry: $x = -\frac{b}{2a} = -\frac{4}{2(1)} = -2$

When $x = -2$: $y = (-2)^2 + 4(-2) + 8 = 4$

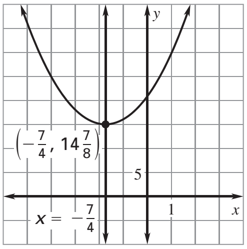
The vertex is $(-2, 4)$.




**17.** $y = 2x^2 + 7x + 21$

Because $a > 0$, the parabola opens up.

Axis of symmetry: $x = -\frac{b}{2a} = -\frac{7}{2(2)} = -\frac{7}{4}$



When $x = -\frac{7}{4}$: $y = 2\left(-\frac{7}{4}\right)^2 + 7\left(-\frac{7}{4}\right) + 21 = \frac{119}{8}$

The vertex is $\left(-\frac{7}{4}, \frac{119}{8}\right)$.

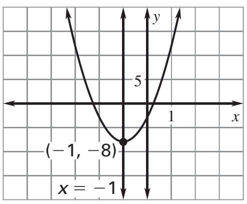
**18.** $y = 5x^2 + 10x - 3$

Because $a > 0$, the parabola opens up.

Axis of symmetry: $x = -\frac{b}{2a} = -\frac{10}{2(5)} = -1$

When $x = -1$: $y = 5(-1)^2 + 10(-1) - 3 = -8$

The vertex is $(-1, -8)$.



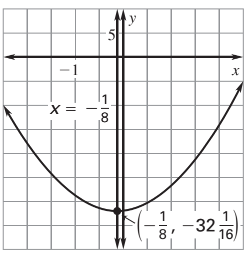

**19.** $y = 4x^2 + x - 32$

Because $a > 0$, the parabola opens up.

Axis of symmery: $x = -\frac{b}{2a} = -\frac{1}{2(4)} = -\frac{1}{8}$

When $x = -\frac{1}{8}$: $y = 4\left(-\frac{1}{8}\right)^2 + \left(-\frac{1}{8}\right) - 32 = -\frac{513}{16}$

The vertex is: $\left(-\frac{1}{8}, -\frac{513}{16}\right)$.



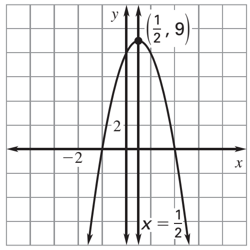
**20.** $y = -4x^2 + 4x + 8$

Because $a < 0$, the parabola opens down.

Axis of symmetry: $x = -\frac{b}{2a} = -\frac{4}{2(-4)} = \frac{1}{2}$

When $x = \frac{1}{2}$: $y = -4\left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right) + 8 = 9$

The vertex is $\left(\frac{1}{2}, 9\right)$.



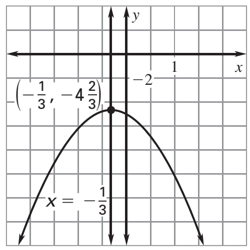
**21.** $y = -3x^2 - 2x - 5$

Because $a < 0$, the parabola opens down.

Axis of symmetry: $x = -\frac{b}{2a} = -\frac{-2}{2(-3)} = -\frac{1}{3}$

When $x = -\frac{1}{3}$: $y = -3\left(-\frac{1}{3}\right)^2 - 2\left(-\frac{1}{3}\right) - 5 = -\frac{14}{3}$

The vertex is $\left(-\frac{1}{3}, -\frac{14}{3}\right)$.



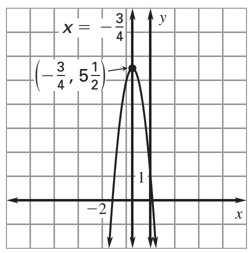
**22.** $y = -8x^2 - 12x + 1$

Because $a < 0$, the parabola opens down.

Axis of symmetry: $x = -\frac{b}{2a} = -\frac{-12}{2(-8)} = -\frac{3}{4}$

When $x = -\frac{3}{4}$: $y = -8\left(-\frac{3}{4}\right)^2 - 12\left(-\frac{3}{4}\right) + 1 = \frac{11}{12}$

The vertex is $\left(-\frac{3}{4}, \frac{11}{2}\right)$.



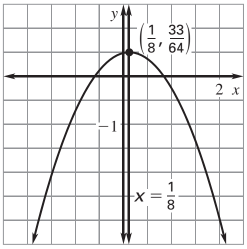
23. $y = -x^2 + \frac{1}{4}x + \frac{1}{2}$

Since $a < 0$, the parabola opens down.

Axis of symmetry: $x = -\frac{b}{2a} = -\frac{\frac{1}{4}}{2(-1)} = \frac{1}{8}$

When $x = \frac{1}{8}$: $y = -\left(\frac{1}{8}\right)^2 + \frac{1}{4}\left(\frac{1}{8}\right) + \frac{1}{2} = \frac{33}{64}$

The vertex is $\left(\frac{1}{8}, \frac{33}{64}\right)$.



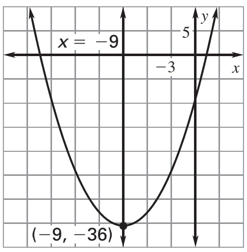

24. $y = \frac{1}{3}x^2 + 6x - 9$

Because $a > 0$, the parabola opens up.

Axis of symmetry: $x = -\frac{b}{2a} = -\frac{6}{2\left(\frac{1}{3}\right)} = -9$

When $x = -9$: $y = \frac{1}{3}(-9)^2 + 6(-9) - 9 = -36$

The vertex is $(-9, -36)$



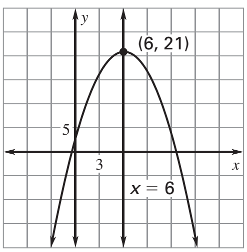

25. $y = -\frac{1}{2}x^2 + 6x + 3$

Since $a < 0$, the parabola opens down.

Axis of symmetry: $x = -\frac{b}{2a} = -\frac{6}{2\left(-\frac{1}{2}\right)} = 6$

When $x = 6$: $y = -\frac{1}{2}(6)^2 + 6(6) + 3 = 21$

The vertex is $(6, 21)$.



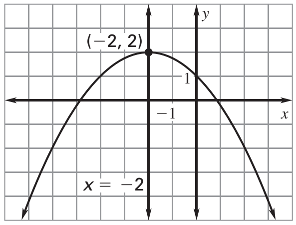

26. $y = -\frac{1}{4}x^2 - x + 1$

Because $a < 0$, the parabola opens down.

Axis of symmetry: $x = -\frac{b}{2a} = -\frac{-1}{2\left(-\frac{1}{4}\right)} = -2$

When $x = -2$: $y = -\frac{1}{4}(-2)^2 - (-2) + 1 = 2$

The vertex is $(-2, 2)$.




27. B; $y = -\frac{1}{2}x^2 + 2x + 3$

Because $a < 0$, the parabola opens down.

Axis of symmetry: $x = -\frac{b}{2a} = -\frac{2}{2\left(-\frac{1}{2}\right)} = 2$

When $x = 2$: $y = -\frac{1}{2}(2)^2 + 2(2) + 3 = 5$

The vertex is $(2, 5)$.

When $x = 0$: $y = -\frac{1}{2}(0)^2 + 2(0) + 3 = 3$

The $y$-intercept is 3.

28. Because $a > 0$, $f(x) = x^2 - 6$ has a minimum value.

$x = -\frac{b}{2a} = -\frac{0}{2(1)} = 0$

The minimum value is: $f(0) = (0)^2 - 6 = -6$

29. Because $a < 0$, $f(x) = -5x^2 + 7$ has a maximum value.

$x = -\frac{b}{2a} = -\frac{0}{2(-5)} = 0$

The maximum value is $f(0) = -5(0)^2 + 7 = 7$

30. Because $a > 0$, $f(x) = 4x^2 + 32x$ has a minimum value.

$x = \frac{b}{2a} = -\frac{32}{2(4)} = -4$

The minimum value is $f(-4) = 4(-4)^2 + 32(-4) = -64$

31. Because a $< 0$, $f(x) = -3x^2 + 12x - 20$ has a maximum value.

$x = -\frac{b}{2a} = -\frac{12}{2(-3)} = 2$

The maximum value is $f(2) = -3(2)^2 + 12(2) - 20 = -8$.

32. Because $a > 0$, $f(x) = x^2 + 7x + 8$ has a minimum value.

$x = -\frac{b}{2a} = -\frac{7}{2(1)} = -\frac{7}{2}$

The minimum value is $f\left(-\frac{7}{2}\right) = \left(-\frac{7}{2}\right)^2 + 7\left(-\frac{7}{2}\right) + 8 = -\frac{17}{4}$

**33.** Because $a < 0$, $f(x) = -2x^2 - x + 10$ has a maximum value.

$$x = -\frac{b}{2a} = -\frac{-1}{2(-2)} = -\frac{1}{4}$$

The maximum value is

$$f\left(-\frac{1}{4}\right) = -2\left(-\frac{1}{4}\right)^2 - \left(-\frac{1}{4}\right) + 10 = \frac{81}{8}.$$

**34.** Because $a > 0$, $f(x) = \frac{1}{2}x^2 - 2x + 5$ has a minimum value.

$$x = -\frac{b}{2a} = -\frac{-2}{2\left(\frac{1}{2}\right)} = 2$$

The minimum value is $f(2) = \frac{1}{2}(2)^2 - 2(2) + 5 = 3$.

**35.** Because $a < 0$, $f(x) = -\frac{3}{8}x^2 + 9x$ has a maximum value.

$$x = -\frac{b}{2a} = -\frac{9}{2\left(-\frac{3}{8}\right)} = 12$$

The maximum value is $f(12) - \frac{3}{8}(12)^2 + 9(12) = 54$.

**36.** Because $a > 0$, $f(x) = \frac{1}{4}x^2 + 7x + 11$ has a minimum value.

$$x = -\frac{b}{2a} = -\frac{7}{2\left(\frac{1}{4}\right)} = -14$$

The minimum value is

$$f(-14) = \frac{1}{4}(-14)^2 + 7(-14) + 11 = -38.$$

**37.** $y = x^2 + 4x + 1$

$$x = -\frac{b}{2a} = -\frac{4}{2(1)} = -2$$

$y = (-2)^2 + 4(-2) + 1 = -3$

Vertex: $(-2, -3)$

$y = x^2 - 4x + 1$

$$x = -\frac{b}{2a} = -\frac{-4}{2(1)} = 2$$

$y = (2)^2 - 4(2) + 1 = -3$

Vertex: $(2, -3)$

The vertex $(-2, -3)$ is 4 units to the left of the vertex $(2, -3)$, so the graph of $y = x^2 + 4x + 1$ is a horizontal translation (of 4 units left) of the graph of $y = x^2 - 4x + 1$.

**38. a.** $y = ax^2 + bx$

$y = x(ax + b)$

$x = 0$ and $ax + b = 0$

$ax = -b$

$x = -\frac{b}{a}$

The $x$-intercepts are 0 and $-\frac{b}{a}$.

**b.** $x$-coordinate of point halfway between $x$-intercepts:

$$\frac{x_1 + x_2}{2} = \frac{0 + \left(-\frac{b}{a}\right)}{2} = -\frac{b}{2a}$$

The equation of the vertical line through this point is

$$x = -\frac{b}{2a}.$$

**39.** $y = ax^2 + bx$

Using (1, 6): $6 = a(1)^2 + b(1) = a + b$

Using (3, 6): $6 = a(3)^2 + b(3) = 9a + 3b$

System of equations:

$a + b = 6 \rightarrow a = 6 - b$

$9a + 3b = 6$

When $a = 6 - b$: $9(6 - b) + 3b = 6 \rightarrow b = 8$

When $b = 8$: $a + 8 = 6 \rightarrow a = -2$

The graph of the function $y = -2x^2 + 8x$ contains the points (1, 6) and (3, 6).

### *Problem Solving*

**40.** The maximum height of the spider is at the vertex of the parabola.

$$x = -\frac{b}{2a} = -\frac{820}{2(-4500)} \approx 0.091$$

The spider's body reaches its maximum height after about 0.091 seconds.

$y = -4500(0.091)^2 + 820(0.091) + 43 \approx 80$

The maximum height is about 80 millimeters.

**41.** The maximum height of the arch is at the vertex of the parabola.

$$x = -\frac{b}{2a} = -\frac{0.71}{2(-0.0019)} \approx 187$$

$h = y = -0.0091(187)^2 + 0.71(187) \approx 66$

The maximum height of the arch is about 66 feet.

**42. a.** $R = (s + n)(150 - 10n)$

$R = 750 - 50n + 150n - 10n^2$

$R = -10n^2 + 100n - 750$

**b.** $n = -\frac{b}{2a} = -\frac{100}{2(-10)} = 5$

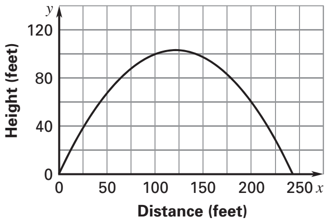
When $n = 5$:

$R = -10(5)^2 + 100(5) + 750 = 1000$

The maximum value of the functions is \$1000.

**c.** The maximum value of the \$1000 occurs when $n = s$, so the students should increase last year's price by 5(\$1) = \$5, for a total of \$5 + \$5 = \$10 per package.

**43.**




Because the $x$-intercepts are 0 and roughly 243, the hanger is about 243 feet wide at its base.

$y = -0.007x^2 + 1.7x$

| **x** | 50 | 100 | 122 | 150 | 200 | 242 | 243 |
|---|---|---|---|---|---|---|---|
| **y** | 67.5 | 100 | 103.212 | 97.5 | 60 | 1.452 | −2.43 |

**44.** No; the maximum value occurs at the vertex, when $t = -\frac{b}{2a} = -\frac{132}{2(-10.4)} \approx 6.35$, which occurred sometime during 1996.

**45.** $y$-intercept (0, 1.5):

$y = ax^2 + bx + c$

$1.5 = a(0)^2 + b(0) + c$

$1.5 = c$

$x$-coordinate of vertex (18, 1.6):

$x = -\frac{b}{2a}$

$18 = -\frac{b}{2a}$

$b = -36a$

$y$-coordinate of vertex (18, 1.6):

$y = a(18)^2 + b(18) + c$

$1.6 = 324a + 18b + c$

When $c = 1.5$ and $b = -36a$:

$1.6 = 324a + 18(-36a) + 1.5$

$1.6 = 324a - 648a + 1.5$

$0.1 = -324a$

$a \approx -0.00031$

When $a \approx -0.00031$: $b = -36a \approx 0.011$

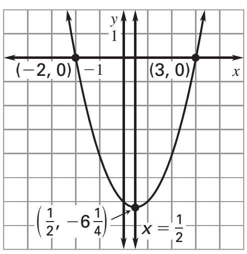
An equation for the path of the arrow is $y = -0.00031x^2 + 0.011x + 1.5$.

### *Extension for the lesson "Graph $y = ax^2 + bx + c$"*

**1.** $y = (x + 2)(x - 3)$

$n = -2, q = 3$

$x$-intercepts: $(-2, 0), (3, 0)$.




Axis of symmetry:

$x = \frac{p + q}{2} = -\frac{2 + 3}{2} = \frac{1}{2}$

When $x = \frac{1}{2}$:

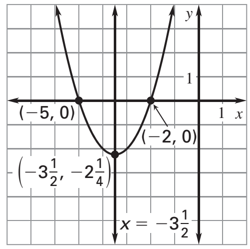
$y = \left(\frac{1}{2} + 2\right)\left(\frac{1}{2} - 3\right) = -\frac{25}{4}$

Vertex: $\left(\frac{1}{2}, -\frac{25}{4}\right)$.

**2.** $y = (x + 5)(x + 2)$

$p = -5, q = -2$

$x$-intercepts: $(-5, 0)(-2, 0)$




Axis of symmetry:

$x = \frac{p + q}{2} = \frac{-5 + (-2)}{2} = \frac{-7}{2}$

When $x = -\frac{7}{2}$:

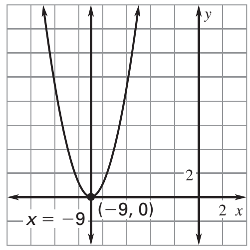
$y = \left(-\frac{7}{2} + 5\right)\left(-\frac{7}{2} + 2\right) = -\frac{9}{4}$

Vertex: $\left(-\frac{7}{2}, -\frac{9}{4}\right)$.

**3.** $y = (x + 9)^2 = (x + 9)(x + 9)$

$p = -9, q = -9$

$x$-intercept: $(-9, 0)$




Axis of symmetry:

$x = \frac{p + q}{2} = \frac{-9 + (-9)}{2} = -9$

When $x = -9$:

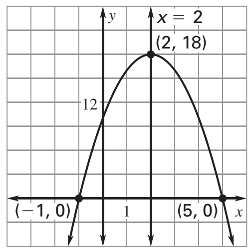
$y = (-9 + 9)^2 = 0$

Vertex: $(-9, 0)$.

**4.** $y = -2(x - 5)(x + 1)$

$p = 5, q = -1$

$x$-intercepts: $(5, 0), (-1, 0)$




Axis of symmetry:

$x = \frac{p + q}{2} = \frac{5 + (-1)}{2} = 2$

When $x = 2$:

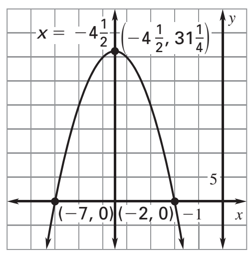
$y = -2(2 - 5)(2 + 1) = 18$

Vertex: $(2, 18)$.

**5.** $y = -5(x + 7)(x + 2)$

$p = -7, q = -2$

$x$-intercepts: $(-7, 0), (-2, 0)$




Axis of symmetry:

$x = \frac{p + q}{2} = \frac{-7 + (-2)}{2} = \frac{-9}{2}$

When $x = -\frac{9}{2}$:

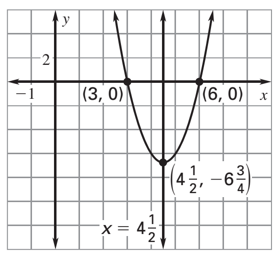
$y = -5\left(-\frac{9}{2} + 7\right)\left(-\frac{9}{2} + 2\right) = \frac{125}{4}$

Vertex: $\left(-\frac{9}{2}, \frac{125}{4}\right)$.

**6.** $y = 3(x - 6)(x - 3)$

$p = 6, q = 3$

$x$-intercepts: $(6, 0), (3, 0)$




Axis of symmetry:

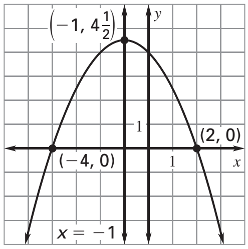
$x = \frac{p + q}{2} = \frac{6 + 3}{2} = \frac{9}{2}$

When $x = \frac{9}{2}$:

$y = 3\left(\frac{9}{2} - 6\right)\left(\frac{9}{2} - 3\right) = -\frac{27}{4}$

Vertex: $\left(\frac{9}{2}, -\frac{27}{4}\right)$.

**7.** $y = \frac{1}{2}(x + 4)(x - 2)$




$p = -4, q = 2$

$x$-intercepts: $(-4, 0), (2, 0)$

Axis of symmetry:

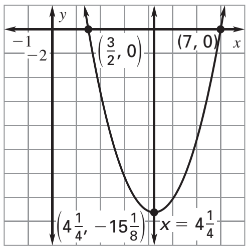
$x = \frac{p + q}{2} = \frac{-4 + 2}{2} = -1$

When $x = -1$:

$y = -\frac{1}{2}(-1 + 4)(-1 - 2) = \frac{9}{2}$

Vertex: $\left(-1, \frac{9}{2}\right)$.

**8.** $y = (x - 7)(2x - 3)$




$p = 7, q = \frac{3}{2}$

$x$-intercepts: $(7, 0), \left(\frac{3}{2}, 0\right)$

Axis of symmetry:

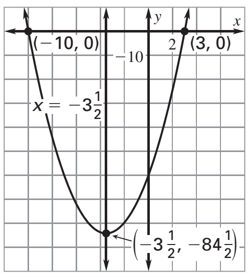
$x = \frac{p + q}{2} = \frac{7 + \frac{3}{2}}{2} = \frac{17}{4}$

When $x = \frac{17}{4}$:

$y = \left(\frac{17}{4} - 7\right)\left(2\left(\frac{17}{4}\right) - 3\right) = -\frac{121}{8}$

Vertex: $\left(\frac{17}{4}, -\frac{121}{8}\right)$.

**9.** $y = 2(x + 10)(x - 3)$




$p = -10, q = 3$

$x$-intercepts: $(-10, 0), (3, 0)$

Axis of symmetry:

$x = \frac{p + q}{2} = \frac{-10 + 3}{2} = \frac{-7}{2}$

When $x = -\frac{7}{2}$:

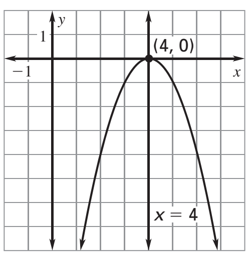
$y = 2\left(-\frac{7}{2} + 10\right)\left(-\frac{7}{2} - 3\right) = -\frac{169}{2}$

Vertex: $\left(-\frac{7}{2}, -\frac{169}{2}\right)$.

**10.** $y = -x^2 + 8x - 16$

$= (-x + 4)(x - 4)$

$= -(x - 4)(x - 4)$




$p = 4, q = 4$

$x$-intercepts: $(4, 0)$

Axis of symmetry:

$x = \frac{p + q}{2} = \frac{4 + 4}{2} = 4$

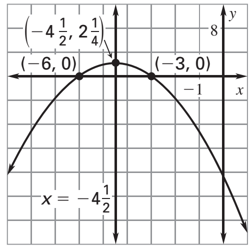
When $x = 4$: $y = -(4)^2 + 8(4) - 16 = 0$

Vertex: $(4, 0)$.

**11.** $y = -x^2 + 9x - 18$

$= (-x - 6)(x + 3)$

$= -(x + 6)(x + 3)$




$p = -6, q = -3$

$x$-intercepts: $(-6, 0), (-3, 0)$

Axis of symmetry:

$x = \frac{p + q}{2} = \frac{-6 + (-3)}{2} = \frac{-9}{2}$

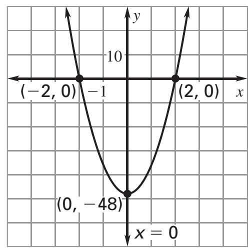
When $x = -\frac{9}{2}$: $y = -\left(-\frac{9}{2}\right)^2 - 9\left(-\frac{9}{2}\right) - 18 = \frac{9}{4}$

Vertex: $\left(-\frac{9}{2}, \frac{9}{4}\right)$.

**12.** $y = 12x^2 - 48$

$= 12(x^2 - 4)$

$= 12(x + 2)(x - 2)$




$p = -2, q = 2$

$x$-intercepts: $(-2, 0), (2, 0)$

Axis of symmetry:

$x = \frac{p + q}{2} = \frac{-2 + 2}{2} = 0$

When $x = 0$: $y = 12(0)^2 - 48 = -48$

Vertex: $(0, -48)$.

**13.** $3x^2 - 12x + 12 = 3(x - 2)(x - 2)$, so the intercept form of the function is $y = 3(x - 2)(x - 2)$, and the graph has one intercept, 2.

**14. a.** The $x$-intercepts are $-3$ and 5.

**b.** The vertex is (1, 6), so $x = 1$ and $y = 6$. From part (a), $p = -3$ and $q = 5$.

$y = a(x - p)(x - q)$

$6 = a(1 - (-3))(1 - 5)$

$6 = -16a$

$-\frac{3}{8} = a$

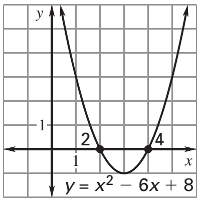
**c.** An equation is $y = -\frac{3}{8}(x - (-3))(x - 5)$

$= -\frac{3}{8}(x + 3)(x - 5)$

## Lesson 9.3 Solve Quadratic Equations by Graphing

### *Guided Practice for the lesson "Solve Quadratic Equations by Graphing"*

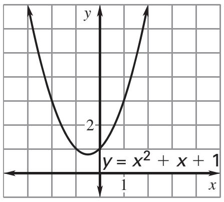
**1.** $x^2 - 6x + 8 = 0$




The $x$-intercepts are 2 and 4. So, the solutions of the equation are 2 and 4.

**2.** $x^2 + x = -1$

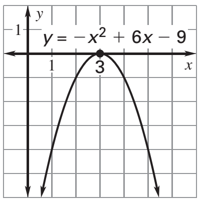
$x^2 + x + 1 = 0$




There are no $x$-intercepts. So, the equation has no solution.

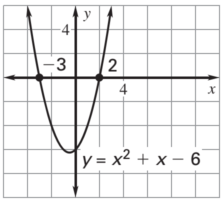
**3.** $-x^2 + 6x = 9$

$-x^2 + 6x - 9 = 0$



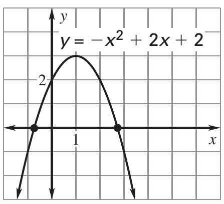

The $x$-intercept is 3. So, the solution of the equation is 3.

**4.** $f(x) = x^2 + x - 6$




The $x$-intercepts are $-3$ and 2. So, the zeros of the function are $-3$ and 2.

**5.** $f(x) = -x^2 + 2x + 2$




$x$-intercepts: between $-1$ and 0 and between 2 and 3.

| **x** | $-1$ | $-0.9$ | $-0.8$ | $\mathbf{-0.7}$ | $-0.6$ | $-0.5$ | $-0.4$ |
|---|---|---|---|---|---|---|---|
| ***f*(*x*)** | $-1$ | $-0.61$ | $-0.24$ | **0.11** | 0.44 | 0.75 | 1.04 |

| **x** | $-0.3$ | $-0.2$ | $-0.1$ | 0 |
|---|---|---|---|---|
| ***f*(*x*)** | 1.31 | 1.56 | 1.79 | 2 |

| **x** | 2 | 2.1 | 2.2 | 2.3 | 2.4 | 2.5 | 2.6 |
|---|---|---|---|---|---|---|---|
| ***f*(*x*)** | 2 | 1.79 | 1.56 | 1.31 | 1.04 | 0.75 | 0.44 |

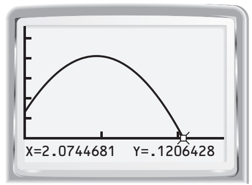
| **x** | **2.7** | 2.8 | 2.9 | 3 |
|---|---|---|---|---|
| ***f*(*x*)** | **0.11** | $-0.24$ | $-0.61$ | $-1$ |

The zeros of $f(x) = -x^2 + 2x + 2$ are about $-0.7$ and 2.7.

**6.** When $v = 30$, $s = 6.5$, and $h = 0$:

$h = -16t^2 + vt + s$

$0 = -16t^2 + 30t + 6.5$




The *trace* feature of a graphing calculator shows that the positive $t$-intercept is about 2.1. The shot put is in the air for about 2.1 seconds.

### *Exercises for the lesson "Solve Quadratic Equations by Graphing"*

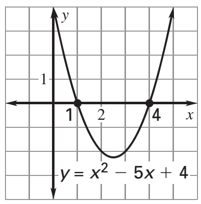
### *Skill Practice*

**1.** $2x^2 + 11 = 9x$

$2x^2 - 9x + 11 = 0$

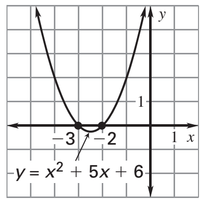
**2.** Yes, $3x^2 - 2 = 0$ is a quadratic equation in the standard form $ax^2 + bx + c = 0$, where $a = 3$, $b = 0$, and $c = -2$.

**3.** $x^2 - 5x + 4 = 0$




The solutions are 1 and 4.

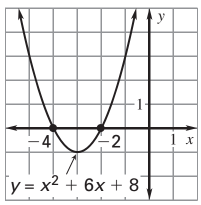
**4.** $x^2 + 5x + 6 = 0$




The solutions are $-3$ and $-2$.

**5.** $x^2 + 6x = -8$

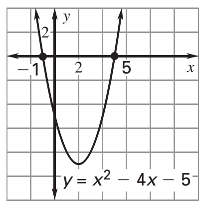
$x^2 + 6x + 8 = 0$




The solutions are $-4$ and $-2$.

**6.** $x^2 - 4x = 5$

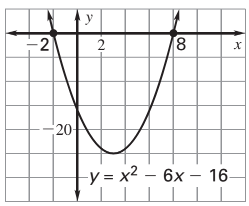
$x^2 - 4x - 5 = 0$




The solutions are $-1$ and 5.

**7.** $x^2 - 16 = 6x$

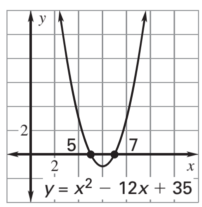
$x^2 - 6x - 16 = 0$




The solutions are $-2$ and 8.

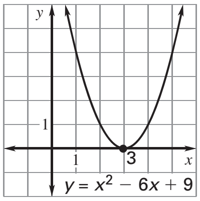
**8.** $x^2 - 12x = -35$

$x^2 - 12x + 35 = 0$



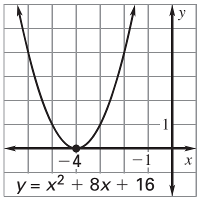

The solutions are 5 and 7.

**9.** $x^2 - 6x + 9 = 0$




The solution is 3.

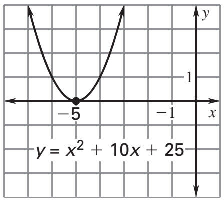
**10.** $x^2 + 8x + 16 = 0$




The solution is $-4$.

**11.** $x^2 + 10x = -25$

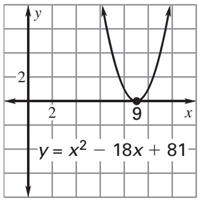
$x^2 + 10x + 25 = 0$




The solution is $-5$.

**12.** $x^2 + 81 = 18x$

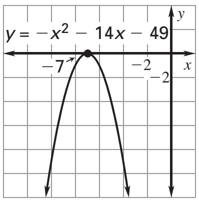
$x^2 - 18x + 81 = 0$




The solution is 9.

**13.** $-x^2 - 14x = 49$

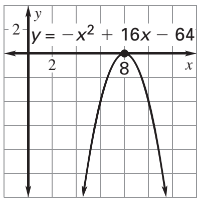
$-x^2 - 14x - 49 = 0$




The solution is $-7$.

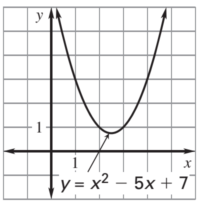
**14.** $-x^2 + 16x = 64$

$-x^2 + 16x - 64 = 0$



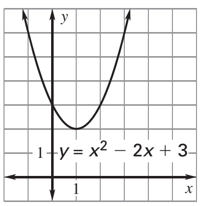

The solution is 8.

**15.** $x^2 - 5x + 7 = 0$




There is no $x$-intercept. The equation has no solution.

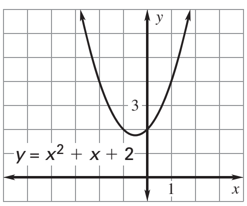
**16.** $x^2 - 2x + 3 = 0$




There is no $x$-intercept. The equation has no solution.

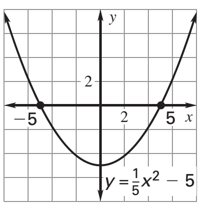
**17.** $x^2 + x = -2$

$x^2 + x + 2 = 0$




There is no $x$-intercept. The equation has no solution.

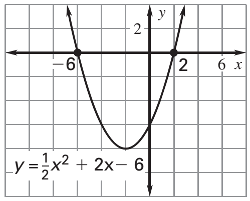
**18.** $\frac{1}{5}x^2 - 5 = 0$




The solutions are $-5$ and 5.

**19.** $\frac{1}{2}x^2 + 2x = 6$

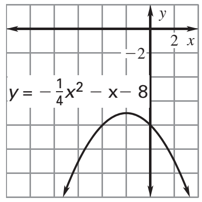
$\frac{1}{2}x^2 + 2x - 6 = 0$




The solutions are $-6$ and 2.

**20.** $-\frac{1}{4}x^2 - 8 = x$

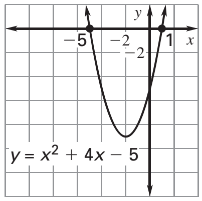
$-\frac{1}{4}x^2 - x - 8 = 0$




There is no $x$-intercept. The equation has no solution.

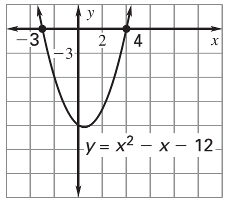
**21.** The solution was incorrectly found to be the $y$-intercept rather than the $x$-intercept. Because the $x$-intercept is 2, the solution of the equation $0 = x^2 - 4x + 4$ is 2.

**22.** $f(x) = x^2 + 4x - 5$



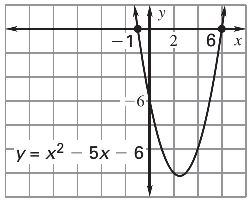

The zeros of the function are $-5$ and 1.

**23.** $f(x) = x^2 - x - 12$



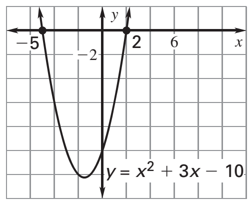

The zeros of the function are $-3$ and 4.

**24.** $f(x) = x^2 - 5x - 6$



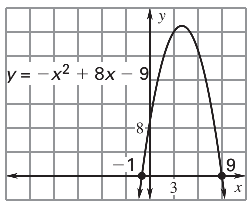

The zeros of the function are $-1$ and 6.

**25.** $f(x) = x^2 + 3x - 10$



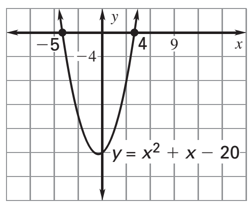

The zeros of the function are $-5$ and 2.

**26.** $f(x) = -x^2 + 8x + 9$



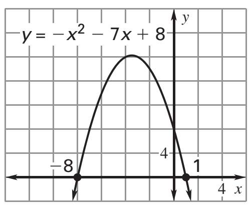

The zeros of the function are $-1$ and 9.

**27.** $f(x) = x^2 + x - 20$



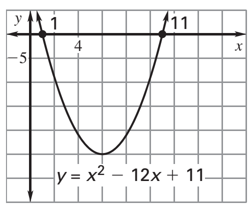

The zeros of the function are $-5$ and 4.

**28.** $f(x) = -x^2 - 7x + 8$



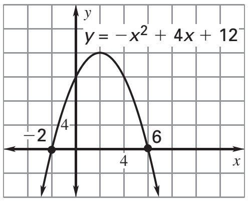

The zeros of the function are $-8$ and 1.

**29.** $f(x) = x^2 - 12x + 11$




The zeros of the function are 1 and 11.

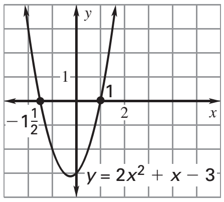
**30.** $f(x) = -x^2 + 4x + 12$




The zeros of the function are −2 and 6.

**31.**

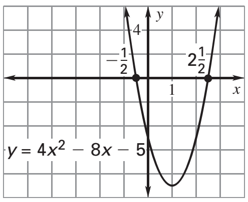
$$2x^2 + x = 3$$
$$2x^2 + x - 3 = 0$$




The solutions are are $-1\frac{1}{2}$ and 1.

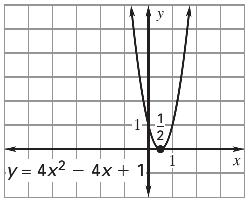
**32.**

$$4x^2 - 5 = 8x$$
$$4x^2 - 8x - 5 = 0$$




The solutions are $-\frac{1}{2}$ and $2\frac{1}{2}$.

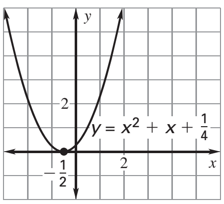
**33.** $4x^2 - 4x + 1 = 0$




The solution is $\frac{1}{2}$.

**34.**

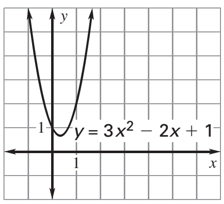
$$x^2 + x = -\frac{1}{4}$$
$$x^2 + x + \frac{1}{4} = 0$$




The solution is $-\frac{1}{2}$.

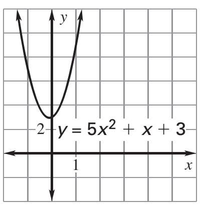
**35.**

$$3x^2 + 1 = 2x$$
$$3x^2 - 2x + 1 = 0$$



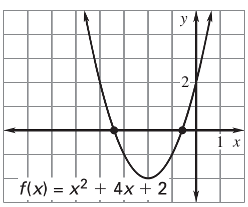

There are no $x$-intercepts. The equation has no solution.

**36.** $5x^2 + x + 3 = 0$




There are no $x$-intercepts. The equation has no solution.

**37.** $f(x) = x^2 + 4x + 2$




$x$-intercepts: between −4 and −3, and between −1 and 0

| | | | | | | |
|---|---|---|---|---|---|---|
| ***x*** | −3.9 | −3.8 | −3.7 | −3.6 | −3.5 | **−3.4** |
| ***f*(*x*)** | 1.61 | 1.24 | 0.89 | 0.56 | 0.25 | **−0.04** |

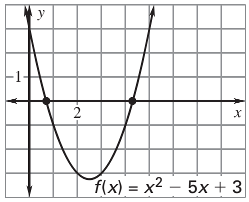
| | | | |
|---|---|---|---|
| ***x*** | −3.3 | −3.2 | −3.1 |
| ***f*(*x*)** | −0.31 | −0.56 | −0.79 |

| | | | | | | |
|---|---|---|---|---|---|---|
| ***x*** | −0.9 | −0.8 | −0.7 | **−0.6** | −0.5 | −0.4 |
| ***f*(*x*)** | −0.79 | −0.56 | −0.31 | **−0.04** | 0.25 | 0.56 |

| | | | |
|---|---|---|---|
| ***x*** | −0.3 | −0.2 | −0.1 |
| ***f*(*x*)** | 0.89 | 1.24 | 1.61 |

The zeros of $f(x) = x^2 + 4x + 2$ are about −3.4 and −0.6

**38.** $f(x) = x^2 - 5x + 3$




$x$-intercepts: between 0 and 1 and between 4 and 5

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| ***x*** | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | **0.7** |
| ***f*(*x*)** | 2.51 | 2.04 | 1.59 | 1.16 | 0.75 | 0.36 | **−0.01** |

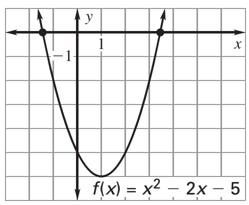
| | | |
|---|---|---|
| ***x*** | 0.8 | 0.9 |
| ***f*(*x*)** | −0.36 | −0.69 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| ***x*** | 4.1 | 4.2 | **4.3** | 4.4 | 4.5 | 4.6 | 4.7 |
| ***f*(*x*)** | −0.69 | −0.36 | **−0.01** | 0.36 | 0.75 | 1.16 | 1.59 |

| | | |
|---|---|---|
| ***x*** | 4.8 | 4.9 |
| ***f*(*x*)** | 2.04 | 2.51 |

The zeros of $f(x) = x^2 - 5x + 3$ are about 0.7 and 4.3.

**39.**




$f(x) = x^2 - 2x - 5$
$x$-intercepts: between −2 and −1 and between 3 and 4.

| | | | | | | |
|---|---|---|---|---|---|---|
| ***x*** | −1.9 | −1.8 | −1.7 | −1.6 | −1.5 | **−1.4** |
| ***f*(*x*)** | 2.41 | 1.84 | 1.29 | 0.76 | 0.25 | **−0.24** |

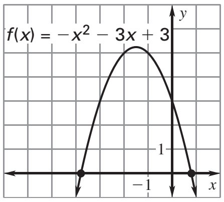
| | | | |
|---|---|---|---|
| ***x*** | −1.3 | −1.2 | −1.1 |
| ***f*(*x*)** | −0.71 | −1.16 | −1.59 |

| | | | | | | |
|---|---|---|---|---|---|---|
| ***x*** | 3.1 | 3.2 | 3.3 | **3.4** | 3.5 | 3.6 |
| ***f*(*x*)** | −1.59 | −1.16 | −0.71 | **−0.24** | 0.25 | 0.76 |

| | | | |
|---|---|---|---|
| ***x*** | 3.7 | 3.8 | 3.9 |
| ***f*(*x*)** | 1.29 | 1.84 | 2.41 |

The zeros of $f(x) = x^2 - 2x - 5$ are about −1.4 and 3.4.

**40.**




$f(x) = -x^2 - 3x + 3$
$x$-intercepts: between −4 and −3 and between 0 and 1.

| **x** | −3.9 | **−3.8** | −3.7 | −3.6 | −3.5 | −3.4 |
|---|---|---|---|---|---|---|
| **f(x)** | −0.51 | **−0.04** | 0.41 | 0.84 | 1.25 | 1.64 |

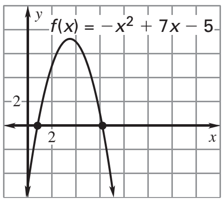
| **x** | −3.3 | −3.2 | −3.1 |
|---|---|---|---|
| **f(x)** | 2.01 | 2.36 | 2.69 |

| **x** | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 |
|---|---|---|---|---|---|---|
| **f(x)** | 2.69 | 2.36 | 2.01 | 1.64 | 1.25 | 0.84 |

| **x** | 0.7 | **0.8** | 0.9 |
|---|---|---|---|
| **f(x)** | 0.41 | **−0.04** | −0.51 |

The zeros of $f(x) = -x^2 - 3x + 3$ are about −3.8 and 0.8.

**41.**




$f(x) = -x^2 + 7x - 5$
$x$-intercepts: between 0 and 1 and between 6 and 7.

| **x** | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 |
|---|---|---|---|---|---|---|
| **f(x)** | −4.31 | −3.64 | −2.99 | −2.36 | −1.75 | −1.16 |

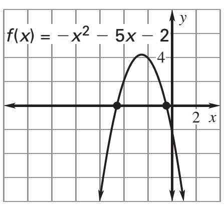
| **x** | 0.7 | **0.8** | 0.9 |
|---|---|---|---|
| **f(x)** | −0.59 | **−0.04** | 0.49 |

| **x** | 6.1 | **6.2** | 6.3 | 6.4 | 6.5 | 6.6 |
|---|---|---|---|---|---|---|
| **f(x)** | 0.49 | **−0.04** | −0.59 | −1.16 | −1.75 | −2.36 |

| **x** | 6.7 | 6.8 | 6.9 |
|---|---|---|---|
| **f(x)** | −2.99 | −3.64 | −4.31 |

The zeros of $f(x) = -x^2 + 7x - 5$ are about 0.8 and 6.2.

**42.**




$f(x) = -x^2 - 5x - 2$
$x$-intercepts: between −5 and −4 and between −1 and 0.

| **x** | −4.9 | −4.8 | −4.7 | **−4.6** | −4.5 | −4.4 |
|---|---|---|---|---|---|---|
| **f(x)** | −1.51 | −1.04 | −0.59 | **−0.16** | 0.25 | 0.64 |

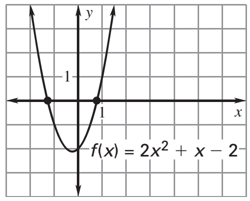
| **x** | −4.3 | −4.2 | −4.1 |
|---|---|---|---|
| **f(x)** | 1.01 | 1.36 | 1.69 |

| **x** | −0.9 | −0.8 | −0.7 | −0.6 | −0.5 | **−0.4** |
|---|---|---|---|---|---|---|
| **f(x)** | 1.69 | 1.36 | 1.01 | 0.64 | 0.25 | **−0.16** |

| **x** | −0.3 | −0.2 | −0.1 |
|---|---|---|---|
| **f(x)** | −0.59 | −1.04 | −1.51 |

The zeros of $f(x) = -x^2 - 5x - 2$ are about −4.6 and −0.4.

**43.**




$f(x) = 2x^2 + x - 2$
$x$-intercepts: between −2 and −1 and between 0 and 1.

| **x** | −1.9 | −1.8 | −1.7 | −1.6 | −1.5 | −1.4 |
|---|---|---|---|---|---|---|
| **f(x)** | 3.32 | 2.68 | 2.08 | 1.52 | 1 | 0.52 |

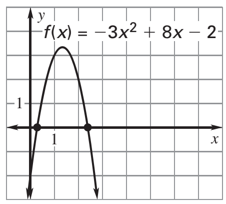
| **x** | **−1.3** | −1.2 | −1.1 |
|---|---|---|---|
| **f(x)** | **0.08** | −0.32 | −0.68 |

| **x** | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 |
|---|---|---|---|---|---|---|
| **f(x)** | −1.88 | −1.72 | −1.52 | −1.28 | −1 | −0.68 |

| **x** | 0.7 | **0.8** | 0.9 |
|---|---|---|---|
| **f(x)** | −0.32 | **0.08** | 0.52 |

The zeros of $f(x) = 2x^2 + x - 2$ are about −1.3 and 0.8.

**44.**




$f(x) = -3x^2 + 8x - 2$
$x$-intercepts: between 0 and 1 and between 2 and 3.

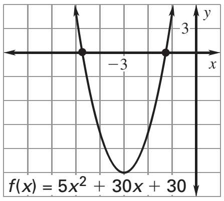
| **x** | 0.1 | 0.2 | **0.3** | 0.4 | 0.5 | 0.6 |
|---|---|---|---|---|---|---|
| **f(x)** | −1.23 | −0.52 | **0.13** | 0.72 | 1.25 | 1.72 |

| **x** | 0.7 | 0.8 | 0.9 |
|---|---|---|---|
| **f(x)** | 2.13 | 2.48 | 2.77 |

| **x** | 2.1 | 2.2 | 2.3 | **2.4** | 2.5 | 2.6 |
|---|---|---|---|---|---|---|
| **f(x)** | 1.57 | 1.08 | 0.53 | **−0.08** | −0.75 | −1.48 |

| **x** | 2.7 | 2.8 | 2.9 |
|---|---|---|---|
| **f(x)** | −2.27 | −3.12 | −4.03 |

The zeros of $f(x) = -3x^2 + 8x - 2$ are about 0.3 and 2.4.

**45.** 


$f(x) = 5x^2 + 30x + 30$ $x$-intercepts: between $-5$ and $-4$ and between $-2$ and $-1$.

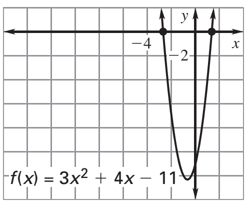
| **x** | −4.9 | −4.8 | **−4.7** | −4.6 | −4.5 | −4.4 |
|---|---|---|---|---|---|---|
| **f(x)** | 3.05 | 1.2 | **−0.55** | −2.2 | −3.75 | −5.2 |

| **x** | −4.3 | −4.2 | −4.1 |
|---|---|---|---|
| **f(x)** | −6.55 | −7.8 | −8.95 |

| **x** | −1.9 | −1.8 | −1.7 | −1.6 | −1.5 | −1.4 |
|---|---|---|---|---|---|---|
| **f(x)** | −8.95 | −7.8 | −6.55 | −5.2 | −3.75 | −2.2 |

| **x** | **−1.3** | −1.2 | −1.1 |
|---|---|---|---|
| **f(x)** | **−0.55** | 1.2 | 3.05 |

The zeros of $f(x) = 5x^2 + 30x + 30$ are about $-4.7$ and $-1.3$.

**46.** C; 

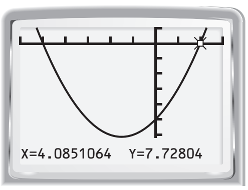

$f(x) = 3x^2 + 4x - 11$ There are two $x$-intercepts: one is between $-3$ and $-2$ and another between 1 and 2.

**47.** $S = 251\text{ft}^2$; $h = 6\text{ft}$

$S = 2\pi rh + 2\pi r^2$

$251 = 2(3.14)r(6) + 2(3.14)r^2$

$0 = 6.28r^2 + 37.68r - 251$




The positive $r$-intercept is about 4.

| ***r*** | 3.8 | 3.9 | **4.0** | 4.1 | 4.2 |
|---|---|---|---|---|---|
| ***S*** | −17.13 | −8.53 | **0.2** | 9.05 | 18.04 |

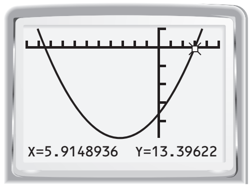
The radius is about about 4.0 feet.

**48.** $S = 716m^2$; $h = 13m$

$S = 2\pi rh + 2\pi r^2$

$716 = 2(3.14)r(13) + 2(3.14)r^2$

$0 = 6.28r^2 + 81.64r - 716$




The positive $r$-intercept is about 6.

| ***r*** | 5.8 | 5.9 | **6.0** | 6.1 | 6.2 |
|---|---|---|---|---|---|
| ***S*** | −31.23 | −15.72 | **−0.08** | 15.68 | 31.57 |

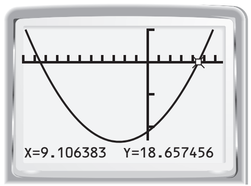
The radius is about 6.0 meters.

**49.** $S = 1074\text{ cm}^2$; $h = 10\text{ cm}$

$S = 2\pi rh + 2\pi r^2$

$1074 = 2(3.14)r(10) + 2(3.14)r^2$

$0 = 6.28r^2 + 62.8r - 1074$




The positive $r$-intercept is about 9.

| ***r*** | 8.8 | 8.9 | **9.0** | 9.1 | 9.2 |
|---|---|---|---|---|---|
| ***S*** | −35.04 | −17.64 | **−0.12** | 17.53 | 35.23 |

The radius is about 9.0 centimeters.

### *Problem Solving*

**50.** $y = -0.04x^2 + 1.2x$

The soccer ball lands when $y = 0$.
The *trace* feature of graphing calculator shows that the $x$-intercepts of $y = -0.04x^2 + 1.2x$ are 0 and 30.
The ball was kicked a distance of 30 feet.

**51.** $y = -0.0017x^2 + 0.041x$

The width of the road is the non-zero $x$-intercept.
The *trace* feature on a graphing calculator shows that the non-zero $x$-intercept is about 24.1.
The width of the road is about 24.1 feet.

**52.** **a.** $h = -16t^2 + vt + s$; $v = 8$; $s = 70$

$h = -16t^2 + 8t + 70$

**b.** The diver reaches the water when $h = 0$. The *trace* feature on a graphing calculator shows that the $t$-intercept is about 2.4. The cliff diver reaches the water about 2.4 seconds after the beginning of the dive.

**53.** $y = -0.75x^2 + 6x$

The arc of water hits the surface of the water when $y = 0$. The *trace* feature on a graphing calculator shows that the $x$-intercept is 8. The distance from the nozzle to one end is 8 feet, so the display diameter is $2(8) = 16$ feet.

**54. a.** $h = -16t^2 + vt + s;\ v = 40;\ s = 5.5$

$h = -16t^2 + 40t + 5.5$

**b.** If the ball lands on the ground, $h = 0$. The *trace* feature on a graphing calculator shows that the $t$-intercept is about 2.6 the ball was in the air about 2.6 seconds.

**c.** If her teammate catches the ball at 5.5 feet, $h = 5.5$. To find the time $t$ when $h = 5.5$, solve $5.5 = -16t^2 + 40t + 5.5$, or $0 = -16t^2 + 40t$, for $t$. Graphing the function $y = -16t^2 + 40t$ on a graphing calculator and using the *trace* feature shows that the $t$-intercept is 2.5. The ball was in the air 2.5 seconds

**55.** $y = -0.003x^2 + 0.58x + 3$

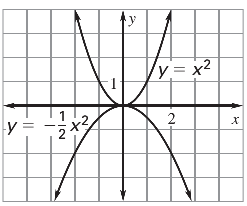
When $x = 137$:

$y = -0.003(137)^2 + 0.58(137) + 3 \approx 26.2$

No; the height of the water at the point 137 feet from the firefighter is about 26.2 feet. Because the top of the window is 26 feet above the ground and the water hits 26.2 feet above the ground, the water will hit the building just above the window.

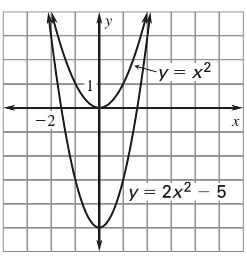
### *Quiz for the lessons "Graph y = ax² + c" "Graph y = ax² + bx + c", and "Solve Quadratic Equations by Graphing"*

**1.**



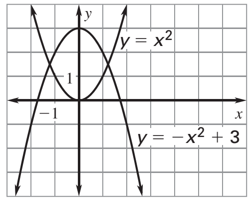

Both graphs have the same vertex, (0, 0), and the same axis of symmetry, $x = 0$. The graph of $y = -\frac{1}{2}x^2$ opens down and is wider because of a reflection in the $x$-axis and a vertical shrink $\left(\text{by a factor of } \frac{1}{2}\right)$.

**2.**




Both graphs open up and have the same axis of symmetry $x = 0$. The graph of $y = 2x^2 - 5$ has a lower vertex and is narrower because of a vertical stretch (by factor of 2) and a vertical translation of 5 units down.

**3.**




Both graphs have the same axis of symmetry, $x = 0$. The graph of $y = -x^2 + 3$ opens down and has a higher vertex because of a reflection in the $x$-axis and a vertical translation of 3 units up.

**4.** $y = x^2 + 5$

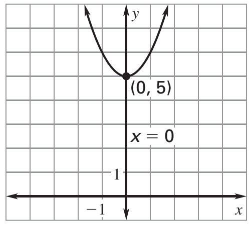
Because $a > 0$, the parabola opens up.

$x = -\frac{b}{2a} = -\frac{0}{2(1)} = 0$

Axis of symmetry: $x = 0$

When $x = 0$: $y = (0)^2 + 5 = 5$

Vertex: (0, 5).




**5.** $y = -5x^2 + 1$

Because $a < 0$, the parabola opens down.

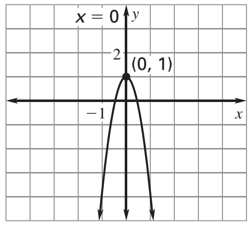
$x = -\frac{b}{2a} = -\frac{0}{2(-5)} = 0$

Axis of symmetry: $x = 0$

When $x = 0$:

$y = -5(0)^2 + 1 = 1$

Vertex: (0, 1).




**6.** $y = x^2 + 4x - 2$

Because $a > 0$, the parabola opens up.

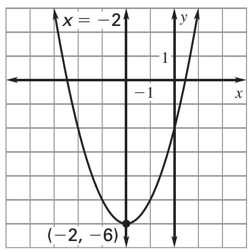
$x = -\frac{b}{2a} = -\frac{4}{2(1)} = -2$

Axis of symmetry: $x = -2$

When $x = -2$:

$y = (-2)^2 + 4(-2) - 2 = -6$

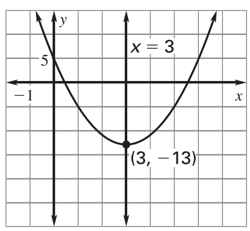
Vertex: $(-2, -6)$.




**7.** $y = 2x^2 - 12x + 5$

Because $a > 0$, the parabola opens up.

$x = -\frac{b}{2a} = -\frac{-12}{2(2)} = 3$



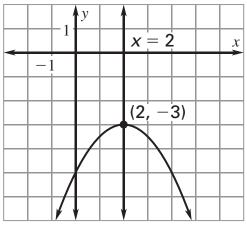

Axis of symmetry: $x = 3$

When $x = 3$:

$y = 2(3)^2 - 12(3) + 5 = -13$

Vertex: $(3, -13)$.

**8.** $y = -\frac{1}{2}x^2 + 2x - 5$




Because $a < 0$, the parabola opens down.

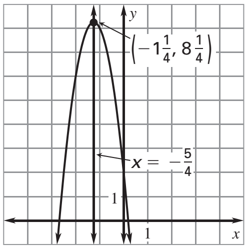
$x = -\frac{b}{2a} = -\frac{2}{2\left(-\frac{1}{2}\right)} = 2$

Axis of symmetry: $x = 2$

When $x = 2$: $y = -\frac{1}{2}(2)^2 + 2(2) - 5 = -3$

Vertex: $(2, -3)$.

**9.** $y = -4x^2 - 10x + 2$




Because $a < 0$, the parabola opens down.

$x = -\frac{b}{2a} = -\frac{-10}{2(-4)} = -\frac{5}{4}$

Axis of symmetry: $x = -\frac{5}{4}$

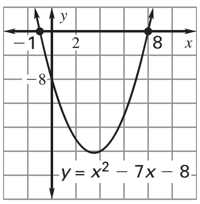
When $x = -\frac{5}{4}$:

$y = -4\left(-\frac{5}{4}\right)^2 - 10\left(-\frac{5}{4}\right) + 2 = \frac{33}{4}$

Vertex: $\left(-\frac{5}{4}, \frac{33}{4}\right)$.

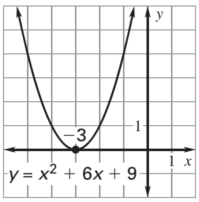
**10.** $x^2 - 7x = 8$

$x^2 - 7x - 8 = 0$




The solutions are $-1$ and 8.

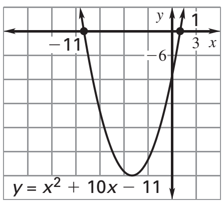
**11.** $x^2 + 6x + 9 = 0$




The solution is $-3$.

**12.** $x^2 + 10x = 11$

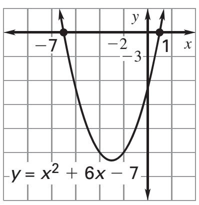
$x^2 + 10x - 11 = 0$




The solutions are $-11$ and 1.

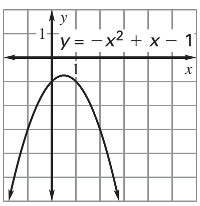
**13.** $x^2 - 7 = -6x$

$x^2 + 6x - 7 = 0$



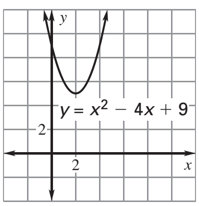

The solutions are $-7$ and 1

**14.** $-x^2 + x - 1 = 0$



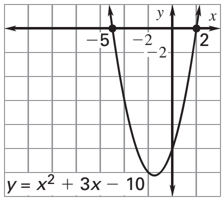

There are no $x$-intercepts. The equation has no solution.

**15.** $x^2 - 4x + 9 = 0$



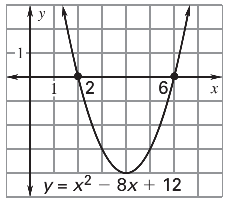

There are no $x$-intercepts. The equation has no solution.

**16.** $f(x) = x^2 + 3x - 10$



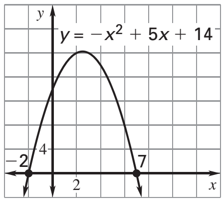

The zeros of the function are $-5$ and 2.

**17.** $f(x) = x^2 - 8x + 12$




The zeros of the function are 2 and 6.

**18.** $f(x) = -x^2 + 5x + 14$




The zeros of the function are $-2$ and 7.

### *Graphing Calculator Activity for the lesson "Solve Quadratic Equations by Graphing"*

**1.** $y = 3x^2 - 8x + 7$; The minimum value of the function is about 1.67.

**2.** $y = -x^2 + 3x + 10$; The maximum value of the function is 12.25.

**3.** $y = -4x^2 - 6x - 6$; The maximum value of the function is $-3.75$.

**4.** $y = 5x^2 + 10x - 8$; The minimum value of the function is $-13$.

**5.** $y = -1.4x^2 + 3.8x - 6.1$; The maximum value of the function is $-3.5$.

**6.** $y = 2.57x^2 - 8.45x - 5.04$; The minimum value of the function is about $-12$.

**7.** $y = 2x^2 - 5x - 8$; The zeros of the function are about $-1.11$ and 3.61.

**8.** $y = -3x^2 + 6x - 2$; The zeros of the function are about 0.42 and 1.58.

**9.** $y = -x^2 + 4x + 9$; The zeros of the function are about $-1.61$ and 5.61.

**10.** $y = 4x^2 - 7x + 1$; The zeros of the function are about 0.16 and 1.59.

**11.** $y = -2.5x^2 + 7.7x - 4.9$; The zeros of the function are about 0.90 and 2.18.

**12.** $y = 1.56x^2 - 5.19x - 2.25$; The zeros of the function are about $-0.39$ and 3.72.

**13.** $y = -0.82x^2 - 4x + 12.4$; The zeros of the function are about $-7.03$ and 2.15.

**14.** $y = 5.36x^2 + 17x + 2.67$; The zeros of the function are about $-3.01$ and $-0.17$.

**15.** If a quadratic function has only one zero, the maximum or minimum value is 0. There is only one zero, so the behavior of the graph changes at that point; therefore making the point a maximum or a minimum.

16. If a quadratic function has a maximum value greater than 0, it has two zeros. The function increases to the maximum and then decreases after the maximum. Because the maximum is greater than 0, the function passes through the $x$-axis twice, creating two zeros.

## Lesson 9.4 Use Square Roots to Solve Quadratic Equations

### *Guided Practice for the lesson "Use Square Roots to Solve Quadratic Equations"*

1. $c^2 - 25 = 0$

$c^2 = 25$

$c = \pm\sqrt{25} = \pm 5$

The solutions are $-5$ and 5.

2. $5w^2 + 12 = -8$

$5w^2 = -20$

$w^2 = -4$

Negative real numbers do not have real square roots. So, there is no solution.

3. $2x^2 + 11 = 11$

$2x^2 = 0$

$x^2 = 0$

$x = 0$

The solution is 0.

4. $25x^2 = 16$

$x^2 = \frac{16}{25}$

$x^2 = \pm\sqrt{\frac{16}{25}} = \pm\frac{4}{5}$

The solutions are $-\frac{4}{5}$ and $\frac{4}{5}$.

5. $9m^2 = 100$

$m^2 = \frac{100}{9}$

$m = \pm\sqrt{\frac{100}{9}} = \pm\frac{10}{3}$

The solutions are $-\frac{10}{3}$ and $\frac{10}{3}$.

6. $49b^2 + 64 = 0$

$49b^2 = -64$

$b^2 = -\frac{64}{49}$

Negative real numbers do not have real square roots. So, there is no solution.

7. $x^2 + 4 = 14$

$x^2 = 10$

$x = \pm\sqrt{10}$

$x \approx \pm 3.16$

The solutions are about $-3.16$ and 3.16.

8. $3k^2 - 1 = 0$

$3k^2 = 1$

$k^2 = \frac{1}{3}$

$k = \pm\sqrt{\frac{1}{3}}$

$k \approx \pm 0.58$

The solutions are about $-0.58$ and 0.58.

9. $2p^2 - 7 = 2$

$2p^2 = 9$

$p^2 = \frac{9}{2}$

$p = \pm\sqrt{\frac{9}{2}}$

$p \approx \pm 2.12$

The solutions are about $-2.12$ and 2.12.

10. $2(x - 2)^2 = 18$

$(x - 2)^2 = 9$

$x - 2 = \pm 3$

$x = 2 \pm 3$

The solutions are $2 - 3 = -1$ and $2 + 3 = 5$.

11. $4(q - 3)^2 = 28$

$(q - 3)^2 = 7$

$q - 3 = \pm\sqrt{7}$

$q = 3 \pm \sqrt{7}$

The solutions are $3 - \sqrt{7} \approx 0.35$ and $3 + \sqrt{7} \approx 5.65$.

12. $3(t + 5)^2 = 24$

$(t + 5)^2 = 8$

$t + 5 = \pm 2\sqrt{2}$

$t = -5 \pm 2\sqrt{2}$

The solutions are $-5 - 2\sqrt{2} \approx -7.83$ and $-5 + 2\sqrt{2} \approx -2.17$.

13. Vertical motion model:

$h = -16t^2 + vt + s$

$h = -16t^2 + 0t + 58$

Substitute 12 for $h$ and solve for $t$.

$h = -16t^2 + 58$

$12 = -16t^2 + 58$

$-46 = -16t^2$

$2.875 = t^2$

$\sqrt{2.875} = t$

$1.70 \approx t$

The ball is in the air for about 1.7 seconds.

### *Exercises for the lesson "Use Square Roots to Solve Quadratic Equations"*

#### *Skill Practice*

1. If $b^2 = a$, then $b$ is $a(n)$ *square root* of $a$.

2. (1) Graph the parabola $y = ax^2 + c$ and find its $x$-intercepts. (2) Write the equation in the form $x^2 = -\frac{c}{a}$ and take the square root of each side.

3. $3x^2 - 3 = 0$

$3x^2 = 3$

$x^2 = 1$

$x = \pm\sqrt{1} = \pm 1$

The solutions are $-1$ and 1.

4. $2x^2 - 32 = 0$

$2x^2 = 32$

$x^2 = 16$

$x^2 = \pm\sqrt{6} = \pm 4$

The solutions are $-4$ and 4.

5. $4x^2 - 400 = 0$

$4x^2 = 400$

$x^2 = 100$

$x = \pm\sqrt{100} = \pm 10$

The solutions are $-10$ and 10.

6. $2m^2 - 42 = 8$

$2m^2 = 50$

$m^2 = 25$

$m = \pm\sqrt{25} = \pm 5$

The solutions are $-5$ and 5.

7. $15d^2 = 0$

$d^2 = 0$

$d = \pm\sqrt{0} = 0$

8. $a^2 + 8 = 3$

$a^2 = -5$

The solution is 0.

Negative real numbers do not have real square roots. So, there is no solution.

**9.** $4g^2 + 10 = 11$

$4g^2 = 1$

$g^2 = \frac{1}{4}$

$g = \pm\sqrt{\frac{1}{4}} = \pm\frac{1}{2}$

The solutions are $-\frac{1}{2}$ and $\frac{1}{2}$.

**10.** $2w^2 + 13 = 11$

$2w^2 = -2$

$w^2 = -1$

Negative real numbers do not have square roots. So, there is no solution.

**11.** $9q^2 - 35 = 14$

$9q^2 = 49$

$q^2 = \frac{49}{9}$

$q = \pm\sqrt{\frac{49}{9}} = \pm\frac{7}{3}$

The solutions are $-\frac{7}{3}$ and $\frac{7}{3}$.

**12.** $25b^2 + 11 = 15$

$25b^2 = 4$

$b^2 = \frac{4}{25}$

$b = \pm\sqrt{\frac{4}{25}} = \pm\frac{2}{5}$

The solutions are $-\frac{2}{5}$ and $\frac{2}{5}$.

**13.** $3z^2 - 18 = -18$

$3z^2 = 0$

$z^2 = 0$

$z = \pm\sqrt{0} = 0$

The solution is 0.

**14.** $5n^2 - 17 = -19$

$5n^2 = -2$

$n^2 = -\frac{2}{5}$

Negative real numbers do not have square roots. So, there is no solution.

**15.** A; $61 - 3n^2 = -14$

$-3n^2 = -75$

$n^2 = 25$

$n^2 = \pm\sqrt{25} = \pm 5$

The solution are $-5$ and 5.

**16.** C; $13 - 36x^2 = -12$

$-36x^2 = -25$

$x^2 = \frac{25}{36}$

$x = \pm\frac{\sqrt{25}}{36} = \pm\frac{5}{6}$

The solutions are $-\frac{5}{6}$ and $\frac{5}{6}$.

**17.** $x^2 + 6 = 13$

$x^2 = 7$

$x = \pm\sqrt{7}$

$x \approx \pm 2.65$

The solutions are about $-2.65$ and 2.65.

**18.** $x^2 + 11 = 24$

$x^2 = 13$

$x = \pm\sqrt{13}$

$x \approx \pm 3.61$

The solutions are about $-3.61$ and 3.61.

**19.** $14 - x^2 = 17$

$-x^2 = 3$

$x^2 = -3$

Negative real numbers do not have real square roots. So, there is no solution.

**20.** $2a^2 - 9 = 11$

$2a^2 = 20$

$a^2 = 10$

$a = \pm\sqrt{10}$

$a \approx \pm 3.16$

The solutions are about $-3.16$ and 3.16.

**21.** $4 - k^2 = 4$

$-k^2 = 0$

$k^2 = 0$

$k = 0$

The solution is 0.

**22.** $5 + 3p^2 = 38$

$3p^2 = 33$

$p^2 = 11$

$p = \pm\sqrt{11}$

$p \approx \pm 3.32$

The solutions are about $-3.32$ and 3.32.

**23.** $53 = 8 + 9m^2$

$45 = 9m^2$

$5 = m^2$

$\pm\sqrt{5} = m$

$\pm 2.24 \approx m$

The solutions are about $-2.24$ and 2.24.

**24.** $-21 = 15 - 2z^2$

$-36 = -2z^2$

$18 = z^2$

$\pm\sqrt{18} = z$

$\pm 4.24 \approx z$

The solutions are about $-4.24$ and 4.24.

**25.** $7c^2 = 100$

$c^2 = \frac{100}{7}$

$c = \pm\sqrt{\frac{100}{7}}$

$c \approx \pm 3.78$

The solutions are about $-3.78$ and 3.78.

**26.** $5d^2 + 2 = 6$

$5d^2 = 4$

$d^2 = \frac{4}{5}$

$d = \pm\sqrt{\frac{4}{5}}$

$d \approx \pm 0.89$

The solutions are about $-0.89$ and 0.89.

**27.** $4b^2 - 5 = 2$

$4b^2 = 7$

$b^2 = \frac{7}{4}$

$b = \pm\sqrt{\frac{7}{4}}$

$b \approx \pm 1.32$

The solutions are about $-1.32$ and 1.32.

**28.** $9n^2 - 14 = -3$

$9n^2 = 11$

$n^2 = \frac{11}{9}$

$n = \pm\sqrt{\frac{11}{9}}$

$n \approx \pm 1.11$

The solutions are about $-1.11$ and 1.11.

**29.** D; $17 - \frac{1}{4}x^2 = 12$

$-\frac{1}{4}x^2 = -5$

$x^2 = 20$

$x = \pm\sqrt{20}$

$x \approx \pm 4.47$

4.47 is between 4 and 5.

**30.** $x^2 = 36$ has two solutions 6 and $-6$, so both numbers should be given as solutions of the equation.

$x = \pm\sqrt{36}$

$x = \pm 6$

The solutions are $-6$ and 6.

**31.** Negative numbers do not have real number square roots, so $\pm\sqrt{-\frac{11}{7}}$ are not real numbers. There is no solution.

**32.** $(x - 7)^2 = 6$

$x - 7 = \pm\sqrt{6}$

$x = 7 \pm\sqrt{6}$

The solutions are $7 - \sqrt{6} \approx 4.55$ and $7 + \sqrt{6} \approx 9.45$.

**33.** $7(x-3)^2 = 35$

$(x-3)^2 = 5$

$x - 3 = \pm\sqrt{5}$

$x = 3 \pm\sqrt{5}$

The solutions are $3-\sqrt{5} \approx 0.76$ and $3 + \sqrt{5} \approx 5.24$.

**34.** $6(x+4)^2 = 18$

$(x+4)^2 = 3$

$x + 4 = \pm\sqrt{3}$

$x = -4 \pm\sqrt{3}$

The solutions are $-4 - \sqrt{3} \approx -5.73$ and $-4 + \sqrt{3} \approx -2.27$.

**35.** $20 = 2(m+5)^2$

$10 = (m+5)^2$

$\pm\sqrt{10} = m + 5$

$-5 \pm\sqrt{10} = m$

The solutions are $-5 - \sqrt{10} \approx -8.16$ and $-5 + \sqrt{10} \approx -1.84$.

**36.** $5(a-2)^2 = 70$

$(a-2)^2 = 14$

$a - 2 = \pm\sqrt{14}$

$a = 2 \pm\sqrt{14}$

The solutions are $2 - \sqrt{14} \approx -1.74$ and $2 + \sqrt{14} \approx 5.74$.

**37.** $21 = 3(z+14)^2$

$7 = (z+14)^2$

$\pm\sqrt{7} = z + 14$

$-14 \pm\sqrt{7} = z$

The solutions are $-14 - \sqrt{7} \approx -16.65$ and $-14 + \sqrt{7} \approx -11.35$

**38.** $\frac{1}{2}(c-8)^2 = 3$

$(c-8)^2 = 6$

$c - 8 = \pm\sqrt{6}$

$c = 8 \pm\sqrt{6}$

The solutions are $8 - \sqrt{6} \approx 5.55$ and $8 + \sqrt{6} \approx 10.45$.

**39.** $\frac{3}{2}(n+1)^2 = 33$

$(n+1)^2 = 22$

$n + 1 = \pm\sqrt{22}$

$n = -1 \pm\sqrt{22}$

The solutions are $-1 - \sqrt{22} \approx -5.69$ and $-1 + \sqrt{22} \approx 3.69$.

**40.** $\frac{4}{3}(k-6)^2 = 20$

$(k-6)^2 = 15$

$k - 6 = \pm\sqrt{15}$

$k = 6 \pm\sqrt{15}$

The solutions are $6 - \sqrt{15} \approx 2.13$ and $6 + \sqrt{15} \approx 9.87$.

**41.** $3x^2 - 35 = 45 - 2x^2$

$5x^2 - 35 = 45$

$5x^2 = 80$

$x^2 = 16$

$x = \pm\sqrt{16} = \pm 4$

The solutions are $-4$ and 4.

**42.** $42 = 3(x^2 + 5)$

$14 = x^2 + 5$

$9 = x^2$

$\pm\sqrt{9} = x$

$\pm 3 = x$

The solutions are $-3$ and 3.

**43.** $11x^2 + 3 = 5(4x^2 - 3)$

$11x^2 + 3 = 20x^2 - 15$

$3 = 9x^2 - 15$

$18 = 9x^2$

$2 = x^2$

$\pm\sqrt{2} = x$

$\pm 1.41 \approx x$

The solutions are $-1.41$ and 1.41.

**44.** $\left(\frac{t-5}{3}\right)^2 = 49$

$\frac{t-5}{3} = \pm 7$

$t - 5 = \pm 21$

$t = 5 \pm 21$

The solutions are $5 - 21 = -16$ and $5 + 21 = 26$.

**45.** $11\left(\frac{w-7}{2}\right)^2 - 20 = 101$

$11\left(\frac{w-7}{2}\right)^2 = 121$

$\left(\frac{w-7}{2}\right)^2 = 11$

$\frac{w-7}{2} = \pm\sqrt{11}$

$w - 7 = \pm 2\sqrt{11}$

$w = 7 \pm 2\sqrt{11}$

The solutions are $7 - 2\sqrt{11} \approx 0.37$ and $7 + \sqrt{11} \approx 13.63$.

**46.** $(4m^2 - 6)^2 = 81$

$4m^2 - 6 = \pm 9$

$4m^2 = 6 \pm 9$

$m^2 = \frac{6 \pm 9}{4}$

$m = \pm\sqrt{\frac{6 \pm 9}{4}}$

$m = \pm\sqrt{\frac{6 + 9}{4}}$

$m = \pm\sqrt{\frac{15}{4}}$

$m \approx \pm 1.94$

The solutions are $-1.94$ and 1.94.

**47.** $A = \pi r^2$

$144\pi = \pi r^2$

$144 = r^2$

$\pm 12 = r$

The radius is 12 inches.

**48.** $A = \pi r^2$

$21\pi = \pi r^2$

$21 = r^2$

$\pm\sqrt{21} = r$

$\pm 4.58 \approx r$

The radius is about 4.58 meters.

**49.** $A = \pi r^2$

$34\pi = \pi r^2$

$34 = r^2$

$\pm\sqrt{34} = r$

$\pm 5.83 \approx r$

$d = 2r$

$d = 2(5.83)$

$d \approx 11.66$

The diameter is about 11.66 feet.

**50.** $y = \frac{1}{2}(x - 2)^2 + 1$

$9 = \frac{1}{2}(x - 2)^2 + 1$

$8 = \frac{1}{2}(x - 2)^2$

$16 = (x - 2)^2$

$\pm 4 = x - 2$

$2 \pm 4 = x$

The $x$-coordinates are $2 - 4 = -2$ and $2 + 4 = 6$.

**51.** $x^2 = 1.44$

$100x^2 = 144$

$x^2 = \frac{144}{100}$

$x = \pm\sqrt{\frac{144}{100}}$

$x = \pm\frac{12}{10}$

$x = \pm 1.2$

When you multiply each side by 100, you can eliminate the decimal leaving two perfect squares.

**52.** **a.** *Sample answer:* $a = 5, c = -1$

$5x^2 - 1 = 0$

$5x^2 = 1$

$x^2 = \frac{1}{5}$

$x = \pm\sqrt{\frac{1}{5}} = \pm 0.45$

**b.** *Sample answer:* $a = 3, c = 0$

$3x^2 = 0$

$x^2 = 0$

$x = 0$

**c.** *Sample answer:* $a = 2, c = 4$

$2x^2 + 4 = 0$

$2x^2 = -4$

$x^2 = -2$

Negative numbers do not have real solutions. So, there is no solution.

**53.** $x^2 - 12x + 36 = 64$

$x^2 - 12x - 28 = 0$

$(x - 14)(x + 2) = 0$

$x - 14 = 0$ $\quad$ $x + 2 = 0$

$x = 14$ $\quad$ $x = -2$

The solutions are $-2$ and 14.

**54.** $x^2 + 14x + 49 = 16$

$x^2 + 14x + 33 = 0$

$(x + 11)(x + 3) = 0$

$x + 11 = 0$ $\quad$ $x + 3 = 0$

$x = -11$ $\quad$ $x = -3$

The solutions are $-11$ and $-3$.

**55.** $x^2 + 18x + 81 = 25$

$x^2 + 18x + 56 = 0$

$(x + 14)(x + 4) = 0$

$x + 14 = 0$ $\quad$ $x + 4 = 0$

$x = -14$ $\quad$ $x = -4$

The solutions are $-14$ and $-4$.

### *Problem Solving*

**56.** Vertical motion model:

$h = -16t^2 + vt + s$

$h = -16t^2 + 0t + 38$

Substitute 0 for $h$ and solve for $t$.

$h = -16t^2 + 38$

$0 = -16t^2 + 38$

$-38 = -16t^2$

$2.375 = t^2$

$\sqrt{2.375} = t$

$1.54 \approx t$

It took about 1.54 seconds for the sunglasses to hit the ground.

**57.** C; Vertical motion model:

$h = -16t^2 + vt + s$

If something is dropped, $v = 0$. If something hits the ground, $h = 0$. We are given that its initial height is 68 feet, so $s = 68$. Substituting in those values, we get

$0 = -16t^2 + 68$ $\quad$ or $\quad$ $-16t^2 + 68 = 0$.

**58.** $y = 12{,}697x^2 + 55{,}722$

When $y = 100{,}000$: $100{,}000 = 12{,}697x^2 + 55{,}722$

$44{,}278 = 12{,}697x^2$

$3.49 \approx x^2$

$1.87 \approx x$

$1995 + 1.87 = 1996.87$. Between 1996 and 1997 the number of Internet users worldwide reached 100,000,000.

**59.** $w = 0.0018D^2ds$

**a.** When $w = 1$, $d = 4.5$, $s = 2.65$:

$1 = 0.0018D^2(4.5)(2.65)$

$1 = 0.021465D^2$

$46.6 \approx D^2$

$\pm 6.8 \approx D$

The diameter of the Amethyst is about 6.8 mm.

**b.** When $w = 1$, $d = 4.5$, $s = 3.52$:

$1 = 0.0018D^2(4.5)(3.52)$

$1 = 0.028512D^2$

$35.1 \approx D^2$

$5.9 \approx D$

The diameter of the Diamond is about 5.9 mm.

**c.** When $w = 1$, $d = 4.5$, $s = 4.00$:

$1 = 0.0018D^2(4.5)(4)$

$1 = 0.0324D^2$

$30.9 \approx D^2$

$5.6 \approx D$

The diameter of the Ruby is about 5.6 mm.

**60.** $2\pi s^2 = 9.8L$

**a.** When $L = 6$: $2\pi s^2 = 9.8(6)$

$s^2 \approx 9.36$

$s \approx \pm 3.06$

The speed is about 3.06 m/sec for a 6 m wavelength.

When $L = 10$: $2\pi s^2 = 9.8(10)$

$s^2 \approx 15.61$

$s \approx \pm 3.95$

The speed is about 3.95 m/sec for a 10 m wavelength.

When $L = 25$: $2\pi s^2 = 9.8(25)$

$s^2 \approx 39.01$

$s \approx \pm 6.25$

The speed is about 6.25 m/sec for a 25 m wavelength.

**b.** As the wavelength in part (a) increased, the speed of the respective series of waves increased.

**61. a.** $V = \frac{L(D-4)^2}{16}$

$16V = L(D-4)^2$

$\frac{16V}{L} = (D-4)^2$

$\pm\sqrt{\frac{16V}{L}} = D - 4$

$4 \pm\sqrt{\frac{16V}{L}} = D$

**b.** When $V = 50$ and

$L = 16$: $D = 4 + \sqrt{\frac{16(50)}{16}} = 4 + 5\sqrt{2} \approx 11.1$

The diameter is about 11.1 inches when the length of a log is 16 feet.

$L = 18$: $D = 4 + \sqrt{\frac{16(50)}{18}} = 4 + \sqrt{44.44} \approx 10.7$

The diameter is about 10.7 inches when the length of a log is 18 feet.

$L = 20$: $D = 4 + \sqrt{\frac{16(50)}{20}} = 4 + 2\sqrt{10} \approx 10.3$

The diameter is about 10.3 inches when the length of a log is 20 feet.

$L = 22$: $D = 4 + \sqrt{\frac{16(50)}{22}} = 4 + \sqrt{36.36} \approx 10.0$

The diameter is about 10.0 inches when the length of a log is 22 feet.

**62. a.** $h = -16t^2 + vt + s$

$h = -16t^2 + 250$

**b.** When $V = 0$ and $s = 250$:

| **time, seconds, *s*** | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| **height, feet, *h*** | 250 | 234 | 186 | 106 | −6 |

The riders experience free fall for about 3 seconds.

**c.** $105 = -16t^2 + 250$

$-145 = -16t^2$

$9.0625 = t^2$

$3.0 \approx t$

The riders experience free fall for about 3.0 seconds.

**63.** Earth

When $g = 32$ and $h = 0$:

$0 = \frac{-32}{2}t_E^{\,2} + s$

$-s = -16t_E^{\,2}$

$\frac{s}{16} = t_E^{\,2}$

$\sqrt{\frac{s}{16}} = t_E$

Mars

When $g = 12$ and $h = 0$:

$0 = -\frac{12}{2}t^2M + s$

$-s = -6t^2M$

$\frac{s}{6} = t_m^{\,2}$

$\sqrt{\frac{s}{6}} = t_M$

The object will hit the ground on Earth first because $t_E$ is always less than $t_M$.

## *Problem Solving Workshop for the lesson "Use Square Roots to Solve Quadratic Equations"*

**1.** Method 1: Use factoring. Vertical motion model:

$$h = -16t^2 + vt + s$$

When $v = 0$ and $s = 45$: $h = -16t^2 + 0t + 45$

Substitute 10 for $h$.

$10 = -16t^2 + 45$

$0 = -16t^2 + 35$

Solve the equation by factoring. Replace 35 with the closest perfect square, 36.

$0 = -16t^2 + 36$

$0 = -(16t^2 - 36)$

$0 = -(4t - 6)(4t + 6)$

$4t - 6 = 0$ or $4t + 6 = 0$

$t = \frac{6}{4}$ or $t = -\frac{6}{4}$

The ball is in the air about $\frac{6}{4}$, or 1.5 seconds.

Method 2: Make and use a table. Substitute values in the function $h = -16t^2 + 45$. Use increments of 1 second.

| Time $t$ (seconds) | Height $h$ (feet) |
|---|---|
| 0 | 45 |
| 1 | 29 |
| 2 | $-19$ |

Identify the time interval in which the height of the ball is 10 feet. This happens between 1 and 2 seconds. Make a second table using increments of 0.1 second to get a closer approximation.

| Time $t$ (seconds) | Height $h$ (feet) |
|---|---|
| 1.3 | 17.96 |
| 1.4 | 13.64 |
| **1.5** | **9** |
| 1.6 | 4.04 |
| 1.7 | $-1.24$ |

The ball is in the air about 1.5 seconds.

**2.** Answers will vary.

**3. a.** $V = \ell wh$

When $\ell = 5x$, $w = 5$, $h = x$:

$V = (5x)(5)(x)$

$V = 25x^2$

**b.** $83 = 25x^2$

$0 = 25x^2 - 83$

Replace 83 with the closest perfect square, 81.

$0 = 25x^2 - 81$

$0 = (5x - 9)(5x + 9)$

$5x - 9 = 0$ $\quad 5x + 9 = 0$

$x = \frac{9}{5}$ $\quad x = -\frac{9}{5}$

The dimensions of the box are about 9 inches by 5 inches by 1.8 inches.

**c.**

| $x$, (inches) | $V$, (cubic inches) |
|---|---|
| 1.6 | 64 |
| 1.7 | 72.25 |
| 1.8 | 81 |
| 1.9 | 90.25 |
| 2 | 100 |

$x$ is approximately 1.8 inches, so the dimensions of the box are about 9 inches by 5 inches by 1.8 inches.

**4.** Vertical motion model: $h = -16t^2 + vt + s$

When $h = 6$, $v = 0$, and $s = 54$: $6 = -16t^2 + 0t + 54$

$6 = -16t^2 + 54$

$-48 = -16t^2$

$3 = t^2$

$\sqrt{3} = t$

$1.73 \approx t$

It takes about 1.73 seconds for your shoe to hit the net.

**5.** The error occurs when the student adds 6 to each side in the second step instead of subtracting 6 from each side.

$6 = -16t^2 + 54$

$0 = -16t^2 + 48$

Replace 48 with the closest perfect square, 49.

$0 = -16t^2 + 49$

$0 = -(4t + 7)(4t - 7)$

$t = -\frac{7}{4}$ or $t = \frac{7}{4}$

It takes about $\frac{7}{4}$ or 1.75 seconds.

## *Mixed Review of Problem Solving for the "Graph y = ax² + c", "Graph y = ax² + bx + c", "Solve Quadratic Equations by Graphing", and "Use Square Roots to Solve Quadratic Equations"*

**1. a.** The lowest yearly profit is at the vertex of the parabola.

When $a = 1$ and $b = -8$

$x = -\frac{b}{2a} = -\frac{-8}{2(1)} = 4$

The lowest yearly profit in 1996 + 4, or 2000.

**b.** Substitute 4 for $x$ to find the $y$-coordinate of the vertex.

$y = (4)^2 - 8(4) + 80 = 64$

The lowest yearly profit was $64,000.

**2. a.** $A = \ell \cdot w$

When $\ell = 2x$ and $w = 14 - x$:

$A = (2x)(14 - x) = 28x - 2x^2$

The greatest possible area is at the vertex of the parabola.

When $a = -2$ and $b = 28$:

$$x = -\frac{b}{2a} = -\frac{28}{2(-2)} = 7$$

When $x = 7$, the area is the greatest.

**b.** Substitute 7 for $x$ to find the $y$-coordinate of the vertex.

$A = 28(7) - 2(7)^2 = 98$

The greatest possible area is 98 square feet.

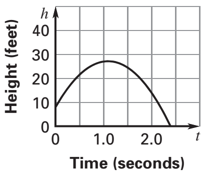
**3. a.** $h = -16t^2 + vt + s$

When $y = 35$ and $s = 8$: $h = -16t^2 + 35t + 8$

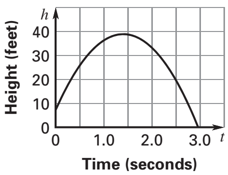
**b.** $h = -16t^2 + vt + s$

When $v = 45$ and $s = 7$: $h = -16t^2 + 45t + 7$

**c.** First throw: $-16t^2 + 35t + 8$




Second throw: $h = -16t^2 + 45t + 7$




From the graph, the first throw takes about 2.4 seconds to hit the ground. The second throw takes about 3 seconds to hit the ground. So, the ball is in the air longer during the second throw.

**4.** Answers will vary.

**5.** Yes; The height of the football 45 yards from where is was kicked can be found by the formula $y = -0.03x^2 + 1.8x$ when $x = 45$, $y = -0.03(45^2) + 1.8(45) = 20.25$ ft. The ball will be above the 10-foot high goal post.

**6.** $F = \frac{mv^2}{r}$

When $m = 75$, $F = 18{,}150$, and $r = 8$:

$$18{,}150 = \frac{75v^2}{8}$$

$$1936 = v^2$$

$$\sqrt{1936} = v$$

$$44 = v$$

The velocity of the train around the curve is 44 meters per second.

**7. a.** The maximum height is at the vertex of the parabola.

When $a = -0.18$ and $b = 4.4$:

$$x = -\frac{b}{2a} = -\frac{4.4}{2(-0.18)} \approx 12.2$$

Substitute 12.2 for $x$ to find the $y$-coordinate of the vertex.

$y = -0.18(12.2)^2 + 4.4(12.2) - 12 \approx 14.9$

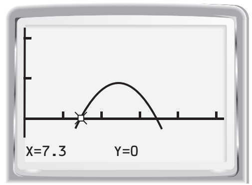
The maximum height of the tunnel is about 14.9 feet.

**b.** Yes. Find the $x$-coordinates of the points corresponding to a height of 10.5 feet:

$10.5 = -0.18x^2 + 4.4x - 12$

$0 = -0.18x^2 + 4.4x - 22.5$

Graph $y = -0.18x^2 + 4.4x - 22.5$ on a graphing calculator. Use the *trace* feature to find the $x$-intercepts, about 7.3 and 17.2. The distance between these two points is about 17.2 − 7.3, or 9.9 feet. So, the semi trailer will be able to fit through the tunnel.




## Lesson 9.5 Solve Quadratic Equations by Completing the Square

### *Investigating Algebra Activity for the lesson "Solve Quadratic Equations by Completing the Square"*

**1.** $x^2 + 6x + 9 = (x + 3)^2$

| $x^2$ | $x$ | $x$ | $x$ |
|---|---|---|---|
| $x$ | 1 | 1 | 1 |
| $x$ | 1 | 1 | 1 |
| $x$ | 1 | 1 | 1 |

$x^2 + 8x + 16 = (x + 4)^2$

| $x^2$ | $x$ | $x$ | $x$ | $x$ |
|---|---|---|---|---|
| $x$ | 1 | 1 | 1 | 1 |
| $x$ | 1 | 1 | 1 | 1 |
| $x$ | 1 | 1 | 1 | 1 |
| $x$ | 1 | 1 | 1 | 1 |

$x^2 + 10x + 25 = (x + 5)^2$

| $x^2$ | $x$ | $x$ | $x$ | $x$ | $x$ |
|---|---|---|---|---|---|
| $x$ | 1 | 1 | 1 | 1 | 1 |
| $x$ | 1 | 1 | 1 | 1 | 1 |
| $x$ | 1 | 1 | 1 | 1 | 1 |
| $x$ | 1 | 1 | 1 | 1 | 1 |
| $x$ | 1 | 1 | 1 | 1 | 1 |

| Expression | Number of 1-tiles needed to complete the square | Expression written as a square |
|---|---|---|
| $x^2 + 4x$ | 4 | $x^2 + 4x + 4 = (x + 2)^2$ |
| $x^2 + 6x$ | 9 | $x^2 + 6x + 9 = (x + 3)^2$ |
| $x^2 + 8x$ | 16 | $x^2 + 8x + 16 = (x + 4)^2$ |
| $x^2 + 10x$ | 25 | $x^2 + 10x + 25 = (x + 5)^2$ |

**2.** From the table, you can see that $b = 2d$ and $c = d^2$ in the equation $x^2 + bx + c = (x + d)^2$.

**3.** When $b = 18$, $d = 9$.

When $d = 9$, $c = 81$.

So, you need 81 1-tiles to complete the square.

### *Guided Practice for the lesson "Solve Quadratic Equations by Completing the Square"*

**1.** $c = \left(\frac{8}{2}\right)^2 = \frac{64}{4} = 16$

$x^2 + 8x + c = x^2 + 8x + 16 = (x + 4)^2$

The value of $c$ is 16 and the expression is $(x + 4)^2$.

**2.** $c = \left(-\frac{12}{2}\right)^2 = \frac{144}{4} = 36$

$x^2 - 12x + c = x^2 - 12x + 36 = (x - 6)^2$

The value of $c$ is 36 and the expression is $(x - 6)^2$.

**3.** $c = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

$x^2 + 3x + c = x^2 + 3x + \frac{9}{4} = \left(x + \frac{3}{2}\right)^2$

The value of $c$ is $\frac{9}{4}$ and the expression is $\left(x + \frac{3}{2}\right)^2$.

**4.**
$$x^2 - 2x = 3$$
$$x^2 - 2x + (-1)^2 = 3 + (-1)^2$$
$$(x - 1)^2 = 4$$
$$x - 1 = \pm\sqrt{4}$$
$$x = 1 \pm 2$$

The solutions are $1 - 2 = -1$ and $1 + 2 = 3$.

**5.**
$$m^2 + 10m = -8$$
$$m^2 + 10m + 5^2 = -8 + 5^2$$
$$(m + 5)^2 = 17$$
$$m + 5 = \pm\sqrt{17}$$
$$m = -5 \pm\sqrt{17}$$

The solutions are $-5 - \sqrt{17} \approx -9.12$ and $-5 + \sqrt{17} \approx -0.88$.

**6.**
$$3g^2 - 24g + 27 = 0$$
$$3g^2 - 24g = -27$$
$$g^2 - 8g = -9$$
$$g^2 - 8g + (-4)^2 = -9 + (-4)^2$$
$$(g - 4)^2 = 7$$
$$g - 4 = \pm\sqrt{7}$$
$$g = 4 \pm\sqrt{7}$$

The solutions are $4 - \sqrt{7} \approx 1.35$ and $4 + \sqrt{7} \approx 6.65$.

**7.**

| Area of chalkboard (square feet) | = | Length of chalkboard (feet) | • | Width of chalkboard (feet) |
|---|---|---|---|---|
| 4 | = | $(7 - 2x)$ | • | $(3 - 2x)$ |

$$4 = (7 - 2x)(3 - 2x)$$
$$4 = 21 - 20x + 4x^2$$
$$-17 = 4x^2 - 20x$$
$$-\frac{17}{4} = x^2 - 5x$$
$$-\frac{17}{4} + \frac{25}{4} = x^2 - 5x + \frac{25}{4}$$
$$-\frac{17}{4} + \frac{25}{4} = \left(x - \frac{5}{2}\right)^2$$
$$2 = \left(x - \frac{5}{2}\right)^2$$
$$\pm\sqrt{2} = x - \frac{5}{2}$$
$$\frac{5}{2} \pm\sqrt{2} = x$$

The solutions of the equation are $\frac{5}{2} - \sqrt{2} \approx 1.09$ and $\frac{5}{2} + \sqrt{2} \approx 3.91$. It is not possible for the width of the border to be 3.91 feet because the width of the door is 3 feet. So, the border is 1.09 feet.

$$1.09\ \cancel{\text{feet}} \cdot \frac{12 \text{ inches}}{1\ \cancel{\text{foot}}} = 13.08 \text{ in.}$$

The width of the border should be about 13 inches.

### *Exercises for the lesson "Solve Quadratic Equations by Completing the Square"*

#### *Skill Practice*

**1.** The process of writing an expression of the form $x^2 + bx$ as a perfect square trinomial is called *completing the square*.

**2.** *Sample answer:* $x^2 + 14x + 49$; 49 is the square of one half of 14, which makes it the square of a binomial $(x + 7)^2$.

**3.** $c = \left(\frac{6}{2}\right)^2 = \frac{36}{4} = 9$

$x^2 + 6x + c = x^2 + 6x + 9 = (x + 3)^2$

The value of $c$ is 9 and the expression is $(x + 3)^2$.

**4.** $c = \left(\frac{12}{2}\right)^2 = \frac{144}{4} = 36$

$x^2 + 12x + c = x^2 + 12x + 36 = (x + 6)^2$

The value of $c$ is 36 and the expression is $(x + 6)^2$.

**5.** $c = \left(\frac{-4}{2}\right)^2 = \frac{16}{4} = 4$

$x^2 - 4x + c = x^2 - 4x + 4 = (x - 2)^2$

The value of $c$ is 4 and the expression is $(x - 2)^2$.

6. $c = \left(\frac{-8}{2}\right)^2 = \frac{64}{4} = 16$

$x^2 - 8x + c = x^2 - 8x + 16 = (x - 4)^2$

The value of $c$ is 16 and the expression is $(x - 4)^2$.

7. $c = \left(\frac{-3}{2}\right)^2 = \frac{9}{4}$

$x^2 - 3x + c = x^2 - 3x + \frac{9}{4} = \left(x - \frac{3}{2}\right)^2$

The value of $c$ is $\frac{9}{4}$ and the expression is $\left(x - \frac{3}{2}\right)^2$.

8. $c = \left(\frac{5}{2}\right)^2 = \frac{25}{4}$

$x^2 + 5x + c = x^2 + 5x + \frac{25}{4} = \left(x + \frac{5}{2}\right)^2$

The value of $c$ is $\frac{25}{4}$ and the expression is $\left(x + \frac{5}{2}\right)^2$.

9. $c = \left(\frac{2.4}{2}\right)^2 = \frac{5.76}{4} = 1.44$

$x^2 + 2.4x + c = x^2 + 2.4x + 1.44 = (x + 1.2)^2$

The value of $c$ is 1.44 and the expression is $(x + 1.2)^2$.

10. $c = \left(\frac{-\frac{1}{2}}{2}\right)^2 = \frac{\frac{1}{4}}{4} = \frac{1}{16}$

$x^2 - \frac{1}{2}x + c = x^2 - \frac{1}{2}x + \frac{1}{16} = \left(x - \frac{1}{4}\right)^2$

The value of $c$ is $\frac{1}{16}$ and the expression is $\left(x - \frac{1}{4}\right)^2$.

11. $c = \left(\frac{-\frac{4}{3}}{2}\right)^2 = \frac{\frac{16}{9}}{4} = \frac{4}{9}$

$x^2 - \frac{4}{3}x + c = x^2 - \frac{4}{3}x + \frac{4}{9} = \left(x - \frac{2}{3}\right)^2$

The value of $c$ is $\frac{4}{9}$ and the expression is $\left(x - \frac{2}{3}\right)^2$.

12. $x^2 + 2x = 3$

$x^2 + 2x + 1^2 = 3 + 1^2$

$(x + 1)^2 = 4$

$x + 1 = \pm\sqrt{4}$

$x = -1 \pm 2$

The solutions are $-1 - 2 = -3$ and $-1 + 2 = 1$.

13. $x^2 + 10x = 24$

$x^2 + 10x + 5^2 = 24 + 5^2$

$(x + 5)^2 = 49$

$x + 5 = \pm\sqrt{49}$

$x = -5 \pm 7$

The solutions are $-5 - 7 = -12$ and $-5 + 7 = 2$.

14. $c^2 - 14c = 15$

$c^2 - 14c + (-7)^2 = 15 + (-7)^2$

$(c - 7)^2 = 64$

$c - 7 = \pm\sqrt{64}$

$c = 7 \pm 8$

The solutions are $7 - 8 = -1$ and $7 + 8 = 15$.

15. $n^2 - 6n = 72$

$n^2 - 6n + (-3)^2 = 72 + (-3)^2$

$(n - 3)^2 = 81$

$n - 3 = \pm\sqrt{81}$

$n = 3 \pm 9$

The solutions are $3 - 9 = -6$ and $3 + 9 = 12$.

16. $a^2 - 8a + 15 = 0$

$a^2 - 8a = -15$

$a^2 - 8a + (-4)^2 = -15 + (-4)^2$

$(a - 4)^2 = 1$

$a - 4 = \pm\sqrt{1}$

$a = 4 \pm 1$

The solutions are $4 - 1 = 3$ and $4 + 1 = 5$.

17. $y^2 + 4y - 21 = 0$

$y^2 + 4y = 21$

$y^2 + 4y + 2^2 = 21 + 2^2$

$(y + 2)^2 = 25$

$y + 2 = \pm\sqrt{25}$

$y = -2 \pm 5$

The solutions are $-2 - 5 = -7$ and $-2 + 5 = 3$.

18. $w^2 - 5w = \frac{11}{4}$

$w^2 - 5w + \left(-\frac{5}{2}\right)^2 = \frac{11}{4} + \left(\frac{5}{2}\right)^2$

$\left(w - \frac{5}{2}\right)^2 = 9$

$w - \frac{5}{2} = \pm\sqrt{9}$

$w = \frac{5}{2} \pm 3$

The solutions are $\frac{5}{2} - 3 = -0.5$ and $\frac{5}{2} + 3 = 5.5$.

19. $z^2 + 11z = \frac{-21}{4}$

$z^2 + 11z + \left(\frac{11}{2}\right)^2 = \frac{-21}{4} + \left(\frac{11}{2}\right)^2$

$\left(z + \frac{11}{2}\right)^2 = 25$

$z + \frac{11}{2} = \pm\sqrt{25}$

$z = \frac{-11}{2} \pm 5$

The solutions are $\frac{-11}{2} - 5 = -10.5$ and $\frac{-11}{2} + 5 = -0.5$.

**20.** $g^2 - \frac{2}{3}g = 7$

$g^2 - \frac{2}{3}g + \left(-\frac{1}{3}\right)^2 = 7 + \left(-\frac{1}{3}\right)^2$

$\left(g - \frac{1}{3}\right)^2 = \frac{64}{9}$

$g - \frac{1}{3} = \pm\sqrt{\frac{64}{9}}$

$g = \frac{1}{3} \pm \frac{8}{3}$

The solutions are $\frac{1}{3} - \frac{8}{3} \approx -2.33$ and $\frac{1}{3} + \frac{8}{3} = 3$.

**21.** $k^2 - 8k - 7 = 0$

$k^2 - 8k = 7$

$k^2 - 8k + (-4)^2 = 7 + (-4)^2$

$(k - 4)^2 = 23$

$k - 4 = \pm\sqrt{23}$

$k = 4 \pm\sqrt{23}$

The solutions are $4 - \sqrt{23} \approx -0.80$ and $4 + \sqrt{23} \approx 8.80$.

**22.** $v^2 - 7v + 1 = 0$

$v^2 - 7v = -1$

$v^2 - 7v + \left(\frac{-7}{2}\right)^2 = -1 + \left(\frac{-7}{2}\right)^2$

$\left(v - \frac{7}{2}\right)^2 = 11.25$

$v - \frac{7}{2} = \pm\sqrt{11.25}$

$v = \frac{7}{2} \pm\sqrt{11.25}$

The solutions are $\frac{7}{2} - \sqrt{11.25} \approx 0.15$ and $\frac{7}{2} + \sqrt{11.25} \approx 6.85$.

**23.** $m^2 + 3m + \frac{5}{4} = 0$

$m^2 + 3m = \frac{-5}{4}$

$m^2 + 3m + \left(\frac{3}{2}\right)^2 = \frac{-5}{4} + \left(\frac{3}{2}\right)^2$

$\left(m + \frac{3}{2}\right)^2 = 1$

$m + \frac{3}{2} = \pm\sqrt{1}$

$m = \frac{-3}{2} \pm 1$

The solutions are $\frac{-3}{2} - 1 = -2.5$ and $\frac{-3}{2} + 1 = -0.5$.

**24.** C; $4x^2 + 16x = 9$

$x^2 + 4x = \frac{9}{4}$

$x^2 + 4x + 2^2 = \frac{9}{4} + 2^2$

$(x + 2)^2 = \frac{25}{4}$

$x + 2 = \pm\sqrt{\frac{25}{4}}$

$x = -2 \pm \frac{5}{2}$

The solutions are $-2 - \frac{5}{2} = \frac{-9}{2}$ and $-2 + \frac{5}{2} = \frac{1}{2}$.

**25.** B; $x^2 + 12x + 10 = 0$

$x^2 + 12x = -10$

$x^2 + 12x + 6^2 = -10 + 6^2$

$(x + 6)^2 = 26$

$x + 6 = \pm\sqrt{26}$

$x = -6 \pm\sqrt{26}$

**26.** The error occurs in the second step. 49 should be added to both sides not just the left side.

$x^2 - 14x = 11$

$x^2 - 14x + 49 = 11 + 49$

$(x - 7)^2 = 60$

$x - 7 = \pm\sqrt{60}$

$x = 7 \pm 2\sqrt{15}$

**27.** The error occurs when factoring $x^2 - 2x + 1$. It should be $(x - 1)^2$, not $(x + 1)^2$.

$x^2 - 2x - 4 = 0$

$x^2 - 2x = 4$

$x^2 - 2x + 1 = 4 + 1$

$(x - 1)^2 = 5$

$x - 1 = \pm\sqrt{5}$

$x = 1 \pm\sqrt{5}$

28. $2x^2 - 8x - 14 = 0$

$$2x^2 - 8x = 14$$
$$x^2 - 4x = 7$$
$$x^2 - 4x + (-2)^2 = 7 + (-2)^2$$
$$(x-2)^2 = 11$$
$$x - 2 = \pm\sqrt{11}$$
$$x = 2 \pm\sqrt{11}$$

The solutions are $2 - \sqrt{11} \approx -1.32$ and $2 + \sqrt{11} \approx 5.32$.

29. $2x^2 + 24x + 10 = 0$

$$2x^2 + 24x = -10$$
$$x^2 + 12x = -5$$
$$x^2 + 12x + 6^2 = -5 + 6^2$$
$$(x+6)^2 = 31$$
$$x + 6 = \pm\sqrt{31}$$
$$x = -6 \pm\sqrt{31}$$

The solutions are $-6 - \sqrt{31} \approx -11.57$ and $-6 + \sqrt{31} \approx -0.43$.

30. $3x^2 - 48x + 39 = 0$

$$3x^2 - 48x = -39$$
$$x^2 - 16x = -13$$
$$x^2 - 16x + (-8)^2 = -13 + (-8)^2$$
$$(x-8)^2 = 51$$
$$x - 8 = \pm\sqrt{51}$$
$$x = 8 \pm\sqrt{51}$$

The solutions are $8 - \sqrt{51} \approx 0.86$ and $8 + \sqrt{51} \approx 15.14$.

31. $4y^2 + 4y - 7 = 0$

$$4y^2 + 4y = 7$$
$$y^2 + y = \frac{7}{4}$$
$$y^2 + 1 + \left(\frac{1}{2}\right)^2 = \frac{7}{4} + \left(\frac{1}{2}\right)^2$$
$$\left(y + \frac{1}{2}\right)^2 = \pm\sqrt{2}$$
$$y + \frac{1}{2} = \pm\sqrt{2}$$
$$y = \frac{-1}{2} \pm\sqrt{2}$$

The solutions are $\frac{-1}{2} - \sqrt{2} \approx -1.91$ and $\frac{-1}{2} + \sqrt{2} \approx 0.91$.

32. $9n^2 + 36n + 11 = 0$

$$9n^2 + 36n = -11$$
$$n^2 + 4n = \frac{-11}{9}$$
$$n^2 + 4n + 2^2 = \frac{-11}{9} + 2^2$$
$$(n+2)^2 = \frac{25}{9}$$
$$n + 2 = \pm\sqrt{\frac{25}{9}}$$
$$n = -2 \pm \frac{5}{3}$$

The solutions are $-2 - \frac{5}{3} \approx -3.67$ and $-2 + \frac{5}{3} \approx -0.33$.

33. $3w^2 - 18w - 20 = 0$

$$3w^2 - 18w = 20$$
$$w^2 - 6w = \frac{20}{3}$$
$$w^2 - 6w + (-3)^2 = \frac{20}{3} + (-3)^2$$
$$(w-3)^2 = \frac{47}{3}$$
$$w - 3 = \pm\sqrt{\frac{47}{3}}$$
$$w = 3 \pm\sqrt{\frac{47}{3}}$$

The solutions are $3 - \sqrt{\frac{47}{3}} \approx -0.96$ and $3 + \sqrt{\frac{47}{3}} \approx 6.96$.

34. $3p^2 - 30p - 11 = 6p$

$$3p^2 - 36p = 11$$
$$p^2 - 12p = \frac{11}{3}$$
$$p^2 - 12p + (-6)^2 = \frac{11}{3} + (-6)^2$$
$$(p-6)^2 = \frac{119}{3}$$
$$p - 6 = \pm\sqrt{\frac{119}{3}}$$
$$p = 6 \pm\sqrt{\frac{119}{3}}$$

The solutions are $6 - \sqrt{\frac{119}{3}} \approx -0.30$ and $6 + \sqrt{\frac{119}{3}} \approx 12.30$.

35. $3a^2 - 12a + 3 = -a^2 - 4$

$$4a^2 - 12a = -7$$
$$a^2 - 3a = \frac{-7}{4}$$
$$a^2 - 3a + \left(-\frac{3}{2}\right)^2 = \frac{-7}{4} + \left(-\frac{3}{2}\right)^2$$
$$\left(a - \frac{3}{2}\right)^2 = \frac{1}{2}$$
$$a - \frac{3}{2} = \pm\sqrt{\frac{1}{2}}$$
$$a = \frac{3}{2} \pm\sqrt{\frac{1}{2}}$$

The solutions are $\frac{3}{2} - \sqrt{\frac{1}{2}} \approx 0.79$ and $\frac{3}{2} + \sqrt{\frac{1}{2}} \approx 2.21$.

36. $15c^2 - 51c - 30 = 9c + 15$

$$15c^2 - 60c = 45$$
$$c^2 - 4c = 3$$
$$c^2 - 4c + (-2)^2 = 3 + (-2)^2$$
$$(c-2)^2 = 7$$
$$c - 2 = \pm\sqrt{7}$$
$$c = 2 \pm\sqrt{7}$$

The solutions are $2 - \sqrt{7} \approx -0.65$ and $2 + \sqrt{7} \approx 4.65$.

**37.** $7m^2 + 24m - 2 = m^2 - 9$

$$6m^2 + 24m = -7$$
$$m^2 + 4m = \frac{-7}{6}$$
$$m^2 + 4m + 2^2 = \frac{-7}{6} + 2^2$$
$$(m + 2)^2 = \frac{17}{6}$$
$$m + 2 = \pm\sqrt{\frac{17}{6}}$$
$$m = -2 \pm\sqrt{\frac{17}{6}}$$

The solutions are $-2 - \sqrt{\frac{17}{6}} \approx -3.68$ and $-2 + \sqrt{\frac{17}{6}} \approx -0.32$.

**38.** $g^2 + 2g + 0.4 = 0.9g^2 + g$

$$0.1g^2 + g = -0.4$$
$$g^2 + 10g = -4$$
$$g^2 + 10g + 5^2 = -4 + 5^2$$
$$(g + 5)^2 = 21$$
$$g + 5 = \pm\sqrt{21}$$
$$g = -5 \pm\sqrt{21}$$

The solutions are $-5 - \sqrt{21} \approx -9.58$ and $-5 + \sqrt{21} \approx -0.42$.

**39.** $11z^2 - 10z - 3 = -9z^2 + \frac{3}{4}$

$$20z^2 - 10z = \frac{15}{4}$$
$$z^2 - \frac{1}{2}z = \frac{3}{16}$$
$$z^2 - \frac{1}{2}z + \left(-\frac{1}{4}\right)^2 = \frac{3}{16} + \left(-\frac{1}{4}\right)^2$$
$$\left(z - \frac{1}{4}\right)^2 = \frac{1}{4}$$
$$z - \frac{1}{4} = \pm\sqrt{\frac{1}{4}}$$
$$z = \frac{1}{4} \pm \frac{1}{2}$$

The solutions are $\frac{1}{4} - \frac{1}{2} = -\frac{1}{4}$ and $\frac{1}{4} + \frac{1}{2} = \frac{3}{4}$.

**40.** $A = \frac{1}{2}bh$

When $A = 108$, $b = x + b$, and $h = x$:

$$108 = \frac{1}{2}(x + b)(x)$$
$$108 = \frac{1}{2}(x^2 + bx)$$
$$216 = x^2 + bx$$
$$216 + 3^2 = x^2 + 6x + 3^2$$
$$225 = (x + 3)^2$$
$$\pm\sqrt{225} = x + 3$$
$$-3 \pm 15 = x$$

The solutions are $-3 - 15 = -18$ and $-3 + 15 = 12$.

Because distance cannot be negative, $x = 12$.

**41.** $A = \ell w$

When $A = 288$, $\ell = 2x + 10$, and $w = 3x$:

$$288 = (2x + 10)(3x)$$
$$288 = 6x^2 + 30x$$
$$48 = x^2 + 5x$$
$$48 + \left(\frac{5}{2}\right)^2 = x^2 + 5x + \left(\frac{5}{2}\right)^2$$
$$\frac{217}{4} = \left(x + \frac{5}{2}\right)^2$$
$$\pm\sqrt{\frac{217}{4}} = x + \frac{5}{2}$$
$$-\frac{5}{2} \pm \frac{\sqrt{217}}{2} = x$$

The solutions are $-\frac{5}{2} - \frac{\sqrt{217}}{2} \approx -9.87$ and $-\frac{5}{2} + \frac{\sqrt{217}}{2} \approx 4.87$. Because distance cannot be negaive, $x \approx 4.87$.

**42.** $x^2 + bx = c$ has no solutions when $c < -\left(\frac{b}{2}\right)^2$. When $c = -\left(\frac{b}{2}\right)^2$, $x^2 + bx = c$ has one solution. When $c < -\left(\frac{b}{2}\right)^2$, the graph is translated vertically upward resulting in no real solutions.

**43.** Let $n$ and $n - 1$ be two consecutive negative integers.

$$(n)(n - 1) = 210$$
$$n^2 - n = 210$$
$$n^2 - n + \left(-\frac{1}{2}\right)^2 = 210 + \left(-\frac{1}{2}\right)^2$$
$$\left(n - \frac{1}{2}\right)^2 = \frac{841}{4}$$
$$n - \frac{1}{2} = \pm\sqrt{\frac{841}{4}}$$
$$n = \frac{1}{2} \pm \frac{29}{2}$$

$n = \frac{1}{2} - \frac{29}{2} = -14$ or $n = \frac{1}{2} + \frac{29}{2} = 15$

The two integers are $n = -14$ because $n$ must be negative, and $n - 1 = -15$.

**44.** Let $2n$ and $2n + 2$ be two consecutive positive even integers.

$$(2n)(2n + 2) = 288$$
$$4n^2 + 4n = 288$$
$$n^2 + n = 72$$
$$n^2 + n + \left(\frac{1}{2}\right)^2 = 72 + \left(\frac{1}{2}\right)^2$$
$$\left(n + \frac{1}{2}\right)^2 = \frac{289}{4}$$
$$n + \frac{1}{2} = \pm\sqrt{\frac{289}{4}}$$
$$n = -\frac{1}{2} \pm \frac{17}{2}$$

$$n = -\frac{1}{2} - \frac{17}{2} = -9 \quad \text{or} \quad n = -\frac{1}{2} + \frac{17}{2} = 8$$

The two integers are 2(8) = 16 because 2$n$ must be positive, and 2(8) + 2 = 18.

### Problem Solving

**45.**
$$\begin{aligned} 140 &= (20-2x)(16-2x) \\ 140 &= 320 - 72x + 4x^2 \\ -180 &= 4x^2 - 72x \\ -45 &= x^2 - 18x \\ -45 + 9^2 &= x^2 - 18x + 9^2 \\ 36 &= (x-9)^2 \\ \pm\sqrt{36} &= x - 9 \\ 9 \pm 6 &= x \end{aligned}$$

The solutions are 3 and 15. Because of the dimensions of the patio, the border cannot be 15 feet. The width of the border should be 3 feet.

**46.** $d = 0.05s^2 + 1.1s$

When $d = 78$:

$$\begin{aligned} 78 &= 0.05s^2 + 1.1s \\ 1560 &= s^2 + 22s \\ 1560 + 11^2 &= s^2 + 22s + 11^2 \\ 1681 &= (s+11)^2 \\ \pm\sqrt{1681} &= s + 11 \\ -11 \pm\sqrt{1681} &= s \\ -11 \pm 41 &= s \end{aligned}$$

The solutions are −52 and 30. Because speed cannot be negative, the maximum speed at which the car can travel is 30 miles per hour.

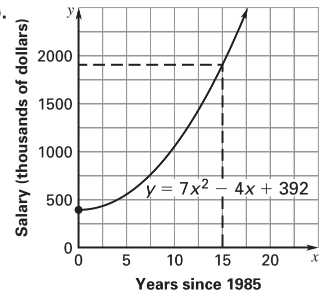
**47. a.** $y = 7x^2 - 4x + 392$

When $y = 1904$: $1904 = 7x^2 - 4x + 392$

$$\begin{aligned} 1512 &= 7x^2 - 4x \\ 216 &= x^2 - \frac{4}{7}x \\ 216 + \left(\frac{-2}{7}\right)^2 &= x^2 - \frac{4}{7}x + \left(\frac{-2}{7}\right)^2 \\ \frac{10{,}588}{49} &= \left(x - \frac{2}{7}\right)^2 \\ \pm\sqrt{\frac{10{,}588}{49}} &= x - \frac{2}{7} \\ \frac{2}{7} \pm \frac{\sqrt{10{,}588}}{7} &= x \end{aligned}$$

The solutions are about −14.4 and 15. Because $x$ cannot be negative, the average salary was $1,904,000 15 years after 1985, or in 2000.

**b.**




Because the graph has an $x$-intercept of approximately 15, the solution checks.

**48. a.** Perimeter = $\ell + 2w$

When the perimeter is 80: $80 = \ell + 2w$

Area = $\ell \cdot w$

When the area is 750: $750 = \ell \cdot w$

**b.** $\ell + 2w = 80 \rightarrow \ell = 80 - 2w$

$$\begin{aligned} \ell \cdot w &= 750 \\ (80 - 2w) \cdot w &= 750 \\ 80w - 2w^2 &= 750 \\ w^2 - 40w &= -375 \\ w^2 - 40w + (-20)^2 &= -375 + (-20)^2 \\ (w-20)^2 &= 25 \\ w - 20 &= \pm\sqrt{25} \\ w &= 20 \pm 5 \end{aligned}$$

$w = 15$ or $w = 25$

When $w = 15$: $\ell = 80-2(15) = 50$

When $w = 25$: $\ell = 80 - 2(25) = 30$

The possible dimensions are 50 feet by 15 feet or 30 feet by 25 feet.

**49.** $y = -0.025x^2 + x + 16$

When $y = 23.50$: $23.50 = -0.025x^2 + x + 16$

$$\begin{aligned} 7.50 &= -0.025x^2 + x \\ -300 &= x^2 - 40x \\ -300 + (-20)^2 &= x^2 - 40x + (-20)^2 \\ 100 &= (x-20)^2 \\ \pm\sqrt{100} &= x - 20 \\ 20 \pm 10 &= x \end{aligned}$$

The solutions are 10 and 30. You could of sold the stock for $23.50 per share 10 days after the stock was purchased.

**50. a.** The vertical motion model: $h = -16t^2 + vt + s$

When $v = 24$ and $s = 16.4$: $h = -16t^2 + 24t + 16.4$

**b.** When $h = 13.2$: $13.2 = -16t^2 + 24t + 16.4$

$$\begin{aligned} -3.2 &= -16t^2 + 24t \\ 0.2 &= t^2 - 1.5t \\ 0.2 + (-0.75)^2 &= t^2 - 1.5t + (-0.75)^2 \\ 0.7625 &= (t - 0.75)^2 \\ \pm\sqrt{0.7625} &= t - 0.75 \\ 0.75 \pm 0.873 &\approx t \end{aligned}$$

The solutions are about −0.12 and 1.6. Because time cannot be negative, she was in the air for about 1.6

seconds.

**51.** Let $x$ be increase in width.

$A = \ell \cdot w$

When $A = 480$, $\ell = 60 + 10x$, and $w = 4 + x$:

$$480 = (60 + 10x)(4 + x)$$
$$480 = 240 + 100x + 10x$$
$$240 = 10x^2 + 100x$$
$$24 = x^2 + 10x$$
$$5^2 + 24 = x^2 + 10x + 5^2$$
$$49 = (x + 5)^2$$
$$\pm\sqrt{49} = x + 5$$
$$-5 \pm 7 = x$$

The solutions are $-12$ and 2. Because the increase must be positive, the length is $60 + 10(2) = 80$ inches and the width is $4 + 2 = 6$ inches.

### *Extension for the lesson "Solve Quadratic Equations by Completing the Square"*

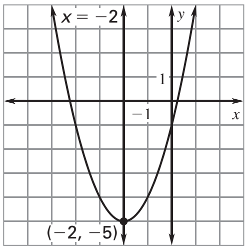
**1.** $y = (x + 2)^2 - 5$

$a = 1, h = -2, k = -5$

Because $a > 0$, the parabola opens up.

The vertex: $(-2, -5)$.

The axis of symmetry: $x = -2$.



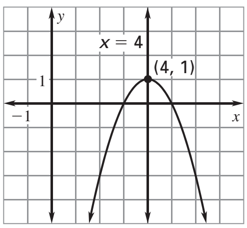

**2.** $y = -(x - 4)^2 + 1$

$a = -1, h = 4, k = 1$

Because $a < 0$, the parabola opens down.

Vertex: $(4, 1)$.

Axis of symmetry: $x = 4$.




**3.** $y = x^2 + 3$

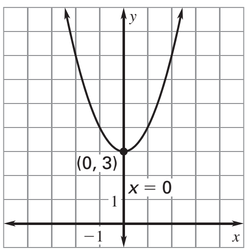
$y = (x + 0)^2 + 3$

$a = 1, h = 0, k = 3$

Because $a > 0$, the parabola opens up.

Vertex: $(0, 3)$.

Axis of symmetry: $x = 0$.



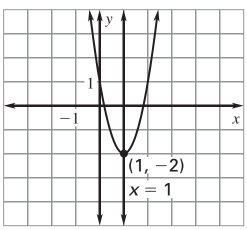

**4.** $y = 3(x - 1)^2 - 2$

$a = 3, h = 1, k = -2$

Because $a > 0$, the parabola opens up.

Vertex: $(1, -2)$.

Axis of symmetry: $x = 1$.



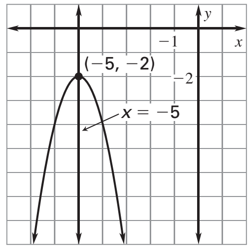

**5.** $y = -2(x + 5)^2 - 2$

$a = -2, h = -5, k = -2$

Because $a < 0$, the parabola opens down.

Vertex: $(-5, -2)$.

Axis of symmetry: $x = -5$.



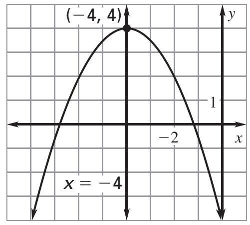

**6.** $y = -\frac{1}{2}(x + 4)^2 + 4$

$a = -\frac{1}{2}, h = -4, k = 4$

Because $a < 0$, the parabola opens down.

Vertex: $(-4, 4)$.

Axis of symmetry: $x = -4$.



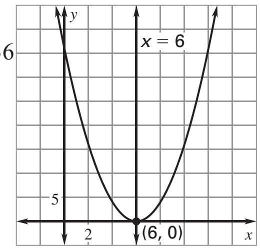

**7.**
$$y = x^2 - 12x + 36$$
$$y + 36 = (x^2 - 12x + 36) + 36$$
$$y + 36 = (x - 6)^2 + 36$$
$$y = (x - 6)^2$$

$a = 1, h = 6, k = 0$

Because $a > 0$, the parabola opens up.

Vertex: $(6, 0)$.

Axis of symmetry: $x = 6$.



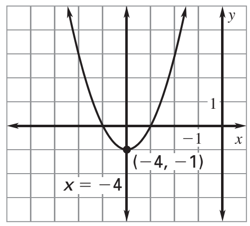

**8.**
$$y = x^2 + 8x + 15$$
$$y + 16 = (x^2 + 8x + 16) + 15$$
$$y + 16 = (x + 4)^2 + 15$$
$$y = (x + 4)^2 - 1$$

$a = 1, h = -4, k = -1$

Because $a > 0$, the parabola opens up.

Vertex: $(-4, -1)$.

Axis of symmetry: $x = -4$.



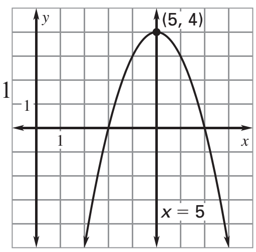

**9.**
$$y = -x^2 + 10x - 21$$
$$y = -(x^2 - 10x) - 21$$
$$y - 25 = -(x^2 - 10x + 25) - 21$$
$$y = -(x - 5)^2 + 4$$

$a = -1, h = 5, k = 4$

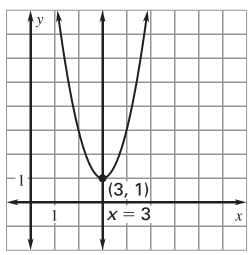
Because $a < 0$, the parabola opens down.

Vertex: $(5, 4)$.

Axis of symmetry: $x = 5$.




**10.**
$$y = 2x^2 - 12x + 19$$
$$\frac{y}{2} = x^2 - 6x + \frac{19}{2}$$
$$\frac{y}{2} + 9 = (x^2 - 6x + 9) + \frac{19}{2}$$
$$\frac{y}{2} + 9 = (x - 3)^2 + \frac{19}{2}$$
$$\frac{y}{2} = (x - 3)^2 + 1$$



$y = 2(x - 3)^2 + 1$

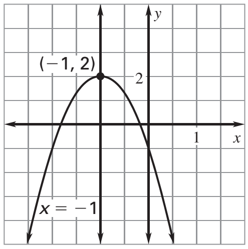
$a = 1, h = 3, k = 1$

Because $a > 0$, the parabola opens up.

Vertex: (3, 1).

Axis of symmetry: $x = 3$.

**11.** $y = -3x^2 - 6x - 1$




$\frac{y}{3} = -\left(x^2 + 2x + \frac{1}{3}\right)$

$\frac{y}{3} - 1 = -(x + 2x + 1) - \frac{1}{3}$

$\frac{y}{3} - 1 = -(x + 1)^2 - \frac{1}{3}$

$\frac{y}{3} = -(x + 1)^2 + \frac{2}{3}$

$y = -3(x + 1)^2 + 2$

$a = -3, h = -1, k = 2$

Because $a < 0$, the parabola opens down.

Vertex: $(-1, 2)$.

Axis of symmetry: $x = -1$.

**12.** $y = \frac{-1}{2}x^2 - 6x - 21$

$2y = -x^2 - 12x - 42$

$2y = -\left(x^2 + 12x + 42\right)$

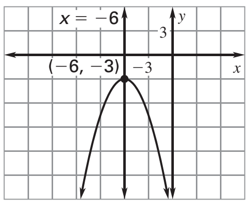
$2y - 36 = -\left(x^2 + 12 + 36\right) - 42$

$2y - 36 = -(x + 6)^2 - 42$

$2y = -(x + 6)^2 - 6$

$y = \frac{-1}{2}(x + 6)^2 - 3$

$a = \frac{-1}{2}, h = -6, k = -3$




Because $a < 0$, the parabola opens down.

Vertex: $(-6, -3)$.

Axis of symmetry: $x = -6$.

**13.** $y = a(x - h)^2 + k$

From the graph, $h = -6$ and $k = 1$.

$y = a(x + 6)^2 + 1$

Use $(-2, 5)$ to find $a$.

$5 = a(-2 + 6)^2 + 1$

$5 = 16a + 1$

$\frac{1}{4} = a$

So, $y = \frac{1}{4}(x + 6)^2 + 1$.

## Lesson 9.6 Solve Quadratic Equations by the Quadratic Formula

### *Guided Practice for the lesson "Solve Quadratic Equations by the Quadratic Formula"*

**1.** $x^2 - 8x + 16 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(16)}}{2(1)} = \frac{8 \pm 0}{2} = 4$$

The solution is 4.

**2.** $3n^2 - 5n = -1$

$3n^2 - 5n + 1 = 0$

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(1)}}{2(3)} = \frac{5 \pm \sqrt{13}}{6}$$

The solutions are $\frac{5 + \sqrt{13}}{6} \approx 1.43$ and $\frac{5 - \sqrt{13}}{6} \approx 0.23$.

**3.** $4z^2 = 7z + 2$

$4z^2 - 7z - 2 = 0$

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-7) \pm \sqrt{(-7)^2 - 4(4)(-2)}}{2(4)}$$

$$= \frac{7 \pm \sqrt{81}}{8} = \frac{7 \pm 9}{8}$$

The solutions are $\frac{7 + 9}{8} = 2$ and $\frac{7 - 9}{8} = -\frac{1}{4}$.

**4.** $y = 10x^2 - 94x + 3900$

When $y = 4750$:

$4750 = 10x^2 - 94x + 3900$

$0 = 10x^2 - 94x - 850$

$$x = \frac{-(-94) \pm \sqrt{(-94)^2 - 4(10)(-850)}}{2(10)}$$

$$= \frac{94 \pm \sqrt{42{,}836}}{20}$$

The solutions are $\frac{94 + \sqrt{42{,}836}}{20} \approx 15$ and $\frac{94 - \sqrt{42{,}836}}{20} \approx -5.6$. The solution $x \approx -5.6$ can be ignored. There were 4750 films producted about 15 years after 1971, or in 1986.

**5.** *Sample answer:* The quadratic can be solved by factoring, $x^2 + x - 6 = 0$ can easlily be factored to $(x + 3)(x - 2) = 0$.

**6.** *Sample answer:* The quadratic equation can be solved using square roots because the equation can be written as $x^2 = d$.

**7.** *Sample answer:* The quadratic equation can be solved by completing the square because it can be written in the form $ax^2 + bx + c = 0$ where $a = 1$ and $b$ is an even number.

### *Exercises for the lesson "Solve Quadratic Equations by the Quadratic Formula"*

#### *Skill Practice*

**1.** The quadratic formula can be used to solve any quadratic equation.

**2.** The quadratic equation can be solved by completing the square because it can easily be put in the form $ax^2 + bx + c = 0$ where $a = 1$ and $b$ is an even number. Also, the quadratic formula can be used because the equation does not factor easily.

**3.** $x^2 + 5x - 104 = 0$

$$x = \frac{-(5) \pm \sqrt{(5)^2 - 4(1)(-104)}}{2(1)}$$

$$= \frac{-5 \pm \sqrt{441}}{2} = \frac{-5 \pm 21}{2}$$

The roots are $\frac{-5 + 21}{2} = 8$ and $\frac{-5 - 21}{2} = -13$.

**4.** $4x^2 - x - 18 = 0$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(4)(-18)}}{2(4)}$$

$$= \frac{1 \pm \sqrt{289}}{8} = \frac{1 \pm 17}{8}$$

The roots are $\frac{1 + 17}{8} = 2.25$ and $\frac{1 - 17}{8} = -2$.

**5.** $6x^2 - 2x - 28 = 0$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(6)(-28)}}{2(6)}$$

$$= \frac{2 \pm \sqrt{676}}{12} = \frac{2 \pm 26}{12}$$

The roots are $\frac{2 + 26}{12} = 2.33$ and $\frac{2 - 26}{12} = -2$.

**6.** $m^2 + 3m + 1 = 0$

$$m = \frac{-(3) \pm \sqrt{(3)^2 - 4(1)(1)}}{2(1)} = \frac{-3 \pm \sqrt{5}}{2}$$

The roots are $\frac{-3 + 5}{2} \approx -0.38$ and $\frac{-3 - 5}{2} \approx -2.62$.

**7.** $-z^2 + z + 14 = 0$

$z^2 - z - 14 = 0$

$$z = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-14)}}{2(1)} = \frac{1 \pm \sqrt{57}}{2}$$

The roots are $\frac{1 + \sqrt{57}}{2} \approx 4.27$ and $\frac{1 - \sqrt{57}}{2} \approx -3.27$.

**8.** $-2n^2 - 5n + 16 = 0$

$2n^2 + 5n - 16 = 0$

$$n = \frac{-(5) \pm \sqrt{(5)^2 - 4(2)(-16)}}{2(2)}$$

$$= \frac{-5 \pm \sqrt{153}}{4}$$

The roots are $\frac{-5 + \sqrt{153}}{4} \approx 1.84$ and $\frac{-5 - \sqrt{153}}{4}$ $\approx -4.34$.

**9.** $4w^2 + 20w + 25 = 0$

$$w = \frac{-(20) \pm \sqrt{(20)^2 - 4(4)(25)}}{2(4)}$$

$$= \frac{-20 \pm \sqrt{0}}{8} = \frac{-20}{8}$$

The root is $\frac{-20}{8} = -2.5$.

**10.** $2t^2 + 3t - 11 = 0$

$$t = \frac{-(3) \pm \sqrt{(3)^2 - 4(2)(-11)}}{2(2)}$$

$$= \frac{-3 \pm \sqrt{97}}{4}$$

The roots are $\frac{-3 + \sqrt{97}}{4} \approx 1.71$ and $\frac{-3 - \sqrt{97}}{4} \approx -3.21$.

**11.** $-6g^2 + 9g + 8 = 0$

$6g^2 - 9g - 8 = 0$

$$g = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(6)(-8)}}{2(6)}$$

$$= \frac{9 \pm \sqrt{273}}{12}$$

The roots are $\frac{9 + \sqrt{273}}{12} \approx 2.13$ and $\frac{9 - \sqrt{273}}{12} \approx -0.63$.

**12.** B; $10x^2 - 3x - 1 = 0$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(10)(-1)}}{2(10)}$$

$$= \frac{3 \pm \sqrt{49}}{20} = \frac{3 \pm 7}{20}$$

The solutions are $\frac{3 + 7}{20} = \frac{1}{2}$ and $\frac{3 - 7}{20} = -\frac{1}{5}$.

**13.** $x^2 - 5x = 14$

$x^2 - 5x - 14 = 0$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-14)}}{2(1)}$$

$$= \frac{5 \pm \sqrt{81}}{2} = \frac{5 \pm 9}{2}$$

The solutions are $\frac{5 + 9}{2} = 7$ and $\frac{5 - 9}{2} = -2$.

**14.** $3x^2 - 4 = 11x$

$3x^2 - 11x - 4 = 0$

$$x = \frac{-(-11) \pm \sqrt{(-11)^2 - 4(3)(-4)}}{2(3)}$$

$$= \frac{11 \pm \sqrt{169}}{6} = \frac{11 \pm 13}{6}$$

The solutions are $\frac{11 + 13}{6} = 4$ and $\frac{11 - 13}{6} = -0.33$.

**15.** $9 = 7x^2 - 2x$

$0 = 7x^2 - 2x - 9$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(7)(-9)}}{2(7)}$$

$$= \frac{2 \pm \sqrt{256}}{14} = \frac{2 \pm 16}{14}$$

The solutions are $\frac{2 + 16}{14} \approx 1.29$ and $\frac{2 - 16}{14} = -1$.

**16.** $2m^2 + 9m + 7 = 3$

$2m^2 + 9m + 4 = 0$

$$m = \frac{-(9) \pm \sqrt{(9)^2 - 4(2)(4)}}{2(2)}$$

$$= \frac{-9 \pm \sqrt{49}}{4} = \frac{-9 \pm 7}{4}$$

The solutions are $\frac{-9 + 7}{4} = -0.5$ and $\frac{-9 - 7}{4} = -4$.

**17.** $-10 = r^2 - 10r + 12$

$0 = r^2 - 10r + 22$

$$r = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(22)}}{2(1)} = \frac{10 \pm \sqrt{12}}{2}$$

The solutions are $\frac{10 + \sqrt{12}}{2} \approx 6.73$ and $\frac{10 - \sqrt{12}}{2} \approx 3.27$.

**18.** $3g^2 - 6g - 14 = 3g$

$3g^2 - 9g - 14 = 0$

$$g = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(3)(-14)}}{2(3)}$$

$$= \frac{9 \pm \sqrt{249}}{6}$$

The solutions are $\frac{9 + \sqrt{249}}{6} \approx 4.13$ and $\frac{9 - \sqrt{249}}{6} \approx -1.13$.

**19.** $6z^2 = 2z^2 + 7z + 5$

$4z^2 - 7z - 5 = 0$

$$z = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(4)(-5)}}{2(4)}$$

$$= \frac{7 \pm \sqrt{129}}{8}$$

The solutions are $\frac{7 + \sqrt{129}}{8} \approx 2.29$ and $\frac{7 - \sqrt{129}}{8} \approx -0.54$.

**20.** $8h^2 + 8 = 6 - 9h$

$8h^2 + 9h + 2 = 0$

$$= \frac{-(9) \pm \sqrt{(9)^2 - 4(8)(2)}}{2(8)}$$

$$= \frac{-9 \pm \sqrt{17}}{16}$$

The solutions are $\frac{-9 + \sqrt{17}}{16} \approx -0.30$ and $\frac{-9 - \sqrt{17}}{16} \approx -0.82$.

**21.** $4t^2 - 3t = 5 - 3t^2$

$7t^2 - 3t - 5 = 0$

$$t = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(7)(-5)}}{2(7)} = \frac{3 \pm \sqrt{149}}{14}$$

The solutions are $\frac{3 + \sqrt{149}}{14} \approx 1.09$ and $\frac{3 - \sqrt{149}}{14} \approx -0.66$.

**22.** $-4y^2 - 3y + 3 = 2y + 4$

$0 = 4y^2 + 5y + 1$

$$y = \frac{-5 \pm \sqrt{(5)^2 - 4(4)(1)}}{2(4)}$$

$$= \frac{-5 \pm \sqrt{9}}{8} = \frac{-5 \pm 3}{8}$$

The solutions are $\frac{-5 + 3}{8} = -0.25$ and $\frac{-5 - 3}{8} = -1$.

**23.** $7n + 5 = -3n^2 + 2$

$3n^2 + 7n + 3 = 0$

$$n = \frac{-(7) \pm \sqrt{(7)^2 - 4(3)(3)}}{2(3)} = \frac{-7 \pm \sqrt{13}}{6}$$

The solutions are $\frac{-7 + \sqrt{13}}{6} \approx -0.57$ and $\frac{-7 - \sqrt{13}}{6} \approx -1.77$.

**24.** $5w^2 + 4 = w + 6$

$5w^2 - w - 2 = 0$

$$w = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(5)(-2)}}{2(5)} = \frac{1 \pm \sqrt{41}}{10}$$

The solutions are $\frac{1 + \sqrt{41}}{10} \approx 0.74$ and $\frac{1 - \sqrt{41}}{10} \approx -0.54$.

**25.** B; $x^2 + 14x = 2x - 11$

$x^2 + 12x + 11 = 0$

$(x + 1)(x + 11) = 0$

$x + 1 = 0$ or $x + 11 = 0$

$x = -1$ or $x = -11$

The solutions are $-1$ and $-11$.

**26.** When $-5$ was substituted in the formula for $b$, it was not negated in the numerator.

$7x^2 - 5x - 1 = 0$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(7)(-1)}}{2(7)}$$

$$= \frac{5 \pm \sqrt{53}}{14}$$

$x \approx -0.16$ and $0.88$

**27.** The equation was not in standard form when the quadratic formula was used, so the wrong coefficients were substituted in the formula.

$-2x^2 + 3x = 1$

$0 = 2x^2 - 3x + 1$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(1)}}{2(2)}$$

$$= \frac{3 \pm \sqrt{1}}{7} = \frac{3 \pm 1}{4}$$

$x = \frac{1}{2}$ and $1$

**28.** *Sample answer:* The quadratic equation can be solved using square roots because the equation can be written in the form $x^2 = d$.

**29.** *Sample answer:* The quadratic equation can be solved using square roots because the equation can be written in the form $x^2 = d$.

**30.** *Sample answer:* The equation can be solved by factoring because $2x^2 - 12x = 0$ can be factored easily to $2x(x - 6) = 0$.

**31.** *Sample answer:* The quadratic equation can be solved by factoring because $m^2 + 5m + 6 = 0$ factors easily to $(m + 2)(m + 3) = 0$.

**32.** *Sample answer:* The equation can be solved by completing the square because the equation is written in the form $ax^2 + bx + c = 0$ where $a = 1$ and $b$ is an even number.

**33.** *Sample answer:* The quadratic equation can be solved using factoring because $10g^2 - 13g + 4 = 0$ factors to $(5x - 4)(2x - 1) = 0$.

**34.** $-2x^2 = -32$

$x^2 = 16$

$\sqrt{x^2} = \pm\sqrt{16}$

$x = \pm 4$

**35.** $x^2 - 8x = -16$

$x^2 - 8x + 16 = 0$

$(x - 4)^2 = 0$

$x - 4 = 0$

$x = 4$

The solutions are $-4$ and 4. The solution is 4.

**36.** $x^2 + 2x - 6 = 0$

$$x = \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(-6)}}{2(1)} = \frac{-2 \pm \sqrt{28}}{2}$$

The solutions are $\frac{-2 + \sqrt{28}}{2} \approx 1.65$ and $\frac{-2 - \sqrt{28}}{2} \approx -3.65$.

**37.** $x^2 = 12x - 36$

$x^2 - 12x + 36 = 0$

$(x - 6)^2 = 0$

$x - 6 = 0$

$x = 6$

The solution is 6.

**38.** $x^2 + 4x = 9$

$x^2 + 4x - 9 = 0$

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(-9)}}{2(1)} = \frac{-4 \pm \sqrt{52}}{2}$$

The solutions are $\frac{-4 + \sqrt{52}}{2} \approx 1.61$ and $\frac{-4 - \sqrt{52}}{2} \approx -5.61$.

**39.** $-4x^2 + x = -17$

$0 = 4x^2 - x - 17$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(4)(-17)}}{2(4)} = \frac{1 \pm \sqrt{273}}{8}$$

The solutions are $\frac{1 + \sqrt{273}}{8} \approx 2.19$ and $\frac{1 - \sqrt{273}}{8} \approx -1.94$.

**40.** $11x^2 - 1 = 6x^2 + 2$

$5x^2 - 3 = 0$

$5x^2 = 3$

$x^2 = \frac{3}{5}$

$x = \pm\sqrt{\frac{3}{5}}$

The solutions are $\sqrt{\frac{3}{5}} \approx 0.77$ and $-\sqrt{\frac{3}{5}} \approx -0.77$.

**41.** $-2x^2 + 5 = 3x^2 - 10x$

$0 = 5x^2 - 10x - 5$

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(5)(-5)}}{2(5)}$$

$$= \frac{10 \pm \sqrt{200}}{10}$$

The solutions are $\frac{10 + \sqrt{200}}{10} \approx 2.41$ and $\frac{10 - \sqrt{200}}{10} \approx -0.41$.

**42.** $(x + 13)^2 = 25$

$\sqrt{(x + 13)^2} = \pm\sqrt{25}$

$x + 13 = \pm 5$

$x = -13 \pm 5$

The solutions are $-13 + 5 = -8$ and $-13 - 5 = -18$.

**43.** $A = \ell w$

When $A = 91$, $\ell = 2x + 3$, and $w = x + 2$:

$91 = (2x + 3)(x + 2)$

$91 = 2x^2 + 7x + 6$

$0 = 2x^2 + 7x - 85$

$0 = (2x + 17)(x - 5)$

$2x + 17 = 0$ or $x - 5 = 0$

$x = \frac{-17}{2}$ or $x = 5$

Since the length cannot be negative, disregard $\frac{-17}{2}$.

The value of $x$ is 5. The width is $5 + 2 = 7$ m. The length is $2(5) + 3 = 13$ m.

**44.** $A = \ell \cdot w$

When $A = 209$, $\ell = 4x + 3$, and $w = 4x - 5$:

$209 = (4x + 3)(4x - 5)$

$209 = 16x^2 - 8x - 15$

$0 = 16x^2 - 8x - 224$

$0 = 2x^2 - x - 28$

$0 = (2x + 7)(x - 4)$

$2x + 7 = 0$ or $x - 4 = 0$

$x = \frac{-7}{2}$ or $x = 4$

A side length of a rectangle cannot be negative so disregard $\frac{-7}{2}$. The value of $x$ is 4. The length is $4(4) + 3 = 19$ feet. The width is $4(4) - 5 = 11$ feet.

**45.** $$\text{Mean} = \frac{1}{2}\left[\frac{-b + \sqrt{b^2 - 4ac}}{2a} + \frac{-b - \sqrt{b^2 - 4ac}}{2a}\right]$$

$$= \frac{1}{2}\left[\frac{-2b}{2a}\right] = \frac{1}{2}\left(-\frac{b}{a}\right) = -\frac{b}{2a}$$

The mean $-\frac{b}{2a}$ is also the axis of symmetry of the graph of $y = ax^2 + bx + c$.

### *Problem Solving*

**46.** $y = 0.93x^2 + 2.2x + 130$

When $y = 164$: $164 = 0.93x^2 + 2.2x + 130$

$0 = 0.93x^2 + 2.2x - 34$

$$x = \frac{-(2.2) \pm \sqrt{(2.2)^2 - 4(0.93)(-34)}}{2(0.93)}$$

$$= \frac{-2.2 \pm \sqrt{131.32}}{1.86}$$

The solutions are $\frac{-2.2 + \sqrt{131.32}}{1.86} \approx 5$ and $\frac{-2.2 - \sqrt{131.32}}{1.86} \approx -7.3$. Ignore the negative solution. So, 164 billion dollars was spent on advertising about 5 years after 1990, or in 1995.

**47.** $y = 0.7x^2 - 4.3x + 5.5$

When $y = 16$: $16 = 0.7x^2 - 4.3x + 5.5$

$0 = 0.7x^2 - 4.3x - 10.5$

$$x = \frac{-(-4.3) \pm \sqrt{(-4.3)^2 - 4(0.7)(-10.5)}}{2(0.7)}$$

$$= \frac{4.3 \pm \sqrt{47.89}}{1.4}$$

The solutions are $\frac{4.3 + \sqrt{47.89}}{1.4} \approx 8$ and $\frac{4.3 - \sqrt{47.89}}{1.4}$ $\approx -1.9$. Ignore the negative solution. There will be about 16 million cell phone subscribers 8 years after 1985, or in 1993.

**48. a.** Vertical motion model: $h = -16t^2 + vt + s$

When $v = 45$ and $s = 2.5$:

$h = -16t^2 + 45t + 2.5$

**b.** When $h = 5.5$: $5.5 = -16t^2 + 45t + 2.5$

$0 = -16t^2 + 45t - 3$

$$t = \frac{-(-45) \pm \sqrt{(45)^2 - 4(-16)(-3)}}{2(-16)}$$

$$= \frac{-45 \pm \sqrt{1833}}{-32}$$

The solutions are $\frac{-45 + \sqrt{1833}}{-32} \approx 0.1$ and $\frac{-45 - \sqrt{1833}}{-32}$ $\approx 2.7$. The diagram shows the ball being caught on its way back to the ground so the football is in the air for about 2.7 seconds.

**49. a.** $y = -46.7x^2 + 169x + 2650$

When $y = 2500$:

$2500 = -46.7x^2 + 169x + 2650$

$0 = -46.7x^2 + 169x + 150$

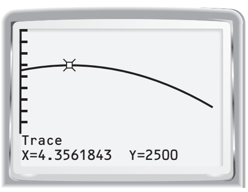
$$x = \frac{-(169) \pm \sqrt{(169)^2 - 4(-46.7)(150)}}{2(-46.7)}$$

$$= \frac{-169 \pm \sqrt{56{,}581}}{-93.4}$$

The solutions are $\frac{-169 + \sqrt{56{,}581}}{-93.4} \approx -0.7$

and $\frac{-169 - \sqrt{56{,}581}}{-93.4} \approx 4.4$. Ignore the negative solution. There were 2,500,000 16 and 17 year olds employed 4 years after 1997, or in 2001.

**b.** Let $y_1 = -46.7x^2 + 169x + 2650$




**50.** $h = -11t^2 + 700t + 21{,}000$

When $h = 30{,}800$

$30{,}800 = -11t^2 + 700t + 21{,}000$

$0 = -11t^2 + 700t - 9800$

$$t = \frac{-(700) \pm \sqrt{(700)^2 - 4(-11)(-9800)}}{2(-11)}$$

$$= \frac{-700 \pm \sqrt{58{,}800}}{-22}$$

The solutions of the equation are $\frac{-700 + \sqrt{58{,}800}}{-22} \approx 20.8$ seconds and $\frac{-700 - \sqrt{58{,}800}}{-22} \approx 42.8$ seconds. The period of weightlessness is the difference between solutions, or about 22 seconds.

**51.** Area of original pipe $= \pi r^2$

Area of current pipe $= \pi(r - 4)^2$

The area of current pipe is 90% of the original area.

$0.9(\pi r^2) = \pi(r - 4)^2$

$0.9\pi r^2 = \pi(r^2 - 8r + 16)$

$0.9r^2 = r^2 - 8r + 16$

$0 = 0.1r^2 - 8r + 16$

$$r = \frac{8 \pm \sqrt{(-8)^2 - 4(.1)(16)}}{2(.1)} = \frac{8 \pm \sqrt{57.6}}{0.2}$$

The solutions are $\frac{8 + \sqrt{57.6}}{0.2} \approx 77.9$ and $\frac{8 - \sqrt{57.6}}{0.2}$ $\approx 2.1$. The solution $r = 2.1$ can be ignored because it would make the radius of the current pipe negative. So, the diameter of the orginal pipe is 2(77.9), or about 156 millimeters.

### *Quiz for the lessons "Use Square Roots to Solve Quadratic Equations", "Solve Quadratic Equations by Completing the Square", and "Solve Quadratic Equations by the Quadratic Formula"*

**1.** $3x^2 - 48 = 0$

$3x^2 = 48$

$x^2 = 16$

$x = \pm\sqrt{16} = \pm 4$

The solutions of the equation are $-4$ and 4.

**2.** $-6x^2 = -24$

$x^2 = 4$

$x = \pm\sqrt{4} = \pm 2$

The solutions of the equation area $-2$ and 2.

**3.** $x^2 + 5 = 16$

$x^2 = 11$

$x = \pm\sqrt{11} \approx \pm 3.32$

The solutions of the equation are about $-3.32$ and 3.32.

**4.** $x^2 + 2x + 6 = 0$

$x^2 + 2x = -6$

$x^2 + 2x + (1)^2 = -6 + (1)^2$

$(x + 1)^2 = -5$

Negative real numbers do not have real square roots. So, there is no solution.

**5.** $x^2 + 10x - 12 = 0$

$x^2 + 10x = 12$

$x^2 + 10x + (5)^2 = 12 + (5)^2$

$(x + 5)^2 = 37$

$x + 5 = \pm\sqrt{37}$

$x = -5 \pm \sqrt{37}$

The solutions are $-5 + \sqrt{37} \approx 1.08$ and $-5 - \sqrt{37} \approx -11.08$.

**6.** $x^2 - 8x = -6$

$x^2 - 8x + (-4)^2 = -6 + (-4)^2$

$(x - 4)^2 = 10$

$x - 4 = \pm\sqrt{10}$

$x = 4 \pm \sqrt{10}$

The solutions are $4 + \sqrt{10} \approx 7.16$ and $4 - \sqrt{10} \approx 0.84$.

**7.** $x^2 - 12x = 30$

$x^2 - 12x + (-6)^2 = 30 + (-6)^2$

$(x - 6)^2 = 66$

$x - 6 = \pm\sqrt{66}$

$x = 6 \pm \sqrt{66}$

The solutions are $6 + \sqrt{66} \approx 14.12$ and $6 - \sqrt{66} \approx -2.12$.

**8.** $x^2 - 5x = -\frac{9}{4}$

$x^2 - 5x + \left(-\frac{5}{2}\right)^2 = -\frac{9}{4} + \left(-\frac{5}{2}\right)^2$

$\left(x - \frac{5}{2}\right)^2 = 4$

$x - \frac{5}{2} = \pm 2$

$x = \frac{5}{2} \pm 2$

The solutions are $\frac{5}{2} + 2 = 4.5$ and $\frac{5}{2} - 2 = 0.5$.

**9.** $x^2 + x = -7.75$

$x^2 + x + \left(\frac{1}{2}\right)^2 = -7.75 + \left(\frac{1}{2}\right)^2$

$\left(x + \frac{1}{2}\right)^2 = -7.5$

Negative real numbers do not have real square roots. So, there is no solution.

**10.** $x^2 + 4x + 1 = 0$

$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(1)}}{2(1)} = \frac{-4 \pm \sqrt{12}}{2}$

The solutions are $\frac{-4 + \sqrt{12}}{2} \approx -0.27$ and $\frac{-4 - \sqrt{12}}{2} \approx -3.73$.

**11.** $-3x^2 + 3x = 1$

$0 = 3x^2 - 3x - 1$

$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(3)(-1)}}{2(3)} = \frac{3 \pm \sqrt{21}}{6}$

The solutions are $\frac{3 + \sqrt{21}}{6} \approx 1.26$ and $\frac{3 - \sqrt{21}}{6} \approx -0.26$.

**12.** $4x^2 - 11x = 3$

$4x^2 - 11x - 3 = 0$

$x = \frac{-(-11) \pm \sqrt{(-11)^2 - 4(4)(-3)}}{2(4)}$

$= \frac{11 \pm \sqrt{169}}{8} = \frac{11 \pm 13}{8}$

The solutions are $\frac{11 + 13}{8} = 3$ and $\frac{11 - 13}{8} = -0.25$.

### *Extension for the lesson "Solve Quadratic Equations by the Quadratic Formula"*

**1.** $x^2 + 4x + 2 = 0$

$x = \frac{-(4) \pm \sqrt{(4)^2 - 4(1)(2)}}{2(1)}$

$= \frac{-4 \pm \sqrt{8}}{2}$

$= \frac{-4 \pm \sqrt{4 \cdot 2}}{2}$

$= \frac{-4 \pm 2\sqrt{2}}{2} = -2 \pm \sqrt{2}$

Solutions: $-2 + \sqrt{2}$ and $-2 - \sqrt{2}$

Check $x = -2 + \sqrt{2}$:

$x^2 + 4x + 2 = 0$

$(-2 + \sqrt{2})^2 + 4(-2 + \sqrt{2}) + 2 \stackrel{?}{=} 0$

$4 - 4\sqrt{2} + 2 - 8 + 4\sqrt{2} + 2 \stackrel{?}{=} 0$

$0 = 0$ ✓

Check: $x = -2 - \sqrt{2}$:

$x^2 + 4x + 2 = 0$

$(-2 - \sqrt{2})^2 + 4(-2 - \sqrt{2}) + 2 \stackrel{?}{=} 0$

$4 + 4\sqrt{2} + 2 - 8 - 4\sqrt{2} + 2 \stackrel{?}{=} 0$

$0 = 0$ ✓

**2.** $x^2 + 6x - 1 = 0$

$x = \frac{-(6) \pm \sqrt{(6)^2 - 4(1)(-1)}}{2(1)}$

$= \frac{-6 \pm \sqrt{40}}{2}$

$= \frac{-6 \pm \sqrt{4 \cdot 10}}{2}$

$= \frac{-6 \pm 2\sqrt{10}}{2} = -3 \pm \sqrt{10}$

Solutions: $-3 + \sqrt{10}$ and $-3 - \sqrt{10}$

Check: $x = -3 + \sqrt{10}$:

$x^2 + 6x - 1 = 0$

$(-3 + \sqrt{10})^2 + 6(-3 + \sqrt{10}) - 1 \stackrel{?}{=} 0$

$9 - 6\sqrt{10} + 10 - 18 + 6\sqrt{10} - 1 \stackrel{?}{=} 0$

$0 = 0$ ✓

Check: $x = -3 - \sqrt{10}$:

$x^2 + 6x - 1 = 0$

$(-3 - \sqrt{10})^2 + 6(-3 - \sqrt{10}) - 1 \stackrel{?}{=} 0$

$9 + 6\sqrt{10} + 10 - 18 - 6\sqrt{10} - 1 \stackrel{?}{=} 0$

$0 = 0$ ✓

**3.** $x^2 + 8x + 8 = 0$

$x = \frac{-(8) \pm \sqrt{(8)^2 - 4(1)(8)}}{2(1)}$

$= \frac{-8 \pm \sqrt{32}}{2}$

$= \frac{-8 \pm \sqrt{16 \cdot 2}}{2}$

$= \frac{-8 \pm 4\sqrt{2}}{2} = -4 \pm 2\sqrt{2}$

Solutions: $-4 + 2\sqrt{2}$ and $-4 - 2\sqrt{2}$

Check $x = -4 + 2\sqrt{2}$:

$$x^2 + 8x + 8 = 0$$
$$(-4 + 2\sqrt{2})^2 + 8(-4 + 2\sqrt{2}) + 8 \stackrel{?}{=} 0$$
$$16 - 16\sqrt{2} + 8 - 32 + 16\sqrt{2} + 8 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark$$

Check $x = -4 - 2\sqrt{2}$:

$$x^2 + 8x + 8 = 0$$
$$(-4 - 2\sqrt{2})^2 + 8(-4 - 2\sqrt{2}) + 8 \stackrel{?}{=} 0$$
$$16 + 16\sqrt{2} + 8 - 32 - 16\sqrt{2} + 8 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark$$

**4.** $x^2 - 7x + 1 = 0$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(1)}}{2(1)}$$
$$= \frac{7 \pm \sqrt{45}}{2}$$
$$= \frac{7 \pm \sqrt{9 \cdot 5}}{2}$$
$$= \frac{7 \pm 3\sqrt{5}}{2}$$

Solutions: $\frac{7 + 3\sqrt{5}}{2}$ and $\frac{7 - 3\sqrt{5}}{2}$

Check $x = \frac{7 + 3\sqrt{5}}{2}$:

$$x^2 - 7x + 1 = 0$$
$$\left(\frac{7 + 3\sqrt{5}}{2}\right)^2 - 7\left(\frac{7 + 3\sqrt{5}}{2}\right) + 1 \stackrel{?}{=} 0$$
$$\frac{49}{4} + \frac{21\sqrt{5}}{2} + \frac{45}{4} - \frac{49}{2} - \frac{21\sqrt{5}}{2} + 1 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark$$

Check $x = \frac{7 - 3\sqrt{5}}{2}$:

$$x^2 - 7x + 1 = 0$$
$$\left(\frac{7 - 3\sqrt{5}}{2}\right)^2 - 7\left(\frac{7 - 3\sqrt{5}}{2}\right) + 1 = 0$$
$$\frac{49}{4} - \frac{21\sqrt{5}}{2} + \frac{45}{4} - \frac{49}{2} + \frac{21\sqrt{5}}{2} + 1 = 0$$
$$0 = 0 \checkmark$$

**5.** $3x^2 + 6x - 1 = 0$

$$x = \frac{-(6) \pm \sqrt{(6)^2 - 4(3)(-1)}}{2(3)}$$
$$= \frac{-6 \pm \sqrt{48}}{6}$$
$$= \frac{-6 \pm \sqrt{16 \cdot 3}}{6}$$
$$= \frac{-6 \pm 4\sqrt{3}}{6} = -1 \pm \frac{2}{3}\sqrt{3}$$

Solutions: $-1 + \frac{2}{3}\sqrt{3}$ and $-1 - \frac{2}{3}\sqrt{3}$

Check: $x = -1 + \frac{2}{3}\sqrt{3}$:

$$3x^2 + 6x - 1 = 0$$
$$3\left(-1 + \frac{2}{3}\sqrt{3}\right)^2 + 6\left(-1 + \frac{2}{3}\sqrt{3}\right) - 1 \stackrel{?}{=} 0$$
$$3\left(1 + \frac{4}{3}\sqrt{3} + \frac{4}{3}\right) - 6 - 4\sqrt{3} - 1 \stackrel{?}{=} 0$$
$$3 - 4\sqrt{3} + 4 - 6 + 4\sqrt{3} - 1 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark$$

Check $x = -1 - \frac{2}{3}\sqrt{3}$:

$$3x^2 + 6x - 1 = 0$$
$$3\left(-1 - \frac{2}{3}\sqrt{3}\right)^2 + 6\left(-1 - \frac{2}{3}\sqrt{3}\right) - 1 \stackrel{?}{=} 0$$
$$3\left(1 + \frac{4}{3}\sqrt{3} + \frac{4}{3}\right) - 6 - 4\sqrt{3} - 1 \stackrel{?}{=} 0$$
$$3 + 4\sqrt{3} + 4 - 6 - 4\sqrt{3} - 1 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark$$

**6.** $2x^2 - 4x - 3 = 0$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(-3)}}{2(2)}$$
$$= \frac{4 \pm \sqrt{40}}{4}$$
$$= \frac{4 \pm \sqrt{4 \cdot 10}}{4}$$
$$= \frac{4 \pm 2\sqrt{10}}{4} = 1 \pm \frac{1}{2}\sqrt{10}$$

Solutions: $1 + \frac{1}{2}\sqrt{10}$ and $1 - \frac{1}{2}\sqrt{10}$

Check $x = 1 + \frac{1}{2}\sqrt{10}$:

$$2x^2 - 4x - 3 = 0$$
$$2\left(1 + \frac{1}{2}\sqrt{10}\right)^2 - 4\left(1 + \frac{1}{2}\sqrt{10}\right) - 3 \stackrel{?}{=} 0$$
$$2(1 + \sqrt{10} + 2.5) - 4 - 2\sqrt{10} - 3 \stackrel{?}{=} 0$$
$$2 + 2\sqrt{10} + 5 - 4 - 2\sqrt{10} - 3 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark$$

Check $x = 1 - \frac{1}{2}\sqrt{10}$:

$$2x^2 - 4x - 3 = 0$$
$$2\left(1 - \frac{1}{2}\sqrt{10}\right)^2 - 4\left(1 - \frac{1}{2}\sqrt{10}\right) - 3 \stackrel{?}{=} 0$$
$$2(1 - \sqrt{10} + 2.5) - 4 + 2\sqrt{10} - 3 \stackrel{?}{=} 0$$
$$2 - 2\sqrt{10} + 5 - 4 + 2\sqrt{10} - 3 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark$$

**7.** $5x^2 - 2x - 2 = 0$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(5)(-2)}}{2(5)}$$
$$= \frac{2 \pm \sqrt{44}}{10}$$
$$= \frac{2 \pm \sqrt{4 \cdot 11}}{10}$$
$$= \frac{2 \pm 2\sqrt{11}}{10} = \frac{1 \pm \sqrt{11}}{5}$$

Solutions: $\frac{1 + \sqrt{11}}{5}$ and $\frac{1 - \sqrt{11}}{5}$

Check $x = \frac{1}{5} + \frac{\sqrt{11}}{5}$:

$$5x^2 - 2x - 2 = 0$$
$$5\left(\frac{1}{5} + \frac{\sqrt{11}}{5}\right)^2 - 2\left(\frac{1}{5} + \frac{\sqrt{11}}{5}\right) - 2 \stackrel{?}{=} 0$$
$$5\left(\frac{1}{25} + \frac{2\sqrt{11}}{25} + \frac{11}{25}\right) - \frac{2}{5} - \frac{2\sqrt{11}}{5} - 2 \stackrel{?}{=} 0$$
$$\frac{5}{25} + \frac{10\sqrt{11}}{25} + \frac{55}{25} - \frac{2}{5} - \frac{2\sqrt{11}}{5} - 2 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark$$

Check: $x = \frac{1}{5} - \frac{\sqrt{11}}{5}$:

$$5x^2 - 2x - 2 = 0$$
$$5\left(\frac{1}{5} - \frac{\sqrt{11}}{5}\right)^2 - 2\left(\frac{1}{5} - \frac{\sqrt{11}}{5}\right) - 2 \stackrel{?}{=} 0$$
$$5\left(\frac{1}{25} - \frac{2\sqrt{11}}{25} + \frac{11}{25}\right) - \frac{2}{5} + \frac{2\sqrt{11}}{5} - 2 \stackrel{?}{=} 0$$
$$\frac{5}{25} - \frac{10\sqrt{11}}{25} + \frac{55}{25} - \frac{2}{5} + \frac{2\sqrt{11}}{5} - 2 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark$$

**8.** $4x^2 + 10x + 3 = 0$

$$x = \frac{-(10) \pm \sqrt{(10)^2 - 4(4)(3)}}{2(4)}$$
$$= \frac{-10 \pm \sqrt{52}}{8}$$
$$= \frac{-10 \pm \sqrt{4 \cdot 13}}{8}$$
$$= \frac{-10 \pm 2\sqrt{13}}{8} = \frac{-5 \pm \sqrt{13}}{4}$$

Solutions: $-\frac{5}{4} + \frac{\sqrt{13}}{4}$ and $-\frac{5}{4} - \frac{\sqrt{13}}{4}$

Check: $x = -\frac{5}{4} + \frac{\sqrt{13}}{4}$:

$$4x^2 + 10x + 3 = 0$$
$$4\left(\frac{-5}{4} + \frac{\sqrt{13}}{4}\right)^2 + 10\left(-\frac{5}{4} + \frac{\sqrt{13}}{4}\right) + 3 \stackrel{?}{=} 0$$
$$4\left(\frac{25}{16} - \frac{5\sqrt{13}}{8} + \frac{13}{16}\right) - \frac{50}{4} + \frac{10\sqrt{13}}{4} + 3 \stackrel{?}{=} 0$$
$$\frac{25}{4} - \frac{5\sqrt{13}}{2} + \frac{13}{4} - \frac{25}{2} + \frac{5\sqrt{13}}{2} + 3 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark$$

Check: $x = -\frac{5}{4} - \frac{\sqrt{13}}{4}$:

$$4x^2 + 10x + 3 = 0$$
$$4\left(-\frac{5}{4} - \frac{\sqrt{13}}{4}\right)^2 + 10\left(-\frac{5}{4} - \frac{\sqrt{13}}{4}\right) + 3 \stackrel{?}{=} 0$$
$$4\left(\frac{25}{16} + \frac{5\sqrt{13}}{8} + \frac{13}{16}\right) - \frac{50}{4} - \frac{10\sqrt{13}}{4} + 3 \stackrel{?}{=} 0$$
$$\frac{25}{4} + \frac{5\sqrt{13}}{2} + \frac{13}{4} - \frac{25}{2} - \frac{5\sqrt{13}}{2} + 3 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark$$

**9.** $x^2 - x - 3 = 0$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-3)}}{2} = \frac{1 \pm \sqrt{13}}{2}$$

Solutions: $\frac{1}{2} + \frac{\sqrt{13}}{2}$ and $\frac{1}{2} - \frac{\sqrt{13}}{2}$

Check $x = \frac{1}{2} + \frac{\sqrt{13}}{2}$:

$$x^2 - x - 3 = 0$$
$$\left(\frac{1}{2} + \frac{\sqrt{13}}{2}\right)^2 - \left(\frac{1}{2} + \frac{\sqrt{13}}{2}\right) - 3 \stackrel{?}{=} 0$$
$$\frac{1}{4} + \frac{\sqrt{13}}{2} + \frac{13}{4} - \frac{1}{2} - \frac{\sqrt{13}}{2} - 3 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark$$

Check $x = \frac{1}{2} - \frac{\sqrt{13}}{2}$:

$$x^2 - x - 3 = 0$$
$$\left(\frac{1}{2} - \frac{\sqrt{13}}{2}\right)^2 - \left(\frac{1}{2} - \frac{\sqrt{13}}{2}\right) - 3 \stackrel{?}{=} 0$$
$$\frac{1}{4} - \frac{\sqrt{13}}{2} + \frac{13}{4} - \frac{1}{2} + \frac{\sqrt{13}}{2} - 3 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark$$

**10.** $x^2 - 2x - 8 = 0$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-8)}}{2(1)}$$
$$= \frac{2 \pm \sqrt{36}}{2}$$
$$= \frac{2 \pm 6}{2} = 1 \pm 3$$

Solutions: $-2$ and $4$

Check $x = -2$:

$$x^2 - 2x - 8 = 0$$
$$(-2)^2 - 2(-2) - 8 \stackrel{?}{=} 0$$
$$4 + 4 - 8 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark$$

Check $x = 4$:

$$x^2 - 2x - 8 = 0$$
$$(4)^2 - 2(4) - 8 \stackrel{?}{=} 0$$
$$16 - 8 - 8 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark$$

**11.** $-x^2 + 7x + 3 = 0$

$$x = \frac{-(7) \pm \sqrt{(7)^2 - 4(-1)(3)}}{2(-1)} = \frac{-7 \pm \sqrt{61}}{-2}$$

Solutions: $\frac{7}{2} - \frac{\sqrt{61}}{2}$ and $\frac{7}{2} + \frac{\sqrt{61}}{2}$

Check: $x = \frac{7}{2} - \frac{\sqrt{61}}{2}$:

$$-x^2 + 7x + 3 = 0$$
$$-\left(\frac{7}{2} - \frac{\sqrt{61}}{2}\right)^2 + 7\left(\frac{7}{2} - \frac{\sqrt{61}}{2}\right) + 3 \stackrel{?}{=} 0$$
$$-\left(\frac{49}{4} - \frac{7\sqrt{61}}{2} + \frac{61}{4}\right) + \frac{49}{2} - \frac{7\sqrt{61}}{2} + 3 \stackrel{?}{=} 0$$
$$-\frac{49}{4} + \frac{7\sqrt{61}}{2} - \frac{61}{4} + \frac{49}{2} - \frac{7\sqrt{61}}{2} + 3 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark$$

Check $x = \frac{7}{2} + \frac{\sqrt{61}}{2}$:

$$-x^2 + 7x + 3 = 0$$

$$-\left(\frac{7}{2} + \frac{\sqrt{61}}{2}\right)^2 + 7\left(\frac{7}{2} + \frac{\sqrt{61}}{2}\right) + 3 \stackrel{?}{=} 0$$

$$-\left(\frac{49}{4} + \frac{7\sqrt{61}}{2} + \frac{61}{4}\right) + \frac{49}{2} + \frac{7\sqrt{61}}{2} + 3 \stackrel{?}{=} 0$$

$$-\frac{49}{4} - \frac{7\sqrt{61}}{2} - \frac{61}{4} + \frac{49}{2} + \frac{7\sqrt{61}}{2} + 3 \stackrel{?}{=} 0$$

$$0 = 0 ✓$$

**12.** $x^2 + 3x - 9 = 0$

$$x = \frac{-(3) \pm \sqrt{(3)^2 - 4(1)(-9)}}{2(1)}$$

$$= \frac{-3 \pm \sqrt{45}}{2}$$

$$= \frac{-3 \pm \sqrt{9 \cdot 5}}{2}$$

$$= \frac{-3 \pm 3\sqrt{5}}{2} = \frac{3}{2}(-1 \pm \sqrt{5})$$

Solutions: $\frac{3}{2}(-1 + \sqrt{5})$ and $\frac{3}{2}(-1 - \sqrt{5})$

Check $x = \frac{3}{2}(-1 + \sqrt{5})$:

$$x^2 + 3x - 9 = 0$$

$$\left(\frac{3}{2}(-1\sqrt{5})\right)^2 + 3\left(\frac{3}{2}(-1 + \sqrt{5})\right) - 9 \stackrel{?}{=} 0$$

$$\frac{9}{4}(1 - 2\sqrt{5} + 5) + \frac{9}{2}(-1 + \sqrt{5}) - 9 \stackrel{?}{=} 0$$

$$\frac{9}{4} - \frac{9}{2}\sqrt{5} + \frac{45}{4} - \frac{9}{2} + \frac{9}{2}\sqrt{5} - 9 \stackrel{?}{=} 0$$

$$0 = 0 ✓$$

Check $x = \frac{3}{2}(-1 - \sqrt{5})$:

$$x^2 + 3x - 9 = 0$$

$$\left(\frac{3}{2}(-1 - \sqrt{5})\right)^2 + 3\left(\frac{3}{2}(-1 - \sqrt{5})\right) - 9 \stackrel{?}{=} 0$$

$$\frac{9}{4}(1 + 2\sqrt{5} + 5) + \frac{9}{2}(-1 - \sqrt{5}) - 9 \stackrel{?}{=} 0$$

$$\frac{9}{4} + \frac{9}{2}\sqrt{5} + \frac{45}{4} - \frac{9}{2} - \frac{9}{2}\sqrt{5} - 9 \stackrel{?}{=} 0$$

$$0 = 0 ✓$$

**13.** $-\frac{5}{2}x^2 + 10x - 5 = 0$

$$x = \frac{-(10) \pm \sqrt{(10)^2 - 4\left(-\frac{5}{2}\right)(-5)}}{2\left(-\frac{5}{2}\right)}$$

$$= \frac{-10 \pm \sqrt{50}}{-5}$$

$$= \frac{-10 \pm \sqrt{25 \cdot 2}}{-5}$$

$$= \frac{-10 \pm 5\sqrt{2}}{-5} = 2 \pm \sqrt{2}$$

Solutions: $2 + \sqrt{2}$ and $2 - \sqrt{2}$

Check $x = 2 + \sqrt{2}$:

$$-\frac{5}{2}x^2 + 10x - 5 = 0$$

$$-\frac{5}{2}(2 + \sqrt{2})^2 + 10(2 + \sqrt{2}) - 5 \stackrel{?}{=} 0$$

$$-\frac{5}{2}(4 + 4\sqrt{2} + 2) + 20 + 10\sqrt{2} - 5 \stackrel{?}{=} 0$$

$$-10 - 10\sqrt{2} - 5 + 20 + 10\sqrt{2} - 5 \stackrel{?}{=} 0$$

$$0 = 0 ✓$$

Check $x = 2 - \sqrt{2}$:

$$-\frac{5}{2}x^2 + 10x - 5 = 0$$

$$-\frac{5}{2}(2 - \sqrt{2})^2 + 10(2 - \sqrt{2}) - 5 \stackrel{?}{=} 0$$

$$-\frac{5}{2}(4 - 4\sqrt{2} + 2) + 20 - 10\sqrt{2} - 5 \stackrel{?}{=} 0$$

$$-10 + 10\sqrt{2} - 5 + 20 - 10\sqrt{2} - 5 \stackrel{?}{=} 0$$

$$0 = 0 ✓$$

**14.** $\frac{1}{2}x^2 + 3x - 9 = 0$

$$x = \frac{-(3) \pm \sqrt{(3)^2 - 4\left(\frac{1}{2}\right)(-9)}}{2\left(\frac{1}{2}\right)}$$

$$= -3 \pm \sqrt{27}$$

$$= -3 \pm \sqrt{9 \cdot 3}$$

$$= -3 \pm 3\sqrt{3}$$

Solutions: $-3 + 3\sqrt{3}$ and $-3 - 3\sqrt{3}$

Check $x = -3 + 3\sqrt{3}$:

$$\frac{1}{2}x^2 + 3x - 9 = 0$$

$$\frac{1}{2}(-3 + 3\sqrt{3})^2 + 3(-3 + 3\sqrt{3}) - 9 \stackrel{?}{=} 0$$

$$\frac{1}{2}(9 - 18\sqrt{3} + 27) - 9 + 9\sqrt{3} - 9 \stackrel{?}{=} 0$$

$$\frac{9}{2} - 9\sqrt{3} + \frac{27}{2} - 9 + 9\sqrt{3} - 9 \stackrel{?}{=} 0$$

$$0 = 0 ✓$$

Check $x = -3 - 3\sqrt{3}$:

$$\frac{1}{2}x^2 + 3x - 9 = 0$$

$$\frac{1}{2}(-3 - 3\sqrt{3})^2 + 3(-3 - 3\sqrt{3}) - 9 \stackrel{?}{=} 0$$

$$\frac{1}{2}(9 + 18\sqrt{3} + 27) - 9 - 9\sqrt{3} - 9 \stackrel{?}{=} 0$$

$$\frac{9}{2} + 9\sqrt{3} + \frac{27}{2} - 9 - 9\sqrt{3} - 9 \stackrel{?}{=} 0$$

$$0 = 0 ✓$$

**15.** $3x^2 - 2 = 0$

$3x^2 + 0x - 2 = 0$

$$x = \frac{0 \pm \sqrt{(0)^2 - 4(3)(-2)}}{2(3)}$$

$$= \frac{\pm\sqrt{24}}{6}$$

$$= \frac{\pm\sqrt{4 \cdot 6}}{6}$$

$$= \frac{\pm 2\sqrt{6}}{6} = \frac{\pm\sqrt{6}}{3}$$

Solutions: $\frac{\sqrt{6}}{3}$ and $-\frac{\sqrt{6}}{3}$

Check $x = \frac{\sqrt{6}}{3}$:

$$3x^2 - 2 = 0$$
$$3\left(\frac{\sqrt{6}}{3}\right)^2 - 2 \stackrel{?}{=} 0$$
$$3\left(\frac{6}{9}\right) - 2 \stackrel{?}{=} 0$$
$$2 - 2 \stackrel{?}{=} 0$$
$$0 \stackrel{?}{=} 0 \checkmark$$

Check $x = -\frac{\sqrt{6}}{3}$:

$$3x^2 - 2 = 0$$
$$3\left(-\frac{\sqrt{6}}{3}\right)^2 - 2 \stackrel{?}{=} 0$$
$$3\left(\frac{6}{9}\right) - 2 \stackrel{?}{=} 0$$
$$2 - 2 \stackrel{?}{=} 0$$
$$0 \stackrel{?}{=} 0 \checkmark$$

**16.** $-2x^2 - 7x = 0$

$$-2x^2 - 7x + 0 = 0$$
$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(-2)(0)}}{2(-2)} = \frac{7 \pm \sqrt{49}}{-4} = \frac{7 \pm 7}{-4}$$

Solution: $-\frac{7}{2}$ or 0

Check $x = -\frac{7}{2}$:

$$-2x^2 - 7x = 0$$
$$-2\left(-\frac{7}{2}\right)^2 - 7\left(-\frac{7}{2}\right) \stackrel{?}{=} 0$$
$$-2\left(\frac{49}{4}\right) + \frac{49}{2} \stackrel{?}{=} 0$$
$$-\frac{49}{2} + \frac{49}{2} \stackrel{?}{=} 0$$
$$0 = 0 \checkmark$$

Check $x = 0$:

$$-2x^2 - 7x = 0$$
$$-2(0)^2 - 7(0) \stackrel{?}{=} 0$$
$$0 - 0 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark$$

**17.** $3x^2 + x = 6$

$$3x^2 + x - 6 = 0$$
$$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(3)(-6)}}{2(3)} = \frac{-1 \pm \sqrt{73}}{6}$$

Solutions: $-\frac{1}{6} + \frac{\sqrt{73}}{6}$ and $-\frac{1}{6} - \frac{\sqrt{73}}{6}$

Check $x = -\frac{1}{6} + \frac{\sqrt{73}}{6}$:

$$3x^2 + x - 6 = 0$$
$$3\left(-\frac{1}{6} + \frac{\sqrt{73}}{6}\right)^2 + \left(-\frac{1}{6} + \frac{\sqrt{73}}{6}\right) - 6 \stackrel{?}{=} 0$$
$$3\left(\frac{1}{36} - \frac{\sqrt{73}}{18} + \frac{73}{36}\right) - \frac{1}{6} + \frac{\sqrt{73}}{6} - 6 \stackrel{?}{=} 0$$
$$\frac{3}{36} - \frac{3\sqrt{73}}{18} + \frac{219}{36} - \frac{1}{6} + \frac{\sqrt{73}}{6} - 6 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark$$

Check $x = -\frac{1}{6} - \frac{\sqrt{73}}{6}$:

$$3x^2 + x - 6 = 0$$
$$3\left(-\frac{1}{6} - \frac{\sqrt{73}}{6}\right)^2 + \left(-\frac{1}{6} - \frac{\sqrt{73}}{6}\right) - 6 \stackrel{?}{=} 0$$
$$3\left(\frac{1}{36} + \frac{\sqrt{73}}{18} + \frac{73}{36}\right) - \frac{1}{6} - \frac{\sqrt{73}}{6} - 6 \stackrel{?}{=} 0$$
$$\frac{3}{36} + \frac{3\sqrt{73}}{18} + \frac{219}{36} - \frac{1}{6} - \frac{\sqrt{73}}{6} - 6 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark$$

**18.** $x^2 - 4x = -2$

$$x^2 - 4x + 2 = 0$$
$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(2)}}{2(1)}$$
$$= \frac{4 \pm \sqrt{8}}{2}$$
$$= \frac{4 \pm \sqrt{4 \cdot 2}}{2}$$
$$= \frac{4 \pm 2\sqrt{2}}{2} = 2 \pm \sqrt{2}$$

Solutions: $2 + \sqrt{2}$ and $2 - \sqrt{2}$

Check $x = 2 + \sqrt{2}$:

$$x^2 - 4x + 2 = 0$$
$$(2 + \sqrt{2})^2 - 4(2 + \sqrt{2}) + 2 \stackrel{?}{=} 0$$
$$4 + 4\sqrt{2} + 2 - 8 - 4\sqrt{2} + 2 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark$$

Check $x = 2 - \sqrt{2}$:

$$x^2 - 4x + 2 = 0$$
$$(2 - \sqrt{2})^2 - 4(2 - \sqrt{2}) + 2 \stackrel{?}{=} 0$$
$$4 - 4\sqrt{2} + 2 - 8 + 4\sqrt{2} + 2 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark$$

**19.** Check $x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$:

$$ax^2 + bx + c = 0$$
$$a\left(\frac{-b + \sqrt{b^2 - 4ac}}{2a}\right)^2 + b\left(\frac{-b + \sqrt{b^2 - 4ac}}{2a}\right) + c \stackrel{?}{=} 0$$
$$a\left(\frac{b^2}{4a^2} - \frac{2b\sqrt{b^2 - 4ac}}{4a^2} + \frac{b^2 - 4ac}{4a^2}\right) - \frac{b^2}{2a} + \frac{b\sqrt{b^2 - 4ac}}{2a} + c \stackrel{?}{=} 0$$
$$\frac{ab^2}{4a^2} - \frac{2ab\sqrt{b^2 - 4ac}}{4a^2} + \frac{ab^2}{4a^2} - \frac{4a^2c}{4a^2} - \frac{b^2}{2a} + \frac{b\sqrt{b^2 - 4ac}}{2a} + c \stackrel{?}{=} 0$$
$$\frac{b^2}{4a} - \frac{b\sqrt{b^2 - 4ac}}{2a} + \frac{b^2}{4a} - c - \frac{b^2}{2a} + \frac{b\sqrt{b^2 - 4ac}}{2a} + c \stackrel{?}{=} 0$$

Check $x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$:

$$ax^2 + bx + c = 0$$
$$a\left(\frac{-b - \sqrt{b^2 - 4ac}}{2a}\right)^2 + b\left(\frac{-b - \sqrt{b^2 - 4ac}}{2a}\right) + c \stackrel{?}{=} 0$$
$$a\left(\frac{b^2}{4a^2} + \frac{2b\sqrt{b^2 - 4ac}}{4a^2} + \frac{b^2 - 4ac}{4a^2}\right) - \frac{b^2}{2a} - \frac{b\sqrt{b^2 - 4ac}}{2a} + c \stackrel{?}{=} 0$$
$$\frac{ab^2}{4a^2} + \frac{2ab\sqrt{b^2 - 4ac}}{4a^2} + \frac{ab^2}{4a^2} - \frac{4a^2c}{4a^2} - \frac{b^2}{2a} - \frac{b\sqrt{b^2 - 4ac}}{2a} + c \stackrel{?}{=} 0$$
$$\frac{b^2}{4a} + \frac{b\sqrt{b^2 - 4ac}}{2a} + \frac{b^2}{4a} - c - \frac{b^2}{2a} - \frac{b\sqrt{b^2 - 4ac}}{2a} + c \stackrel{?}{=} 0$$

**20.** $ax^2 + x + c = 0$

$$x = \frac{-(1) \pm \sqrt{(1)^2 - 4ac}}{2(a)} = \frac{-1 \pm \sqrt{1 - 4ac}}{2a}$$

$-2x^2 + x + 8 = 0$

$$x = \frac{-1 \pm \sqrt{1 - 4(-2)(8)}}{2(-2)} = \frac{-1 \pm \sqrt{65}}{-4}$$

Solutions: $\frac{1}{4} + \frac{\sqrt{65}}{4}$ and $\frac{1}{4} - \frac{\sqrt{65}}{4}$

Check $x = \frac{1}{4} + \frac{\sqrt{65}}{4}$:

$$-2x^2 + x + 8 = 0$$
$$-2\left(\frac{1}{4} + \frac{\sqrt{65}}{4}\right)^2 + \left(\frac{1}{4} + \frac{\sqrt{65}}{4}\right) + 8 \stackrel{?}{=} 0$$
$$-2\left(\frac{1}{16} + \frac{\sqrt{65}}{8} + \frac{65}{16}\right) + \frac{1}{4} + \frac{\sqrt{65}}{4} + 8 \stackrel{?}{=} 0$$
$$-\frac{2}{16} - \frac{2\sqrt{65}}{8} - \frac{130}{16} + \frac{1}{4} + \frac{\sqrt{65}}{4} + 8 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark$$

Check $x = \frac{1}{4} - \frac{\sqrt{65}}{4}$:

$$-2x^2 + x + 8 = 0$$
$$-2\left(\frac{1}{4} - \frac{\sqrt{65}}{4}\right)^2 + \left(\frac{1}{4} - \frac{\sqrt{65}}{4}\right) + 8 \stackrel{?}{=} 0$$
$$-2\left(\frac{1}{16} + \frac{\sqrt{65}}{8} + \frac{65}{16}\right) + \frac{1}{4} - \frac{\sqrt{65}}{4} + 8 \stackrel{?}{=} 0$$
$$-\frac{2}{16} - \frac{2\sqrt{65}}{8} - \frac{130}{16} + \frac{1}{4} - \frac{\sqrt{65}}{4} + 8 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark$$

**21.** Sum:

$$\frac{-b + \sqrt{b^2 - 4ac}}{2a} + \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{\left(-b + \sqrt{b^2 - 4ac}\right) + \left(-b - \sqrt{b^2 - 4ac}\right)}{2a}$$
$$= \frac{-2b}{2a}$$
$$= \frac{-b}{a}$$

Product:

$$\left(\frac{-b + \sqrt{b^2 - 4ac}}{2a}\right)\left(\frac{-b - \sqrt{b^2 - 4ac}}{2a}\right) = \frac{b^2 - (b^2 - 4ac)}{4a^2}$$
$$= \frac{4ac}{4a^2}$$
$$= \frac{c}{a}$$

Sum $= 2$       product $= \frac{1}{2}$

$-\frac{b}{a} = 2$       $\frac{c}{a} = \frac{1}{2}$

Let $a = -2$; $b = 4$, $c = -1$

Expression: $-2x^2 + 4x - 1 = 0$

**22.** $ax^2 + 12x + 3 = 0$

For one real solution, $a = 12$.

For two real solutions; $a < 12$, $a \neq 0$.

If $a = 12$, $\sqrt{b^2 - 4ac}$ will equal zero which will leave $\frac{-b}{2a}$ as the solution. $a$ must be less than 12 to have two real solutions because if it is more then 12, $\sqrt{b^2 - 4ac}$ will have a negative radicand which is not a real number. Also, $a \neq 0$ or the fractional solution will be undefined.

## Lesson 9.7 Solve Systems with Quadratic Equations

### *Guided Practice for the lesson "Solve Systems with Quadratic Equations"*

**1.** Substitute $2x^2 - 3x - 2$ for $y$ in the equation $y = x + 4$. Then solve for $x$.

$2x^2 - 3x - 2 = x + 4$

$2x^2 - 4x - 6 = 0$

$2(x + 1)(x - 3) = 0$

$x + 1 = 0$    or    $x - 3 = 0$

$x = -1$       $x = 3$

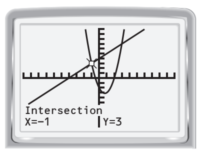
Now substitute $-1$ and 3 for $x$ in the equation $y = x + 4$.

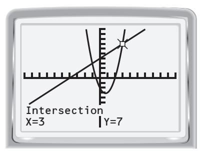
$y = x + 4$       $y = x + 4$

$y = -1 + 4$       $y = 3 + 4$

$y = 3$       $y = 7$

The solutions are $(-1, 3)$ and $(3, 7)$. Verify these solutions using a graphing calculator.




**2.** Substitute $-x^2 + 6x + 1$ for $y$ in the equation $y = x + 1$. Then solve for $x$.

$-x^2 + 6x + 1 = x + 1$

$-x^2 + 5x = 0$

$-x(x - 5) = 0$

$-x = 0$    or    $x - 5 = 0$

$x = 0$       $x = 5$

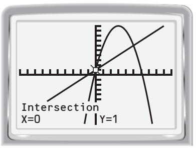
Now substitute 0 and 5 for $x$ in the equation $y = x + 1$.

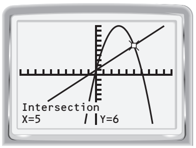
$y = x + 1$       $y = x + 1$

$y = 0 + 1$       $y = 5 + 1$

$y = 1$ $\quad$ $y = 6$

The solutions are (0, 1) and (5, 6). Verify these solutions using a graphing calculator.




**3.** Substitute $x^2 - 6x + 11$ for $y$ in the equation $y = x + 1$. Then solve for $x$.

$x^2 - 6x + 11 = x + 1$

$x^2 - 7x + 10 = 0$

$(x - 2)(x - 5) = 0$

$x - 2 = 0$ or $x - 5 = 0$

$x = 2$ $\quad$ $x = 5$

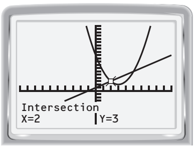
Now substitute 2 and 5 for $x$ in the equation $y = x + 1$.

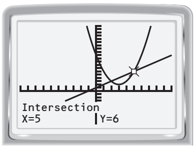
$y = x + 1$ $\quad$ $y = x + 1$

$y = 2 + 1$ $\quad$ $y = 5 + 1$

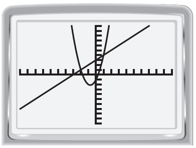
$y = 3$ $\quad$ $y = 6$

The solutions are (2, 3) and (5, 6). Verify these solutions using a graphing calculator.



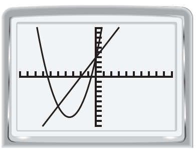




**4.** Graph the equations $y = x + 3$ and $y = 2x^2 + 3x - 1$ on a graphing calculator.



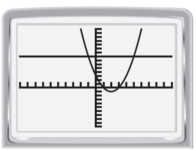
The graphs intersect at $(-2, 1)$ and $(1, 4)$. The $x$-value of each point of intersection is a solution of the original equation.
The solutions are $x = -2$ and $x = 1$.

**5.** Graph the equations $y = x^2 + 7x + 4$ and $y = 2x + 4$ on a graphing calculator.



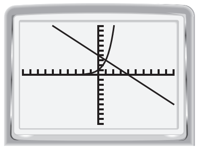
The graphs intersect at $(-5, -6)$ and $(0, 4)$. The $x$-value of each point of intersection is a solution of the original equation.
The solutions are $x = -5$ and $x = 0$.

**6.** Graph the equations $y = 8$ and $y = x^2 - 4x + 3$ on a graphing calculator.



The graphs intersect at $(-1, 8)$ and $(5, 8)$. The $x$-value of each point of intersection is a solution of the original equation.
The solutions are $x = -1$ and $x = 5$.

**7.** Graph the equations $y = -x + 4$ and $y = 3^x$ on a graphing calculator.



The graphs intersect at (1, 3). The $x$-value of the point of intersection is the solution of the original equation.
The solution is $x = 1$.

**8.** The baseball will hit the wall if the value of $y$ is between 0 and 40 when $x = 240$.
Substitute 240 for $x$ in the equation

$y = -0.004x^2 + x + 3$.

$y = -0.004(240)^2 + 240 + 3 = 12.6$

Yes, the baseball does hit the gym wall. It hits the wall at a height of 12.6 feet.

**9.** Less. *Sample answer:* Try graphing an equation with a smaller value such as $-0.005$ as the leading coefficient. If the path of the ball follows this equation, then the ball does not reach as high or travel as far.

### *Exercises for the lesson "Solve Systems with Quadratic Equations"*

### *Skill Practice*

**1.** First solve one of the equations for a variable. Then substitute that expression for the variable in the other equation. Solve the resulting equation in one variable. Use that solution to substitute into one of the original equations to find the value of the other variable.

**2.** The system could have two solutions, one solution, or no solutions.

**3.** Substitute $x^2 - x + 2$ for $y$ in the equation $y = x + 5$. Then solve for $x$.

$x^2 - x + 2 = x + 5$

$x^2 - 2x - 3 = 0$

$(x + 1)(x - 3) = 0$

$x + 1 = 0$ or $x - 3 = 0$

$x = -1$ $\quad$ $x = 4$

Now substitute $-1$ and 3 for $x$ in the equation $y = x + 5$.

$y = x + 5$ $\quad$ $y = x + 5$

$y = -1 + 5$ $\quad$ $y = 3 + 5$

$y = 4$ $\quad$ $y = 8$

The solutions are $(-1, 4)$ and (3, 8).

**4.** Substitute $-x^2 + 4x - 2$ for $y$ in the equation $y = 4x - 6$. Then solve for $x$.

$-x^2 + 4x - 2 = 4x - 6$

$-x^2 + 4 = 0$

$-(x + 2)(x - 2) = 0$

$x + 2 = 0$ or $x - 2 = 0$

$x = -2$ $\quad$ $x = 2$

Now substitute $-2$ and 2 for $x$ in the equation $y = 4x - 6$.

$y = 4x - 6$ $\quad$ $y = 4x - 6$

$y = 4(-2) - 6$ $\quad$ $y = 4(2) - 6$

$y = -14$ $\quad$ $y = 2$

The solutions are $(-2, -14)$ and $(2, 2)$.

**5.** Substitute $x^2 - x$ for $y$ in the equation $y = -\frac{5}{2}x + 1$. Then solve for $x$.

$x^2 - x = -\frac{5}{2}x + 1$

$2x^2 - 2x = -5x + 2$

$2x^2 + 3x - 2 = 0$

$(2x - 1)(x + 2) = 0$

$2x - 1 = 0$ or $x + 2 = 0$

$x = \frac{1}{2}$ $\quad$ $x = -2$

Now substitute $\frac{1}{2}$ and $-2$ for $x$ in the equation $y = -\frac{5}{2}x + 1$.

$y = -\frac{5}{2}x + 1$ $\quad$ $y = -\frac{5}{2}x + 1$

$y = -\frac{5}{2}\left(\frac{1}{2}\right) + 1$ $\quad$ $y = -\frac{5}{2}(-2) + 1$

$y = -\frac{5}{4} + 1$ $\quad$ $y = 5 + 1$

$y = -\frac{1}{4}$ $\quad$ $y = 6$

The solutions are $\left(\frac{1}{2}, -\frac{1}{4}\right)$ and $(-2, 6)$.

**6.** Substitute $2x^2 + x - 1$ for $y$ in the equation $y = -x - 3$. Then solve for $x$.

$2x^2 + x - 1 = -x - 3$

$2x^2 + 2x + 2 = 0$

$2(x^2 + x + 1) = 0$

$x^2 + x + 1 = 0$

The equation $x^2 + x + 1 = 0$ has no real number solutions, so the system of equations has no solution.

**7.** Substitute $3x^2 - 6$ for $y$ in the equation $y = -3x$. Then solve for $x$.

$3x^2 - 6 = -3x$

$3x^2 + 3x - 6 = 0$

$3(x^2 + x - 2) = 0$

$3(x + 2)(x - 1) = 0$

$x + 2 = 0$ or $x - 1 = 0$

$x = -2$ $\quad$ $x = 1$

Now substitute $-2$ and 1 for $x$ in the equation $y = -3x$.

$y = -3x$ $\quad$ $y = -3x$

$y = -3(-2)$ $\quad$ $y = -3(1)$

$y = 6$ $\quad$ $y = -3$

The solutions are $(-2, 6)$ and $(1, -3)$.

**8.** Substitute $-2x^2 - 2x + 3$ for $y$ in the equation $y = \frac{7}{2}$. Then solve for $x$.

$-2x^2 - 2x + 3 = \frac{7}{2}$

$-4x^2 - 4x + 6 = 7$

$-4x^2 - 4x - 1 = 0$

$-(2x + 1)^2 = 0$

$2x + 1 = 0$

$x = -\frac{1}{2}$

From the second equation, $y = \frac{7}{2}$

The solution is $\left(-\frac{1}{2}, \frac{7}{2}\right)$.

**9.** The student substituted incorrectly, replacing $x$ with 4 rather than $y$. Begin by substituting 4 for $y$ in Equation 1:

$4 = 3x^2 - 6x + 4$

$0 = 3x^2 - 6x$ $\quad$ Subtract 4 from each side.

$0 = 3x(x - 2)$ $\quad$ Factor the right side.

$3x = 0$ or $x - 2 = 0$

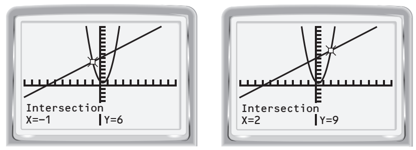
$x = 0$ $\quad$ $x = 2$

From the second equation, $y = 4$. The solutions to the system are $(0, 4)$ and $(2, 4)$.

**10.** When a system of equations includes a linear equation and a quadratic equation, there will *never* be an infinite number of solutions.

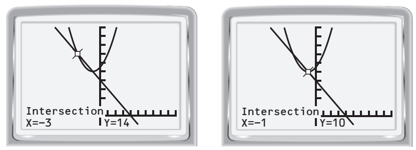
**11.** The graphs of $y = -1$ and $x = 2$ each intersect the graph of $y = x^2 - 4x + 3$ just once, while the graph of $y + x = -1$ does not intersect the graph of $y = x^2 - 4x + 3$ at all. Only the graph of $y + 1 = x$ intersects the graph of $y = x^2 - 4x + 3$ twice. The correct answer is C.

**12.**



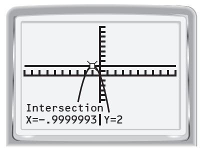

The solutions are $(-1, 6)$ and $(2, 9)$.

**13.**



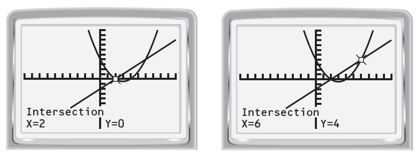

The solutions are $(-1, 10)$ and $(-3, 14)$.

**14.**



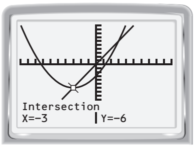

The solution is $(-1, 2)$.

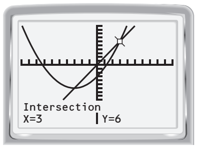
**15.**




The solutions are $(2, 0)$ and $(6, 4)$.

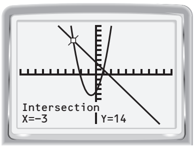
**16.**

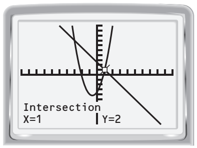



The solutions are $(-3, -6)$ and $(3, 6)$.

**17.**



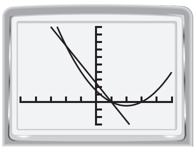




The solutions are $(-3, 14)$ and $(1, 2)$.

**18.** Write a system of two equations by setting both the left and right sides of the equation each equal to $y$.

$y = -5x + 5$
$y = x^2 - 4x + 3$

Graph the two equations on a graphing calculator.

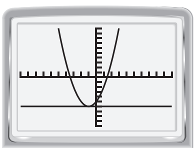


The graphs intersect at $(-2, 15)$ and $(1, 0)$. The $x$-value of each point of intersection is a solution of the original equation.
The solutions are $x = -2$ and $x = 1$.

**19.** Write a system of two equations by setting both the left and right sides of the equation each equal to $y$.

$y = -6$
$y = x^2 + 2x - 5$

Graph the two equations on a graphing calculator.

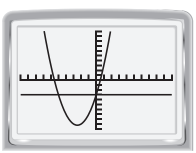


The graphs intersect at $(-1, -6)$. The $x$-value of the point of intersection is the solution of the original equation.
The solution is $x = -1$.

**20.** Write a system of two equations by setting both the left and right sides of the equation each equal to $y$.

$y = -3$
$y = x^2 + 5x - 3$

Graph the two equations on a graphing calculator.



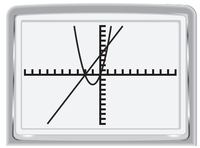
The graphs intersect at $(-5, -3)$ and $(0, -3)$. The $x$-value of each point of intersection is a solution of the original equation.

The solutions are $x = -5$ and $x = 0$.

**21.** Write a system of two equations by setting both the left and right sides of the equation each equal to $y$.

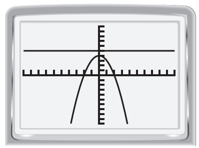
$y = 2x^2 + 4x$
$y = 2x + 4$

Graph the two equations on a graphing calculator.



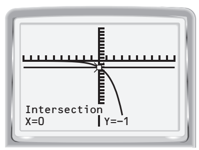
The graphs intersect at $(-2, 0)$ and $(1, 6)$. The $x$-value of each point of intersection is a solution of the original equation.
The solutions are $x = -2$ and $x = 1$.

**22.**



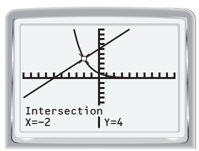
The graphs do not intersect. There is no solution to the system of equations.

**23.**



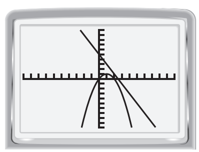

The solution is $(0, -1)$.

**24.**



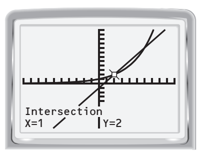

The solution is $(-2, 4)$.

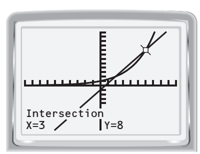
**25.**



The graphs do not intersect. There is no solution to the system of equations.

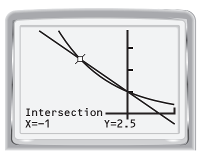
**26.**

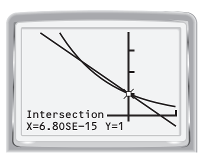



The solutions are $(1, 2)$ and $(3, 8)$.

**27.**



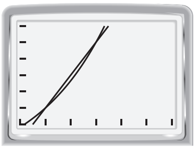

The solutions are $(-1, 2.5)$ and $(0, 1)$.

28. The system of equations could have two solutions, one solution, or no solutions.

29. Write a system of two equations by setting both the left and right sides of the equation each equal to $y$.

$y = 2^x + 1$
$y = 2x + 1$

Graph the two equations on a graphing calculator.

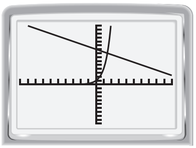


The graphs intersect at (1, 3) and (2, 5). The $x$-value of each point of intersection is a solution of the original equation.
The solutions are $x = 1$ and $x = 2$.

30. Write a system of two equations by setting both the left and right sides of the equation each equal to $y$.

$y = 4^x$

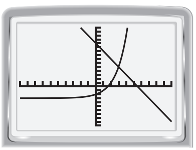
$y = -\frac{2}{3}x + 9$



The graphs intersect at (1.5, 8). The $x$-value of the point of intersection is the solution of the original equation.
The solution is $x = 1.5$.

31. Write a system of two equations by setting both the left and right sides of the equation each equal to $y$.

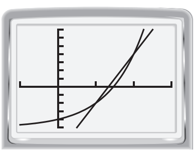
$y = -2x + 11$
$y = 2^x - 3$



The graphs intersect at (3, 5). The $x$-value of the point of intersection is the solution of the original equation.
The solution is $x = 3$.

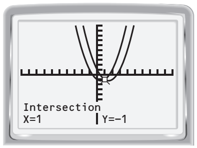
32. Write a system of two equations by setting both the left and right sides of the equation each equal to $y$.

$y = 3^x - 5$
$y = 6x - 8$



The graphs intersect at (1, −2) and (2, 4). The $x$-value of each point of intersection is a solution of the original equation.
The solutions are $x = 1$ and $x = 2$.

33.




The graphs intersect at (1, −1).
The solution is (1, −1).

### Problem Solving

34. Substitute $-x^2 - 4x - 1$ for $y$ in the equation $y = 2x + 8$. Then solve for $x$.

$$-x^2 - 4x - 1 = 2x + 8$$
$$-x^2 - 6x - 9 = 0$$
$$-(x + 3)^2 = 0$$
$$x + 3 = 0$$
$$x = -3$$

Now substitute −3 for $x$ in the equation $y = 2x + 8$.

$y = 2x + 8$
$y = 2(-3) + 8$
$y = 2$

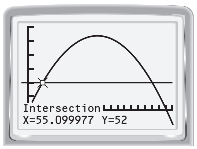
Yes, the paths do cross. The coordinates of the point where they meet are (−3, 2).

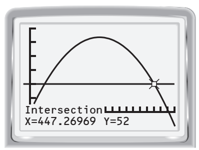
35. Substitute 4 for $y$ in the equation $y = -x^2 + 3$. Then solve for $x$.

$$4 = -x^2 + 3$$
$$0 = x^2 + 1$$

The equation $x^2 + 1 = 0$ has no real number solutions, so the system has no solution.
No, the dogs' paths will not cross.

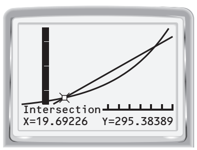
36. Graph the equations $y = -0.00211x^2 + 1.06x$ and $y = 52$ on a graphing calculator.

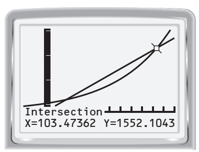



The graphs intersect near the points (55, 52) and (447, 52). The road intersects the arch of the bridge approximately 55 meters and 447 meters from the left pylons.

37.




The graphs intersect when the two girls have the same amount of money saved. Miranda has more money saved for the first 20 months, and then again after 104 months, because the graphs intersect near $x = 20$ and $x = 104$.

38. a. Substitute $2x^2 - 3x + 1$ for $y$ in the equation $y = 8x - 13$. Then solve for $x$.

$$2x^2 - 3x + 1 = 8x - 13$$
$$2x^2 - 11x + 14 = 0$$
$$(2x - 7)(x - 2) = 0$$
$$2x - 7 = 0 \quad \text{or} \quad x - 2 = 0$$
$$x = 3.5 \qquad\qquad x = 2$$

Now substitute 3.5 and 2 for $x$ in the equation $y = 8x - 13$.

$y = 8x - 13$ $\quad$ $y = 8x - 13$
$y = 8(3.5) - 13$ $\quad$ $y = 8(2) - 13$
$y = 15$ $\quad$ $y = 3$

Yes, the paths of the objects do intersect. The coordinates of the points where the paths intersect are (3.5, 15) and (2, 3).
Yes, because the paths intersect it is possible for the two objects to collide.

**b.** In order to know whether the two objects will collide one would have to know their positions at the same time and each of their speeds.

**39. a.** Substitute $2x^2 - 4x + 6$ for $y$ in the equation $y = 3x + 1$. Then solve for $x$.

$2x^2 - 4x + 6 = 3x + 1$
$2x^2 - 7x + 5 = 0$
$(x - 1)(2x - 5) = 0$
$x - 1 = 0$ or $2x - 5 = 0$
$x = 1$ $\quad$ $x = 2.5$

Now substitute 1 and 2.5 for $x$ in the equation $y = 3x + 1$.

$y = 3x + 1$ $\quad$ $y = 3x + 1$
$y = 3(1) + 1$ $\quad$ $y = 3(2.5) + 1$
$y = 4$ $\quad$ $y = 8.5$

The points of intersection are (1, 4) and (2.5, 8.5).

**b.** Substitute $-2x + 6$ for $y$ in the equation $y = 3x + 1$. Then solve for $x$.

$-2x + 6 = 3x + 1$
$-5x = -5$
$x = 1$

Now substitute 1 for $x$ in the equation $y = 3x + 1$.

$y = 3x + 1$
$y = 3(1) + 1$
$y = 4$

The point of intersection is (1, 4).

**c.** Substitute $2x^2 - 4x + 6$ for $y$ in the equation $y = -2x + 6$. Then solve for $x$.

$2x^2 - 4x + 6 = -2x + 6$
$2x^2 - 2x = 0$
$2x(x - 1) = 0$
$2x = 0$ or $x - 1 = 0$
$x = 0$ $\quad$ $x = 1$

Now substitute 0 and 1 for $x$ in the equation $y = -2x + 6$.

$y = -2x + 6$ $\quad$ $y = -2x + 6$
$y = -2(0) + 6$ $\quad$ $y = -2(1) + 6$
$y = 6$ $\quad$ $y = 4$

The points of intersection are (0, 6) and (1, 4).

**d.** Yes; all three graphs pass through the point (1, 4).

**40.** Substitute $-x - 1$ for $y$ in the equation $x^2 + y^2 = 41$. Then solve for $x$.

$x^2 + y^2 = 41$
$x^2 + (-x - 1)^2 = 41$
$x^2 + (x^2 + 2x + 1) = 41$
$2x^2 + 2x - 40 = 0$
$x^2 + x - 20 = 0$
$(x + 5)(x - 4) = 0$
$x + 5 = 0$ or $x - 4 = 0$
$x = -5$ $\quad$ $x = 4$

Now substitute $-5$ and 4 for $x$ in the equation $y = -x - 1$.

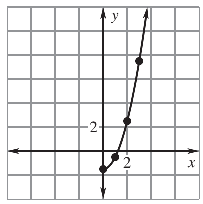
$y = -x - 1$ $\quad$ $y = -x - 1$
$y = -(-5) - 1$ $\quad$ $y = -4 - 1$
$y = 4$ $\quad$ $y = -5$

The points of intersection are $(-5, 4)$ and $(4, -5)$.

## Lesson 9.8 Compare Linear, Exponential, and Quadratic Models

### *Guided Practice for the lesson "Compare Linear, Exponential, and Quadratic Models"*

**1.**




The ordered pairs represent a quadratic function.

**2.**

| $x$ | $-2$ | $-1$ | 0 | 1 |
|---|---|---|---|---|
| $y$ | 0.08 | 0.4 | 2 | 10 |

Ratio: $\frac{0.04}{0.08} = 5$ $\quad$ 5 $\quad$ 5

The table of values represents an exponential function because the ratios are equal.

**3.**

| $x$ | $-3$ | $-2$ | $-1$ | 0 | 1 |
|---|---|---|---|---|---|
| $y$ | $-7$ | $-5$ | $-3$ | $-1$ | 1 |

Differences: 2 $\quad$ 2 $\quad$ 2 $\quad$ 2

The table of values represents a linear function because the differences are equal.

Linear function: $y = ax + b$

Use $(0, -1)$: $-1 = a(0) + b$

$-1 = b$

Use (1, 1): $1 = a(1) - 1$

$2 = a$

The equation is $y = 2x - 1$.

**4.**

| $x$ | $-2$ | $-1$ | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | 8 | 2 | 0 | 2 | 8 |

First differences: $-6$ $\quad$ $-2$ $\quad$ 2 $\quad$ 6

Second differences: 4 $\quad$ 4 $\quad$ 4

The table represents a quadratic function because the second differences are equal.

Quadratic equation: $y = ax^2$

Use (1, 2): $2 = a(1)^2$

$2 = a$

The equation is $y = 2x^2$.

**5.** From example 4, $y = 6.38(1.11)^x$.

When $x = 15$: $y = 6.38(1.11)^{15}$

$= 30.5$

The breathing rate is 30.5 liters of air per minute.

### *Exercises for the lesson "Compare Linear, Exponential, and Quadratic Models"*

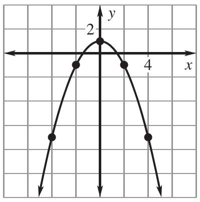
### *Skill Practice*

**1.** A function that is of the for m $y = ab^x$ is an *exponential function.*

**2.** A table of values represents a quadratic function when the second differences are equal.

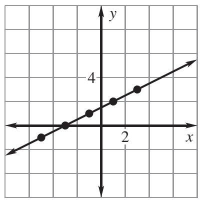
**3.** B **4.** C **5.** A

**6.** $y = -0.5x^2 + 1$



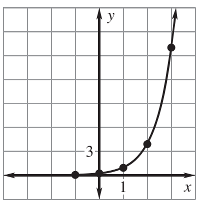

The ordered pairs represent a quadratic function.

**7.** $y = \frac{1}{2}x + \frac{3}{2}$



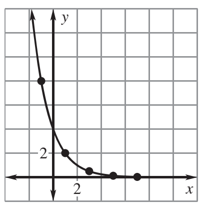

The ordered pairs represent a linear function.

**8.** $y = \frac{1}{2}(4)^x$



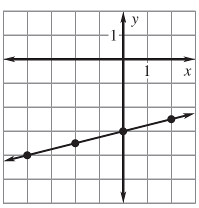

The ordered pairs represent an exponential function.

**9.** $y = 4\left(\frac{1}{2}\right)^x$



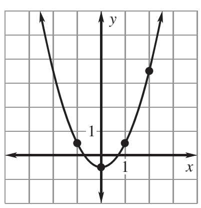

The ordered pairs represent an exponential function.

**10.** $y = \frac{1}{4}x - 3$




The ordered pairs represent a linear function.

**11.** $y = x^2 - \frac{1}{2}$




The ordered pairs represent a quadratic function.

**12.**

| x | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| y | 1 | 0 | −1 | −2 | −3 |

Differences: −1 −1 −1 −1

The table of values represents a linear function because the differences are equal.

Linear function: $y = ax + b$

Use (0, 1): $1 = a(0) + b$

$1 = b$

Use (1, 0): $0 = a(1) + 1$

$-1 = a$

The equation is $y = -x + 1$.

**13.**

| x | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| y | −4 | −1 | 0 | −1 | −4 |

First differences: 3 1 −1 −3

Second differences: −2 −2 −2

The table of values represents a quadratic function because the second differences are equal.

Quadratic function: $y = ax^2$

Use (1,−1): $-1 = a(1)^2$

$-1 = a$

The equation is $y = -x^2$.

**14.**

| x | −3 | −2 | −1 | 0 | 1 |
|---|---|---|---|---|---|
| y | 13.5 | 6 | 1.5 | 0 | 1.5 |

First differences: −7.5 −4.5 −1.5 1.5

Second differences: 3 3 3

The table of values represents a quadratic function because the second differences are equal.

Quadratic function: $y = ax^2$

Use (1, 1.5): $1.5 = a(1)^2$

$1.5 = a$

The equation is $y = 1.5x^2$.

**15.**

| x | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| y | −5 | −2 | 1 | 4 | 7 |

Differences: 3 3 3 3

The table of values represents a linear function because the differences are equal.

Linear function: $y = ax + b$

Use (0, 1): $1 = a(0) + b$

$1 = b$

Use (1, 4): $4 = a(1) + 1$

$3 = a$

The equation is $y = 3x + 1$.

**16.**

| $x$ | $-2$ | $-1$ | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | $\frac{1}{9}$ | $\frac{1}{3}$ | 1 | 3 | 9 |

Ratios: $\frac{\frac{1}{3}}{\frac{1}{9}} = 3 \quad 3 \quad 3 \quad 3$

The table of values represents an exponential function because the ratios are equal.

Exponential function: $y = ab^x$

Use (0, 1): $1 = ab^0$

$1 = a$

Use (1, 3): $3 = b^1$

$3 = b$

The equation is $y = 3^x$.

**17.**

| $x$ | $-1$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|
| $y$ | 16 | 4 | 1 | $\frac{1}{4}$ | $\frac{1}{16}$ |

Ratios: $\frac{4}{16} = \frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4}$

The table of values represents an exponential function because the ratios are equal.

Exponential function: $y = ab^x$

Use (0, 4): $4 = ab^0$

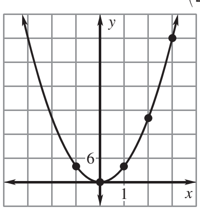
$4 = a$

Use (1, 1): $1 = 4b^1$

$\frac{1}{4} = b$

The equation is $y = 4\left(\frac{1}{4}\right)^x$.

**18.** C;




The ordered pairs represent a quadratic function.

Quadratic function: $y = ax^2$.

Use (1, 4): $4 = a(1)^2$

$4 = a$

The equation is $y = 4x^2$.

**19.** The error occurs in writing the quadratic equation. The point (2, 10) was not substituted correctly into the general equation.

$y = ax^2$

$10 = a(2)^2$

$\frac{5}{2} = a$

So, the equation is $y = \frac{5}{2}x^2$.

**20.** **a.** The graph does not have a parabolic shape, so the graph represents an exponential function.

**b.**

| $x$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $y$ | 2 | 4 | 8 | 16 |

Ratios: $\frac{4}{2} = 2 \quad 2 \quad 2$

The table of values represents an exponential function because the ratios are equal. So, the answer from part (a) checks.

**c.** Exponential function: $y = ab^x$

The value of $y$ increases by a factor of 2, so $b = 2$.

Use (1, 2): $2 = a(2)^1$

$1 = a$.

The equation is $y = 2^x$.

**21.**

| Side length, $s$ (cm) | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| **Area, $A$ (cm$^2$)** | $0.25\sqrt{3}$ | $\sqrt{3}$ | $2.25\sqrt{3}$ | $4\sqrt{3}$ | $6.25\sqrt{3}$ |

First differences: 0.75 1.25 1.75 2.25

Second differences: 0.5 0.5 0.5

The table of values represents a quadratic function because the second differences are equal.

Quadratic function: $A = as^2$

Use $(2, \sqrt{3})$: $\sqrt{3} = a(2)^2$

$\frac{\sqrt{3}}{4} = a$

The equation is $A = \frac{\sqrt{3}}{4}s^2$.

When $s = 10$, $A = \frac{\sqrt{3}}{4}(10)^2 = 25\sqrt{3}$.

The area of the equilateral triangle is $25\sqrt{3}$ square centimeters.

**22.**

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $y$ | $3m - 1$ | $10m + 2$ | $26m$ | $51m - 7$ | $85m - 19$ |

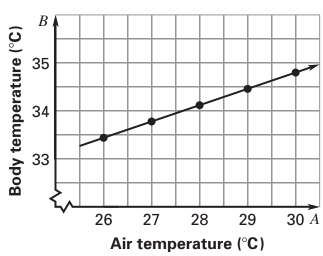
First differences: $7m + 3 \quad 16m - 2 \quad 25m - 7 \quad 34m - 12$

Second differences: $9m - 5 \quad 9m - 5 \quad 9m - 5$

The ordered pairs represent a quadratic function because the second differences are equal.

## *Problem Solving*

**23.**




The graph appears to be a straight line. So, a linear function could model the data.

| Air temperature, $A$ (°C) | 26 | 27 | 28 | 29 | 30 |
|---|---|---|---|---|---|
| **Body temperature, $B$ (°C)** | 33.44 | 33.78 | 34.12 | 34.46 | 34.80 |

Differences: 0.34 0.34 0.34 0.34

The data represent a linear function because the differences are equal.

Linear function: $B = aA + b$

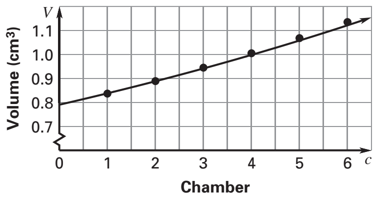
For every 1°C increase in air temperature, the body temperature increases at a rate of 0.34°C, so $a = 0.34$.

Use (26, 33.44): $33.44 = 0.34(26) + b$

$24.6 = b$

The equation is $B = 0.34A + 24.6$.

**24.**




The graph has a slight curve. So, a quadratic function or an exponential function could model the data.

| **Chamber** | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| **Volume ($cm^3$)** | 0.836 | 0.889 | 0.945 | 1.005 | 1.068 | 1.135 |

Ratios: $\frac{0.889}{0.836} \approx 1.06$ 1.06 1.06 1.06 1.06

The data represent an exponential function because the ratios are approximately equal.

Let $c$ represent the chamber and $V$ represent the volume.

Exponential function: $V = ab^2$

The volume increases by a factor of 1.06, so $b = 1.06$.

Use (1, 0.836): $0.836 = a(1.06)^1$

$0.789 \approx a$

The equation is $V = 0.789(1.06)^c$.

**25. a.** The population of Troy doubled every decade, so the ratios of successive $y$-values are equal, and the data can be modeled by an exponential function. The population of Union increased by a fixed amount every decade, and the data can be modeled by a linear function.

**b.**

| **Decades since 1970** | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| **Troy's pop. (thousands)** | 3 | 6 | 12 | 24 | 48 |

Ratios: 2 2 2 2

The data represent an exponential function because the ratios are equal.

| **Decades since 1970** | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| **Union's pop. (thousands)** | 3 | 6 | 9 | 12 | 15 |

Differences: 3 3 3 3

The data represent a linear function because the differences are equal.

**c.** Let $y$ = population and $x$ = number of decades since 1970.

**Troy:** The equation has the form $y = ab^x$. The population increases by a factor of 2, so $b = 2$.

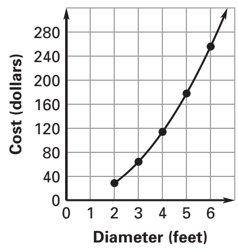
Use (1, 6000): $6000 = a \cdot 2^1$

$3000 = a$

The equation is $y = 3000 \cdot 2^x$. The year 2030 is 6 decades after 1970. When $x = 6$, $3000 \cdot 2^6 = 192{,}000$, so the population of Troy in 2030 is predicted to be 192,000.

**Union:** The equation has the form $y = mx + b$. The change in population per decade is 3000, so $m = 3000$. When $x = 0$, $y = 3000$, so $m = 3000$. The equation is $y = 3000x + 3000$. The year 2030 is 6 decades after 1970. When $x = 6$, $3000(6) + 3000 = 21{,}000$, so the population of Union in 2030 is predicted to be 21,000.

**26.** C;




The graph has a curve. So, a quadratic function or an exponential function could model the data.

| **Diameter (ft)** | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| **Cost (dollars)** | 28.40 | 63.90 | 113.60 | 177.50 | 255.50 |

First differences: 35.5 49.7 63.9 78.1

Second differences: 14.2 14.2 14.2

The data represent a quadratic function because the second differences are equal.

Let $c$ represent the cost and $d$ represent the diameter.

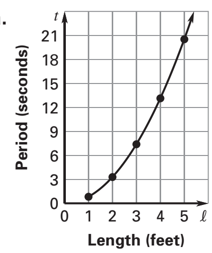
Quadratic function: $c = ad^2$

Use $(2, 28.40)$: $28.40 = a(2)^2$

$7.1 = a$

The equation is $c = 7.1d^2$. When $d = 8$, $c = 7.1(8)^2$, or $454.40.

**27. a.**




The graph has a curve. So, a quadratic function or an exponential function could model the data.

| **Period, $t$ (sec)** | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| **Length, $\ell$ (ft)** | 0.82 | 3.28 | 7.38 | 13.12 | 20.5 |

First differences: 2.46 4.1 5.74 7.38

Second differences: 1.64 1.64 1.64

The data represent a quadratic function because the second differences are equal.

Quadratic function: $\ell = at^2$

Use $(1, 0.82)$: $0.82 = a(1)^2$

$$0.82 = a$$

The equation is $\ell = 0.82t^2$.

**b.** When $t = 0.5$, $\ell = 0.82(0.5)^2$, or 0.205 feet.

**c.** Let $\ell_1 = 0.82(t_1)^2$ and $\ell_2 = 0.82(t_2)^2$.

We know that $\ell_2 = 0.5\ell_1$

$$0.82(t_2)^2 = 0.5[0.82(t_1)^2]$$

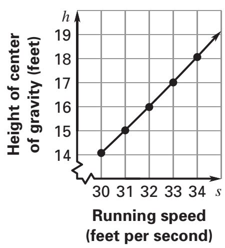
$$\frac{(t_2)^2}{(t_1)^2} = 0.5$$

$$\frac{t_2}{t_1} = \sqrt{0.5}$$

$$t_2 = t_1\sqrt{0.5} \approx 0.707t_1$$

So, decreasing the length of a pendulum by 50% decreases the period by a factor of 0.707. Justifications will vary.

**28. a.**




The graph appears to be linear, but the first differences are not equal. So, a quadratic function or an exponential function could model the data.

| **Running speed, $s$ (ft/sec)** | 30 | 31 | 32 | 33 | 34 |
|---|---|---|---|---|---|
| **Height of center of gravity, $h$ (ft)** | $14\frac{1}{16}$ | $15\frac{1}{64}$ | 16 | $17\frac{1}{64}$ | $18\frac{1}{16}$ |

First differences: $\frac{61}{64}$ $\frac{63}{64}$ $\frac{65}{64}$ $\frac{67}{64}$

Second differences: $\frac{2}{64}$ $\frac{2}{64}$ $\frac{2}{64}$

The data represent a quadratic function because the second differences are equal.

Quadratic model: $h = as^2$

Use $\left(30, \frac{225}{16}\right)$: $\frac{225}{16} = a(30)^2$

$$\frac{225}{16} = 900a$$

$$\frac{1}{64} = a$$

The equation is $h = \frac{1}{64}s^2$.

When $s = \frac{63}{2}$: $h = \frac{1}{64}\left(\frac{63}{2}\right)^2 = \frac{3969}{256}$

The pole vaulter's center of gravity reaches a height of $15\frac{129}{256}$ feet.

**b.** When $h = 19$: $19 = \frac{1}{64}s^2$

$$1216 = s^2$$

$$\pm\sqrt{1216} = s$$

The solutions are $\sqrt{1216} \approx 35$ or $-\sqrt{1216} \approx -35$. Ignore the negative solution. The pole vaulter needs to run at a speed of about 35 feet per second.

### *Quiz for the lessons "Interpret the Discriminant", and "Compare Linear, Exponential, and Quadratic Models"*

**1.** $x^2 + x + 5 = 0$

$b^2 - 4ac = 1^2 - 4(1)(5) = -19$

The value of the discriminant is negative. So, there is no solution.

**2.** $5x^2 + 4x - 1 = 0$

$b^2 - 4ac = 4^2 - 4(5)(-1) = 36$

The value of the discriminant is positive. So, there are two solutions.

**3.** $y = -3x^2 + 4x - 2$

$0 = -3x^2 + 4x - 2$

$b^2 - 4ac = 4^2 - 4(-3)(-2) = -8$

The value of the discriminant is negative. So, there are no $x$-intercepts of the graph of the function.

**4.** $y = \frac{4}{9}x^2 + 4x + 9$

$0 = \frac{4}{9}x^2 + 4x + 9$

$b^2 - 4ac = 4^2 - 4\left(\frac{4}{9}\right)(9) = 0$

The value of the discriminant is 0. So, there is one $x$-intercept of the graph of the function.

**5.**

| **x** | −6 | −3 | 0 | 3 | 6 |
|---|---|---|---|---|---|
| **y** | −9 | −2.25 | 0 | −2.25 | −9 |

First differences: 6.75 2.25 −2.25 −6.75

Second differences: −4.5 −4.5 −4.5

The table of values represents a quadratic function because the second differences are equal.

Quadratic function: $y = ax^2$

Use $(6, -9)$: $-9 = a(6)^2$

$$-\frac{1}{4} = a$$

The equation is $y = -\frac{1}{4}x^2$.

**6.**

| **x** | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| **y** | 5 | 1 | $\frac{1}{5}$ | $\frac{1}{25}$ | $\frac{1}{125}$ |

Ratios: $\frac{1}{5} =$ $\frac{1}{5}$ $\frac{1}{5}$ $\frac{1}{5}$ $\frac{1}{5}$

The table of values represents a exponential function because the ratios are equal.

Exponential function: $y = ab^x$

The value of $y$ decreases by a factor of $\frac{1}{5}$, so $b = \frac{1}{5}$.

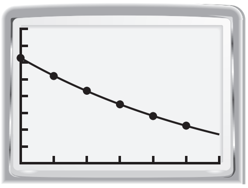
Use (1, 5): $5 = a\left(\frac{1}{5}\right)^1$

$25 = a$

The equation is $y = 25\left(\frac{1}{5}\right)^x$.

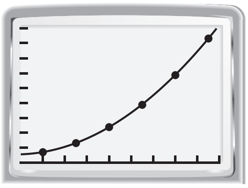
### *Graphing Calculator Activity for the lesson "Compare Linear, Exponential, and Quadratic Models"*

**1.**



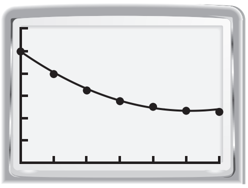
Using the exponential regression feature, the model is $y = 15{,}600(0.866)^x$. Let $x = 7$. Then $15{,}600(0.866)^x = 15{,}600(0.866)^7 \approx 5698$. So, the value of the car after 7 years will be about \$5698.

**2.**



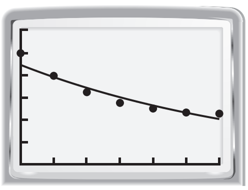
Using the quadratic regression feature, the model is $y = 10.18x^2 - 96.8x + 285$.

**3.** Quadratic Model:



Using the quadratic regression feature, the model is $y = 0.04x^2 - 4.1x + 197$.

Exponential Model:



Using the exponential regression feature, the model is $y = 179(0.987)^x$. By comparing the graphs of the models with the scatter plot of the data, the quadratic model fits the data better than the exponential model.

**4.** Models may vary, but a linear model should fit the data in part (a) best, and a quadratic model should fit the data in part (b) best.

## Lesson 9.9 Modeling Relationships

### *Guided Practice for the lesson "Modeling Relationships"*

**1.** The function is increasing as $x$ increases from 0 to about 70. This is when the water is moving upward until it reaches its maximum height. The function is decreasing as $x$ increases from about 70 to about 140. This is when the water is traveling downward until it reaches the surface of the water that the boat is on.

**2.** The rate of change of $y = 4x + 5$ is 4. The rate of change of $y = 3 - 4x$ is $-4$. The first function is increasing at the same rate that the second function is decreasing.

**3.** The $x$-coordinate of the vertex of Quadratic Function 1 is now $-\frac{b}{2a} = -\frac{-6}{2(1)} = \frac{6}{2} = 3$. Substitute 3 for $x$ in the new equation for Quadratic Function 1.

$y = x^2 - 6x - 7$
$= (3)^2 - 6(3) - 7$
$= -16$

The minimum value of Quadratic Function 1 is now $-16$. As seen in the graph of Quadratic Function 2, its minimum value is –9. Quadratic Function 1 again has the lesser minimum value.

**4.** The intervals of times are equal. Juan's mileage is decreasing each hour. Compare the ratio of miles for consecutive hours.

$\frac{5.4}{6} = 0.9$  $\frac{4.86}{5.4} = 0.9$  $\frac{4.374}{4.86} = 0.9$

The distance Juan travels per hour is decreasing by a constant percent rate per unit interval of time. The decay rate is 10%. You should use an exponential decay model for this situation.

### *Exercises for the lesson "Modeling Relationships"*

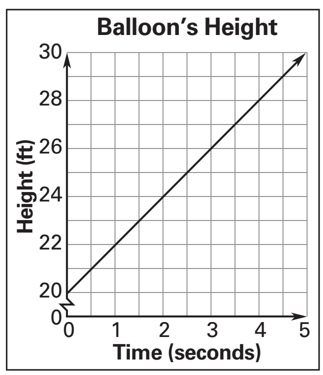
### *Skill Practice*

**1.** A *verbal model* describes a real-world situation using words as labels and using math symbols to relate the words.

**2.** A linear function with a positive slope is an increasing function. A linear function with a negative slope is a decreasing function.

**3. a.** The balloon rises at a steady rate of 2 feet per second. So the height of the balloon is increasing by a constant rate per unit interval of time. A linear function is a good model.

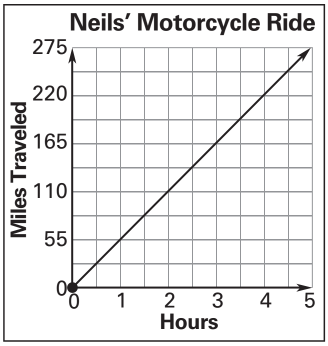
**b.**



**c.** The graph shows that the function is increasing throughout its domain, $x \geq 0$. As the time since it resumed its ascent increases, the altitude of the balloon increases.

**4. a.** Each hour that Neil rides his motorcycle he averages 55 miles. The distance Neil rides his motorcycle is increasing by a constant rate per unit interval of time. A linear function is a good model.

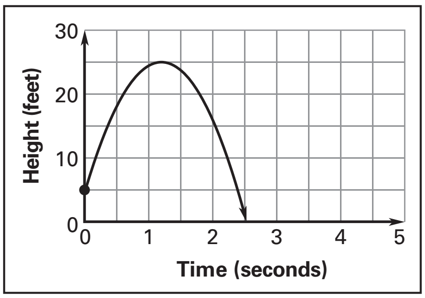
**b.**




**c.** The $x$-intercept and the $y$-intercept are both 0. The intercepts represent the fact that before he starts riding his motorcycle Neil has not left his starting point.

**5. a.** The ball rises to a maximum height of 25 feet and then falls until it is caught 2.5 seconds after it was tossed. A quadratic function is a good model.

**b.**




**c.** The graph is increasing from $x = 0$ to about $x = 1.25$. This is the time during which the ball is traveling upward until it reaches its maximum height at about 1.25 seconds. The graph is decreasing from about $x = 1.25$ to $x = 2.5$. This is the time during which the ball is traveling downward until the juggler catches it at 2.5 seconds.

**6.** The area of a rectangle is found by multiplying length and width. The product of $x$ and $3 - x$ is $3x - x^2$, which is a quadratic expression. The correct answer is B.

**7.** A function that increases by a constant percent rate per unit interval of time should be modeled by an exponential growth function.

**8.** A function that increases by a constant rate per unit interval of time should be modeled by a linear function.

**9.** The slope of Linear Function 1 is given as $-1$. The table for Linear Function 2 shows that for each increase of 2 in the value of $x$ there is a decrease of 4 in the value of $y$, so its slope is $\frac{0 - 4}{0 - (-2)} = \frac{-4}{2} = -2$. So Linear Function 2 is decreasing more rapidly.

**10.** *Sample answer:* The maximum value of Quadratic Function 1 is the $y$-value of the vertex of its parabola.

The $x$-coordinate of the vertex is $-\frac{b}{2a} = -\frac{4}{2(-1)} = \frac{4}{2} = 2$.

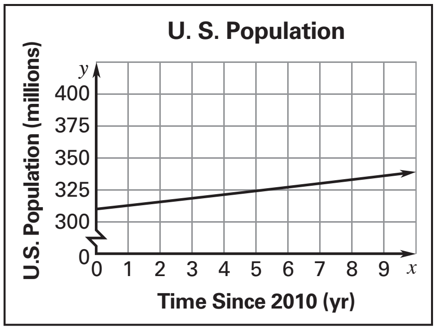
Substitute 2 for $x$ in the equation for Quadratic Function 1.

$$\begin{aligned} y &= -x^2 + 4x + 2 \\ &= -(2)^2 + 4(2) + 2 \\ &= 6 \end{aligned}$$

The maximum value of Quadratic Function 1 is 6. As seen in the graph of Quadratic Function 2, its maximum value is 2. Quadratic Function 2 has the greater maximum value.

**11.** The relationships in choices B and D show decay not growth. In choice A, the number of miles increases by a constant amount per unit of time, so the relationship is linear. In choice C, the number of windows installed doubles each day. This is a 100% growth rate per day. The correct answer is C.

**12. a.**




A function that increases by a constant percent rate per unit interval of time, in this case 0.9% per year, will be best modeled by an exponential growth function.

**b.** The $y$-intercept is 310 million. This represents the population in the year 2010. There is no $x$-intercept.

**c.** Increasing; this is an increasing function because the population continues to grow.

### *Problem Solving*

**13.** The number of songs Celia has downloaded is increasing by a constant amount of 2 songs per week. As seen in the table, the number of songs Connie has downloaded is increasing by a constant amount of 5 songs per week. Connie's playlist is growing faster.

**14.** To find the maximum height reached by the ball Tim threw, find the $y$-value of the vertex of the parabola. The $x$-coordinate of the vertex is $-\frac{b}{2a} = -\frac{40}{2(-16)} = \frac{40}{32}$ $= 1.25$. Substitute 1.25 for $x$ in the equation for Tim's baseball to find the maximum height.

$$\begin{aligned} y &= -16x^2 + 40x + 5 \\ &= -16(1.25)^2 + 40(1.25) + 5 \\ &= 30 \end{aligned}$$

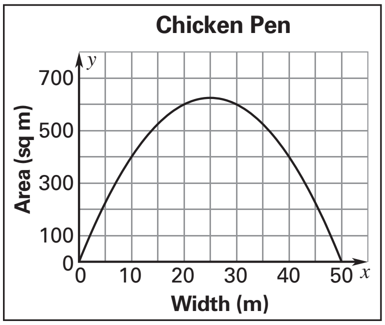
Tim's ball reached a maximum height of 30 feet. As seen in the graph, Matt's ball reached a maximum height of about 19 feet. So Tim's ball reached a greater height.

**15.** Growth. *Sample answer:* The number of spores in the Petri dish represents growth. To determine the rate, find the ratio of the number of spores found in the Petri dish in any given hour to the number of spores found in the Petri dish in the previous hour.

$\frac{24}{16} = 1.5 \quad \frac{36}{24} = 1.5 \quad \frac{54}{36} = 1.5$

Because 1.5 is the growth factor, the growth rate is $1.5 - 1 = 0.5$, or 50%.

**16. a.**




**b.** There are no intercepts. If there was an $x$ intercept there would be no area. If there was a $y$-intercept the pen would have no measurable length or width.

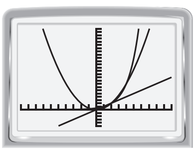
**17.** The number of miles is decreasing for each successive 15-minute interval. To determine the rate, find the ratio of the number of miles for consecutive intervals.

$\frac{1.6}{2} = 0.8 \quad \frac{1.28}{1.6} = 0.8 \quad \frac{1.024}{1.28} = 0.8$

The rowing crew's distances are decreasing by a constant percent rate per unit interval of time. The decay rate is 20%.

**18.** In choice A, the rental charge is \$10 for the first day, \$20 for the second day, \$40 for the third, and so on. The change varies; it is not a constant amount. In choice B, Alexi's stamp collection increases by 2 stamps each time he goes to the post office. The change is a constant amount. The correct answer choice is B.

**19. a.**



the exponential function $y = 2^x$

**b.** For the linear function $y = 3x + 1$:

| **x** | −1 | 0 | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|---|---|
| **y** | −2 | 1 | 7 | 13 | 19 | 25 | 31 |

For the quadratic function $y = 3x^2 + 1$:

| **x** | −1 | 0 | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|---|---|
| **y** | 4 | 1 | 13 | 49 | 109 | 193 | 301 |

For the exponential function $y = 3^x + 1$:

| **x** | −1 | 0 | 2 | 4 | 6 | 8 |
|---|---|---|---|---|---|---|
| **y** | 1.3 | 2 | 10 | 82 | 730 | 6562 |

the exponential function $y = 3^x + 1$

**c.** The exponential growth function; because the $y$-value increases exponentially as $x$ increases, an exponential growth function will eventually exceed both a linear and a quadratic function.

**20.** *Sample answer:* In the case of the linear function $y = ax + b$, successive range values increase/decrease by $a$ units each time. In the case of the exponential function $y = a(b)^x$, the range values increase/decrease by a factor of $b$ units each time. For the family of linear functions, successive range values increase/decrease by the value of the slope of the linear function each time. For the family of exponential functions, successive range values increase/decrease by a factor of the growth/decay factor of the exponential function each time.

### *Quiz for the lessons "Solve Systems with Quadratic Equations," "Compare Linear, Exponential, and Quadratic Models," and "Modeling Relationships"*

**1.** $1 \div 5 = \frac{1}{5}; \frac{1}{5} \div 1 = \frac{1}{5}; \frac{1}{25} \div \frac{1}{5} = \frac{1}{5}; \frac{1}{125} \div \frac{1}{25} = \frac{1}{5};$

The ratios of the $y$-values is constant so the table of values represents an exponential function.

$y = ab^x$

$5 = a\left(\frac{1}{5}\right)^1$

$\frac{5}{\frac{1}{5}} = a$

$25 = a$

The equation is $y = 25\left(\frac{1}{5}\right)^x$.

**2.** Substitute $x - 5$ for $y$ in $y = x^2 - 4x - 11$ and solve for $x$.

$y = x^2 - 4x - 11$

$x - 5 = x^2 - 4x - 11$

$0 = x^2 - 5x - 6$

$0 = (x - 6)(x + 1)$

$0 = x - 6$ or $0 = x + 1$

$x = 6$ or $x = -1$

Substitute both 6 and $-1$ for $x$ in $y = x - 5$.

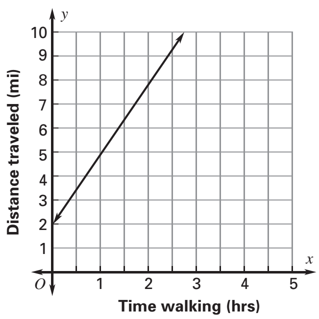
$y = x - 5 \qquad y = x - 5$

$y = 6 - 5 \qquad y = -1 - 5$

$y = 1 \qquad y = -6$

The solutions are $(6, 1)$ and $(-1, -6)$.

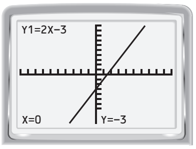
**3.**




The $y$-intercept, 2, indicates the distance traveled before you begin walking which is 2 miles. There is no $x$-intercept.

### *Graphing Calculator Activity for the lesson "Compare Linear, Exponential, and Quadratic Models"*

**1. Step 1**

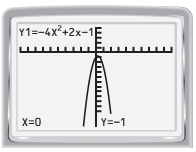



**Step 2**: *Sample answer:*

| Points | Average Rate of Change | Absolute Value of the Average Rate of Change |
|---|---|---|
| (0, −3), (2.34, 1.68) | 2 | 2 |
| (−1.70, −6.40), (2.98, 2.96) | 2 | 2 |

For $y = 2x - 3$, the average rate of change is the constant 2.

**2.** $y =$ Step 1

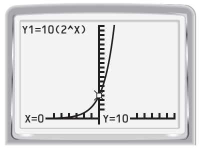



Step 2: *Sample answer:*

| Points | Average Rate of Change | Absolute Value of the Average Rate of Change |
|---|---|---|
| (0, −1), (−1.06, −7.65) | 6.27 | 6.27 |
| (0.85, −2.20), (1.49, −6.89) | −7.33 | 7.33 |

For $y = -4x^2 + 2x - 1$, the average rate of change is positive but decreasing as $x$ increases for $x < 0$ and negative and increasing as $x$ increases for $x > 0$.

**3. Step 1**

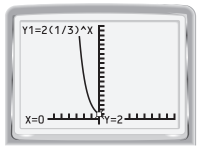



**Step 2**: *Sample answer:*

| Points | Average Rate of Change | Absolute Value of the Average Rate of Change |
|---|---|---|
| (0, 10), (1.28, 4.13) | 4.86 | 4.86 |
| (0.85, 18.04), (1.70, 32.54) | 17.06 | 17.06 |

Depending on the interval, the average rate of change can be very large or very small, but it is always positive.

**4.Step 1**

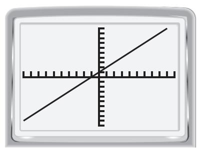



**Step 2**: *Sample answer:*

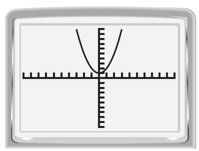
| Points | Average Rate of Change | Absolute Value of the Average Rate of Change |
|---|---|---|
| (0, 2), (1.06, 0.62) | −1.30 | 1.30 |
| (−1.06, 6.44), (−1.91, 16.39) | −11.71 | 11.71 |

For $y = y = 2\left(\frac{1}{3}\right)^x$, the average rate of change is always negative and becomes smaller as $x$ increases.

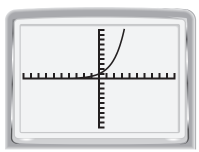
**5.** $y = x + 1$



$y = x^2 + 1$



$y = 2^x$

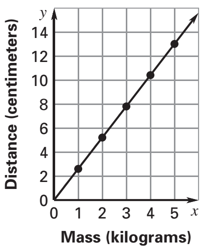


| Function | $y = x + 1$ | $y = x^2 + 1$ | $y = 2^x$ |
|---|---|---|---|
| $\frac{f(10) - f(0)}{10 - 0}$ | 1 | 10 | 102.3 |
| $\frac{f(100) - f(10)}{100 - 10}$ | 1 | 110 | $1.4 \times 10^{28}$ |
| $\frac{f(1000) - f(100)}{1000 - 100}$ | 1 | 1100 | very large |
| $\frac{f(10{,}000) - f(1000)}{10{,}000 - 1000}$ | 1 | 11,000 | extremely large |

*Sample answer:* As $x$ becomes greater, the average rate of change of the exponential function increases much more rapidly than the average rate of change of the quadratic function. The linear function maintains a constant rate of change.

### *Mixed Review of Problem Solving for the lessons "Solve Quadratic Equations by Completing the Square", "Solve Quadratic Equations by the Quadratic Formula", "Interpret the Discriminant", and "Compare Linear, Exponential, and Quadratic Models"*

**1. a.**




The graph appears to be modeled by a linear function.

| Mass (kilograms) | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Distance (centimeters) | 2.6 | 5.2 | 7.8 | 10.4 | 13.0 |

Differences: 2.6 2.6 2.6 2.6

The data represent a linear function because the differences are equal.

**b.** Linear function: $y = ax + 6$

For every 1 kg increase in mass, the distance increases at a rate of 2.6 centimeters, so $a = 2.6$.

Use (1, 2.6): $2.6 = 2.6(1) + b$

$0 = b$

The equation is $y = 2.6x$.

**2. a.** Vertical motion model:

$h = -16t^2 + vt + s$

When $v = 35$ and $s = 2$: $h = -16t^2 + 35t + 2$

The equation is $h = -16t^2 + 35t + 2$.

**b.** When $h = 2.5$:

$2.5 = -16t^2 + 35t + 2$

$0 = -16t^2 + 35t - 0.5$

$$t = \frac{-35 \pm \sqrt{35^2 - 4(-16)(-0.5)}}{2(-16)} = \frac{-35 \pm \sqrt{1193}}{-32}$$

The solutions are $\frac{-35 - \sqrt{1193}}{-32} \approx 2.2$ or $\frac{-35 + \sqrt{1193}}{-32} \approx 0.01$. Because the ball is pitched underhand, the batter hits the ball the second time the ball is 2.5 feet above the ground. So, it takes 2.2 seconds for the ball to be hit.

**3.** Vertical motion model: $h = -16t^2 + vt + s$

When $h = 20$, $v = 30$, and $s = 4.5$:

$20 = -16t^2 + 30t + 4.5$

$0 = -16t^2 + 30t - 15.5.$

$$x = \frac{-30 \pm \sqrt{30^2 - 4(-16)(-15.5)}}{2(-16)}$$

$$= \frac{-30 \pm \sqrt{-92}}{32}$$

The value of the number under the square root is negative, so the equation has no real solution. The flyer's center of gravity never reaches a height of 20 feet.

**4.** *Sample answer:* Let $v = 40$

When $h = 25$, $v = 40$, and $s = 4.5$:

$25 = -16t^2 + 40t + 4.5$

$0 = -16t^2 + 40t - 20.5$

$$x = \frac{-40 \pm \sqrt{40^2 - 4(-16)(-20.5)}}{2(-16)}$$

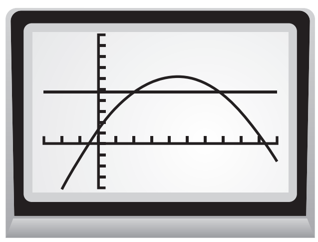
$$= \frac{-30 \pm \sqrt{288}}{-32}$$

$$x = \frac{30 - \sqrt{288}}{32} \approx 0.407 \text{ or } x = \frac{30 + \sqrt{288}}{32} \approx 1.468$$

The equation has two solutions. Using an initial velocity of 40 feet per second, the flyer's center of gravity will reach a height of 25 feet at two points during the stunt.

**5. a.** $y = -0.05x^2 + 2.2x + 7$

$y = 24$



The system of equations has two intersection points, so there are two values that correspond to $y = 24$.

**b.** From part ($a$), when $y = 24$:

$0 = -0.05x^2 + 2.2x - 17$

$$x = \frac{-2.2 \pm \sqrt{2.2^2 - 4(-0.05)(-17)}}{2(-0.05)} = \frac{-2.2 \pm \sqrt{1.44}}{-0.1}$$

The solutions are $\frac{-2.2 - \sqrt{1.44}}{-0.1} = 34$ and $\frac{-2.2 + \sqrt{1.44}}{-0.1} = 10$. The solution $x = 34$, or in 2024, is outside the range of 1990 – 2000, so it can be rejected. Sales reaches \$24 billion 10 years after 1990, or in 2000.

**6.** Area $= \frac{1}{2}h(b_1 + b_2)$

When area $= 54$, $h = x + 1$, $b_1 = x + 3$, and $b_2 = 2x$:

$$54 = \frac{1}{2}(x + 1)[(x + 3) + 2x]$$
$$108 = (x + 1)(3x + 3)$$
$$108 = 3x^2 + 6x + 3$$
$$36 = x^2 + 2x + 1$$
$$0 = x^2 + 2x - 35$$
$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(-35)}}{2(1)} = \frac{-2 \pm \sqrt{144}}{2} = -1 \pm 6$$

The solutions are $-1 + 6 = 5$ and $-1 - 6 = -7$. Distance cannot be negative, so you can reject $x = -7$. The value of $x$ is 5.

**7.** Let $\ell =$ the length of the pen and $w =$ the width of the pen.

Perimeter $= 2\ell + 2w$ and Area $= \ell \cdot w$

When perimeter $= 24$ and Area $= 150$:

$$24 = 2\ell + 2w \rightarrow \ell = 12 - w$$
$$150 = \ell \cdot w$$
$$150 = (12 - w)w$$
$$150 = 12w - w^2$$
$$0 = w^2 - 12w + 150$$

Value of discriminant:

$$b^2 - 4ac = (-12)^2 - 4(1)(150) = -456$$

The value of the discriminant is negative, so the equation had no solution. It is not possible for the 24 feet of fencing to enclose a rectangular area of 150 square feet.

**8. a.**

Area of border (in.$^2$) = Total Area (in.$^2$) − Area of tiled table top (in.$^2$)

$$A = (16 + 2x)(12 + 2x) - (16)(12)$$
$$= 192 + 56x + 4x^2 - 192$$
$$= 4x^2 + 56x$$

So, the equation is $A = 4x^2 + 56x$

**b.** When $A = 130$:

$$130 = 4x^2 + 56x$$
$$32.5 = x^2 + 14x$$
$$32.5 + 7^2 = x^2 + 14x + 7^2$$
$$81.5 = (x + 7)^2$$
$$\pm\sqrt{81.5} = x + 7$$
$$-7 \pm \sqrt{81.5} = x$$

The solutions are $-7 - \sqrt{81.5} \approx -16$ and $-7 + \sqrt{81.5} \approx 2$. The border should be about 2 inches wide.

**c.** A distance cannot be negative, so you can ignore the solution $x = -16$.

### *Chapter Review for the chapter "Quadratic Equations and Functions"*

**1.** The line that passes through the vertex and divides a parabola into two symmetric parts is called the *axis of symmetry*.

**2.** $f(x) = 5x^2 - 4x$

The leading coefficient is positive so the function has a minimum value.

**3.** $f(x) = -x^2 + 6x + 2$

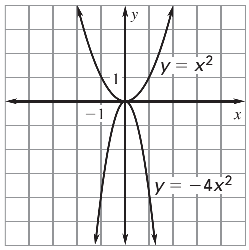
The leading coefficient is negative so the function has a maximum value.

**4.** $f(x) = 0.3x^2 - 7.7x + 1.8$

The leading coefficient is positive so the function has a minimum value.

**5.**

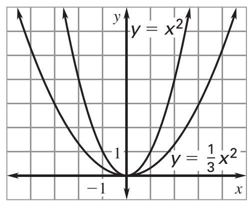
| x | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| y | −16 | −4 | 0 | −4 | −16 |




Both graphs have the same axis of symmetry, $x = 0$ and the same vertex, (0, 0). The graph of $y = -4x^2$ is a vertical stretch and a reflection in the $x$-axis of the graph of $y = x^2$.

**6.**

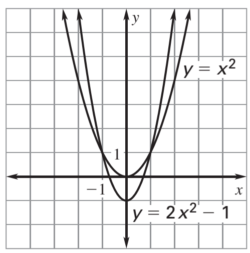
| x | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| y | $\frac{4}{3}$ | $\frac{1}{3}$ | 0 | $\frac{1}{3}$ | $\frac{4}{3}$ |




Both graphs have the same axis of symmetry, $x = 0$ and the same vertex, (0, 0). The graph of $y = \frac{1}{3}x^2$ is a vertical shrink of the graph of $y = x^2$.

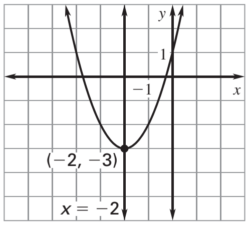
**7.**

| x | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| y | 7 | 1 | −1 | 1 | 7 |




Both graphs have the same axis of symmetry, $x = 0$. The graph of $y = 2x^2 - 1$ has a different vertex than the graph of $y = x^2$ because it is a vertical translation of 1 unit down of $y = x^2$. The graph of $y = 2x^2 - 1$ is also a vertical stretch of the graph of $y = x^2$.

**8.** $y = x^2 + 4x + 1$

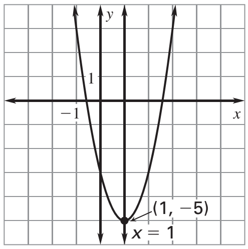



Because $a > 0$, the parabola opens up.

Axis of symmetry: $x = \frac{-b}{2a} = \frac{-4}{2(1)} = -2$.

The $x$-component of the vertex is $-2$. The $y$-component of the vertex is $y = (-2)^2 + 4(-2) + 1 = -3$.

**9.** $y = 2x^2 - 4x - 3$

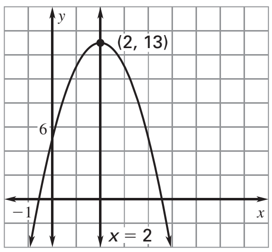



Because $a > 0$, the parabola opens up.

Axis of symmetry: $x = \frac{-b}{2a} = \frac{-(-4)}{2(2)} = 1$.

The $x$-coordinate of the vertex is $\frac{-b}{2a}$ or 1. The $y$-coordinate of the vertex is $y = 2(1)^2 + 4(1) - 3 = -5$.

**10.** $y = 2x^2 + 8x + 5$




Because $a < 0$, the parabola opens down.

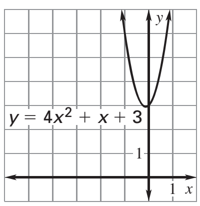
Axis of symmetry: $x = \frac{-b}{2a} = \frac{-8}{2(-2)} = 2$.

The $x$-coordinate of the vertex is $\frac{-b}{2a}$ or 2. The $y$-coordinate of the vertex is $y = -2(2)^2 + 8(2) + 5 = 13$.

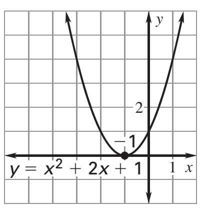
**11.** $4x^2 + x + 3 = 0$

The graph has no $x$-intercepts.

This means there are no solutions.




**12.** $x^2 + 2x = -1$




$x^2 + 2x + 1 = 0$

The $x$-intercept of the graph is $-1$.

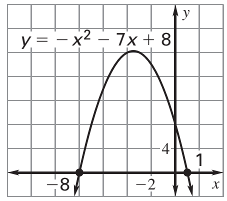
The solution is $-1$.

**13.** $-x^2 + 8 = 7x$

$-x^2 - 7x + 8 = 0$

The $x$-intercepts of the graph are $-8$ and 1.

The solutions are $-8$ and 1.




**14.** $6x^2 - 54 = 0$

$6x^2 = 54$

$x^2 = 9$

$x = \pm 3$

The solutions are $-3$ and 3.

**15.** $3x^2 + 7 = 4$

$3x^2 = -3$

$x^2 = -1$

Negative real numbers do not have real square roots. So, there is no solution.

**16.** $g^2 + 11 = 24$

$g^2 = 13$

$g = \pm\sqrt{13}$

The solutions are $\sqrt{13} \approx 3.61$ and $-\sqrt{13} \approx -3.61$.

**17.** $7n^2 + 5 = 9$

$7n^2 = 4$

$n^2 = \frac{4}{7}$

$n = \pm\sqrt{\frac{4}{7}}$

The solutions are $\sqrt{\frac{4}{7}} \approx 0.76$ and $-\sqrt{\frac{4}{7}} \approx -0.76$.

**18.** $2(a + 7)^2 = 34$

$(a + 7)^2 = 17$

$a + 7 = \pm\sqrt{17}$

$a = -7 \pm\sqrt{17}$

The solutions are $-7 + \sqrt{17} \approx -2.88$ and $-7 - \sqrt{17} \approx -11.12$.

**19.** $3(w - 4)^2 = 5$

$(w - 4)^2 = \frac{5}{3}$

$w - 4 = \pm\sqrt{\frac{5}{3}}$

$w = 4 \pm\sqrt{\frac{5}{3}}$

The solutions are $4 + \sqrt{\frac{5}{3}} \approx 5.29$ and $4 - \sqrt{\frac{5}{3}} \approx 2.71$.

**20.** $x^2 - 14x = 51$

$x^2 - 14x + 7^2 = 51 + 7^2$

$(x - 7)^2 = 100$

$x - 7 = \pm\sqrt{100}$

$x = 7 \pm 10$

The solutions are $7 + 10 = 17$ and $7 - 10 = -3$.

**21.** $2a^2 + 12a - 4 = 0$

$a^2 + 6a - 2 = 0$

$a^2 + 6a = 2$

$a^2 + 6a + 3^2 = 2 + 3^2$

$(a + 3)^2 = 11$

$a + 3 = \pm\sqrt{11}$

$a = -3 \pm \sqrt{11}$

The solutions are $-3 + \sqrt{11} \approx 0.32$ and $-3 - \sqrt{11} \approx -6.32$.

**22.** $2n^2 + 4n + 1 = 10n + 9$

$2n^2 - 6n = 8$

$n^2 - 3n = 4$

$n^2 - 3n + \left(-\frac{3}{2}\right)^2 = 4 + \left(-\frac{3}{2}\right)^2$

$\left(n - \frac{3}{2}\right)^2 = \frac{25}{4}$

$n - \frac{3}{2} = \pm\sqrt{\frac{25}{4}}$

$n = \frac{3}{2} \pm \frac{5}{2}$

The solutions are $\frac{3}{2} + \frac{5}{2} = 4$ and $\frac{3}{2} - \frac{5}{2} = -1$.

**23.** $5g^2 - 3g + 6 = 2g^2 + 9$

$3g^2 - 3g = 3$

$g^2 - g = 1$

$g^2 - g + \left(-\frac{1}{2}\right)^2 = 1 + \left(-\frac{1}{2}\right)^2$

$\left(g - \frac{1}{2}\right)^2 = \frac{5}{4}$

$g - \frac{1}{2} = \pm\sqrt{\frac{5}{4}}$

$g = \frac{1}{2} \pm \sqrt{\frac{5}{4}}$

The solutions are $\frac{1}{2} + \sqrt{\frac{5}{4}} \approx 1.62$ and $\frac{1}{2} - \sqrt{\frac{5}{4}} \approx -0.62$.

**24.** $x^2 - 2x - 15 = 0$

$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-15)}}{2(1)} = \frac{2 \pm \sqrt{64}}{2}$

$= \frac{2 \pm 8}{2}$

The solutions are $\frac{2 + 8}{2} = 5$ and $\frac{2 - 8}{2} = -3$.

**25.** $2m^2 + 7m - 3 = 0$

$m = \frac{-(7) \pm \sqrt{7^2 - 4(2)(-3)}}{2(2)} = \frac{-7 \pm \sqrt{73}}{4}$

The solutions are $\frac{-7 \pm \sqrt{73}}{4} \approx 0.39$ and $\frac{-7 - \sqrt{73}}{4} \approx -3.89$.

**26.** $-w^2 + 5w = 3$

$-w^2 + 5w - 3 = 0$

$w = \frac{-(5) \pm \sqrt{(5)^2 - 4(-1)(-3)}}{2(-1)} = \frac{-5 \pm \sqrt{13}}{-2}$

The solutions are $\frac{-5 \pm \sqrt{13}}{-2} \approx 0.70$ and $\frac{-5 - \sqrt{13}}{-2} \approx 4.30$.

**27.** $5n^2 - 7n = -1$

$5n^2 - 7n + 1 = 0$

$n = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(5)(1)}}{2(5)} = \frac{7 \pm \sqrt{29}}{10}$

The solutions are $\frac{7 + \sqrt{29}}{10} \approx 1.24$ and $\frac{7 - \sqrt{29}}{10} \approx 0.16$.

**28.** $t^2 - 4t = 6t + 8$

$t^2 - 6t - 12 = 0$

$t = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-12)}}{2(1)} = \frac{6 \pm \sqrt{84}}{2}$

The solutions are $\frac{6 + \sqrt{84}}{2} \approx 7.58$ and $\frac{6 - \sqrt{84}}{2} \approx -1.58$.

**29.** $2h^2 - 1 = 10 - 9h^2$

$9h^2 + 2h - 11 = 0$

$h = \frac{-(2) \pm \sqrt{(2)^2 - 4(9)(-11)}}{2(9)} = \frac{-2 \pm \sqrt{400}}{18}$

$= \frac{-2 \pm 20}{18}$

The solutions are $\frac{-2 + 20}{18} = 1$ and $\frac{-2 - 20}{18} \approx -1.22$.

**30.** Substitute $x + 8$ for $y$ in $y = x^2 + 2x + 2$ and solve for $x$.

$y = x^2 + 2x + 2$

$x + 8 = x^2 + 2x + 2$

$0 = x^2 + x - 6$

$0 = (x - 2)(x + 3)$

$0 = x - 2$ or $0 = x + 3$

$x = 2$ or $x = -3$

Substitute both 2 and $-3$ for $x$ in $y = x + 8$.

$y = x + 8$ $\quad$ $y = x + 8$

$y = 2 + 8$ $\quad$ $y = -3 + 8$

$y = 10$ $\quad$ $y = 5$

The solutions are (2, 10) and (−3, 5).

**31.** Substitute $-x$ for $y$ in $y = 2x^2 - 3x - 4$ and solve for $x$.

$y = 2x^2 - 3x - 4$

$-x = 2x^2 - 3x - 4$

$0 = 2x^2 - 2x - 4$

$0 = (2x + 2)(x - 2)$

$0 = 2x + 2$ or $0 = x - 2$

$x = -1$ or $x = 2$

Substitute both $-1$ and 2 for $x$ in $x + y = 0$.

$x + y = 0$ $\quad$ $x + y = 0$

$-1 + y = 0$ $\quad$ $2 + y = 0$

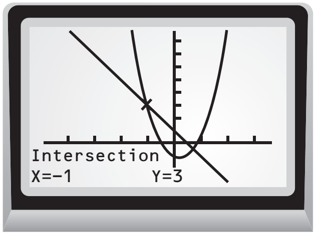
$y = 1$ $\quad$ $y = -2$

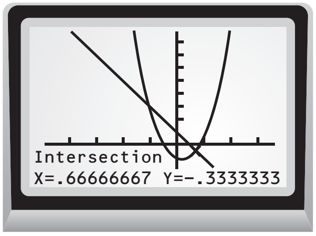
The solutions are (−1, 1) and (2, −2).

**32.** $2x + y = 1$

$y = -2x + 1$

Graph $y = -2x + 1$ and $y = 3x^2 - x - 1$ and find the intersection points.



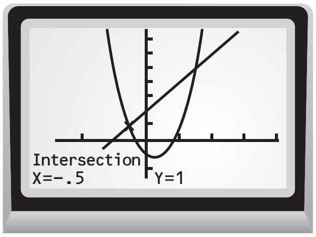



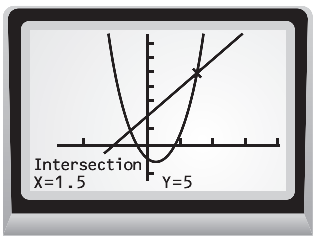

The solutions are $(-1, 3)$ and $\left[\frac{2}{3}, -\frac{1}{3}\right]$.

**33.** $y = 4x^2 - 2x - 1$

$y = 2x + 2$

Graph $y = 2x + 2$ and $y = 4x^2 - 2x - 1$ and find the intersection points.




The solutions are $(-0.5, 1)$ and $(1.5, 5)$.

**34.**

| **x** | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| **y** | 1 | 2 | 4 | 8 | 16 | 32 |

Ratios: $\frac{2}{1} = 2$   2   2   2   2

The table of values represents a exponential function because the ratios are equal.

**35.**

| **x** | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|
| **y** | 0 | 3 | 6 | 9 | 12 | 15 |

Differences: 3   3   3   3   3

The table of values represents a linear function because the differences are equal.

**36.** The points $(4, 0)$ and $(0, 3)$ are on the graph of Linear Function 2, so its slope is $\frac{3-0}{0-4} = -\frac{3}{4}$. Linear Function 1 is decreasing more rapidly.

**37.** The ratios of successive $y$-values would be equal because the rate of increase stays the same. So an exponential function would be a good model.

### *Chapter Test for the chapter "Quadratic Equations and Functions"*

**1.** C; The graph of $y = x^2 - 2$ is a vertical translation (of 2 units down) of the graph of $y = x^2$.

**2.** A; The graph of $y = x^2 + 2$ is a vertical translation (of 2 units up) of the graph of $y = x^2$.

**3.** B; The graph of $y = -2x^2$ is a vertical stretch (by a factor of 2) and a reflection in the $x$-axis of the graph of $y = x^2$.

**4.** $y = 2x^2 + 6x - 5$

Because $a > 0$, the graph opens up.

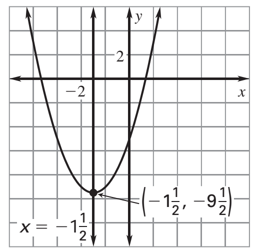
Axis of symmetry:

$$x = -\frac{b}{2a} = -\frac{6}{2(2)} = -\frac{3}{2}$$

When $x = -\frac{3}{2}$:

$$y = 2\left(-\frac{3}{2}\right)^2 + 6\left(-\frac{3}{2}\right) - 5 = -\frac{19}{2}$$

Vertex: $\left(-\frac{3}{2}, -\frac{19}{2}\right)$.




**5.** $y = -4x^2 - 8x + 25$

Because $a < 0$, the graph opens down.

Axis of symmetry:

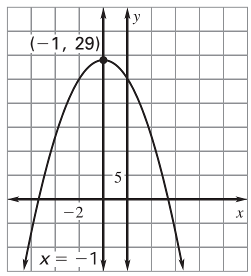
$$x = -\frac{b}{2a} = -\frac{8}{2(-4)} = -1$$

When $x = -1$:

$y = -4(-1)^2 - 8(-1) + 25$

$= 29$

Vertex: $(-1, 29)$.




**6.** $y = -\frac{1}{4}x^2 - x - 7$

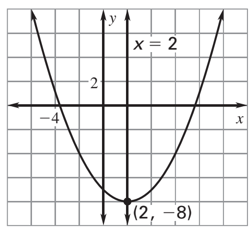
Because $a > 0$, the graph opens up.

Axis of symmetry:

$$x = -\frac{b}{2a} = -\frac{-1}{2\left(\frac{1}{4}\right)} = 2$$

When $x = 2$: $y = \frac{1}{4}(2)^2 - (2) - 7 = -8$

Vertex: $(2, -8)$.




**7.** $f(x) = x^2 + 5x + 1$

$0 = x^2 + 5x + 1$

$$x = \frac{-(5) \pm \sqrt{5^2 - 4(1)(1)}}{2(1)} = \frac{-5 \pm \sqrt{21}}{2}$$

The zeros are $\frac{-5 + \sqrt{21}}{2} \approx -0.2$ and $\frac{-5 - \sqrt{21}}{2} \approx -4.8$.

**8.** $f(x) = x^2 - 8x + 3$

$0 = x^2 - 8x + 3$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(3)}}{2(1)} = \frac{8 \pm \sqrt{52}}{2}$$

The zeros are $\frac{8 + \sqrt{52}}{2} \approx 7.6$ and $\frac{8 - \sqrt{52}}{2} \approx 0.4$.

**9.** $f(x) = -3x^2 - 2x + 5$

$0 = -3x^2 - 2x + 5$

$0 = 3x^2 + 2x - 5$

$0 = (3x + 5)(x - 1)$

$3x + 5 = 0$ or $x - 1 = 0$

$x = -\frac{5}{3}$ or $x = 1$

The zeros are $-\frac{5}{3} \approx -1.7$ and 1.

**10.** $3x^2 = 108$

$x^2 = 36$

$x = \pm\sqrt{36}$

$x = \pm 6$

The solutions of the equation are $-6$ and 6.

**11.** $-5w^2 + 51 = 6$

$-5w^2 = -45$

$w^2 = 9$

$w = \pm\sqrt{9}$

$w = \pm 3$

The solutions of the equation are $-3$ and 3.

**12.** $-p^2 + 2p + 3 = 0$

$p^2 - 2p - 3 = 0$

$(p - 3)(p + 1) = 0$

$p - 3 = 0$ or $p + 1 = 0$

$p = 3$ or $p = -1$

The solutions of the equation are $-1$ and 3.

**13.** $-2t^2 + 6t + 9 = 0$

$$t = \frac{-(6) \pm \sqrt{6^2 - 4(-2)(9)}}{2(-2)} = \frac{-6 \pm \sqrt{108}}{-4}$$

$$= \frac{-3 \pm 3\sqrt{3}}{2}$$

The solutions of the equation are $\frac{-3 + 3\sqrt{3}}{2} \approx -1.10$ and $\frac{-3 - 3\sqrt{3}}{2} \approx 4.10$.

**14.** $5m^2 - m = 5$

$5m^2 - m - 5 = 0$

$$m = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(5)(-5)}}{2(5)}$$

$$= \frac{1 \pm \sqrt{101}}{10}$$

The solutions of the equation are $\frac{1 + \sqrt{101}}{10} \approx 1.10$ and $\frac{1 - \sqrt{101}}{10} \approx -0.90$.

**15.** $2x^2 - 12x - 1 = -7x + 6$

$2x^2 - 5x - 7 = 0$

$(2x - 7)(x + 1) = 0$

$2x - 7 = 0$ or $x + 1 = 0$

$x = 3.5$ or $x = -1$

The solutions of the equation are 3.5 and $-1$.

**16.** Substitute $3x - 2$ for $y$ in $y = -x^2 + 2x + 4$ and solve for $x$.

$y = -x^2 + 2x + 4$

$3x - 2 = -x^2 + 2x + 4$

$0 = -x^2 - x + 6$

$0 = x^2 + x - 6$

$0 = (x - 2)(x + 3)$

$0 = x - 2$ or $0 = x + 3$

$x = 2$ or $x = -3$

Substitute both 2 and $-3$ for $x$ in $y = 3x - 2$.

$y = 3x - 2$ $\quad$ $y = 3x - 2$

$y = 3(2) - 2$ $\quad$ $y = 3(-3) - 2$

$y = 4$ $\quad$ $y = -11$

The solutions are (2, 4) and $(-3, -11)$.

**17.** $x + y = -3$

$y = -x - 3$

Substitute $-x - 3$ for $y$ in $y = 2x^2 + x - 7$ and solve for $x$.

$y = 2x^2 + x - 7$

$-x - 3 = 2x^2 + x - 7$

$0 = 2x^2 + 2x - 4$

$0 = (2x - 2)(x + 2)$

$0 = 2x - 2$ or $0 = x + 2$

$x = 1$ or $x = -2$

Substitute both 1 and $-2$ for $x$ in $y = -x - 3$.

$y = -x - 3$ $\quad$ $y = -x - 3$

$y = -1 - 3$ $\quad$ $y = -(-2) - 3$

$y = -4$ $\quad$ $y = -1$

The solutions are $(1, -4)$ and $(-2, -1)$.

**18.** $2x - y = 2$

$-y = -2x + 2$

$y = 2x - 2$

Substitute $2x - 2$ for $y$ in $y = -2x^2 - 4x + 6$ and solve for $x$.

$y = -2x^2 - 4x + 6$

$2x - 2 = -2x^2 - 4x + 6$

$0 = -2x^2 - 6x + 8$

$0 = 2x^2 + 6x - 8$

$0 = (2x - 2)(x + 4)$

$0 = 2x - 2$ or $0 = x + 4$

$x = 1$ or $x = -4$

Substitute both 1 and $-4$ for $x$ in $y = 2x - 2$.

$y = 2x - 2$ $\quad$ $y = 2x - 2$

$y = 2(1) - 2$ $\quad$ $y = 2(-4) - 2$

$y = 0$ $\quad$ $y = -10$

The solutions are (1, 0) and $(-4, -10)$.

**19.**

| x | −3 | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|---|
| y | 18 | 8 | 2 | 0 | 2 | 8 |

First differences: −10 −6 −2 +2 +6

Second differences: +4 +4 +4 +4

The second differences are constant, so the table of values represents a quadratic function of the form $y = ax^2$.

Using points(1, 2): $y = ax^2$

$2 = a(1)^2$

$2 = a$

So, an equation for the function is $y = 2x^2$.

**20.**

| x | −4 | 0 | 4 | 8 | 12 | 16 |
|---|---|---|---|---|---|---|
| y | 1 | 2 | 3 | 4 | 5 | 6 |

First differences: 1 1 1 1 1

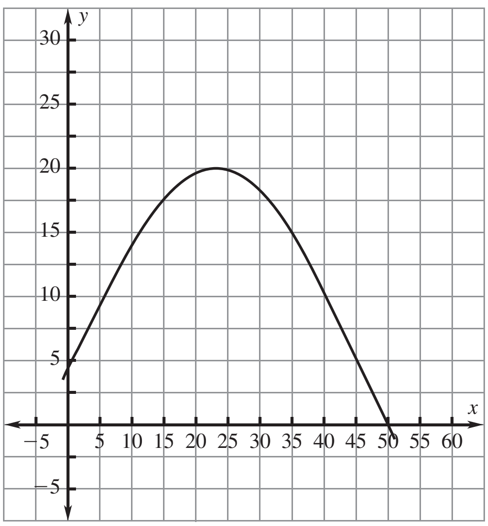
The first differences are constant, so the table of values represents a linear function. The equation has the form $y = mx + b$. Because the point (0, 2) is on the line, the $y$-intercept, $b$, is 2.

Using points (0, 2) and (4, 3):

$$m = \frac{3 - 2}{4 - 0} = \frac{1}{4}$$

So, an equation for the function is $y = \frac{1}{4}x + 2$.

**21. a.** The ball follows the path of a parabola, so a quadratic function should be used. Let $x$ represent the horizontal distance in feet and let $y$ represent the vertical distance in feet.



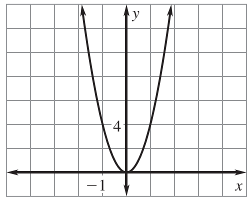

**b.** Because the ball is hit from above the ground, the graph has only one $x$-intercept where the ball hits the ground. The $y$-intercept represents the height from which the ball was hit. The maximum point is where the ball reaches its maximum height.



## Extra Practice for the chapter "Quadratic Equations and Functions"

**1.** $y = 4x^2$

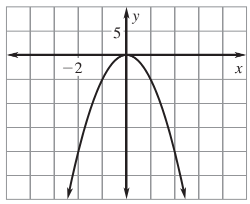
| x | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| y | 16 | 4 | 0 | 4 | 16 |




Both graphs open up and have the same vertex (0, 0) and the same axis of symmetry, $x = 0$. The graph of $y = 4x^2$ is narrower than the graph $y = x^2$ because the graph of $y = 4x^2$ is a vertical stretch by a factor of 4 of the graph of $y = x^2$.

**2.** $y = -5x^2$

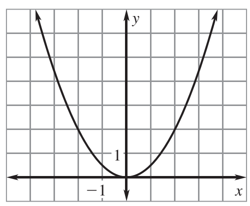
| x | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| y | −20 | −5 | 0 | −5 | −20 |




Both graphs have the same vertex (0, 0), and the same axis of symmetry, $x = 0$. However, the graph of $y = -5x^2$ is narrower than the graph of $y = x^2$ and it opens down. This is because the graph of $y = -5x^2$ is a vertical stretch by a factor of 5 with a reflection in the $x$-axis of the graph of $y = x^2$.

**3.** $y = \frac{1}{2}x^2$

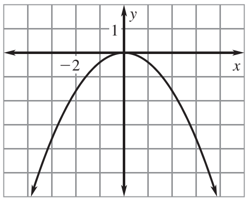
| x | −4 | −2 | 0 | 2 | 4 |
|---|---|---|---|---|---|
| y | 8 | 2 | 0 | 2 | 8 |




Both graphs open up and have the same vertex, (0, 0) and the same axis of symmetry $x = 0$. The graph of $y = \frac{1}{2}x^2$ is wider than the graph of $y = x^2$ because it is a vertical stretch by a factor of $\frac{1}{2}$ of the graph of $y = x^2$.

**4.** $y = -\frac{2}{5}x^2$

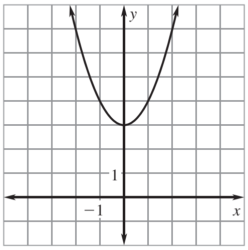
| x | −3 | −1 | 0 | 1 | 3 |
|---|---|---|---|---|---|
| y | $-\frac{18}{5}$ | $-\frac{2}{5}$ | 0 | $-\frac{2}{5}$ | $-\frac{18}{5}$ |




Both graphs have the same vertex (0, 0), and the same axis of symmetry, $x = 0$. The graph of $y = -\frac{2}{5}x^2$ opens down. Also, the graph of $y = -\frac{2}{3}x^2$ is a reflection in the $x$-axis of the graph of $y = x^2$ and it is wider because it is a vertical shrink by a factor of $\frac{2}{5}$ of the graph of $y = x^2$.

**5.** $y = x^2 + 3$

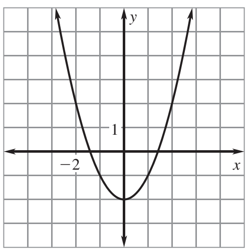
| **x** | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| **y** | 7 | 4 | 3 | 4 | 7 |




Both graphs open up and have the same axis of symmetry, $x = 0$. However, the vertex of the graph of $y = x^2$, (0, 3), is different than the vertex of the graph of $y = x^2$, (0, 0), because the graph of $y = x^2 + 3$ is a vertical translation 3 units up of the graph of $y = x^2$.

**6.** $y = x^2 - 2$

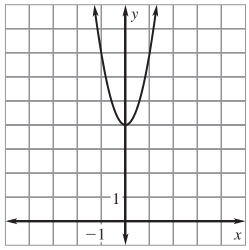
| **x** | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| **y** | 2 | −1 | −2 | −1 | 2 |




Both graphs open up and have the same axis of symmetry, $x = 0$. However, the vertex of the graph of $y = x^2 - 2$, (0, −2) is different than the vertex of the graph of $y = x^2$, (0, 0), because the graph of $y = x^2 - 2$ is a vertical translation 2 units down of the graph of $y = x^2$.

**7.** $y = 3x^2 + 4$

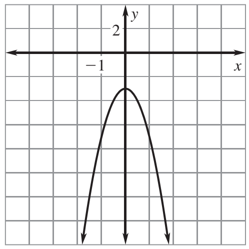
| **x** | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| **y** | 16 | 7 | 4 | 7 | 16 |




Both graphs open up and have the same axis of symmetry, $x = 0$. However the graph of $y = 3x^2 + 4$ is narrower and has a higher vertex than the graph of $y = x^2$ because the graph fo $y = 3x^2 + 4$ is a vertical stretch by a factor of 3 and a vertical translation 4 units up of the graph of $y = x^2$.

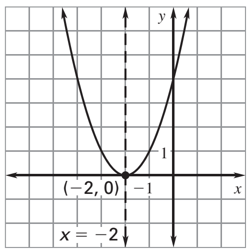
**8.** $y = -4x^2 - 3$

| **x** | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| **y** | −19 | −7 | −3 | −7 | −19 |




The graph of $y = -4x^2 + 3$ is a vertical stretch by a factor of 4 with a vertical translation 3 units down and a reflection in the x-axis of the graph of $y = x^2$.

**9.**




$y = x^2 + 4x + 4$

Because $a > 0$, the parabola opens up.

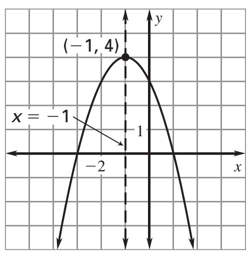
Axis of symmetry: $x = \frac{-b}{2a} = \frac{-4}{2(1)} = -2$

$x$-coordinate of vertex: $-2$

$y$-coordinate of vertex: $y = (-2)^2 + 4(-2) + 4 = 0$

So, the vertex is $(-2, 0)$.

**10.**




$y = -x^2 - 2x + 3$

Because $a < 0$, the parabola opens down.

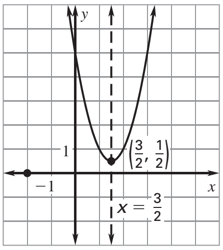
Axis of symmetry: $x = \frac{-b}{2a} = -\frac{-2}{2(-1)} = -1$

$x$-coordinate of vertex: $-1$

$y$-coordinate of vertex: $y = -(-1)^2 - 2(-1) + 3 = 4$

So, the vertex is $(-1, 4)$.

**11.**




$y = 2x^2 - 6x + 5$

Because $a > 0$, the parabola opens up.

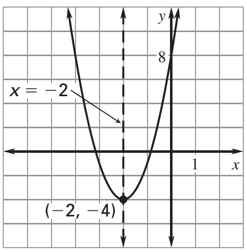
Axis of symmetry: $x = -\frac{b}{2a} = -\frac{-6}{2(2)} = \frac{3}{2}$

$x$-coordinate of vertex: $\frac{3}{2}$

$y$-coordinate of vertex: $y = 2\left(\frac{3}{2}\right)^2 - 6\left(\frac{3}{2}\right) + 5 = \frac{1}{2}$

So, the vertex is $\left(\frac{3}{2}, \frac{1}{2}\right)$.

**12.**




$y = 3x^2 + 12x + 8$

Because $a > 0$, the parabola opens up.

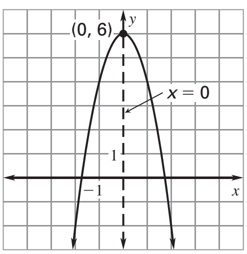
Axis of symmetry: $x = \frac{-b}{2a} = -\frac{12}{2(3)} = -2$

$x$-coordinate of vertex: $-2$

$y$-coordinate of vertex: $y = 3(-2)^2 + 12(-2) + 8 = -4$

So, the vertex is $(-2, -4)$.

**13.**




$y = -2x^2 + 6$

Because $a < 0$, the parabola opens down.

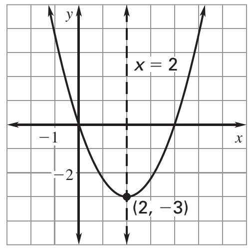
Axis of symmetry: $x = \frac{-b}{2a} = -\frac{0}{2(-2)} = 0$

$x$-coordinate of vertex: 0

$y$-coordinate of vertex: $y = -2(0)^2 + 6 = 6$

So, the vertex is $(0, 6)$.

**14.**




$y = \frac{3}{4}x^2 - 3x$

Because $a > 0$, the parabola opens up.

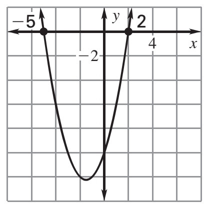
Axis of symmetry: $x = \frac{-b}{2a} = -\frac{-3}{2\left(\frac{3}{4}\right)} = 2$

$x$-coordinate of vertex: 2

$y$-coordinate of vertex: $y = \frac{3}{4}(2)^2 - 3(2) = -3$

So, the vertex is $(2, -3)$.

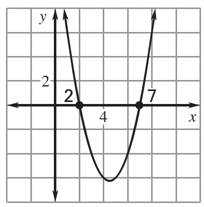
**15.**




$x^2 + 3x - 10 = 0$

The $x$-intercepts of the function $y = x^2 - 3x - 10$ are $-5$ and 2. So, the solutions are $-5$ and 2.

**16.**

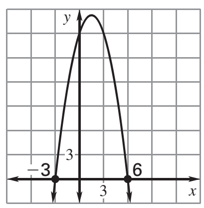



$x^2 + 14 = 9x$

$x^2 - 9x + 14 = 0$

The $x$-intercepts of the function $y = x^2 - 19x + 14$ are 2 and 7. So, the solutions are 2 and 7.

**17.**

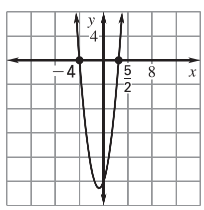



$-x^2 + 3x = -18$

$-x^2 + 3x + 18 = 0$

The $x$-intercepts of the function $y = -x^2 + 3x + 18$ are $-3$ and 6. So, the solutions are $-3$ and 6.

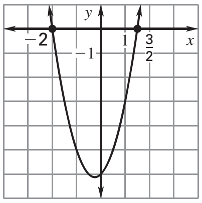
**18.**



$2x^2 + 3x - 20 = 0$

The $x$-intercepts of the function $y = 2x^2 + 3x - 20$ are $-4$ and $\frac{5}{2}$. So, the solutions are $-4$ and $\frac{5}{2}$.

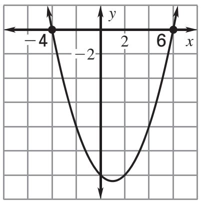
**19.**




$$2x^2 + x = 6$$
$$2x^2 + x - 6 = 0$$

The $x$-intercepts of the function $y = 2x^2 + x - 6$ are $-2$ and $\frac{3}{2}$. So, the solutions are $-2$ and $\frac{3}{2}$.

**20.**




$$\frac{1}{2}x^2 - x = 12$$
$$\frac{1}{2}x^2 - x - 12 = 0$$

The $x$-intercepts of the function $y = \frac{1}{2}x^2 - x - 12$ are $-4$ and 6. So, the solutions are $-4$ and 6.

**21.** $2x^2 - 20 = 78$
$$2x^2 = 98$$
$$x^2 = 49$$
$$x = \pm\sqrt{49} = \pm 7$$

The solutions are $-7$ and 7.

**22.** $3y^2 + 16 = 4$
$$3y^2 = -12$$
$$y^2 = -4$$

Negative real numbers do not have real square roots. So, there is no solution.

**23.** $16y^2 - 6 = 3$
$$16y^2 = 9$$
$$y^2 = \frac{9}{16}$$
$$y = \pm\sqrt{\frac{9}{16}} = \pm\frac{3}{4}$$

The solutions are $-\frac{3}{4}$ and $\frac{3}{4}$.

**24.** $48 - x^2 = -52$
$$-x^2 = -100$$
$$x^2 = 100$$
$$x = \pm\sqrt{100} = \pm 10$$

The solutions are $-10$ and 10.

**25.** $5m^2 - 5 = 10$
$$5m^2 = 15$$
$$m^2 = 3$$
$$m = \pm\sqrt{3} = \pm 1.73$$

The solutions are $-1.73$ and 1.73.

**26.** $2 - 5t^2 = 4$
$$-5t^2 = 2$$
$$t^2 = -\frac{2}{5}$$

Negative real numbers do not have real square roots. So, there is no solution.

**27.** $x^2 + 4x - 21 = 0$
$$x^2 + 4x = 21$$
$$x^2 + 4x + 2^2 = 21 + 2^2$$
$$(x + 2)^2 = 25$$
$$x + 2 = \pm\sqrt{25}$$
$$x = -2 \pm 5$$

The solutions are $-2 - 5 = -7$ and $-2 + 5 = 3$.

**28.** $g^2 - 10g = 24$
$$g^2 - 10g + 5^2 = 24 + 5^2$$
$$(g - 5)^2 = 49$$
$$g - 5 = \pm\sqrt{49}$$
$$g = 5 \pm 7$$

The solutions are $5 - 7 = -2$ and $5 + 7 = 12$.

**29.** $w^2 - 7w + 6 = 0$
$$w^2 - 7w = -6$$
$$w^2 - 7w + \left(\frac{7}{2}\right)^2 = -6 + \left(\frac{7}{2}\right)^2$$
$$\left(w - \frac{7}{2}\right)^2 = \frac{25}{4}$$
$$w - \frac{7}{2} = \pm\sqrt{\frac{25}{4}}$$
$$w = \frac{7}{2} \pm \frac{5}{2}$$

The solutions are $\frac{7}{2} - \frac{5}{2} = 1$ and $\frac{7}{2} + \frac{5}{2} = 6$.

**30.** $y^2 - \frac{3}{4} = \frac{1}{4}$
$$y^2 - \frac{3}{4} + \left(\frac{3}{8}\right)^2 = \frac{1}{4} + \left(\frac{3}{8}\right)^2$$
$$\left(y - \frac{3}{8}\right)^2 = \frac{25}{64}$$
$$y - \frac{3}{8} = \pm\sqrt{\frac{25}{64}}$$
$$y = \frac{3}{8} \pm \frac{5}{8}$$

The solutions are $\frac{3}{8} - \frac{5}{8} = -\frac{1}{4}$ and $\frac{3}{8} + \frac{5}{8} = 1$.

**31.** $x^2 - 6x + 3 = 0$
$$x^2 - 6x = -3$$
$$x^2 - 6x + 3^2 = -3 + 3^2$$
$$(x - 3)^2 = 6$$
$$x - 3 = \pm\sqrt{6}$$
$$x = 3 \pm \sqrt{6}$$

The solutions are $3 - \sqrt{6} \approx 0.55$ and $3 + \sqrt{6} \approx 5.45$.

**32.** $4m^2 + 8m - 7 = 0$

$4m^2 + 8m = 7$

$m^2 + 2m = 1.75$

$m^2 + 2m + 1^2 = 1.75 + 1^2$

$(m + 1)^2 = 2.75$

$m + 1 = \pm\sqrt{2.75}$

$m = -1 \pm \sqrt{2.75}$

The solutions are $-1 - \sqrt{2.75} \approx -2.66$ and $-1 + \sqrt{2.75} \approx 0.66$.

**33.** $h^2 + 6h - 72 = 0$

$$h = \frac{-6 \pm \sqrt{6^2 - 4(1)(-72)}}{2(1)} = \frac{-6 \pm \sqrt{324}}{2} = \frac{-6 \pm 18}{2}$$

The solutions are $\frac{-6 - 18}{2} = -12$ and $\frac{-6 + 18}{2} = 6$.

**34.** $3x^2 - 7x + 2 = 0$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(2)}}{2(3)} = \frac{7 \pm \sqrt{25}}{6} = \frac{7 \pm 5}{6}$$

The solutions are $\frac{7 - 5}{6} \approx 0.33$ and $\frac{7 + 5}{6} = 2$.

**35.** $2k^2 - 5k + 2 = 0$

$$k = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(2)}}{2(2)} = \frac{5 \pm \sqrt{9}}{4} = \frac{5 \pm 3}{4}$$

The solutions are $\frac{5 - 3}{4} = 0.5$ and $\frac{5 + 3}{4} = 2$.

**36.** $n^2 + 1 = 5n$

$n^2 - 5n + 1 = 0$

$$n = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(1)}}{2(1)} = \frac{5 \pm \sqrt{21}}{2}$$

The solutions are $\frac{5 - \sqrt{21}}{2} \approx 0.21$ and $\frac{5 + \sqrt{21}}{2} \approx 4.79$.

**37.** $2z + 4 = 3z^2$

$-3z^2 + 2z + 4 = 0$

$$z = \frac{-2 \pm \sqrt{2^2 - 4(-3)(4)}}{2(-3)} = \frac{-2 \pm \sqrt{52}}{-6}$$

The solutions are $\frac{-2 - \sqrt{52}}{-6} \approx 1.54$ and

$\frac{-2 + \sqrt{52}}{-6} \approx -0.87$.

**38.** $5x^2 - 4x = 2$

$5x^2 - 4x - 2 = 0$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(5)(-2)}}{2(5)} = \frac{4 \pm \sqrt{56}}{10}$$

The solutions are $\frac{4 - \sqrt{56}}{10} \approx -0.35$ and $\frac{4 + \sqrt{56}}{10} \approx 1.15$.

**39.** $m^2 - 2m + 1 = 0$

$b^2 - 4ac = (-2)^2 - 4(1)(1) = 0$

The discriminant is zero, so the equation has 1 solution.

**40.** $3x^2 + 6x + 2 = 0$

$b^2 - 4ac = 6^2 - 4(3)(2) = 12$

The discriminant is positive, so the equation has the two solutions.

**41.** $2q^2 + 3q + 5 = 0$

$b^2 - 4ac = 3^2 - 4(2)(5) = -31$

The discriminant is negative, so the equation has no solution.

**42.** $\frac{3}{4}x^2 - x + 2 = 0$

$b^2 - 4ac = (-1)^2 - 4\left(\frac{3}{4}\right)(2) = -5$

The discriminant is negative, so the equation has no solution.

**43.** $2w^2 - 5w + 6 = 8$

$2w^2 - 5w - 2 = 0$

$b^2 - 4ac = (-5)^2 - 4(2)(-2) = 41$

The discriminant is positive, so the equation has two solutions.

**44.** $2y^2 + 10y - 5 = 3y^2 - 30$

$-y^2 + 10y + 25 = 0$

$b^2 - 4ac = 10^2 - 4(-1)(25) = 200$

The discriminat is positive, so the equation has two solutions.

**45.**

| *x* | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|
| *y* | 3 | 0 | 3 | 12 | 27 |

First differences: −3  3  9  15

Second differences: 6  6  6

The table of values represents a quadratic function because the second differences are equal. The equation has the form $y = ax^2$.

Use (1, 3): $y = ax^2$

$3 = a(1)2$

$3 = a$

The equation is $y = 3x^2$.

**46.**

| *x* | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| *y* | −5 | −2 | 1 | 4 | 7 |

First differences: 3  3  3  3

The table of values represents a linear function because the first differences are equal. The equation has the form $y = mx + b$. The common difference is 3, so the slope of the line is 3. When $x = 0$, $y = -5$ so the $y$-intercept is $-5$. The equation is $y = 3x - 5$.

**47.**

| *x* | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| *y* | 1 | 2 | 4 | 8 | 16 |

Ratios: 2  2  2  2

The table of values represents an exponential function because the ratios between successive $y$-values are equal. The equation has the form $y = ab^x$. The value of $y$ increases by a factor of 2, so $b = 2$.

Use (1, 1): $y = ab^x$

$$1 = a(2)^1$$

$$\frac{1}{2} = a$$

The equation is $y = \frac{1}{2} \cdot 2^x$.

**48.**

| $x$ | $-2$ | $-1$ | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | 18 | 14 | 10 | 6 | 2 |

First differences: $-4$ $-4$ $-4$ $-4$

The table of values represents a linear function because the first differences are equal. The equation has the form $y = mx + b$. The common difference is $-4$, so the slope of the line is $-4$. When $x = 0$, $y = 10$ so the $y$-intercept is 10. The equation is $y = -4x + 10$.