

# Geometric Figures

## Solutions Key

### ARE YOU READY?

- |  |   |
|--|---|
| 1. proportion  | 2. ordered pair   |
| 3. percent   | 4. integer  |
| 5. $0.77 = 77\%$                                       | 6. $0.06 = 6\%$   |
| 7. $0.9 = 90\%$  | 8. $1.04 = 104\%$   |
| 9. $42\% = 0.42$                                       | 10. $80\% = 0.8$  |
| 11. $1\% = 0.01$                                       | 12. $131\% = 1.31$  |
| 13. 10% of 40<br>$0.10 \cdot 40 = 4$                   | 14. 12% of 100<br>$0.12 \cdot 100 = 12$                               |
| 15. 99% of 60<br>$0.99 \cdot 60 = 59.4$                | 16. 100% of 81<br>$1.00 \cdot 81 = 81$                                |
| 17. 45% of 360<br>$0.45 \cdot 360 = 162$               | 18. 55% of 1,024<br>$0.55 \cdot 1,024 = 563.2$                        |
| 19. $45 + n = 97$<br>$97 - 45 = n$<br>$n = 52$         | 20. $n - 18 = 100$<br>$100 + 18 = n$<br>$n = 118$                     |
| 21. $n - 72 = 91$<br>$91 + 72 = n$<br>$n = 163$        | 22. $n + 23 = 55$<br>$55 - 23 = n$<br>$n = 32$                        |
| 23. $5 \times t = 105$<br>$105 \div 5 = t$<br>$t = 21$ | 24. $b \div 13 = 8$<br>$13 \cdot b \div 13 = 8 \cdot 13$<br>$b = 104$ |
| 25. $k \times 18 = 90$<br>$90 \div 18 = k$<br>$k = 5$  | 26. $\frac{d}{7} = 8$<br>$8 \times 7 = d$<br>$d = 56$                 |
| 27. point A = (4, 7)                                   | 28. point B = (3, 3)  |
| 29. point C = (7, 2)                                   | 30. point D = (5, 1)  |
| 31. point E = (2, 5)                                   | 32. point F = (6, 4)  |

### LESSON 1

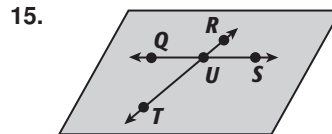
#### Think and Discuss

- Possible answer: You can name a line using any 2 points on the line and name a plane using any of its 3 points that are not on the same line. A line segment can be named using only its 2 endpoints.
- Possible answer: Because a line has only 1 dimension, you need 3 points not on the same line to show the 2 dimensions of a plane.

#### Exercises

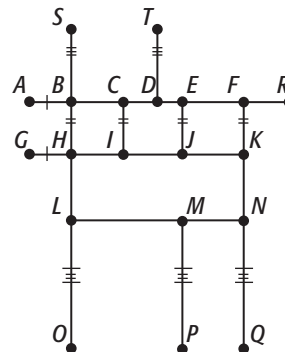
- Possible answer: Q, R, S
- Choose any two points on a line to name the line.  
Possible answer:  $\overleftrightarrow{QS}$ ,  $\overleftrightarrow{RT}$
- Choose any three points on a plane to name the plane. Possible answer: plane QRS
- Name the endpoint of a ray first.  
Possible answer:  $\overrightarrow{UQ}$ ,  $\overrightarrow{UT}$ ,  $\overrightarrow{US}$

- Use the endpoints in any order to name a segment.  
Possible answer:  $\overline{QU}$ ,  $\overline{RU}$ ,  $\overline{SU}$
- $\overline{BA}$  and  $\overline{BC}$  One tick mark  
 $\overline{AE}$  and  $\overline{CE}$  Two tick marks  
 $\overline{AD}$  and  $\overline{CD}$  Three tick marks
- D, E, F
- Possible answer:  $\overleftrightarrow{DE}$ ,  $\overleftrightarrow{EF}$
- Possible answer: plane DEF
- Possible answer  $\overleftrightarrow{DE}$ ,  $\overleftrightarrow{FD}$ ,  $\overleftrightarrow{EF}$
- Possible answer:  $\overleftrightarrow{DE}$ ,  $\overleftrightarrow{EF}$ ,  $\overleftrightarrow{DF}$
- $\overline{AF}$  and  $\overline{BC}$ ,  
 $\overline{AE}$  and  $\overline{BD}$ ,  
 $\overline{AB}$  and  $\overline{ED}$ ,  
 $\overline{FE}$  and  $\overline{CD}$
- Plane  $\overleftrightarrow{ABC}$  contains points A, B, and C; lines  $\overleftrightarrow{AB}$ , and  $\overleftrightarrow{BC}$ ; line segments  $\overline{AB}$ ,  $\overline{AC}$ , and  $\overline{BC}$ ; and rays  $\overrightarrow{AB}$ ,  $\overrightarrow{BA}$ ,  $\overrightarrow{BC}$ ,  $\overrightarrow{CB}$ , and  $\overrightarrow{CA}$ . Plane  $\overleftrightarrow{ACD}$  contains points A, C, and D; line segments  $\overline{AC}$ ,  $\overline{AD}$ , and  $\overline{CD}$ ; and ray  $\overrightarrow{CA}$
- 6 line segments:  $\overline{WX}$ ,  $\overline{WY}$ ,  $\overline{WZ}$ ,  $\overline{XY}$ ,  $\overline{XZ}$ , and  $\overline{YZ}$



Possible answer: plane QRS;  
points Q, R, S, T, and U;  
lines,  $\overleftrightarrow{QS}$ ,  $\overleftrightarrow{RT}$ ;  
and rays  $\overrightarrow{UQ}$ ,  $\overrightarrow{UT}$ ,  $\overrightarrow{US}$ ,  $\overrightarrow{UR}$

16. a-b.

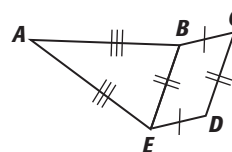


$$\overline{HK} \cong \overline{LN}$$

b. Possible answer:

$$\overline{AB} \cong \overline{GH}, \overline{CI} \cong \overline{EJ}, \overline{HK} \cong \overline{LN}, \overline{MP} \cong \overline{NQ}$$

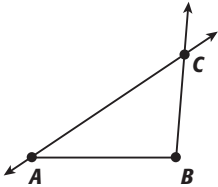
17. Possible answer:



18. No; two endpoints cannot be shared by two different line segments. If two line segments share two endpoints, then the line segments are the same.



19. A line is a straight path that extends forever in two directions; a ray is a straight path that extends forever in one direction from an endpoint; and a segment is a straight path from one endpoint to another. It is possible to estimate the length of a segment, because it does not extend forever in any direction.
20. Yes; since a plane extends forever, it is possible that two faces could be on the same plane and still not touch.
21. C, II and IV; both sets of these segments are congruent.
22. Check students' work. Possible answer:



## LESSON 2

### Think and Discuss

- Possible answer: A right angle measures  $90^\circ$ . An acute angle is less than  $90^\circ$ . An obtuse angle is greater than  $90^\circ$ , and a straight angle measures  $180^\circ$ .
- Possible answer: Since the sum of the measures of complementary angles is  $90^\circ$ , the missing measure can be found using the equation  $m\angle P + 25^\circ = 90^\circ$ . Therefore,  $m\angle P = 65^\circ$ .

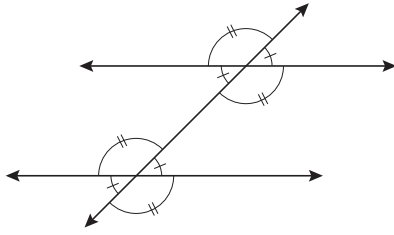
### Exercises

- right angle
- acute angle
- straight angle
- $m\angle AXB = 65^\circ$  and  $m\angle BXC = 20^\circ$   
Since  $65^\circ + 20^\circ = 85^\circ$ ,  $\angle AXB$  and  $\angle BXC$  are neither.
- $m\angle BXC = 20^\circ$  and  $m\angle DXE = 70^\circ$   
Since  $20^\circ + 70^\circ = 90^\circ$ ,  $\angle BXC$  and  $\angle DXE$  are complementary.
- $m\angle DXE = 70^\circ$  and  $m\angle AXD = 110^\circ$   
Since  $70^\circ + 110^\circ = 180^\circ$ ,  $\angle DXE$  and  $\angle AXD$  are supplementary.
- $m\angle CXD = 25^\circ$  and  $m\angle AXB = 65^\circ$   
Since  $25^\circ + 65^\circ = 90^\circ$ ,  $\angle CXD$  and  $\angle AXB$  are complementary.
- $m\angle L = 90 - 34 = 56^\circ$
- $m\angle C = 180 - 119 = 61^\circ$
- obtuse angle
- right angle
- acute angle
- $m\angle NZO = 45^\circ$  and  $m\angle MZN = 45^\circ$   
Since  $45^\circ + 45^\circ = 90^\circ$ ,  $\angle NZO$  and  $\angle MZN$  are complementary.
- $m\angle MZN = 45^\circ$  and  $m\angle OZP = 56^\circ$   
Since  $45^\circ + 56^\circ = 101^\circ$ ,  $\angle MZN$  and  $\angle OZP$  are neither.
- $m\angle LZN = 79^\circ$  and  $m\angle NZP = 101^\circ$   
Since  $79^\circ + 101^\circ = 180^\circ$ ,  $\angle LZN$  and  $\angle NZP$  are supplementary.
- $m\angle NZO = 45^\circ$  and  $m\angle LZN = 34^\circ$   
Since  $45^\circ + 34^\circ = 79^\circ$ ,  $\angle NZO$  and  $\angle LZN$  are neither.
- $m\angle O = 180 - 85 = 95^\circ$
- $m\angle J = 90 - 22 = 68^\circ$
- supplementary  
 $x + 28^\circ = 180^\circ$   
 $x = 152^\circ$
- complementary  
 $x + 66^\circ = 90^\circ$   
 $x = 24^\circ$
- supplementary  
 $x + 134^\circ = 180^\circ$   
 $x = 46^\circ$
- The hands form a straight angle at 6:00.  
The hands form a right angle at 3:00.  
The hands form an obtuse angle at 5:00.
- a. Lines of latitude and longitude cross to form right angles.  
b. Washington, D.C., is about  $39^\circ\text{N}$ ,  $77^\circ\text{W}$ .
- A straight angle measures  $180^\circ$ , so the two angles must be supplementary. Two angles whose sum equals  $180^\circ$  are supplementary.
- Two angles are supplementary when their sum equals  $180^\circ$ . Since obtuse angles measure greater than  $90^\circ$ , the sum of two obtuse angles is greater than  $180^\circ$ .
- Find the  $m\angle BAC$  in the figure.  
 $m\angle CAD + m\angle DAF = 180^\circ$   
 $m\angle CAD + 160^\circ = 180^\circ$   
 $m\angle CAD = 20^\circ$   
 $m\angle BAC + m\angle CAE = 180^\circ$   
 $m\angle BAC + 115^\circ = 180^\circ$   
 $m\angle BAC = 65^\circ$
- C;  $\angle FAE$  and  $\angle EAD$  are complementary angles.  
 $m\angle FAE = 30^\circ$ ,  $m\angle EAD = 90^\circ$   
 $30^\circ + 90^\circ = 120^\circ$ , so this pair of angles is not complementary.
- G;  $120^\circ$   
 $m\angle FAE = 30^\circ$ ,  $m\angle EAD = 90^\circ$   
So,  $m\angle FAD = 30^\circ + 90^\circ = 120^\circ$

## LESSON 3

### Think and Discuss

1. Possible answer:



2. Possible answer: parallel—railroad tracks, sides of a ladder; perpendicular—side and bottom edges of a desk drawer; skew—a telephone pole and the edge of the curb of sidewalk

### Exercises

1.  $\overleftrightarrow{JL}$  and  $\overleftrightarrow{KM}$

The lines are in the same plane and do not intersect.

$$\overleftrightarrow{JL} \parallel \overleftrightarrow{KM}$$

2.  $\overleftrightarrow{LM}$  and  $\overleftrightarrow{KN}$

The lines are in different places and do not intersect.

$\overleftrightarrow{LM}$  and  $\overleftrightarrow{KN}$  are skew.

3.  $\overleftrightarrow{LM}$  and  $\overleftrightarrow{KM}$

The lines appear to intersect to form right angles.

$$\overleftrightarrow{LM} \perp \overleftrightarrow{KM}$$

4.  $\angle 5$  and the  $115^\circ$  angle are obtuse angles. Since all of the obtuse angles in the figure are congruent,  $m\angle 5 = 115^\circ$ .

5.  $\angle 2$  and the  $115^\circ$  angle are vertical angles. Since all of the vertical angles are congruent,  $m\angle 2 = 115^\circ$ .

6.  $\angle 6$  and the  $115^\circ$  angle are congruent because they are alternate exterior angles.

$$m\angle 6 = 115^\circ$$

7.  $\overleftrightarrow{UX}$  and  $\overleftrightarrow{YZ}$

The lines are in different places and do not intersect.

$\overleftrightarrow{UX}$  and  $\overleftrightarrow{YZ}$  are skew.

8.  $\overleftrightarrow{YZ}$  and  $\overleftrightarrow{XY}$

The lines appear to intersect to form right angles.

$$\overleftrightarrow{YZ} \perp \overleftrightarrow{XY}$$

9.  $\overleftrightarrow{UX}$  and  $\overleftrightarrow{VW}$

The lines are in the same plane and do not intersect.

$$\overleftrightarrow{UX} \parallel \overleftrightarrow{VW}$$

10.  $\angle 1$  and the  $30^\circ$  angle are acute angles. Since all of the acute angles in the figure are congruent,  $m\angle 1 = 30^\circ$ .

11.  $\angle 4$  and the  $30^\circ$  angle are congruent because they are alternate interior angles.  
 $m\angle 4 = 30^\circ$

12.  $\angle 6$  is an obtuse angle.

In the figure, the acute and obtuse angles are supplementary.

$$\begin{array}{r} 30^\circ + m\angle 6 = 180^\circ \\ -30 \quad \quad \quad -30 \\ \hline m\angle 6 = 150^\circ \end{array}$$

13. parallel

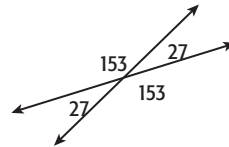
14. perpendicular

15. supplementary; adjacent

16. alternate exterior; congruent

17. The sum of the measures of two angles that are complementary is 90 degrees. If the angles are congruent, each one must measure 45 degrees.

- 18.



Since vertical angles are congruent, the angle opposite the given angle has the same measure as the given angle,  $27^\circ$ . The remaining angles are both adjacent to the angles that measure  $27^\circ$ . Since adjacent angles are supplementary, both remaining angles measure  $180^\circ - 27^\circ = 153^\circ$ .

19. sometimes

20. never

21. always

22. never

23. a.  $\overline{OR}$  and  $\overline{RS}$  are perpendicular.

b.  $\overline{PT}$  is a transversal.

c.  $\angle 1$  and  $\angle 2$  are corresponding angles.

24. If two intersecting lines form congruent, adjacent angles, the lines are perpendicular.

25. C; 10

Draw a Diagram or Make a Model

26. The nonadjacent sides of a pair of adjacent angles are formed by one of the two straight lines. A line is a straight angle, which, by definition, has a measure of  $180^\circ$ . Therefore, the measures of the adjacent angles must add up to the  $180^\circ$ ; that is, they are supplementary.

27. Draw a transversal that intersects two or more lines in the parking lot, and measure a pair of angles that should be congruent, such as corresponding angles. If you can assume that your measurements are correct and the angles are congruent, then you can conclude that the lines in the parking lot are likely to be parallel.

28. D;  $55^\circ$

$$\begin{array}{r} 125^\circ + m\angle 3 = 180^\circ \\ -125 \quad \quad \quad -125 \\ \hline m\angle 3 = 55^\circ \end{array}$$

29.  $F$ ;  $125^\circ$   
 $\angle 2$  and the  $125^\circ$  are vertical angles, which makes them congruent. So,  $m\angle 2 = 125^\circ$ . Since  $r$  and  $s$  are parallel,  $\angle 2$  and  $\angle 6$  become corresponding angles by the transversal line. Therefore,  $\angle 2$  and  $\angle 6$  are congruent, making  $m\angle 6 = 125^\circ$ .

### READY TO GO ON?

- Possible answer:  $A, B, C$
- Choose any two points on a line to name the line.  
Possible answer:  $\overline{AG}, \overline{CE}, \overline{BF}$
- Choose any three points on a plane to name the plane.  
Possible answer: plane  $BCD$
- Use the endpoints in any order to name a segment.  
Possible answer:  $\overline{BD}, \overline{BF}, \overline{DE}$
- Name the endpoint of a ray first.  
Possible answer:  $\overrightarrow{BD}, \overrightarrow{CE}, \overrightarrow{DE}$
- $\overline{AB}$  and  $\overline{ED}$ ,  $\overline{BC}$  and  $\overline{FE}$ ,  $\overline{AF}$  and  $\overline{CD}$ ,  $\overline{BF}$  and  $\overline{CE}$
- right
- obtuse
- acute
- straight
- $\angle DXE$  and  $\angle AXD$   
 $m\angle DXE = 25^\circ$  and  $m\angle AXD = 155^\circ$   
 $25^\circ + 155^\circ = 180^\circ$ , so  $\angle DXE$  and  $\angle AXD$  are supplementary.
- $\angle AXB$  and  $\angle CXD$   
 $m\angle AXB = 48^\circ$  and  $m\angle CXD = 42^\circ$   
 $48^\circ + 42^\circ = 90^\circ$ , so  $\angle AXB$  and  $\angle CXD$  are complementary.
- $\angle DXE$  and  $\angle AXB$   
 $m\angle DXE = 25^\circ$  and  $m\angle AXB = 48^\circ$   
 $25^\circ + 48^\circ = 73^\circ$ , so  $\angle DXE$  and  $\angle AXB$  are neither.
- $\angle BXC$  and  $\angle DXE$   
 $m\angle BXC = 65^\circ$  and  $m\angle DXE = 25^\circ$   
 $65^\circ + 25^\circ = 90^\circ$ , so  $\angle BXC$  and  $\angle DXE$  are complementary.
- $m\angle R + 17 = 90^\circ$   
 $m\angle R = 73^\circ$
- $m\angle F + 45 = 180^\circ$   
 $m\angle F = 135^\circ$
- The lines are in different planes and do not intersect.  $\overleftrightarrow{KL}$  and  $\overleftrightarrow{MN}$  are skew.
- The lines are in the same plane and do not intersect.  $\overleftrightarrow{JL}$  and  $\overleftrightarrow{MN}$  are parallel.
- The lines appear to intersect to form right angles.  $\overleftrightarrow{KL}$  and  $\overleftrightarrow{JL}$  are perpendicular.
- The lines are in different planes and do not intersect.  $\overleftrightarrow{IJ}$  and  $\overleftrightarrow{MN}$  are skew.
- $\angle 3$  and the  $39^\circ$  angle are vertical angles. Since vertical angles are congruent,  $m\angle 3 = 39^\circ$ .

22.  $\angle 4$  is an obtuse angle. In the figure, the acute and obtuse angles are supplementary.  
 $m\angle 3 + m\angle 4 = 180^\circ$   
 $39^\circ + m\angle 4 = 180^\circ$   
 $-39 \quad -39$   
 $\hline m\angle 4 = 141^\circ$
23.  $\angle 8$  and  $\angle 4$  are obtuse angles. Since all of the obtuse angles in the figure are congruent,  $m\angle 8 = 141^\circ$ .
24.  $\angle 6$  and the  $39^\circ$  angle are acute angles. Since all of the acute angles in the figure are congruent,  $m\angle 6 = 39^\circ$ .
25.  $\angle 1$  and  $\angle 4$  are vertical angles. Since vertical angles are congruent,  $m\angle 1 = 141^\circ$ .
26.  $\angle 5$  and  $\angle 8$  are obtuse and vertical angles. Since all of the vertical, obtuse angles in the figure are congruent,  $m\angle 5 = 141^\circ$ .

### LESSON 4

#### Think and Discuss

- Possible answer: Subtract the sum of the measures of the two angles from  $180^\circ$ .
- Possible answer: The octagon has more triangles when the diagonals are drawn. Since there are more triangles, there are more degrees. The octagon has  $1080^\circ$ , while the pentagon has only  $540^\circ$ .
- Possible answer: The size of each angle in a regular polygon increases as the number of the sides increase. For example, each angle of an equilateral triangle is  $60^\circ$ , and each angle of a square is  $90^\circ$ .

#### Exercises

- $43^\circ + 60^\circ + x = 180^\circ$   
 $103^\circ + x = 180^\circ$   
 $-103^\circ \quad -103^\circ$   
 $\hline x = 77^\circ$   
The measure of the unknown angle is  $77^\circ$ .
- $40^\circ + 30^\circ + x = 180^\circ$   
 $70^\circ + x = 180^\circ$   
 $-70^\circ \quad -70^\circ$   
 $\hline x = 110^\circ$   
The measure of the unknown angle is  $110^\circ$ .
- $90^\circ + 35^\circ + x = 180^\circ$   
 $125^\circ + x = 180^\circ$   
 $-125^\circ \quad -125^\circ$   
 $\hline x = 55^\circ$   
The measure of the unknown angle is  $55^\circ$ .
- $90^\circ + 90^\circ + 127^\circ + x = 360^\circ$   
 $307^\circ + x = 360^\circ$   
 $-307^\circ \quad -307^\circ$   
 $\hline x = 53^\circ$   
The measure of the unknown angle is  $53^\circ$ .

$$\begin{array}{r} 5. \quad 76^\circ + 39^\circ + 135^\circ + x = 360^\circ \\ \quad \quad 250^\circ + x = 360^\circ \\ \quad \quad \underline{-250^\circ} \quad \underline{-250^\circ} \\ \quad \quad \quad \quad x = 110^\circ \end{array}$$

The measure of the unknown angle is  $110^\circ$ .

$$\begin{array}{r} 6. \quad 44^\circ + 118^\circ + 109^\circ + x = 360^\circ \\ \quad \quad 271^\circ + x = 360^\circ \\ \quad \quad \underline{-271^\circ} \quad \underline{-271^\circ} \\ \quad \quad \quad \quad x = 89^\circ \end{array}$$

The measure of the unknown angle is  $89^\circ$ .

7. There are 4 triangles.

$$4 \cdot 180^\circ = 720^\circ$$

The sum of the angle measures of a hexagon is  $720^\circ$ .

8. There are 3 triangles.

$$3 \cdot 180^\circ = 540^\circ$$

The sum of the angle measures of a pentagon is  $540^\circ$ .

9. There are 2 triangles.

$$2 \cdot 180^\circ = 360^\circ$$

The sum of the angle measures of a quadrilateral is  $360^\circ$ .

$$\begin{array}{r} 10. \quad 60^\circ + 60^\circ + x = 180^\circ \\ \quad \quad 120^\circ + x = 180^\circ \\ \quad \quad \underline{-120^\circ} \quad \underline{-120^\circ} \\ \quad \quad \quad \quad x = 60^\circ \end{array}$$

The measure of the unknown angle is  $60^\circ$ .

$$\begin{array}{r} 11. \quad 78^\circ + 65^\circ + x = 180^\circ \\ \quad \quad 143^\circ + x = 180^\circ \\ \quad \quad \underline{-143^\circ} \quad \underline{-143^\circ} \\ \quad \quad \quad \quad x = 37^\circ \end{array}$$

The measure of the unknown angle is  $37^\circ$ .

$$\begin{array}{r} 12. \quad 120^\circ + 28^\circ + x = 180^\circ \\ \quad \quad 148^\circ + x = 180^\circ \\ \quad \quad \underline{-148^\circ} \quad \underline{-148^\circ} \\ \quad \quad \quad \quad x = 32^\circ \end{array}$$

The measure of the unknown angle is  $32^\circ$ .

$$\begin{array}{r} 13. \quad 77^\circ + 82^\circ + 113^\circ + x = 360^\circ \\ \quad \quad 272^\circ + x = 360^\circ \\ \quad \quad \underline{-272^\circ} \quad \underline{-272^\circ} \\ \quad \quad \quad \quad x = 88^\circ \end{array}$$

The measure of the unknown angle is  $88^\circ$ .

$$\begin{array}{r} 14. \quad 48^\circ + 135^\circ + 105^\circ + x = 360^\circ \\ \quad \quad 288^\circ + x = 360^\circ \\ \quad \quad \underline{-288^\circ} \quad \underline{-288^\circ} \\ \quad \quad \quad \quad x = 72^\circ \end{array}$$

The measure of the unknown angle is  $72^\circ$ .

$$\begin{array}{r} 15. \quad 61^\circ + 90^\circ + 108^\circ + x = 360^\circ \\ \quad \quad 259^\circ + x = 360^\circ \\ \quad \quad \underline{-259^\circ} \quad \underline{-259^\circ} \\ \quad \quad \quad \quad x = 101^\circ \end{array}$$

The measure of the unknown angle is  $101^\circ$ .

16. There are 7 triangles.

$$7 \cdot 180^\circ = 1,260^\circ$$

The sum of the angle measures of a nonagon is  $1,260^\circ$ .

17. There are 6 triangles.

$$6 \cdot 180^\circ = 1,080^\circ$$

The sum of the angle measures of an octagon is  $1,080^\circ$ .

18. There are 2 triangles.

$$2 \cdot 180^\circ = 360^\circ$$

The sum of the angle measures of a quadrilateral is  $360^\circ$ .

$$\begin{array}{r} 19. \quad 90^\circ + 52^\circ + x = 180^\circ \\ \quad \quad 142^\circ + x = 180^\circ \\ \quad \quad \underline{-142^\circ} \quad \underline{-142^\circ} \\ \quad \quad \quad \quad x = 38^\circ \end{array}$$

The measure of the other acute angle is  $38^\circ$ .

$$\begin{array}{r} 20. \quad 56^\circ + 101^\circ + x = 180^\circ \\ \quad \quad 157^\circ + x = 180^\circ \\ \quad \quad \underline{-157^\circ} \quad \underline{-157^\circ} \\ \quad \quad \quad \quad x = 23^\circ \end{array}$$

The measure of the unknown angle is  $23^\circ$ . One obtuse angle makes this an obtuse triangle.

$$\begin{array}{r} 21. \quad 18^\circ + 63^\circ + x = 180^\circ \\ \quad \quad 81^\circ + x = 180^\circ \\ \quad \quad \underline{-81^\circ} \quad \underline{-81^\circ} \\ \quad \quad \quad \quad x = 99^\circ \end{array}$$

The measure of the unknown angle is  $99^\circ$ . One obtuse angle makes this an obtuse triangle.

$$\begin{array}{r} 22. \quad 62^\circ + 58^\circ + x = 180^\circ \\ \quad \quad 120^\circ + x = 180^\circ \\ \quad \quad \underline{-120^\circ} \quad \underline{-120^\circ} \\ \quad \quad \quad \quad x = 60^\circ \end{array}$$

The measure of the unknown angle is  $60^\circ$ . Three acute angles make this an acute triangle.

$$\begin{array}{r} 23. \quad 41^\circ + 49^\circ + x = 180^\circ \\ \quad \quad 90^\circ + x = 180^\circ \\ \quad \quad \underline{-90^\circ} \quad \underline{-90^\circ} \\ \quad \quad \quad \quad x = 90^\circ \end{array}$$

The measure of the unknown angle is  $90^\circ$ . One right angle makes this a right triangle.

24. There are 3 triangles.

$$3 \cdot 180^\circ = 540^\circ$$

The Pentagon has 5 congruent angles, so divide by 5.  $540^\circ \div 5 = 108^\circ$

Each angle made by the Pentagon's outer walls is  $108^\circ$ .

25. Since it is a right triangle, it has one  $90^\circ$  angle.

Since it is isosceles, the two acute angles are congruent. They have a sum of  $90^\circ$ , so each measures  $45^\circ$ .

26. A polygon has 2 fewer interior triangles than its number of sides. Subtract 2 from the number of sides,  $s$ , to find the number of triangles the polygon can be divided into. Multiply  $(s - 2)$  by  $180^\circ$  to find the sum of the angle measures;  $(s - 2) \cdot 180^\circ =$  the sum of the interior angle measures in a polygon with  $s$  sides.

27. Possible answer: Six triangles can be drawn inside an octagon, not seven.

28. Divide the quadrilateral into two triangles, and then find the sum of the angles measures of each triangle.
29.  $180^\circ - 27^\circ = 153^\circ$   
 $x + 2x = 3x$   
 $3x = 153^\circ$   
 $\frac{3x}{3} = \frac{153^\circ}{3}$   
 $x = 51^\circ$   
 $2x = 2(51^\circ) = 102^\circ$   
 The angles at the tugboat and cargo ship are  $51^\circ$  and  $102^\circ$ .

30. B;  $60^\circ$   
 $180^\circ \div 3 = 60^\circ$

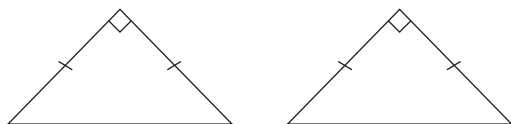
31.  $58^\circ + 42^\circ + x = 180^\circ$   
 $100^\circ + x = 180^\circ$   
 $\frac{-100^\circ}{\quad} \quad \frac{-100^\circ}{\quad}$   
 $x = 80^\circ$

The measure of the unknown angle is  $80^\circ$ .

## LESSON 5

### Think and Discuss

1. Possible answer: If the sides and angles of the isosceles triangle are congruent to the sides and angles of the right triangle, then, an isosceles triangle can be congruent to a right triangle.



2. Possible answer: The corresponding angle measures of congruent figures are always equal, so the figures are similar.

### Exercises

1. The triangles on the game board are congruent, and the holes on the game board are also congruent. The pairs of dragons and words “Chinese Checkers” may also be considered to be congruent.
2. None. The triangles in the kite’s design are not congruent.
3. Possible answer: There are two shapes in the middle that do not have a congruent shape: the kite in the upper middle and the triangle at the bottom in the middle. Otherwise, each shape on the left side has a congruent shape on the right side: two triangles, two trapezoids, two parallelograms, two other triangles, two pentagons, two other pentagons, two rectangles, two other pentagons, and two kites.
4.  $AC = 6$  mm                       $DF = 6$  mm  
 $AB = 5$  mm                         $DE = 5$  mm  
 $CB = 4$  mm                          $FE = 4$  mm

By the Side-by-Side Rule,  $\triangle ABC$  is congruent to  $\triangle DEF$ , or  $\triangle ABC \cong \triangle DEF$ . If you rotate it, it will fit exactly over the other.

5. The corresponding sides are not congruent; therefore, the triangles are not congruent.
6. The corresponding angles in congruent polygons are congruent. The unknown angle measure is  $36^\circ$ .
7. The corresponding sides of congruent polygons are congruent. The unknown side measure is 2.5.
8. None. The gears are not congruent.
9. The triangles in the kite’s design are congruent.
10. 10 squares; rectangles (2 tan; 12 brown; 2 yellow, small; 3 yellow, large)
11. The corresponding sides are not congruent; therefore, the triangles are not congruent.
12.  $JK = 5$  m                               $AB = 5$  m  
 $KL = 13$  m                              $BC = 13$  m  
 $JL = 12$  m                                $AC = 12$  m
- By the Side-by-Side Rule,  $\triangle KJL$  is congruent to  $\triangle BAC$ , or  $\triangle KJL \cong \triangle BAC$ . If you rotate it, it will fit exactly over the other.

13. The corresponding angles in congruent polygons are congruent. The unknown angle measure is  $112^\circ$ . The corresponding sides in congruent polygons are congruent. The unknown side measure is 8 cm.
14. The corresponding angles of congruent polygons are congruent. The unknown angle measure is  $90^\circ$ . The corresponding sides of congruent polygons are also congruent. The unknown side measure is 4 in.
15. the lengths of all of the sides
16. the length of one side in each square
17. the length of adjacent sides in each rectangle
18. the length of each side and the measure of each angle in each pentagon
19. The corresponding sides of congruent triangles are congruent.  $AB \cong DE$  and  $DE = 40$  m. Therefore,  $AB = 40$  m and the distance between the trees is 40 m.

20. The squares appear to be congruent and the triangles appear to be congruent.
21. C; They arrive at the same time.

Use Logical Reasoning

Anji’s vertical distances, when combined, are equal to Art’s vertical distance. Anji’s horizontal distances, when combined, are equal to Art’s horizontal distance. They are walking the same distance.

22. Possible answer: Compare the measures of the three sides. If the corresponding sides are congruent, the triangles are congruent.
23. No. The three angles in each triangle are congruent, but the sides in one triangle can be of different length than those in the other triangle.
24. C; the lightning bolts  
 The lightning bolts are the same size and same shape.



25. G; 5 mm

The corresponding sides of congruent triangles are congruent.  $\overline{AB} \cong \overline{DE}$  and  $\overline{DE} = 5$  mm. Therefore,  $AB = 5$  mm.

### READY TO GO ON?

$$\begin{array}{r} 1. \quad 37^\circ + 65^\circ + x = 180^\circ \\ \quad 102^\circ + x = 180^\circ \\ \quad \underline{-102^\circ} \quad \underline{-102^\circ} \\ \quad \quad \quad x = 78^\circ \end{array}$$

The measure of the unknown angle is  $78^\circ$ .

$$\begin{array}{r} 2. \quad 25^\circ + 90^\circ + x = 180^\circ \\ \quad 115^\circ + x = 180^\circ \\ \quad \underline{-115^\circ} \quad \underline{-115^\circ} \\ \quad \quad \quad x = 65^\circ \end{array}$$

The measure of the unknown angle is  $65^\circ$ .

$$\begin{array}{r} 3. \quad 87^\circ + 138^\circ + 90^\circ + x = 360^\circ \\ \quad 315^\circ + x = 360^\circ \\ \quad \underline{-315^\circ} \quad \underline{-315^\circ} \\ \quad \quad \quad x = 45^\circ \end{array}$$

The measure of the unknown angle is  $45^\circ$ .

$$\begin{array}{r} 4. \quad 56^\circ + 118^\circ + 61^\circ + x = 360^\circ \\ \quad 235^\circ + x = 360^\circ \\ \quad \underline{-235^\circ} \quad \underline{-235^\circ} \\ \quad \quad \quad x = 125^\circ \end{array}$$

The measure of the unknown angle is  $125^\circ$ .

5. A square has 4 sides. Substitute 4 for  $s$ .

$$\begin{aligned} (s - 2) \cdot 180^\circ &= (4 - 2) \cdot 180^\circ \\ &= 2 \cdot 180^\circ \\ &= 360^\circ \end{aligned}$$

6. A 10-sided figure has 10 sides. Substitute 10 for  $s$ .

$$\begin{aligned} (s - 2) \cdot 180^\circ &= (10 - 2) \cdot 180^\circ \\ &= 8 \cdot 180^\circ \\ &= 1,440^\circ \end{aligned}$$

7. A diagonal is a line segment that connects two non-adjacent vertices of a polygon.

The figure has 4 diagonals.

8. Each of the corresponding sides of these two triangles is congruent, so the triangles are congruent.

9. The corresponding sides of these two triangles are not congruent, so the triangles are not congruent.

10. The corresponding sides of congruent triangles are congruent.  $\overline{YZ} \cong \overline{CD}$  and  $YZ = 20$  m. Therefore,  $CD = 20$  m

### STUDY GUIDE: REVIEW

- |  |  |
|--|--|
| 1. supplementary                                 | 2. parallel lines  |
| 3. perpendicular lines                           | 4. straight  |
| 5. $D, E, F$                                     | 6. $\overleftrightarrow{DF}$   |
| 7. plane $DEF$                                   | 8. $\overleftrightarrow{ED}, \overleftrightarrow{FD}, \overleftrightarrow{DF}$ |
| 9. $\overline{DE}, \overline{DF}, \overline{EF}$ | 10. acute  |
| 11. straight                                     |  |

12. Let  $x$  represent the measure of the other angle.

Complementary angles have a sum of  $90^\circ$ .

$$x + 55^\circ = 90^\circ$$

$$x + 55^\circ - 55^\circ = 90^\circ - 55^\circ$$

$$x = 35^\circ$$

13. Let  $x$  represent the measure of the other angle.

Supplementary angles have a sum of  $180^\circ$ .

$$x + 107^\circ = 180^\circ$$

$$x + 107^\circ - 107^\circ = 180^\circ - 107^\circ$$

$$x = 73^\circ$$

14. skew

15. parallel

16.  $\angle 2$  and the  $74^\circ$  angle are congruent because they are vertical angles.

$$m\angle 2 = 74^\circ$$

17.  $\angle 3$  and the  $74^\circ$  angle are supplementary.

$$m\angle 3 + 74^\circ = 180^\circ$$

$$m\angle 3 + 74^\circ - 74^\circ = 180^\circ - 74^\circ$$

$$m\angle 3 = 106^\circ$$

18.  $\angle 5$  and  $\angle 3$  are congruent because they are alternate interior angles. From question 14,

$$m\angle 3 = 106^\circ$$

$$m\angle 5 = 106^\circ$$

19.  $\angle 6$  and the  $74^\circ$  angle are congruent because they are alternate exterior angles.

$$m\angle 6 = 74^\circ$$

20. The corresponding angles in congruent polygons are congruent. The unknown angle measure is  $133^\circ$ .

The corresponding sides in congruent polygons are congruent. The unknown side measure is 10 cm.

21. The corresponding sides of these two triangles are not congruent, so the triangles are not congruent.

22. Each of the corresponding sides of these two triangles is congruent, so the triangles are congruent.

### CHAPTER TEST

1. Possible answers:  $A, B, C, D, E$

2. Possible answer:  $\overleftrightarrow{AB}, \overleftrightarrow{CD}, \overleftrightarrow{CB}$

3. Possible answer: plane  $ABC$

4. Possible answer:  $\overline{AB}, \overline{CD}, \overline{EC}, \overline{EB}, \overline{CB}$

5. Possible answer:  $\overleftrightarrow{AB}, \overleftrightarrow{BA}, \overleftrightarrow{BC}, \overleftrightarrow{CB}, \overleftrightarrow{CD}, \overleftrightarrow{DC}$

6.  $\angle ABC$  and the  $42^\circ$  angle are acute angles. Since all of the acute angles in the figure are congruent,  $m\angle ABC = 42^\circ$ .

7.  $m\angle BCE = 180^\circ$  because it is a straight angle.

8.  $\angle DCE$  is an obtuse angle. In the figure, the acute and obtuse angles are supplementary.

$$42^\circ + m\angle DCE = 180^\circ$$

$$\underline{-42^\circ} \quad \underline{-42^\circ}$$

$$m\angle DCE = 138^\circ$$

9. The lines  $\overleftrightarrow{MN}$  and  $\overleftrightarrow{PO}$  are in the same plane and do not intersect.  $\overleftrightarrow{MN}$  and  $\overleftrightarrow{PO}$  are parallel.

10. The lines  $\overleftrightarrow{LM}$  and  $\overleftrightarrow{PO}$  are in different planes and do not intersect.  $\overleftrightarrow{LM}$  and  $\overleftrightarrow{PO}$  are skew.
11. The lines  $\overleftrightarrow{NO}$  and  $\overleftrightarrow{MN}$  appear to intersect to form right angles.  $\overleftrightarrow{NO}$  and  $\overleftrightarrow{MN}$  are perpendicular.
12. The corresponding sides of congruent polygons are congruent. The missing measure is 6 in.
13. The given angle and angle 1 are alternate exterior angles.  
Alternate exterior angles are congruent.  
The measure of angle 1 is  $31^\circ$ .
14. The given angle and angle 6 are supplementary angles.  
Supplementary angles have a sum of  $180^\circ$ .  
Let  $x$  represent the measure of angle 6.  
 $x + 31^\circ = 180^\circ$   
 $x + 31^\circ - 31^\circ = 180^\circ - 31^\circ$   
 $x = 149^\circ$   
The measure of angle 6 is  $149^\circ$ .

15. The given angle and angle 4 are corresponding angles.  
Corresponding angles are congruent.  
The measure of angle 4 is  $31^\circ$ .  
Complementary angles have a sum of  $90^\circ$ .  
Let  $x$  represent the measure of the complement of angle 4.  
 $x + 31^\circ = 90^\circ$   
 $x + 31^\circ - 31^\circ = 90^\circ - 31^\circ$   
 $x = 59^\circ$   
The measure of the complement of angle 4 is  $59^\circ$ .