Chapter 11 Measurement of Figures and Solids

Prerequisite Skills for the chapter "Measurement of Figures and Solids"

1. radius = 3 **2.** diameter = 6

3. $m\widehat{ADB} = 360° - 70° = 290°$

4. $P = 2\ell + 2w$

$24 = 2(9) + 2w$

$24 = 18 + 2w$

$6 = 2w$

$3 = w$

The width is 3 centimeters.

5. $\frac{UV}{XY} = \frac{VW}{YZ}$

$\frac{5}{XY} = \frac{8}{12}$

$60 = 8XY$

$\frac{15}{2} = XY$ or $7.5 = XY$

6. Because the ratio of the perimeters of the triangles is equal to the ratio of the side lengths of the triangles, the ratio is $\frac{8}{12} = \frac{2}{3}$.

7. $BC = x$ and $AC = x\sqrt{3}$ in a 30°-60°-90° triangle. Because $BC = 5$, $AC = 5\sqrt{3}$.

8. $BC = AC$ in a 45°-45°-90° triangle.

Because $BC = 8$, $AC = 8$.

9. $\cos A = \frac{AC}{AB}$

$\cos 60° = \frac{AC}{13}$

$13 \cos 60° = AC$

$6.5 = AC$

Lesson 11.1 Circumference and Arc Length

Guided Practice for the lesson "Circumference and Arc Length"

1. Circle with diameter of 5 inches:

$C = \pi d = \pi(5) \approx 15.71$

The circumference is about 15.71 inches.

Circle with circumference of 17 feet: $C = \pi d$

$17 = \pi d$

$5.41 \approx d$

The diameter is about 5.41 feet.

2. Circumference: $C = \pi d = \pi(28) \approx 87.96$ in.

Distance traveled = Number of revolutions • circumference

500 ft = Number of revolutions • 87.96 in. • $\frac{1 \text{ ft}}{12 \text{ in.}}$

500 ft = Number of revolutions • 7.33 ft

68 ≈ Number of revolutions

The tire makes about 68 revolutions while traveling 500 feet.

3. Arc length of $\widehat{DQ} = \frac{75°}{360°} \cdot \pi(9) \approx 5.89$ yards

4. $\frac{\text{Arc length of } \widehat{LM}}{c} = \frac{m\widehat{LM}}{360°}$

$\frac{61.26 \text{ m}}{c} = \frac{270°}{360°}$

$\frac{61.26 \text{ m}}{c} = \frac{3}{4}$

$81.68 \text{ m} = c$

5. $\frac{\text{Arc length of } \widehat{EF}}{2\pi r} = \frac{m\widehat{EF}}{360°}$

$\frac{10.5 \text{ ft}}{2\pi r} = \frac{150°}{360°}$

$3780 = 300\pi r$

$4.01 \text{ ft} \approx r$

6. Distance $= 2(84.39) + 2 \cdot \left(\frac{1}{2} \cdot 2\pi \cdot 44.02\right)$

Distance ≈ 445.4 meters

The runner on the blue path travels about 445.4 meters.

Exercises for the lesson "Circumference and Arc Length"

Skill Practice

1. $\frac{\text{Arc length of } \widehat{AB}}{2\pi r} = \frac{m\widehat{AB}}{360°}$

2. The arc measure is the number degrees of a circle the arc is bounded by. The arc length is the part of the circumference of the circle that the arc occupies.

3. $C = 2\pi r = 2\pi(6) \approx 37.70$ in.

4. $C = \pi d = \pi(17) \approx 53.41$ cm

5. $C = 2\pi r$

$63 = 2\pi r$

$10.03 \text{ ft} \approx r$

6. $C = \pi d = \pi(5) = 5\pi$ in.

7. $C = 2\pi r$

$28\pi = 2\pi r$

$14 \text{ m} = r$

8. $C = \pi d = \pi(14) \approx 43.98$ units

9. $C = 2\pi r = 2\pi(3 + 2) = 2\pi(5) \approx 31.42$ units

10. $C = \pi d = \pi(10 \div 2) = \pi(5) \approx 15.71$ units

11. Arc length of $\widehat{AB} = \frac{m\widehat{AB}}{360°} \cdot 2\pi r = \frac{40°}{360°} \cdot 2\pi(6)$

≈ 4.19 m

12. Arc length of $\widehat{AB} = \frac{m\widehat{AB}}{360°} \cdot 2\pi r = \frac{120°}{360°} \cdot 2\pi(14)$

≈ 29.32 cm

13. Arc length of $\widehat{AB} = \frac{m\widehat{AB}}{360°} \cdot \pi d = \frac{45°}{360°} \cdot \pi(8) \approx 3.14$ ft

14. Two arcs from different circles have the same length only if the circles have the same circumference.

15. $m\widehat{QRS} = 360° - m\widehat{QS}$

$m\widehat{QRS} = 360° - 60°$

$m\widehat{QRS} = 300°$

16. Arc length of $\widehat{QRS} = \frac{m\widehat{QRS}}{360°} \cdot 2\pi r = \frac{300°}{360°} \cdot 2\pi(8)$

≈ 41.89 ft

17. $\angle QPR \cong \angle RPS$

$m\angle QPR = \frac{360° - 60°}{2} = \frac{300°}{2} = 150°$

$m\widehat{QR} = 150°$

18. $m\widehat{RS} = 150°$ (from Exercise 17)

$m\widehat{RSQ} = m\widehat{RS} + m\widehat{SQ} = 150° + 60° = 210°$

19. Arc length of $\widehat{QR} = \frac{m\widehat{QR}}{360°} \cdot 2\pi r = \frac{150°}{360°} \cdot 2\pi(8)$

≈ 20.94 ft

20. Arc length of $\widehat{RSQ} = \frac{m\widehat{RSQ}}{360°} \cdot 2\pi r = \frac{210°}{360°} \cdot 2\pi(8)$

≈ 29.32 ft

21. $\frac{\text{Arc length of } \widehat{AB}}{2\pi r} = \frac{m\widehat{AB}}{360°}$

$\frac{8.73}{2\pi(10)} = \frac{m\widehat{AB}}{360°}$

$360° \cdot \frac{8.73}{20\pi} = m\widehat{AB}$

$50° \approx m\widehat{AB}$

22. $\frac{\text{Arc length of } \widehat{CD}}{C} = \frac{m\widehat{CD}}{360°}$

$\frac{7.5}{C} = \frac{76°}{360°}$

$\frac{7.5}{C} = \frac{19}{90}$

35.53 units $\approx C$

23. $\frac{\text{Arc length } \widehat{LM}}{2\pi r} = \frac{m\widehat{LM}}{360°}$

$\frac{38.95}{2\pi r} = \frac{260°}{360°}$

$\frac{38.95}{2\pi r} = \frac{13}{18}$

$701.1 = 26\pi r$

8.58 units $\approx r$

24. $P = 2$ lengths + circumference of the circle

$= 2(13) + \pi(6) \approx 44.85$ units

25. $P = 2$ lengths + $\frac{1}{2}$(circumference of the circle)

$= 2(6) + \frac{1}{2}(2\pi(3)) \approx 21.42$ units

26. $x^2 + y^2 = 16$ has a radius of $\sqrt{16} = 4$.

$C = 2\pi r = 2\pi(4) = 8\pi$

27. $(x + 2)^2 + (y - 3)^2 = 9$ has a radius of $\sqrt{9} = 3$.

$C = 2\pi r = 2\pi(3) = 6\pi$

28. $x^2 + y^2 = 18$ has a radius of $\sqrt{18} = 3\sqrt{2}$.

$C = 2\pi r = 2\pi(3\sqrt{2}) = 6\sqrt{2}\pi$

29. $C = 2\pi r$ | $C = \pi d$

$\frac{C}{2\pi} = r$ | $\frac{C}{\pi} = d$

When $C = 26\pi$: | When $C = 26\pi$:

$\frac{26\pi}{2\pi} = r$ | $\frac{26\pi}{\pi} = d$

$13 = r$ | $26 = d$

30. When $m\widehat{AB} = 45°$ and length of $\widehat{AB} = 4$:

$4 = \frac{45°}{360°} \cdot 2\pi r$

$5.09 \approx r$

When $r = 2$ and $m\widehat{AB} = 60°$:

Arc length of $\widehat{AB} = \frac{60°}{360°} \cdot 2\pi(2) \approx 2.09$

When $r = 0.8$ and length of $\widehat{AB} = 0.3$:

$\frac{0.3}{2\pi(0.8)} = \frac{m\widehat{AB}}{360°}$

$360° \cdot \frac{0.3}{2\pi(0.8)} = m\widehat{AB}$

$21.49° \approx m\widehat{AB}$

When $r = 4.2$ and $m\widehat{AB} = 183°$:

Arc length of $\widehat{AB} = \frac{183°}{360°} \cdot 2\pi(4.2) \approx 13.41$

When $m\widehat{AB} = 90°$ and length of $\widehat{AB} = 3.22$:

$3.22 = \frac{90°}{360°} \cdot 2\pi r$

$2.05 \approx r$

When $r = 4\sqrt{2}$ and length of $\widehat{AB} = 2.86$:

$\frac{2.86}{2\pi(4\sqrt{2})} = \frac{m\widehat{AB}}{360°}$

$360° \cdot \frac{2.86}{2\pi(4\sqrt{2})} = m\widehat{AB}$

$28.97° \approx m\widehat{AB}$

Radius	5.09	2	0.8	4.2	2.05	$4\sqrt{2}$
$m\widehat{AB}$	45°	60°	21.49°	183°	90°	28.98°
Length of $\widehat{AB}$	4	2.09	0.3	13.41	3.22	2.86

31. When $m\widehat{EF} = x°$ and $r = r$: Arc length of $\widehat{EF} = \frac{x°\pi r}{180°}$

a. double the radius: Arc length of $\widehat{EF} = \frac{x°}{360°} \cdot 2\pi(2r)$

Arc length of $EF = \frac{x°\pi r}{90°}$

Because $\frac{x°\pi r}{90°}$ is twice as large as $\frac{x°\pi r}{180°}$, the length of $\widehat{EF}$ is twice as large when the radius is doubled.

b. If you double the measure of $\widehat{EF}$:

Arc length of $\widehat{EF} = \frac{2x°}{360} \cdot 2\pi r$

Arc length of $\overset{\frown}{EF} = \frac{x°\pi r}{90°}$

Because $\frac{x°\pi r}{90°}$ is twice as large as $\frac{x°\pi r}{180°}$, the length of $\overset{\frown}{EF}$ is twice as large when the measure of $\overset{\frown}{EF}$ is doubled.

32. A;

$m\overset{\frown}{XY} + m\overset{\frown}{YZ} = 180°$ because $\overline{XZ}$ is the diameter of the circle.

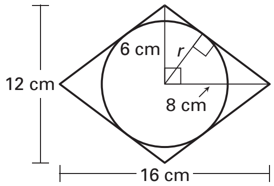
$140° + m\overset{\frown}{YZ} = 180°$

$m\overset{\frown}{TZ} = 40°$

The diameter is 6, so the radius is $6 \div 2 = 3$.

Arc length of $\overset{\frown}{YZ} = \frac{m\overset{\frown}{YZ}}{360°} \cdot 2\pi r = \frac{40°}{360°} \cdot 2\pi(3) = \frac{2}{3}\pi$

33.

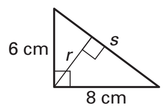


A rhombus has diagonals that bisect each other. The rhombus can also be split into 4 congruent triangles. Look at one such triangle.
$\left(\text{with legs } \frac{16}{2} = 8 \text{ and } \frac{12}{2} = 6\right)$. Use the Pythagorean Theorem to find the side length.

$6^2 + 8^2 = s^2$

$100 = s^2$

$10 = s$



Now look at the radius of the circle and break the original right triangle into two smaller right triangles. Use the Pythagorean Theorem and a system of equations to find the length of the radius.

$r^2 + (10 - x)^2 = 64$

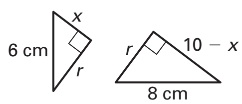
$\underline{- r^2 + x^2 \qquad = 36}$

$(10 - x)^2 - x^2 = 28$

$100 - 20x + x^2 - x^2 = 28$

$72 = 20x$

$3.6 = x$



Now substitute 3.6 for x to find r.

$r^2 + (3.6)^2 = 36$

$r^2 + 12.96 = 36$

$r^2 = 23.04$

$r = 4.8$

You have the radius, so you can now find the circumference.

$C = 2\pi r = 2\pi(4.8) \approx 30.16$

The circumference of a circle inscribed in a rhombus with diagonals that are 12 centimeters and 16 cenitmeters long is about 30.16 centimeters.

34. Because you know the measure of the shaded red angle is 30° and the arc length is 2, you can set up a proportion for the circumference of the blue circle.

$\frac{\text{Arc length } \overset{\frown}{\text{blue}}}{C \text{ blue}} = \frac{m\overset{\frown}{\text{blue}}}{360°}$

In Exercise 31, you found that when you double the radius you double the arc length. Because the radius of the blue circle is double that of the red circle, the arc length is twice as big, or $2(2) = 4$. Because the angle measure of the blue angle and the shaded red angle are the same, $m\overset{\frown}{\text{blue}} = 30°$. So, substituting those values in the proportion, you get:

$\frac{4}{C \text{ blue}} = \frac{30°}{360°}$

$48 = C \text{ blue}$

So, the circumference of the blue circle is 48 units.

Problem Solving

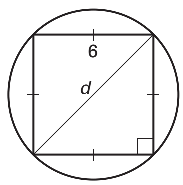
35. The rope length, 21 feet 8 inches, represents the circumferencec of the tree. If you divide the circumference by π, you will get the diameter of the tree.

21 feet 8 inches $= 21\frac{2}{3}$ feet

$21\frac{2}{3}$ feet $\div \pi \approx 6.9$

The diameter of the tree is about 7 feet.

36.



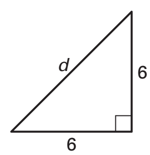
To find the diameter, use the Pythagorean Theorem.

$6^2 + 6^2 = d^2$

$72 = d^2$

$6\sqrt{2} = d$

$C = \pi d = \pi(6\sqrt{2}) \approx 26.66$



The circumference of the circle is about 26.66 units.

37. Find the circumference of the tire:

$C = \pi d = \pi(8 \text{ in.}) \approx 25.13$ in.

length of path = number of rotations • circumference

$= 87 \cdot 25.113 \approx 2186.31$

The length of the path is about 2186 inches long.

38. **a.** The chain touches about half of each sprocket and has an additional two lengths of $6\frac{9}{16}$ inches. Find half of each sprocket's circumference, add them together and then add $2\left(6\frac{9}{16}\right)$ to that.

larger sprocket: $C = \pi d = \pi\left(6\frac{1}{8}\right) \approx 19.24$

So, half of the circumference of the larger sprocket is about $\frac{1}{2}(19.24) = 9.62$ in.

smaller sprocket: $C = \pi d = \pi\left(1\frac{7}{16}\right) \approx 4.52$

So, half of the circumference of the smaller sprocket is about $\frac{1}{2}(4.52) = 2.26$ in.

length of chain $= 9.62 + 2.26 + 2\left(6\frac{9}{16}\right) \approx 25$

The length of the chain is about 25 inches.

b. The chain touches about half of each sprocket, so only half of the teeth on each sprocket are gripping the chain at any given time.

Half of the teeth on the larger sprocket: $\frac{1}{2}(76) = 38$

Half of the teeth on the smaller sprocket: $\frac{1}{2}(15) = 7.5$

Total teeth $= 38 + 7.5 = 45.5$

There are about 46 teeth gripping the chain at any given time.

39. Because $\ell_1 \parallel \ell_2$, $\angle 1 \cong \angle 2$ by the Alternate Interior Angles Theorem. So, $m\angle 1 = 7.2°$.

$$\frac{\text{distance from Alexandra to Syene}}{C} = \frac{m\angle 1}{360°}$$

$$\frac{575}{C} = \frac{7.2°}{360°}$$

$$28{,}750 \approx C$$

The Earth's circumference is about 28,750 miles.

40. Because each arc is a half-circle, the measure of each arc is 180°.

length of each arc $= \frac{m \text{ each arc}}{360°} \cdot 2\pi r$

length of each arc $= \frac{180°}{360°} \cdot 2\pi r$

length of each arc $= \pi r$

Because there are 4 arcs, the sum of the arc lengths is $4\pi r$.

41. 8 segments: length of each arc $= \frac{m \text{ each arc}}{360°} \cdot \pi d$ where $d = r$

length of each arc $= \frac{180°}{360°} \cdot \pi r$

length of each arc $= \frac{1}{2}\pi r$

Because there are 8 arcs, the sum of all of their arc lengths is $8\left(\frac{1}{2}\pi r\right)$ or $4\pi r$.

16 segments: lengths of each arc $= \frac{m \text{ each arc}}{360°} \cdot \pi d$ where $d = \frac{1}{2}r$

length of each arc $= \frac{180°}{360°} \cdot \pi\left(\frac{1}{2}r\right)$

length of each arc $= \frac{1}{4}\pi r$

Because there are 16 arcs, the sum of all their arc lengths is $\left(16\frac{1}{4}\pi\right)r$, or $4\pi r$.

The sum of the arc lengths for 8 segments is $4\pi r$, the sum for the arc lengths for 16 segments is $4\pi r$, and the sum of the arc legnths for n segments is $4\pi r$. The length will be the same, you just have to allocate the radius differently according to how many segments you have.

Extension for the lesson "Circumference and Arc Length"

Activity

Step 1.

Radius	Arc Length Calculation
1	$\frac{m\widehat{AB}}{360°} \cdot 2\pi r = \frac{60°}{360°} \cdot 2\pi \cdot 1 = \frac{\pi}{3}$
2	$\frac{m\widehat{AB}}{360°} \cdot 2\pi r = \frac{60°}{360°} \cdot 2\pi \cdot 2 = \frac{2\pi}{3}$
3	$\frac{m\widehat{AB}}{360°} \cdot 2\pi r = \frac{60°}{360°} \cdot 2\pi \cdot 3 = \pi$
4	$\frac{m\widehat{AB}}{360°} \cdot 2\pi r = \frac{60°}{360°} \cdot 2\pi \cdot 4 = \frac{4\pi}{3}$

Step 2. The arc length y is of the form $y = k \cdot r$, with $k = \frac{60°}{360°} \cdot 2\pi$.

Step 3. Since all circles are similar, if the radius is doubled, the arc length doubles.

Practice

1. arc length of $\widehat{AB} = \frac{m\widehat{AB}}{360°} \cdot 2\pi r$

$= \frac{120°}{360°} \cdot 2\pi(4)$

$= \frac{1}{3} \cdot 8\pi$

$= \frac{8\pi}{3}$

2. arc length of $\widehat{AB} = \frac{m\widehat{AB}}{360°} \cdot 2\pi r$

$= \frac{135°}{360°} \cdot 2\pi(1.5)$

$= \frac{3}{8} \cdot 3\pi$

$= \frac{9\pi}{8}$

3. arc length of $\widehat{AB} = \frac{m\widehat{AB}}{360°} \cdot 2\pi r$

$= \frac{240°}{360°} \cdot 2\pi(3)$

$= \frac{2}{3} \cdot 6\pi$

$= 4\pi$

4. degree measure $\cdot \frac{2\pi}{360°} =$ radian measure

$15° \cdot \frac{2\pi}{360°} = \frac{30\pi}{360}$

$= \frac{\pi}{12}$ radians

5. degree measure $\cdot \frac{2\pi}{360°} =$ radian measure

$70° \cdot \frac{2\pi}{360°} = \frac{140\pi}{360}$

$= \frac{7\pi}{18}$ radians

6. degree measure $\cdot \frac{2\pi}{360°}$ = radian measure

$$300° \cdot \frac{2\pi}{360°} = \frac{600\pi}{360}$$
$$= \frac{5\pi}{3} \text{ radians}$$

7. radian measure $\cdot \frac{360°}{2\pi}$ = degree measure

$$\frac{4\pi}{3} \cdot \frac{360°}{2\pi} = \frac{1440°}{6}$$
$$= 240°$$

8. radian measure $\cdot \frac{360°}{2\pi}$ = degree measure

$$\frac{11\pi}{12} \cdot \frac{360°}{2\pi} = \frac{3960°}{24}$$
$$= 165°$$

9. radian measure $\cdot \frac{360°}{2\pi}$ = degree measure

$$\frac{\pi}{8} \cdot \frac{360°}{2\pi} = \frac{360°}{16}$$
$$= 22.5°$$

10. $30° \cdot \frac{2\pi}{360°} = \frac{60\pi}{360} = \frac{\pi}{6}$

$$45° \cdot \frac{2\pi}{360°} = \frac{90\pi}{360} = \frac{\pi}{4}$$
$$\frac{\pi}{3} \cdot \frac{360°}{2\pi} = \frac{360}{6} = 60°$$
$$\frac{\pi}{2} \cdot \frac{360°}{2\pi} = \frac{360}{4} = 90°$$
$$120° \cdot \frac{2\pi}{360°} = \frac{240\pi}{360} = \frac{2\pi}{3}$$
$$\frac{3\pi}{4} \cdot \frac{360°}{2\pi} = \frac{1080}{8} = 135°$$
$$\pi \cdot \frac{360°}{2\pi} = \frac{360}{2} = 180°$$
$$\frac{3\pi}{2} \cdot \frac{360°}{2\pi} = \frac{1080}{4} = 270°$$
$$360° \cdot \frac{2\pi}{360°} = \frac{720\pi}{360} = 2\pi$$

Degrees	30°	45°	60°	90°	120°
Radians	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$

Degrees	135°	180°	270°	360°
Radians	$\frac{3\pi}{4}$	π	$\frac{3\pi}{2}$	2π

11. $s = r \cdot \theta$

$s = 4 \cdot \left(\frac{3\pi}{4}\right) = 3\pi$ in. or about 9.42 in.

Lesson 11.2 Areas of Circles and Sectors

Guided Practice for the lesson "Areas of Circles and Sectors"

1. $A = \pi r^2 = \pi(14)^2 \approx 615.75$

   The area of $\odot D$ is about 615.75 square feet.

2. Area of red sector $= \frac{m\widehat{FE}}{360°} \cdot \pi r^2$

$$= \frac{120°}{360°} \cdot \pi \cdot 14^2 \approx 205.25$$

   The area of the red sector is about 205.25 square feet.

3. Area of the blue sector $= \frac{m\widehat{FGE}}{360°} \cdot \pi r^2$

$$= \frac{360° - 120°}{360°} \cdot \pi \cdot 14^2 \approx 410.50$$

   The area of the blue sector is about 410.50 square feet.

4. Area of sector $FJG = \frac{m\widehat{FG}}{360°} \cdot$ Area of $\odot H$

$$214.37 = \frac{85°}{360°} \cdot \text{Area of } \odot H$$
$$907.92 = \text{Area of } \odot H$$

   The area of $\odot H$ is 907.92 square centimeters.

5. Area of figure = Area of semicircle + Area of triangle

$$= \frac{180°}{360°} \cdot \left(\pi \cdot \left(\frac{7}{2}\right)^2\right) + \frac{1}{2}(7)(7)$$
$$= 6.125\pi + 24.5 \approx 43.74$$

   The area of the figure is about 43.74 square meters.

6. Yes, you can find the measure of the intercepted arc if you know the area and radius of a sector of the circle. The formula for the area of a sector is

   Area of sector $= \frac{\text{measure of arc}}{360°} \cdot \pi r^2$. If you solve for the measure of the arc, you get:

$$\frac{\text{Area of circle}}{\pi r^2} = \frac{\text{measure of arc}}{360°}$$
$$\frac{360° \cdot \text{Area of circle}}{\pi r^2} = \text{measure of arc}$$

Exercises for the lesson "Areas of Circles and Sectors"

Skill Practice

1. A *sector* of a circle is the region bounded by two radii of the circle and their intercepted arc.
2. Doubling the arc measure of a sector in a given circle will double its area. When you double the arc measure, you make the sector twice as big, which means that the area also doubles.
3. $A = \pi r^2 = \pi \cdot 5^2 = 25\pi \approx 78.54$

   The area of the circle is exactly 25π square inches, which is about 78.54 square inches.
4. The radius of a circle is half of the diameter, so the radius is $\frac{1}{2}(16) = 8$ ft.

   $A = \pi r^2 = \pi \cdot 8^2 = 64\pi \approx 201.06$

   The area of the circle is exactly 64π square feet, which is about 201.06 square feet.

5. The radius of a circle is half of the diameter, so the radius is $\frac{1}{2}(23) = 11.5$ cm.

$A = \pi r^2 = \pi(11.5)^2 = 132.25\pi \approx 415.48$

The area of the circle is exactly 132.25π square centimeters, which is about 415.48 square centimeters.

6. $A = \pi r^2 = \pi \cdot (1.5)^2 = 2.25\pi \approx 7.07$

The area of the circle is exactly 2.25π square kilometers, which is about 7.07 square kilometers.

7. $A = \pi r^2$

$154 = \pi r^2$

$\frac{154}{\pi} = r^2$

$7.00 \approx r$

The radius is about 7 meters.

8. $A = \pi r^2$

$380 = \pi r^2$

$\frac{380}{\pi} = r^2$

$11.00 \approx r$

The radius is about 11 meters.

9. $A = \pi r^2$

$676\pi = \pi r^2$

$\frac{676\pi}{\pi} = r^2$

$26 = r$

The radius is 26 centimeters, so the diameter is 52 centimeters.

10. The area of sector XZY should be divided by the area of the circle, not 360°. Also, the right side should be the measure of the sector divided by 360°, or $\frac{75^\circ}{360^\circ}$.

$\frac{n}{48} = \frac{75^\circ}{360^\circ}$

$n = 10$

The area of sector XZY is 10 square feet.

11. Area of small sector $= \frac{m\widehat{ED}}{360^\circ} \cdot \pi r^2$

$= \frac{60^\circ}{360^\circ} \cdot \pi \cdot 10^2 \approx 52.36$

Area of large sector $= \frac{m\widehat{EGD}}{360^\circ} \cdot \pi r^2$

$= \frac{360^\circ - 60^\circ}{360^\circ} \cdot \pi \cdot 10^2 \approx 261.80$

The areas of the small and large sectors are about 52.36 square inches and 261.80 square inches, respectively.

12. Area of small sector $= \frac{m\widehat{ED}}{360^\circ} \cdot \pi r^2$

$= \frac{360^\circ - 256^\circ}{360^\circ} \cdot \pi \cdot 14^2$

≈ 177.88

Area of large sector $= \frac{m\widehat{EGD}}{360^\circ} \cdot \pi r^2$

$= \frac{256^\circ}{360^\circ} \cdot \pi \cdot 14^2 \approx 437.87$

The areas of the small and large sectors are about 177.88 square centimeters and 437.87 square centimeters, respectively.

13. Area of small sector $= \frac{m\widehat{DE}}{360^\circ} \cdot \pi r^2$

$= \frac{137^\circ}{360^\circ} \cdot \pi \cdot 28^2$

≈ 937.31

Area of large sector $= \frac{m\widehat{EGD}}{360^\circ} \cdot \pi r^2$

$= \frac{360^\circ - 137^\circ}{360^\circ} \cdot \pi \cdot 28^2 \approx 1525.70$

The areas of the small and large sectors are about 937.31 square meters and 1525.70 square meters, respectively.

14. Area of sector $JK = \frac{m\widehat{JK}}{360^\circ} \cdot$ Area of $\odot M$

$38.51 = \frac{165^\circ}{360^\circ} \cdot$ Area of $\odot M$

$84.02 \approx$ Area of $\odot M$

The area of $\odot M$ is about 84.02 square meters.

15. Area of sector $KLJ = \frac{m\widehat{JLK}}{360^\circ} \cdot$ Area of $\odot M$

$56.87 = \frac{360^\circ - 50^\circ}{360^\circ} \cdot$ Area of $\odot M$

$66.04 \approx$ Area of $\odot M$

The area of $\odot M$ is about 66.04 square centimeters.

16. Area of sector $JK = \frac{m\widehat{JK}}{360^\circ} \cdot \pi r^2$

$12.36 = \frac{89^\circ}{360^\circ} \cdot \pi r^2$

$\frac{50}{\pi} = r^2$

$3.99 \approx r$

The radius of $\odot M$ is about 3.99 meters.

17. Area of shaded region = Area of square − 2(Area of semicircle)

$= 6(6) - 2\left(\frac{180^\circ}{360^\circ}(\pi \cdot 3^2)\right)$

$= 36 - 9\pi \approx 7.73$

The area of the shaded region is about 7.73 square meters.

18. Area of shaded region = Area of trapezoid − Area of semicircle

$= \frac{1}{2}(8)(16 + 20)$

$- \left(\frac{180^\circ}{360^\circ} \cdot (\pi \cdot 4^2)\right)$

$= 144 - 8\pi \approx 118.87$

The area of the shaded region is about 118.87 square inches.

19. A;

Area of the putting green = Area of square + Area of sector of circle + Area of rectangle

$= 3.5^2 + \left(\frac{90^\circ}{360^\circ} \cdot (\pi(3.5)^2)\right)$

$+ 7(3.5)$

$\approx 12.25 + 9.62 + 24.5 \approx 46.37$

The area of the putting green is about 46 square feet.

20. $A = \pi r^2$

$260.67 = \pi r^2$

$\frac{260.67}{\pi} = r^2$

$9.11 \approx r$

The radius of $\odot M$ is about 9.11 inches.

21. $C = 2\pi r \approx 2\pi(9.11) \approx 57.24$

The circumference of $\odot M$ is about 57.24 inches.

22. Area of sector $KML = \frac{m\widehat{KL}}{360°} \cdot$ Area of $\odot M$

$42 = \frac{m\widehat{KL}}{360°} \cdot 260.67$

$58° \approx m\widehat{KL}$

$m\widehat{KL}$ is about 58°.

23. Perimeter of blue region = arc length of $\widehat{KNL}$ + 2(radius)

$= \frac{m\widehat{KNL}}{360°} \cdot 2\pi r + 2r$

$\approx \frac{360° - 58°}{360°} \cdot 2\pi(9.11) + 2(9.11)$

$\approx 48.02\pi + 18.22 \approx 66.24$

The perimeter of the blue region is about 66.24 inches.

24. Arc length of $KL = \frac{m\widehat{KL}}{360°} \cdot 2\pi r$

$\approx \frac{58°}{360°} \cdot 2\pi(9.11) \approx 9.22$

The length of $\widehat{KL}$ is about 9.22 inches.

25. Perimeter of red region = length of $\widehat{KL}$ + 2(radius)

$\approx 9.22 + 2(9.11) \approx 27.44$

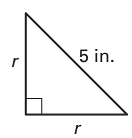
The perimeter of the red region is about 27.44 inches.

26. Use the Pythagorean Theorem to find the radius of the circle and base and height of the triangle.

$r^2 + r^2 = 5^2$

$2r = 25$

$r = \frac{5\sqrt{7}}{2}$



Area of shaded region = Area of circle − Area of triangle

$= \pi \cdot \left(\frac{5\sqrt{2}}{2}\right)^2 - \frac{1}{2}\left(\frac{5\sqrt{2}}{2}\right)$

$\approx 12.5\pi - 12.5 \approx 26.77$

The area of the shaded region is about 26.77 square inches.

27. Area of shaded region = 2(Area of 1 shaded sector)

$= 2\left(\frac{m\widehat{\text{shaded arc}}}{360°} \cdot \pi r^2\right)$

$= 2\left(\frac{180° - 109°}{360°} \cdot \pi \cdot (5.2)^2\right)$

$\approx 2(16.75) \approx 33.51$

The area of the shaded region is about 33.51 square feet.

28. Area of shaded region = Area of square − 4(Area of 1 circle)

$= 20^2 - 4(\pi \cdot (5)^2)$

$= 400 - 100\pi \approx 85.84$

The area of the shaded region is about 85.84 square inches.

29. Area of shaded region = Area of bigger semicircle − Area of smaller semicircle

$= \left[\frac{180°}{360°} \cdot (\pi \cdot (17 + 17)^2\right]$

$- \left[\frac{180°}{360°} \cdot (\pi \cdot 17^2)\right]$

$= 578\pi - 144.5\pi \approx 1361.88$

The area of the shaded region is about 1361.88 square centimeters.

30. Area of shaded region = Area of largest circle

− Area of second largest circle

+ Area of third largest circle

− Area of smallest circle

$= \pi \cdot (2 + 2 + 2 + 2)^2$

$- \pi \cdot (2 + 2 + 2)^2$

$+ \pi \cdot (2 + 2)^2 - \pi(2)^2$

$= 64\pi - 36\pi + 16\pi$

$- 4\pi \approx 125.66$

The area of the shaded region is about 125.66 square feet.

31. Both triangles are right triangles by Theorem 10.9.

Use the Pythagorean Theorem to find the length of the diameter of the circle

$3^2 + 4^2 = d^2$

$25 = d^2$

$5 = d$

Area of shaded region = Area of circle • 2(Area of triangles)

$= \pi\left(\frac{5}{2}\right)^2 - 2\left(\frac{1}{2}(3)(4)\right)$

$= 6.25\pi - 12 \approx 7.63$

The area of the shaded region is about 7.63 square meters.

32. Area of the red region $= \frac{m\widehat{RS}}{360°} \cdot \pi r^2$

$= \frac{108°}{360°} \cdot \pi \cdot (4 + 4)^2 = 19.2\pi$

Area of the blue region $= \pi r^2 = \pi \cdot 4^2 = 16\pi$

Area of the yellow region = Area of $\odot P$

− Area of red region

− Area of blue region

$= \pi \cdot (4 + 4)^2 - 19.2\pi$

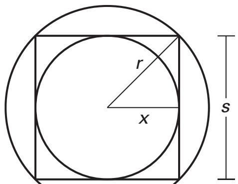
$- 16\pi = 28.8\pi$

The area of the red region is 19.2π square units, the area of the blue region is 16π square units, and the area of the yellow region is 28.8π square units.

33. Rewrite of the Perimeters of Similar Polygons Theorem: For any two circles, the ratio of their circumferences is equal to the ratio of their corresponding radii. Rewrite of the Area of Similar Polygons Theorem: For any two circles, if the length of their radii is in the ratio of $a:b$, then the ratio of their areas is $a^2:b^2$. All circles are similar, so you do not need to add that the circles must be similar.

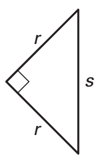
34. These sectors are not similar, therefore the student shouldn't have used Areas of Similar Polygons Theorem. The correct ratio of the arcs of the sectors bounded by these areas is 2 : 1.

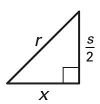
35.



radius of large circle = r, radius of small circle = x, side of square = s

Use the Pythagorean Theorem to find the radius of the small circle in terms of r.





$$r^2 + r^2 = s^2$$

$$2r^2 = s^2$$

$$r\sqrt{2} = s$$

$$\left(\frac{r\sqrt{2}}{2}\right)^2 + x^2 = r^2$$

$$x^2 = r^2 - \frac{r^2}{2}$$

$$x^2 = \frac{1}{2}r^2$$

$$x = \sqrt{\frac{1}{2}r^2}$$

$$x = \frac{r\sqrt{2}}{2}$$

$$\frac{\text{Area of large circle}}{\text{Area of small circle}} = \frac{\pi r^2}{\pi x^2} = \frac{\pi r^2}{\pi\left(\frac{r\sqrt{2}}{2}\right)^2} = \frac{\pi r^2}{\frac{1}{2}\pi r^2} = \frac{1}{\frac{1}{2}} = \frac{2}{1}$$

The ratio of the area of the large circle to the area of the small circle is 2 : 1.

36. Let r be the radius of the smaller circle.

$$\text{Length of } \overset{\frown}{FG} = \frac{m\overset{\frown}{FG}}{360^\circ} \cdot 2\pi r$$

$$10 = \frac{m\overset{\frown}{FG}}{360^\circ} \cdot 2\pi r$$

$$\frac{5}{\pi r} = \frac{m\overset{\frown}{FG}}{360^\circ}$$

$$\text{Length of } \overset{\frown}{EH} = \frac{m\overset{\frown}{EH}}{360^\circ} \cdot 2\pi r$$

$$30 = \frac{m\overset{\frown}{EH}}{360^\circ} \cdot 2\pi(8 + r)$$

$$\frac{15}{\pi(8 + r)} = \frac{m\overset{\frown}{EH}}{360^\circ}$$

Because $m\overset{\frown}{FG} = m\overset{\frown}{EH}$, set their equations equal to each other to find the length of the radius.

$$\frac{5}{\pi r} = \frac{15}{\pi(8 + r)}$$

$$5\pi(8 + r) = 15\pi r$$

$$8 + r = 3r$$

$$8 = 2r$$

$$4 = r$$

Now substitute $4 = r$ to find $m\overset{\frown}{FG}$.

$$10 = \frac{m\overset{\frown}{FG}}{360^\circ} \cdot 2\pi(4)$$

$$\frac{10}{2(\pi)4} = \frac{m\overset{\frown}{FG}}{360^\circ}$$

$$143.2^\circ \approx m\overset{\frown}{FG}$$

So, $m\overset{\frown}{FD} = m\overset{\frown}{EH} \approx 143.2^\circ$.

Area of shaded region = Area of larger sector − Area of smaller sector

$$= \frac{143.2^\circ}{360^\circ} \cdot \pi \cdot (8 + 4)^2 - \frac{143.2^\circ}{360^\circ} \cdot \pi \cdot 4^2$$

$$\approx 160$$

So, the area of the shaded region is about 160 square meters.

Problem Solving

37. The radius for the eye of the hurricane is $\frac{1}{2}(20) = 10$ miles.

$A = \pi r^2 = \pi \cdot 10^2 \approx 314.16$

The area of land underneath the eye is about 314.16 square miles.

38. $A = \pi r^2$

$$13{,}656 = \pi r^2$$

$$\frac{13{,}656}{\pi} = r^2$$

$$210.08 \approx r$$

The distance you walk around the edge is the circumference of the pond.

$C = 2\pi r \approx 2\pi(210.08) \approx 1319.97$

You walk about 1320 feet.

39. **a.** A circle graph is appropriate because the data is already in percentages that add up to 100%.

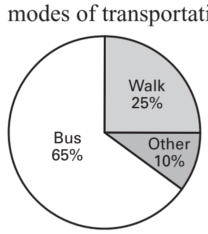
b. percentage • (360°) = central angle

Bus: 0.65 • (360°) = 234°

Walk: 0.25 • (360°) = 90°

Other: 0.10 • (360°) = 36°

The central angle for the bus is 234°, the central angle for walking is 90°, and the central angle for other modes of transportation is 36°.



c. Area of sector $= \frac{\text{Measure of arc}}{360°} \cdot \pi r^2$

Bus: $\frac{234°}{360°} \cdot \pi(2)^2 \approx 8.17$

Walk: $\frac{90°}{360°} \cdot \pi(2)^2 = \pi$

Other: $\frac{36°}{360°} \cdot \pi(2)^2 \approx 1.26$

The area of the bus sector is about 8.17 square inches, the area of the walking sector is π square inches, and the area of the sector for other modes of transportation is about 1.26 square inches.

40. The area of a tortilla with a 6 inch diameter is $\pi\left(\frac{6}{2}\right)^2 = 9\pi$. The area of a tortilla with a 12 inch diameter is $\pi\left(\frac{12}{2}\right)^2 = 36\pi$. Because the 12 inch tortilla is 4 times larger, you need 4 times as much dough. So, you need $4\left(\frac{1}{4}\right) = 1$ cup of dough to make a tortilla with a 12 inch diameter.

41. a. *Sample answer*:

old "a" $= \frac{1}{2}(\pi \cdot 8^2) + \frac{1}{2}(12 + 19)(10 + 16) \approx 374$

new "a" $= \pi(14)^2 + 3(22) \approx 682$

The area of the old "a" is about 374 square units, the area of the new "a" is about 682 square units, and the percentage increase in interior area is about 82%.

b. area of old "a": $75.5(66) = 4983$

area of new "a": $85(76) = 6460$

percent increase $= \frac{6460 - 4983}{4983} \approx 0.296 \approx 30\%$

Sample answer: No; the percent increase of the area of "a" is about 30%, which is much less than the percent increase of the interior area.

42. Area of 12-inch diameter pizza: $\pi\left(\frac{12}{2}\right)^2 = 36\pi \approx 113.10$

Because a 12-inch diameter pizza has an area of about 113 square inches and feeds you and two friends, each person eats about $113 \div 3 = 38$ square inches of pizza.

Area of 10-inch diameter pizza: $\pi\left(\frac{10}{2}\right)^2 = 25\pi \approx 78.54$

Area of 14-inch diameter pizza: $\pi\left(\frac{14}{2}\right)^2 = 49\pi \approx 153.94$

If you want to feed yourself and 7 friends, you need about $8(38) = 304$ square inches of pizza.

a. In order to have 304 square inches of pizza you could buy four 10-inch pizzas, two 14-inch pizzas, or two 10-inch pizzas and one 14-inch pizza.

Price of four 10-inch pizzas = 4($6.99) = $27.96

Price of two 14-inch pizzas = 2($12.99) = $25.98

Price of two 10-inch pizzas and one 14-inch pizza = 2($6.99) + $12.99 = $26.97

To spend as little money as possible, buy two 14-inch pizzas.

b. You should buy two 10-inch pizzas and one 14-inch pizza. Two 10-inch pizzas and one 14-inch pizza is three pizzas total, the amount of pizza you need, and is cheaper than than three 14-inch pizzas.

c. Circumference of four 10-inch pizzas $= 4\left(2 \cdot \pi \cdot \frac{10}{2}\right) = 40\pi \approx 125.66$

Circumference of two 14-inch pizzas $= 2\left(2 \cdot \pi \cdot \frac{14}{2}\right) = 28\pi \approx 87.96$

Circumference of two 10-inch pizzas and one 14-inch pizza $= 2\left(2 \cdot \pi \cdot \frac{10}{2}\right) + \left(2\pi \cdot \frac{14}{2}\right) = 34\pi \approx 106.81$

You should buy four 10-inch pizzas because the circumference is greatest, which means more crust.

43. a. The height is equal to the radius, the base is equal to half of the circumference and the area is the base times the height.

$h = r,\ b = \frac{1}{2}(2\pi r) = \pi r,\ r \cdot \pi r = \pi r^2$

b. *Sample answer:* When you cut the circle into 16 congruent sectors, you are not losing area. When you rearrange the 16 pieces of the circle to make a parallelogram, you can determine that the area is πr^2. Because no area is lost when you make the circle into a parallelogram, the area of the circle will be the same as the area of the parallelogram, or πr^2.

44. Let black diameter $= x$, blue diameter $= y$, and red diameter $= z$. Using Pythagorean Theorem, you can say $x^2 + y^2 = z^2$.

Area of triangle $= \frac{1}{2}xy$

Area of semicircle $z = \frac{180°}{360°} \cdot \left(\pi \cdot \left(\frac{z}{2}\right)^2\right) = \frac{\pi z^2}{8}$

Area of semicircle $y = \frac{180°}{360°} \cdot \left(\pi \cdot \left(\frac{y}{2}\right)^2\right) = \frac{\pi y^2}{8}$

Area of semicircle $x = \frac{180°}{360°} \cdot \left(\pi \cdot \left(\frac{x}{2}\right)^2\right) = \frac{\pi x^2}{8}$

Area of non-shaded areas = Area of semicircle z − Area of triangle

$$= \frac{\pi z^2}{8} - \frac{1}{2}xy$$

Area of shaded crescent = Area of semicircle y + Area of semicircle x − Area of non-shaded areas

$$= \frac{\pi y^2}{8} + \frac{\pi x^2}{8} - \left(\frac{\pi z^2}{8} - \frac{1}{2}xy\right)$$

$$= \frac{\pi(y^2 + x^2)}{8} - \left(\frac{\pi z^2}{8} + \frac{1}{2}xy\right)$$

$$= \frac{\pi z^2}{8} - \frac{\pi z^2}{8} + \frac{1}{2}xy = \frac{1}{2}xy$$

So, because $\frac{1}{2}xy = \frac{1}{2}xy$, the sum of the area of the two shaded crescents equals the area of the triangle.

Quiz for the lessons "Circumference and Arc Length" and "Areas of Circles and Sectors"

1. Arc length of $\widehat{AB} = \frac{m\widehat{AB}}{360°} \cdot 2\pi r = \frac{78°}{360°} \cdot 2\pi\left(\frac{14}{2}\right) \approx 9.53$

The length of $\widehat{AB}$ is about 9.53 meters.

2. $\frac{\text{Arc length of } \widehat{GE}}{C} = \frac{m\widehat{GE}}{360°}$

$\frac{36}{C} = \frac{102°}{360°}$

$127.06 \approx C$

The circumference of $\odot F$ is about 127.06 inches.

3. $\frac{\text{Arc length of } \widehat{JK}}{2\pi r} = \frac{m\widehat{JK}}{360°}$

$\frac{29}{2\pi r} = \frac{65°}{360°}$

$10{,}440 = 130\pi r$

$25.56 \approx r$

The radius of $\odot L$ is about 25.56 feet.

4. Area of shaded region = Area of larger circle − Area of smaller circle

$= \pi \cdot 33^2 - \pi \cdot 11^2$

$= 968\pi \approx 3041.06$

The area of the shaded region is about 3041.06 square meters.

5. Area of shaded region = 2(Area of each shaded sector)

$= 2\left(\frac{63°}{360°} \cdot \pi \cdot (8.7)^2\right) \approx 83.23$

The area of the shaded region is about 83.23 square inches.

6. Area of shaded region = Area of circle − Area of rhombus

$= \pi \cdot \left(\frac{6}{2}\right)^2 - \frac{1}{2}(3)(6)$

$= 9\pi - 9 \approx 19.27$

The area of the shaded region is about 19.27 square centimeters.

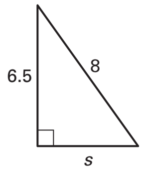
Lesson 11.3 Areas of Regular Polygons

Guided Practice for the lesson "Areas of Regular Polygons"

1. The center of the polygon is P, a radius is $\overline{PY}$ or $\overline{PX}$, the apothem is $\overline{PQ}$, and the central angle is $\angle XPY$.

2. $m\angle XPY = \frac{360°}{4} = 90°$, $m\angle XPQ = \frac{1}{2} m\angle XPY = 45°$, and $m\angle PXQ = 180° - 90° - 45° = 45°$

3. Use the Pythagorean Theorem to find the base of the triangle.



$s^2 + 6.5^2 = 8^2$

$s^2 = 21.75$

$s \approx 4.664$

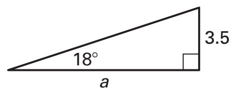
The base of the triangle is about 4.664, so the length of one side of the pentagon is about 2(4.664) = 9.328. A pentagon has 5 sides, so $P \approx 5(9.328) = 46.64$ units.

$A = \frac{1}{2}aP \approx \frac{1}{2}(6.5)(46.64) = 151.58 \approx 151.6$

The perimeter of the pentagon is about 46.6 units and the area is about 151.6 square units.

4. A decagon has 10 sides, so $P = 10(7) = 70$ units. Because each of the 10 triangles that make up the polygon is isosceles, each apothem bisects the side length and central angle. Each side length is 7 units, so the bisected side length is $\frac{1}{2}(7) = 3.5$ units. Each central angle is is $\frac{360°}{10} = 36°$, so the bisected angle is $\frac{1}{2}(36°) = 18°$.

To find the length of the apothem a use the sine function with one of the bisected isosceles triangles.



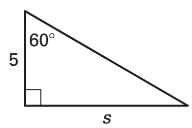
$\tan 18° = \frac{3.5}{a}$

$a \approx 10.77$

$A = \frac{1}{2}aP \approx \frac{1}{2}(10.77)(70) \approx 376.95$

The perimeter for the regular decagon is 70 units and the area is about 377.0 square units.

5. The measure of the central angle is $\frac{360°}{3} = 120°$. The bisected angle is 60°. To find the side length of the triangle, use trigonometric ratios.



$\tan 60° = \frac{s}{5}$

$s \cdot \tan 60° = s$

$5\sqrt{3} = 5$

The regular triangle has side length $2(5\sqrt{3}) = 10\sqrt{3}$. So, the perimeter is $P = 3s = 3(10\sqrt{3}) = 30\sqrt{3}$.

$A = \frac{1}{2}aP \approx \frac{1}{2}(5)(30\sqrt{3}) \approx 129.9$ square units.

The perimeter for the regular triangle is $30\sqrt{3}$ units and the area is about 129.9 square units.

6. You can use a special right triangle to solve Exercise 5.

Exercises for the lesson "Areas of Regular Polygons"

Skill Practice

1. F

2. $\angle AFE$

3. 6.8 units

4. 5.5 units

5. To find the measure of a central angle of a regular polygon with n sides, divide 360° by the number of sides n of the polygon.

6. $\frac{360°}{10} = 36°$

7. $\frac{360°}{18} = 20°$

8. $\frac{360°}{24} = 15°$

9. $\frac{360°}{7} \approx 51.4°$

10. $\angle GJH$ is a central angle. So $m\angle GJH = \frac{360°}{8} = 45°$.

11. $\overline{JK}$ is an apothem, which makes it an altitude of isosceles $\triangle GJH$. So, $\overline{JK}$ bisects $\angle GJH$ and

$m\angle GJK = \frac{1}{2} m\angle GJH = 22.5°$.

12. $m\angle KGJ = 180° - 90° - 22.5° = 67.5°$

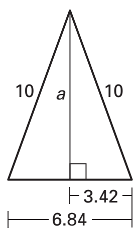
13. $\angle EJH$ is 3 central angles combined. So $m\angle EJH = 3(45°) = 135°$.

14. $P = 3(12) = 36$ units

$A = \frac{1}{2}aP = \frac{1}{2}(2\sqrt{3})(36) \approx 62.4$

The area of the regular triangle is about 62.4 square units.

15. $P = 9(6.84) = 61.56$ units. Use the Pythagorean Theorem to find the apothem a.



$a^2 + 3.42^2 = 10^2$

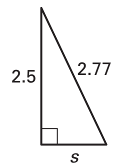
$a^2 = 88.3036$

$a \approx 9.4$

Area $= \frac{1}{2}aP \approx \frac{1}{2}(9.4)(61.56) \approx 289.3$

The area of the regular nonagon is about 289.3 square units.

16. Use the Pythagoream Theorem to find the length of a side of the regular heptagon.



$s^2 + 2.5^2 = 2.77^2$

$s^2 = 1.4229$

$s \approx 1.19$

The length of the base of the triangle is about 1.19, so the length of one side of the polygon is about $2(1.19) = 2.38$.

$A = \frac{1}{2}a \cdot ns \approx \frac{1}{2}(2.5)(7)(2.38) \approx 20.8$

The area of the regular polygon is about 20.8 square units.

17. 7.5 is not the length of one side of the hexagon, it is the length of half of one side (or the base of the triangle). 7.5 should be doubled to get the actual side of the hexagon.

$A = \frac{1}{2}a \cdot ns = \frac{1}{2}(13)(6)(15) = 585 \text{ units}^2$

18. B; The measure of the central angle of a dodecagon is $\frac{360°}{12} = 30°$. The bisected angle is $\frac{1}{2}(30°) = 15°$. You know the side length is 8, so the base of the triangle is $\frac{1}{2}(8) = 4$. Use a trigonomic ratio to find the apothem.

$\tan 15° = \frac{4}{a}$

$a = \frac{4}{\tan 15°}$

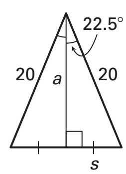
19. The measure of the central angle is $\frac{360°}{8} = 45°$. The bisected angle is $\frac{1}{2}(45°) = 22.5°$. Use trigonometric ratios to find the apothom a and the side length s of the triangle.

$\sin 22.5° = \frac{s}{20}$

$20 \cdot \sin 22.5° = s$

$\cos 22.5° = \frac{a}{20}$

$20 \cdot \cos 22.5° = a$



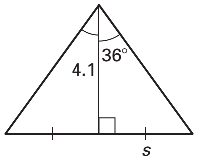
The regular octagon has side length $2(20 \cdot \sin 22.5°) = 40 \sin 22.5°$.

$P = 8(40 \cdot \sin 22.5°) \approx 122.5$ units

$A = \frac{1}{2}aP = \frac{1}{2}(20 \cdot \cos 22.5°)(122.5) \approx 1131.8 \text{ units}^2$

The perimeter of the regular octagon is about 122.5 units and the area is about 1131.8 square units.

20. The measure of the central angle is $\frac{360°}{5} = 72°$. The bisected angle is $\frac{1}{2}(72°) = 36°$. To find the side length of the pentagon, use a trigonometric ratio.

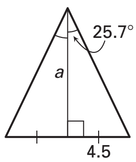


$\tan 36° = \frac{s}{4.1}$

$4.1 \cdot \tan 36° = s$

The regular pentagon has side length $= 2(4.1 \cdot \tan 36°) = 8.2 \cdot \tan 36°$. So, the perimeter is $5(8.2 \cdot \tan 36°) \approx 29.8$ units, and the area is $A = \frac{1}{2}aP = \frac{1}{2}(4.1)(29.8) \approx 61.1$ square units.

21. The measure of the central angle is $\frac{360°}{7} \approx 51.4°$. The bisected angle is $\frac{1}{2}(51.4) \approx 25.7°$. The side of the heptagon is 9, so the base of the triangle is $\frac{1}{2}(9) = 4.5$. To find the apothem use a trigonometric ratio.



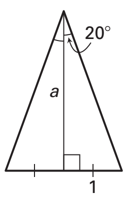
$\tan 25.7° = \frac{4.5}{a}$

$a \approx 9.35$

So, the perimeter is $7(a) = 63$ units, and the area is

$A = \frac{1}{2}aP = \frac{1}{2}(9.35)(63) \approx 294.5$ square units.

22. There is enough information to find the area. The measure of the central angle is $\frac{360°}{9} = 40°$. The bisected angle is $\frac{1}{2}(40°) = 20°$. Because the perimeter is 18 inches and a nonagon has 9 sides, each side length is $18 \div 9 = 2$ inches. The side length of the nonagon is 2 inches, so the base of the triangle is $\frac{1}{2}(2) = 1$ inch. To find the apothem, use a trigonometric ratio.

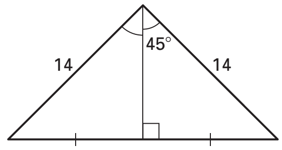


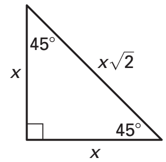
$\tan 20° = \frac{1}{a}$

The area is $A = \frac{1}{2}aP = \frac{1}{2} \cdot \frac{1}{\tan 20°} \cdot 18 \approx 24.7$ square inches.

23. You need to know the apothem and side length to find the area of the regular square. You can use the methods of special right triangles or trigonometry to find the apothem length. The measure of the central angle is $\frac{360°}{4} = 90°$ and the bisected angle is $\frac{1}{2}(90°) = 45°$.

To find the length of the apothem and side length, use a special triangle





$x\sqrt{2} = 14$

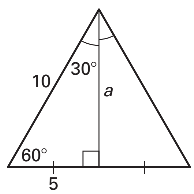
$x = \frac{14}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{14\sqrt{2}}{2} = 7\sqrt{2}$

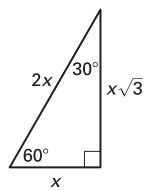
The regular square has side length $s = 2(7\sqrt{2}) = 14\sqrt{2}$.

So, the area is $A = \frac{1}{2}a \cdot ns = \frac{1}{2}(7\sqrt{2})(4)(14\sqrt{2})$

$= 392$ square units.

24. You need to know the apothem to find the area of the regular hexagon. You can use the methods of the Pythagorean Theorem, special triangles, or trigonometry. The measure of the central angle is $\frac{360°}{6} = 60°$ and the bisected angle is $\frac{1}{2}(60°) = 30°$. The side of the hexagon is 10, so the base of the triangle is $\frac{1}{2}(10) = 5$. To find the apothem, use a special triangle.

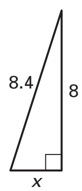




So, $x = 5$. The apothem is $5\sqrt{3}$.

The area is $A = \frac{1}{2}a \cdot ns = \frac{1}{2}(5\sqrt{3})(6)(10) \approx 259.8$ square units.

25. You need to know the side length to find the area of the decagon. You can use the methods of the Pythagorean Theorem or trigonometry. To find the side length, use the Pythagorean Theorem.



$x^2 + 8^2 = 8.4^2$

$x^2 = 6.56$

$x = \sqrt{6.56}$

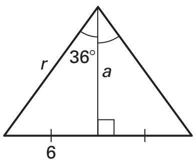
The regular decagon has side length $s = 2\sqrt{656}$.
So, the area is
$A = \frac{1}{2}a \cdot ns = \frac{1}{2}(8)(10)(2\sqrt{6.56}) \approx 204.9$ square units.

26. Area of unshaded region = Area of circle − Area of square

$$= \pi \cdot 14^2 - 392$$
$$= 196\pi - 392 \approx 223.8$$

The area of the unshaded region in Exercise 23 is about 223.8 square units.

27. The measure of the central angle is $\frac{360^\circ}{5} = 72^\circ$ and the measure of the bisected angle is $\frac{1}{2}(72^\circ) = 36^\circ$. The side length of the pentagon is 12, so the base of the triangle is $\frac{1}{2}(12) = 6$. Use trigonometric ratios to find the length of the apothem and the radius of the circle.



$\sin 36^\circ = \frac{6}{r}$ $\qquad$ $\tan 36^\circ = \frac{6}{a}$

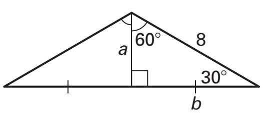
$r = \frac{6}{\sin 36^\circ}$ $\qquad$ $a = \frac{6}{\tan 36^\circ}$

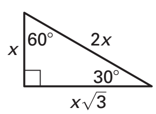
Area of shaded region = Area of circle − Area of pentagon

$$= \pi \cdot r^2 - \left(\frac{1}{2}a \cdot ns\right)$$
$$= \pi \cdot \left(\frac{6}{\sin 36^\circ}\right)^2 - \left[\frac{1}{2}\left(\frac{6}{\tan 36^\circ}\right)(5)(12)\right]$$
$$\approx 79.6 \text{ square units}$$

The area of the shaded region is about 79.6 square units.

28. The measure of the central angle is $\frac{360^\circ}{3} = 120^\circ$ and the measure of the bisected angle is $\frac{1}{2}(120^\circ) = 60^\circ$. Use a special triangle to find the apothem and size length of the triangle.





So, $x = 4$. The apothem a is 4 and the side length b of the triangle is $4\sqrt{3}$.

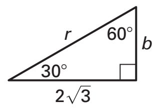
The length of a side of the equilateral triangle is $2(4\sqrt{3}) = 8\sqrt{3}$.

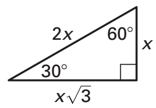
Area of shaded region = Area of circle − Area of triangle

$$= \pi \cdot r^2 - \frac{1}{2} \cdot a \cdot ns$$
$$= \pi \cdot 8^2 - \frac{1}{2}(4)(3)(8\sqrt{3})$$
$$= 64\pi - 48\sqrt{3}$$
$$\approx 117.9 \text{ square units}$$

The area of the shaded region is about 117.9 square units.

29. The measure of the central angle is 60°, so the bisected angle is $\frac{1}{2}(60^\circ) = 30^\circ$. Use a special triangle to find the radius of the circle and base of the smaller triangle.





So, the base b of the triangle is 2 and the radius r is $2(2) = 4$.

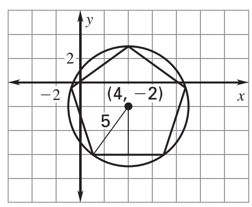
The base of the larger triangle is $2(2) = 4$.

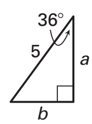
Area of shaded region = Area of sector − Area of triangle

$$= \frac{m\,\widehat{arc}}{360^\circ} \cdot \pi r^2 - \left[\frac{1}{2}(\text{base of larger } \triangle)(a)\right]$$
$$= \frac{60^\circ}{360^\circ} \cdot \pi \cdot 4^2 - \left[\frac{1}{2}(4)(2\sqrt{3})\right]$$
$$= \frac{8}{3}\pi - 4\sqrt{3}$$
$$\approx 1.4 \text{ square units}$$

The area of the shaded region is about 1.4 square units.

30. The circle has a radius of $\sqrt{25} = 5$. The central angle of the pentagon is $\frac{360^\circ}{5} = 72^\circ$ and the bisected angle is $\frac{1}{2}(72^\circ) = 36^\circ$. Use trigonometric ratios to find the side length and apothem of the pentagon.





$\cos 36^\circ = \frac{a}{5}$ $\qquad$ $\sin 36^\circ = \frac{b}{5}$

$5 \cos 36^\circ = a$ $\qquad$ $5 \sin 36^\circ = b$

The regular pentagon has side length

$2b = 2(5 \cdot \sin 36^\circ) = 10 \cdot \sin 36^\circ$. So, the area is

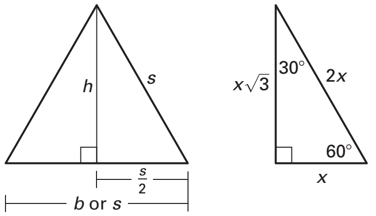
$$A = \frac{1}{2}a \cdot ns = \frac{1}{2}(5 \cdot \cos 36^\circ)(5)(10 \cdot \sin 36^\circ)$$
$$\approx 59.4 \text{ square units.}$$

31. True. Because the radius is fixed and the circle around the n-gons also stays the same, more and more of the circle gets covered up as n gets larger.

32. True. Because the hypotenuse of the right triangle represents the radius and the leg represents the apothem, the apothem must always be less than the radius.

33. False. The radius can be equal to the side length, like in a regular hexagon.

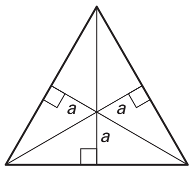
34.



Use a special triangle to find the height of the triangle.

So, $x = \frac{b}{2}$, $s = b$, and $h = \frac{b\sqrt{3}}{2}$.

Because $b = s$, substitute that into the height you just found: $h = \frac{b\sqrt{3}}{2} = \frac{s\sqrt{3}}{2}$. Use the formula $A = \frac{1}{2}bh$ to find the area in terms of s. $A = \frac{1}{2}(s)\left(\frac{s\sqrt{3}}{2}\right) = \frac{\sqrt{3}s^2}{4}$.



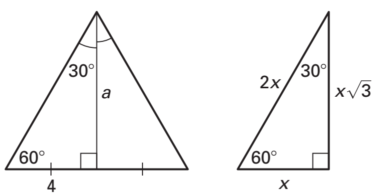
In an equilateral triangle, each altitude creates two congruent triangles. So, each altitude is also a median. Looking at the figure the apothem is $\frac{1}{3}$ the altitude of the equilateral triangle. You found the height of the triangle above, $\frac{s\sqrt{3}}{2}$, so the apothem is $\frac{1}{3}\left(\frac{s\sqrt{3}}{2}\right)$.

Substituting into the formula $A = \frac{1}{2}a \cdot ns$, you get

$A = \frac{1}{2}\left(\frac{1}{\cancel{3}}\left(\frac{s\sqrt{3}}{2}\right)\right) \cdot \cancel{3} \cdot s = \frac{\sqrt{3}s^2}{4}$ square units.

35. Shaded area = Area of hexagon − Area of pentagon + Area of square − Area of triangle

Hexagon: The central angle measures $\frac{360°}{6} = 60°$ and the bisected angle measures $\frac{1}{2}(60°) = 30°$. The side length of the hexagon is 8, so the base of the triangle is $\frac{1}{2}(8) = 4$. Use a special triangle to find the apothem.

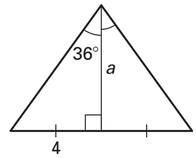


So, $x = 4$ and $a = 4\sqrt{3}$.

So, the area of the hexagon is

$A = \frac{1}{2}a \cdot ns = \frac{1}{2}(4\sqrt{3})(6)(8) = 96\sqrt{3}$ square units.

Pentagon: The central angle measures $\frac{360°}{s} = 72°$ and the bisected angle measures $\frac{1}{2}(72°) = 36°$. Use a trigonometric ratio to find the apothem.



$\tan 36° = \frac{4}{a}$

$a = \frac{4}{\tan 36°}$

So, the area of the pentagon is

$A = \frac{1}{2}a \cdot ns = \frac{1}{2}\left(\frac{4}{\tan 36°}\right)(5)(8) \approx 110$ square units.

Square: The area of the square is $A = s^2 = 8^2 = 64$ square units.

Triangle: Look at the smaller triangle used for the hexagon. The height of the triangle is the apothem of the smaller triangle used for the hexagon. So, the

area $= A = \frac{1}{2}bh = \frac{1}{2}(8)(4\sqrt{3})$

$= 16\sqrt{3}$ square units.

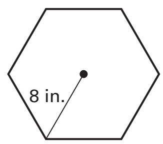
Shaded area $= 96\sqrt{3} - 110 + 64 - 16\sqrt{3}$

≈ 92.6

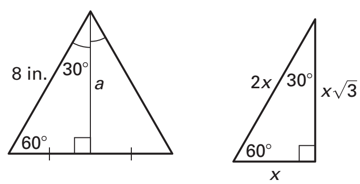
The area of the shaded region is about 93 square units.

Problem Solving

36.



a. The measure of the central angle is $\frac{360°}{6} = 60°$ and the bisected angle measures $\frac{1}{2}(60°) = 30°$. Use a special triangle to find the apothem.

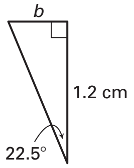


So, $x = 4$. The apothem is $4\sqrt{3}$ in.

b. A regular hexagon is made up of 6 equilateral triangles, so the side length of the hexagon is the same as the radius of the hexagon, or 8. So, the perimeter is $P = 6(8) = 48$ inches and the area is

$A = \frac{1}{2}aP = \frac{1}{2}(4\sqrt{3})(48) \approx 166$ square inches.

37. The apothem of the octagon is $1 + 0.2 = 1.2$ centimeters. The measure of the central angle is $\frac{360°}{8} = 45°$ and the bisected angle measures $\frac{1}{2}(45°) = 22.5°$. Use a trigonometric function to find the side length of the octagon.



$$\tan 22.5° = \frac{b}{1.2}$$

$1.2 \cdot \tan 22.5° = b$

The regular octagon has side length

$s = 2b = 2(1.2 \cdot \tan 22.5°) = 2.4 \cdot \tan 22.5°$.

$$\text{Area of silver border} = \text{Area of octagon} - \text{Area of circle}$$
$$= \frac{1}{2}a \cdot ns - \pi r^2$$
$$= \frac{1}{2}(1.2)(8)(2.4 \cdot \tan 22.5°) - \pi(1)^2$$
$$\approx 1.6 \text{ square centimeters}$$

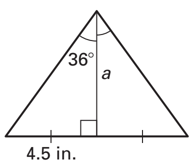
The apothem is 1.2 centimeters, the area of the octagon is about 4.8 square centimeters, and the area of the silver border is about 1.6 square centimeters.

38. *Sample prediction:* The pentagon will have the greatest area and the circle will have the smallest area.

a. Area of circle $= \pi r^2 = \pi\left(\frac{13}{2}\right)^2 \approx 132.7$ in.^{2}

b. Area of triangle $= \frac{1}{2}bh = \frac{1}{2}(18)(15) = 135$ in.^{2}

c. The measure of the central angle is $\frac{360°}{5} = 72°$ and the bisected angle measures $\frac{1}{2}(72°) = 36°$. The side length of the pentagon is 9 inches, so the base of the triangle is $\frac{1}{2}(9) = 4.5$ inches. Use a trigonomic ratio to find the apothem.



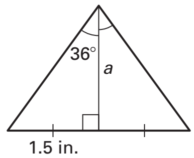
$$\tan 36° = \frac{4.5}{a}$$
$$a = \frac{4.5}{\tan 36°}$$

So, the area is $A = \frac{1}{2}a \cdot ns = \frac{1}{2}\left(\frac{4.5}{\tan 36°}\right)(5)(9)$

$$\approx 139.4 \text{ in.}^2$$

The area of the circle is about 132.7 square inches, the area of the triangle is 135 square inches, and the area of the pentagon is about 139.4 square inches.

39. A regular pentagon has 5 sides of equal length. So, the side length of the smaller pentagon is $15 \div 5 = 3$ inches. The measure of the central angle is $\frac{360°}{5} = 72°$ and the bisected angle measures $\frac{1}{2}(72°) = 36°$. Because the side length of the smaller pentagon is 3 inches, the base of the triangle is $\frac{1}{2}(3) = 1.5$ inches. Use a trigonometric ratio to find the apothem.



$$\tan 36° = \frac{1.5}{a}$$
$$a = \frac{1.5}{\tan 36°}$$

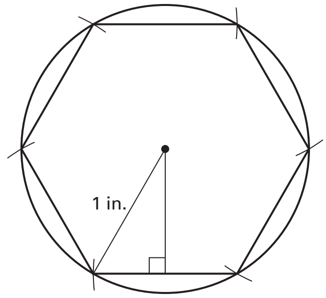
So, the area of the small trivet is

$A = \frac{1}{2}aP = \frac{1}{2}\left(\frac{1.5}{\tan 36°}\right)(15) \approx 15.5$ square inches.

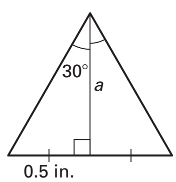
$$\frac{\text{area of small trivet}}{\text{area of large trivet}} = \frac{(\text{perimeter of small trivet})^2}{(\text{perimeter of large trivet})^2}$$
$$\frac{15.5}{\text{area of large trivet}} = \frac{15^2}{25^2}$$
$$43.0 \approx \text{area of large trivet}$$

The area of the small trivet is about 15.5 square inches and the area of the large trivet is about 43.0 square inches.

40. a.



b. The measure of the central angle is $\frac{360°}{6} = 60°$ and the bisected angle measures $\frac{1}{2}(60°) = 30°$. Because the side length is 1 inch, the base of the triangle is $\frac{1}{2}(1) = 0.5$ inch. Use a trigonometric ratio to find the apothem.



$$\tan 30° = \frac{0.5}{a}$$
$$a = \frac{0.5}{\tan 30°}$$

So, the area of the hexagon is

$A = \frac{1}{2}a \cdot ns = \frac{1}{2}\left(\frac{0.5}{\tan 30°}\right)(6)(1) \approx 2.6$ square inches.

Area of shaded region = Area of circle − Area of hexagon

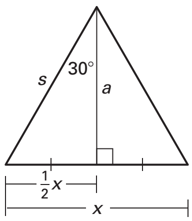
$\approx \pi \cdot 1^2 - 2.6$

≈ 0.54 square inch

The area of the hexagon is about 2.6 square inches and the area of the shaded region is about 0.54 square inches.

c. Draw $\overline{AB}$ with length 1 inch. Open compass to 1 inch and draw a circle with that radius. Using the same compass setting, mark off equal parts along the circle. Connect every other intersection or connect 2 adjacent points with the center of the circle.

41. Let x be the side length of a regular hexagon. A hexagon has 6 central angles, so 6 traingles are formed. The measure of a central angle is $\frac{360°}{6} = 60°$. The bisected angle has a measure of $\frac{1}{2}(60°) = 30°$. Draw one of the 6 triangles formed by the central angles and find the side length s and an expression in terms of a for x.



$\cos 30° = \frac{a}{s}$

$s = \frac{a}{\cos 30°}$

$= \frac{a}{\left(\frac{\sqrt{3}}{2}\right)}$

$= a \cdot \frac{2}{\sqrt{3}}$

$= \frac{2a\sqrt{3}}{3}$

$\tan 30° = \frac{\frac{1}{2}x}{a}$

$2a \cdot \tan 30° = x$

$2a\left(\frac{\sqrt{3}}{3}\right) = x$

$\frac{2a\sqrt{3}}{3} = x$

The side length x of the hexagon is equal to the distance s from the center of the hexagon to a vertex. So, each of the 6 triangles is an equilateral triangle.

42. *Sample answer:* a regular hexagon can be broken into 6 equilateral triangles, 2 isosceles trapezoids, and 3 parallelograms.

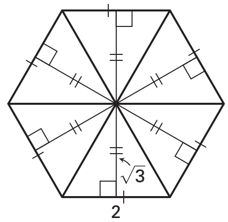
Area of hexagon $= \frac{1}{2}a \cdot ns = \frac{1}{2}(\sqrt{3})(6)(2) = 6\sqrt{3}$

Area of 6 equilateral triangles $= 6\left[\frac{1}{2}bh\right]$

$= 6\left(\left(\frac{1}{2}\right)(2)(\sqrt{3})\right)$

$= 6[\sqrt{3}]$

$= 6\sqrt{3}$

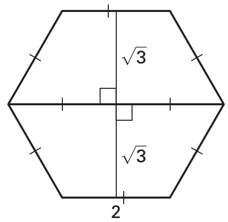


Area of 2 isosceles trapezoids $= 2\left[\frac{1}{2}h(b_1 + b_2)\right]$

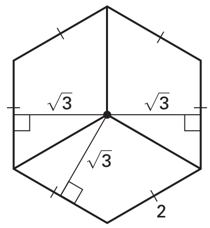
$= 2\left[\frac{1}{2}(\sqrt{3})(2 + (2 + 2))\right]$

$= 2[3\sqrt{3}]$

$= 6\sqrt{3}$



Area of 3 parallelograms $= 3(bh) = 3(2)(\sqrt{3}) = 6\sqrt{3}$



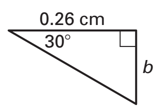
43. Because PC is a radius, then P is the circumcenter of $\triangle ABC$. Let E be the midpoint of $\overline{AB}$. Because P is the circumcenter, $\overline{CE}$ and $\overline{BD}$ are perpendicular bisectors and medians of $\triangle ABC$. By the Concurrency of Medians of a Triangle Theorem, $CP = \frac{2}{3}CE$ and $BP = \frac{2}{3}BD$. So, $DP = \frac{1}{3}BD$. Because $\overline{CE}$ and $\overline{BD}$ are perpendicular bisectors, $BP = CP$. So, $CP = \frac{2}{3}BD$ and $2DP = \frac{2}{3}BD$. So, radius CP is twice the apothem PD.

44. a. The average distance across a cell is $\frac{2.6}{5}$ = 0.52 centimeter.

b. The apothem is $\frac{1}{2}$ diameter $= \frac{1}{2}(0.52)$ = 0.26 centimeter. The measure of the central angle is $\frac{3600}{6} = 60°$ and the bisected angle measures $\frac{1}{2}(60°) = 30°$. Use a trigonometric ratio to find the length of the base of the triangle.

$\tan 30° = \frac{b}{0.26}$

$0.26 \cdot \tan 30° = b$



The length of a side of the hexagon is $2b = 2(0.26 \cdot \tan 30°) = 0.52 \cdot \tan 30° \approx$ 0.3 centimeter. So, the area of the cell is

$$A = \frac{1}{2}a \cdot ns = \frac{1}{2}(0.26)(6)(0.3)$$

$$\approx 0.234 \text{ square centimeter.}$$

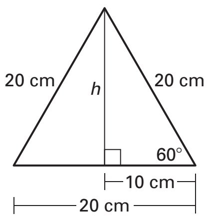
c. 100 cells • 0.234 square centimeter/cell = 23.4 square centimeters

$$23.4 \text{ square centimeters} \cdot \frac{(1 \text{ decimeter})^2}{(10 \text{ centimeter})^2}$$

$$= 0.234 \text{ square decimeter}$$

d. From part (c), 100 cells have an area of about 0.234 square decimeter. To find how many cells are in 1 square decimeter, divide 100 by 0.234. So, there are approximately 427 cells per square decimeter.

45. a. *triangle:*

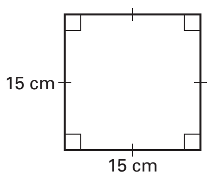


$$\tan 60° = \frac{h}{10}$$

$$10 \cdot \tan 60° = h$$

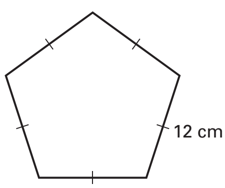
$$A = \frac{1}{2}bh \approx \frac{1}{2}(20)(10 \cdot \tan 60°) \approx 173.2 \text{ cm}^2$$

square:



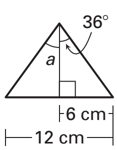
$$A = s^2 = 15^2 = 225 \text{ cm}^2$$

regular pentagon:



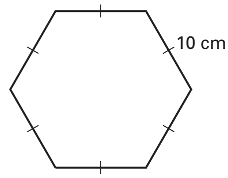
$$\tan 36° = \frac{6}{a}$$

$$a = \frac{6}{\tan 36°}$$



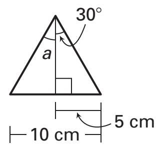
$$A = \frac{1}{2}aP \approx \frac{1}{2}\left(\frac{6}{\tan 36°}\right)(60) \approx 247.7 \text{ cm}^2$$

regular hexagon:



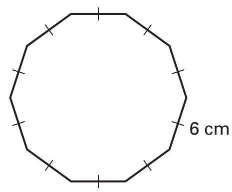
$$\tan 30° = \frac{5}{a}$$

$$a = \frac{5}{\tan 30°}$$



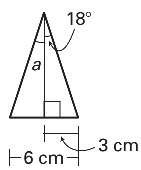
$$A = \frac{1}{2}aP \approx \frac{1}{2}\left(\frac{5}{\tan 30°}\right)(60) \approx 259.8 \text{ cm}^2$$

regular decagon:



$$\tan 18° = \frac{3}{a}$$

$$a = \frac{3}{\tan 18°}$$

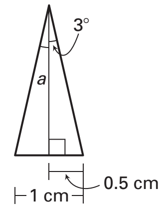


$$A = \frac{1}{2}aP \approx \frac{1}{2}\left(\frac{3}{\tan 18°}\right)(60) \approx 277$$

The area of the equilateral triangle is about 173.2 square centimeters. The area of the square is 225 square centimeters. The area of the regular pentagon is about 247.7 square centimeters. The area of the regular hexagon is about 259.8 square centimeters. The area of the regular decagon is about 277 square centimeters.

The area increases as the number of sides increase.

b. 60-gon: The central angle for a 60-gon measures $\frac{360°}{60} = 6°$, the bisected angle measures $\frac{1}{2}(6°) = 3°$, and the side length is $60 \div 60 = 1$ centimeter. Because the side length is 1 centimeter, the base of the triangle is $\frac{1}{2}(1) = 0.5$ centimeter. Use a trigonometric ratio to find the apothem.



Not drawn to scale

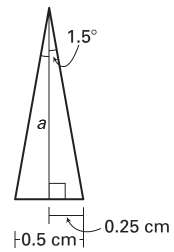
$$\tan 3° = \frac{0.5}{a}$$

$$a = \frac{0.5}{\tan 3°}$$

The area of a 60-gon is $A = \frac{1}{2}aP = \frac{1}{2}\left(\frac{0.5}{\tan 3°}\right)(60)$

≈ 286.2 square centimeters.

120-gon: The central angle for 120-gon measures $\frac{360°}{12°} = 3°$, the bisected angle measures $\frac{1}{2}(3°) = 1.5°$, and the side length is $60 \div 120 = 0.5$ centimeter. Because the side length is 0.5 centimeter, the base of the triangle is $\frac{1}{2}(0.5) = 0.25$ centimeter. Use a trigonometric ratio to find the apothem.



Not drawn to scale

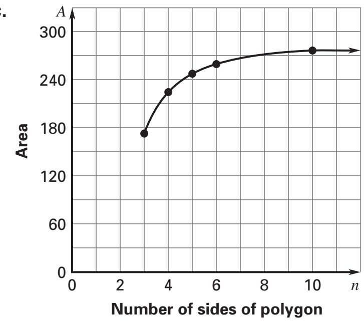
$$\tan 1.5° = \frac{0.25}{a}$$

$$a = \frac{0.25}{\tan 1.5°}$$

The area of a 120-gon is $A = \frac{1}{2}aP = \frac{1}{2}\left(\frac{0.25}{\tan 1.5°}\right)(60)$

≈ 286.4 square centimeters.

c.



A circle will have the greatest area.

$$C = 2\pi r$$

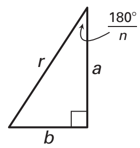
$$60 = 2\pi r$$

$$\frac{30}{\pi} = r$$

$$A = \pi r^2 = \pi\left(\frac{30}{\pi}\right)^2 = \frac{900}{\pi} \approx 286.5 \text{ cm}^2$$

The area of the circle will be about 286.5 square centimeters.

46. The inscribed n-gon would have a central angle of $\frac{360°}{n}$ and a bisected angle of $\frac{1}{2}\left(\frac{360°}{n}\right)$, or $\frac{180°}{n}$. When broken down into reference triangles, the radius of the polygon would equal the radius of the circle.



$$\sin\left(\frac{180°}{n}\right) = \frac{b}{r} \qquad \cos\left(\frac{180°}{n}\right) = \frac{a}{r}$$

$$r \bullet \sin\left(\frac{180°}{n}\right) = b \qquad r \bullet \cos\left(\frac{180}{n}\right) = a$$

The side length of the inscribed n-gon is

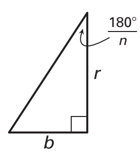
$$s = 2b = 2r \bullet \left(\sin\frac{180°}{n}\right).$$

So, the area of the inscribed n-gon is

$$A = \frac{1}{2}a \bullet ns = \frac{1}{2}\left(r \bullet \cos\left(\frac{180°}{n}\right)\right)(n)\left(2r \bullet \sin\left(\frac{180°}{n}\right)\right)$$

$$= r^2 n \cos\left(\frac{180°}{n}\right)\sin\left(\frac{180°}{n}\right).$$

The circumscribed n-gon would have a central angle of $\frac{360°}{n}$ and a bisected angle of $\frac{1}{2}\left(\frac{360°}{n}\right)$, or $\frac{180°}{n}$. When broken down into reference triangles the apothem would equal the radius of the circle.



$$\tan\left(\frac{180°}{n}\right) = \frac{b}{r}$$

$r \cdot \tan\left(\frac{180°}{n}\right) = b$

The side length of the circumscribed n-gon is

$s = 2b = 2r \cdot \tan\left(\frac{180°}{n}\right)$. So, the area of the

circumscribed n-gon is

$A = \frac{1}{2}a \cdot ns = \frac{1}{2}(r)(n)\left(2r \cdot \tan\left(\frac{180°}{n}\right)\right) = r^2 n \tan\left(\frac{180°}{n}\right)$.

The area between the two polygons is equal to the area of the inscribed polygon subtracted from the area of the circumscribed polygon. So, the area between the polygons is

$r^2 n \tan\left(\frac{180°}{n}\right) - r^2 n \cos\left(\frac{180°}{n}\right) \sin\left(\frac{180°}{n}\right)$, which factors

to $r^2 n\left[\tan\left(\frac{180°}{n}\right) - \cos\left(\frac{180°}{n}\right) \sin\left(\frac{180°}{n}\right)\right]$.

Lesson 11.4 Use Geometric Probability

Guided Practice for the lesson "Use Geometric Probability"

1. $P(\text{point is on } \overline{RT}) = \frac{\text{Length of } \overline{RT}}{\text{Length of } \overline{PQ}} = \frac{|-1-(-2)|}{|5-(-5)|}$

$= \frac{1}{10}$; 0.1, 10%

2. $P(\text{Point is on } \overline{TS}) = \frac{\text{Length of } \overline{TS}}{\text{Length of } \overline{PQ}} = \frac{|4-(1)|}{|5-(-5)|} = \frac{5}{10}$

$= \frac{1}{2}$; 0.5; 50%

3. $P(\text{Point is on } \overline{PT}) = \frac{\text{Length of } \overline{PJ}}{\text{Length of } \overline{PQ}} = \frac{|-1-(-5)|}{|5-(-5)|} = \frac{4}{10}$

$= \frac{2}{5}$; 0.4, 40%

4. $P(\text{Point is on } \overline{RQ}) = \frac{\text{Length of } \overline{RQ}}{\text{Length of } \overline{PQ}} = \frac{|5-(-2)|}{|5-(-5)|}$

$= \frac{7}{10}$; 0.7, 70%

5. The longest you can wait is 8:49 − 8:43 = 6 minutes.

P(You get to the station by 8:58)

$= \frac{\text{Favorable waiting time}}{\text{maximum waiting time}} = \frac{6}{12} = \frac{1}{2}$

The probability that you will get to the station by 8:58 is $\frac{1}{2}$, or 50%.

6. Area of black region = Area of larger black circle − Area of large white circle + Area of smaller black circle − Area of smaller white circle

$= \pi(8+8+8+8+8)^2 - \pi(8+8+8+8)^2 + \pi(8+8+8)^2 - \pi(8+8)^2$

$= 1600\pi - 1024\pi + 576\pi - 256\pi = 896\pi$

$P(\text{arrow lands in black region}) = \frac{\text{Area of black region}}{\text{Area of target}}$

$= \frac{896\pi}{1600\pi} = \frac{56}{100} = \frac{14}{25}$

The probability that the arrow lands in the black region is $\frac{14}{25}$ or 56%.

7. Area of woods = Total area of park − Area of field

$= 58.5 - 30 = 28.5$

$P(\text{ball in woods}) = \frac{\text{Area of woods}}{\text{Total area of park}} \approx \frac{28.5}{58.5} = \frac{285}{585} = \frac{19}{39}$

The probability that the ball is in the field is about $\frac{19}{39}$, or 48.7%.

Exercises for the lesson "Use Geometric Probability"

Skill Practice

1. If an event cannot occur, its probability is 0. If an event is certain to occur, its probability is 1.

2. Geometric probabilities are determined by comparing geometric measures. A probability, found by dividing the number of favorable outcomes by the total number of possible outcomes, deals with events, not geometric measures. In a geometric probablity, you divide the "favorable measure" by the total measure.

3. $P(\text{Point } k \text{ is on } \overline{AD}) = \frac{\text{Length of } \overline{AD}}{\text{Length of } \overline{AE}}$

$= \frac{|3-(-12)|}{|12-(-12)|} = \frac{15}{24}$

$= \frac{5}{8}$; 0.625; 62.5%

4. $P(\text{Point } k \text{ is on } \overline{BC}) = \frac{\text{Length of } \overline{BC}}{\text{Length of } \overline{AE}} = \frac{|-3-(-6)|}{|12-(-12)|}$

$= \frac{3}{24} = \frac{1}{8}$; 0.125; 12.5%

5. $P(\text{Point } k \text{ is on } \overline{DE}) = \frac{\text{Length of } \overline{DE}}{\text{Length of } \overline{AE}} = \frac{|12-3|}{|12-(-12)|}$

$= \frac{9}{24} = \frac{3}{8}$; 0.375; 37.5%

6. $P(\text{Point } k \text{ is on } \overline{AE}) = \frac{\text{Length of } \overline{AE}}{\text{Length of } \overline{AE}} = \frac{|12-(-12)|}{|12-(-12)|}$

$= \frac{24}{24} = 1$; 1.0; 100%

7. $AD + DE = AE$, so $\frac{5}{8} + \frac{3}{8} = 1$. The sum of their probabilities will add up to 1, or 100%.

8. Area of shaded region = Area of circle − Area of square

$= \pi r^2 - s^2 = \pi \cdot 2^2 - (2\sqrt{2})^2$

$= 4\pi - 8$

$P(\text{Point is in shaded region}) = \frac{\text{Area of shaded region}}{\text{Area of entire figure}}$

$= \frac{4\pi - 8}{4\pi} \approx 0.36$ or 36%.

The probability that a randomly chosen point lies in the shaded region is about 36%.

9. $P(\text{Point is in shaded region}) = \dfrac{\text{Area of smaller triangle}}{\text{Area of larger triangle}}$

$$= \frac{\frac{1}{2}(6)(7)}{\frac{1}{2}(12)(14)} = \frac{21}{84} = \frac{1}{4}$$

The probability that a randomly chosen point lies in the shaded region is $\frac{1}{4}$, or 25%.

10. Area of shaded region = Area of trapezoid − Area of rectangle

$$= \frac{1}{2}h(b_1 + b_2) - b_1h$$

$$= \frac{1}{2}(5)(8 + 20) - (8)(5)$$

$$= 70 - 40 = 30$$

$P(\text{Point is in shaded region}) = \dfrac{\text{Area of shaded region}}{\text{Area of trapezoid}}$

$$= \frac{30}{70} = \frac{3}{7}$$

The probability that a randomly chosen point lies in the shaded region is $\frac{3}{7}$, or about 43%.

11. In the numerator, the area of the unshaded rectangle at the bottom should be subtracted. This rectangle has a height of $7 - 5 = 2$.

$$\frac{10(7) - \frac{1}{2}\pi(5)^2 - 2(10)}{10(7)} = \frac{50 - 12.5\pi}{70} \approx 0.153$$

The probability that a randomly chosen point in the figure lies in the shaded region is about 15.3%.

12. *Sample answer:*

The area of the north side of the island is about 31.5 square units.

The area of the south side of the island is about 34.5 square units.

The area of the whole island is about 66 square units.

13. $P(\text{north side}) = \dfrac{\text{Area of north side}}{\text{Total area}} = \dfrac{31.5}{66} = \dfrac{21}{44}$

The probability that a randomly chosen location on the island lies on the north side is about $\frac{21}{44}$, or 47.7%.

14. $P(\text{south side}) = \dfrac{\text{Area of south side}}{\text{Total area}} = \dfrac{34.5}{66} = \dfrac{23}{44}$

The probability that a randomlyi chosen location on the island lies on the south side is about $\frac{23}{44}$, or 52.3%.

15. The shaded triangle is similar to the whole triangle by the AA Similarity Postulate. The ratio of sides is $\frac{6}{12} = \frac{1}{2}$ and $\frac{7}{14} = \frac{1}{2}$. Because you know the ratio of sides, you can use the Area of Similar Polygons Theorem to find the desired probability. The ratio of areas is $1^2 : 2^2 = 1 : 4$, which is the desired probability.

16. $x - 6 \le 1$

$x \le 7$

$P(x \le 7) = \frac{5}{7}$

17. $1 \le 2x - 3 \le 5$

$4 \le 2x \le 8$

$2 \le x \le 4$

$P(2 \le x \le 4) = \frac{2}{7}$

18. $\frac{x}{2} \ge 7$

$x \ge 14$

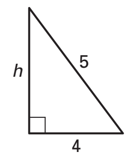
$P(x \ge 14) = 0$

19. $3x \le 27$

$x \le 9$

$P(x \le 9) = 1$

20. Use The Pythagorean Theorem to find the height of the triangle.



$3^2 + h^2 = 5^2$

$h^2 = 16$

$h = 4$

Find the area of the entire figure.

Area of entire figure = Area of shaded region + Area of trapezoid

$$= \frac{1}{2}b_{\pi}h_t + \frac{1}{2}h_2(b_1 + b_2)$$

$$= \frac{1}{2}(3)(4) + \frac{1}{2}(3)(5 + 7)$$

$$= 6 + 18 = 24$$

Write a ratio of the areas to find the probability.

$P(\text{Point is in shaded region}) = \dfrac{\text{Area of shaded region}}{\text{Area of entire figure}}$

$$= \frac{6}{24} = \frac{1}{4}$$

The probability that a randomly chosen point in the figure lies in the shaded region is $\frac{1}{4}$, or 25%.

21. Set up a ratio of side lengths to find the base length of the smaller triangle.

$$\frac{\text{base of smaller triangle}}{\text{base of larger triangle}} = \frac{\text{height of smaller triangle}}{\text{height of larger triangle}}$$

$$\frac{\text{base of smaller triangle}}{14} = \frac{(12 - 8)}{12}$$

$$\text{base of smaller triangle} = \frac{14}{3}$$

Find the area of the shaded region.

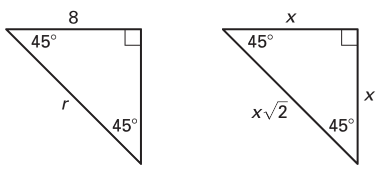
$$\begin{aligned}\text{Area of shaded region} &= \text{Area of larger triangle} \\ &\quad - \text{Area of smaller triangle} \\ &= \tfrac{1}{2}b_l h_l - \tfrac{1}{2}b_s h_s \\ &= \tfrac{1}{2}(14)(12) - \tfrac{1}{2}\left(4\tfrac{2}{3}\right)(4) \\ &= 84 - \tfrac{28}{3} = \tfrac{224}{3}\end{aligned}$$

Write a ratio of the areas to find the probability.

$$P(\text{Point lies in shaded region}) = \frac{\text{Area of shaded region}}{\text{Area of larger triangle}} = \frac{\frac{224}{3}}{84} = \frac{8}{9}$$

The probability that a randomly chosen point in the figure lies in the shaded region is $\frac{8}{9}$, or about 88.9%.

22. Use a special triangle to find the radius of the circle.



So, $r = 8\sqrt{2}$. The base of the triangle formed by the radii is $8 + 8 = 16$.

Find the area of the shaded region.

$$\begin{aligned}\text{Area of shaded region} &= \text{Area of sector} \\ &\quad - \text{Area of triangle} \\ &= \frac{m\,\overset{\frown}{\text{arc}}}{360^\circ} \cdot \pi r^2 - \frac{1}{2}bh \\ &= \frac{90^\circ}{360^\circ} \cdot \pi \cdot (8\sqrt{2})^2 - \frac{1}{2}(16)(8) \\ &= 32\pi - 64\end{aligned}$$

Write a ratio of the areas to find the probability.

$$P(\text{Point lies in shaded region}) = \frac{\text{Area of shaded region}}{\text{Area of circle}} = \frac{32\pi - 64}{\pi(8\sqrt{2})^2} = \frac{32\pi - 64}{128\pi} \approx 0.091$$

The probability that a randomly chosen point in the figure lies in the shaded region is about 0.091, or 9.1%

23. D;

$$P(x \text{ not in } A) = \frac{\text{Area not in } A}{\text{Total Area}} = \frac{\text{Area of } U - \text{Area of } A}{\text{Area of } U}$$

24. $P(\text{Point is an arc}) = \dfrac{\text{Circumference of } \overset{\frown}{\text{arc}}}{\text{circumference of cirlce}}$

$$= \frac{\frac{m\,\overset{\frown}{\text{arc}}}{360^\circ} \cdot \cancel{2\pi r}}{\cancel{2\pi r}} = \frac{80^\circ}{360^\circ} = \frac{2}{9}$$

$$P(\text{Point is in sector}) = \frac{\text{Area of sector}}{\text{Area of circle}} = \frac{\frac{m\,\overset{\frown}{\text{arc}}}{360^\circ} \cdot \cancel{\pi r^2}}{\cancel{\pi r^2}}$$

$$= \frac{80^\circ}{360^\circ} = \frac{2}{9}$$

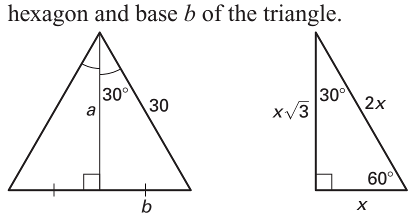
The probability that a randomly chosen point on the circle lies on the arc is $\frac{2}{9}$, or about 22.2%. The probability that a randomly chosen point in the circle lies in the sector is $\frac{2}{9}$, or about 22.2%. The probabilities do not depend on the radius because the circumference and the area of a circle end up canceling with the values in the denominator.

25.

$$\begin{aligned}C &= 2\pi r \\ 188.5 &= 2\pi r \\ 30 &\approx r\end{aligned}$$

Area of circle: $A = \pi r^2 = \pi(30)^2 = 900\pi$

The measure of the central angle of a regular hexagon is $\frac{360^\circ}{6} = 60^\circ$. The bisected angle measures $\frac{1}{2}(60^\circ) = 30^\circ$. Use a 30°-60°-90° triangle to find the apothem a of the hexagon and base b of the triangle.



So, $a = 15\sqrt{3}$ and $b = 15$. The side length of the hexagon is $s = 2b = 2(15) = 30$. The area of the hexagon is

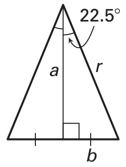
$$A = \frac{1}{2}a \cdot ns = \frac{1}{2}(15\sqrt{3})(6)(30) = 1350\sqrt{3}.$$

$$P(\text{Point is in the hexagon}) = \frac{\text{Area of hexagon}}{\text{Area of circle}} = \frac{1350\sqrt{3}}{900\pi} \approx 0.827$$

The probability that a randomly chosen point in the circle lies in the inscribed regular hexagon is about 0.827, or 82.7%.

26. Area of circle $= \pi r^2$

The measure of the central angle for a regular octagon is $\frac{360°}{8} = 45°$ and the bisected angle measures $\frac{1}{2}(45°) = 22.5°$. Use trigonometric ratios to find the apothem and side length.



$\sin 22.5° = \frac{b}{r}$ $\qquad\qquad$ $\cos 22.5° = \frac{a}{r}$

$r \cdot \sin 22.5° = b$ $\qquad\qquad$ $r \cdot \cos 22.5° = a$

The side length of the regular octagon is $s = 2b = 2r \cdot \sin 22.5°$. So, the area of the octagon is

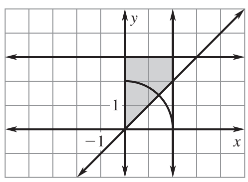
$$A = \frac{1}{2}a \cdot ns = \frac{1}{2}(r \cdot \cos 22.5)(8)(2r \cdot \sin 22.5°)$$
$$= 8r^2 \cdot \cos 22.5° \cdot \sin 22.5°$$

$$P(\text{Point is in polygon}) = \frac{\text{Area of octagon}}{\text{Area of circle}}$$
$$= \frac{8r^2 \cos 22.5° \cdot \sin 22.5°}{\pi r^2} \approx 0.90$$

The probability that a randomly chosen point in the circle lies in the inscribed polygon is about 0.90, or 90%.

27. Because points A and B are end points of a diameter of $\odot D$, the angle they form when connected to point C will always be a right angle, by Theorem 10.9. So, the probability that $\triangle ABC$ is a right triangle is 100%. $m\angle CAB$ will be less than or equal to 45° half the time, so the probability that $m\angle CAB \le 45°$ is 50%.

28.



$b_1 = |3 - 2| = 1, b_2 = |3 - 0| = 3,$

and $h = |2 - 0| = 2$

Area of trapezoid $= \frac{1}{2}h(b_1 + b_2) = \frac{1}{2}(2)(1 + 3) = 4$

$x^2 + y^2 \ge 4$ is the equation of a circle with $r = \sqrt{4} = 2$.

$\frac{1}{8}$ of the circle is in the shaded region, so the area of the shaded circle is $\frac{1}{8}(\pi(2)^2) = \frac{1}{2}\pi$.

$P(\text{the point } (x, y) \text{ in the solution region is in } x^2 + y^2 \ge 4)$

$$= \frac{\text{Area of trapezoid} - \text{Area of shaded circle}}{\text{Area of trapezoid}}$$
$$= \frac{4 - \frac{1}{2}\pi}{4} \approx 0.607 \text{ or } 60.7\%$$

The probability that a point (x, y) in the solution region is in $x^2 + y^2 \ge 4$ is about 60.7%.

29. An expression for each step is

$\text{step}_{n-1} + 0.5(1 - \text{step}_{n-1})$, which simplifies to $0.5\ \text{step}_{n-1} + 0.5$, where n is the current step number.

Step 1: 0.5 painted

Step 2: $0.5(0.5) + 0.5 = 0.75 = 75\%$

Step 3: $0.5(0.75) + 0.5 = 0.875 = 87.5\%$

Step 4: $0.5(0.875) + 0.5 = 0.9375 = 93.75\%$

Step 5: $0.5(0.9375) + 0.5 = 0.96875 = 96.875\%$

Step 6: $0.5(0.96875) + 0.5 = 0.984375 = 98.4375\%$

Step 7: $0.5(0.984375 + 0.5 = 0.9921875 = 99.21875\%$

Step 8: $0.5(0.9921875) + 0.5 = 0.99609375$
$= 99.609375\%$

Step 9: $0.5(0.99609375) + 0.5 = 0.998046875$
$= 99.8046875\%$

Step 10: $0.5(0.998046875) + 0.5 = 0.9990234375$
$= 99.90234375\%$

Step 11: $0.5(0.9990234375) + 0.5 = 0.99951171875$
$= 99.951171875\%$

After 11 steps, the painted portion of the stick is greater than 99.95%. So, $n \ge 11$ steps.

Problem Solving

30. $P(\text{Dart hits inner square}) = \frac{\text{Area of inner square}}{\text{Total area of board}} = \frac{6^2}{18^2}$

$$= \frac{36}{324} = \frac{1}{9}$$

Area outside inner square but inside circle

$=$ Area of circle $-$ Area of inner square

$= \pi \cdot \left(\frac{18}{2}\right)^2 - 6^2 = 81\pi - 36$

$P(\text{Dart hits outside inner square but inside circle})$

$$= \frac{\text{Area outside inner square but inside circle}}{\text{Total area of board}}$$
$$= \frac{81\pi - 36}{324} \approx 0.674$$

The probability that it hits inside the inner square is $\frac{1}{9}$, or about 11.1%. The probability that it hits outside the inner square but inside the circle is about 0.674 or 67.4%.

31. a. $P(\text{bus waiting}) = \frac{\text{wait time}}{\text{time between arrivals}} = \frac{4}{10} = \frac{2}{5}$

The probability that there is a bus waiting when a passenger arrives at a random time is $\frac{2}{5}$, or 40%.

b. $P(\text{bus not waiting}) = \frac{\text{time between arrivals} - \text{waiting time}}{\text{time between arrivals}}$

$$= \frac{10 - 4}{10} = \frac{6}{10} = \frac{3}{5}$$

The probability that there is not a bus waiting when a passenger arrives at a random time is $\frac{3}{5}$, or 60%.

32. $P(\text{fire drill for lunch}) = \frac{\text{amount of time before lunch}}{\text{total amount of time at school}} = \frac{4}{7}$

The probability that the fire drill begins before lunch is $\frac{4}{7}$, or about 57.1%.

33. $P(\text{You miss the call})$

$= \frac{\text{amount of overlap while practicing drums}}{\text{Interval of time for phone call}}$

$= \frac{|7{:}10 - 7{:}00|}{|8{:}00 - 7{:}00|}$

$= \frac{10 \text{ minutes}}{60 \text{ minutes}} = \frac{1}{6}$

The probability that you missed your friend's call is $\frac{1}{6}$, or about 16.7%.

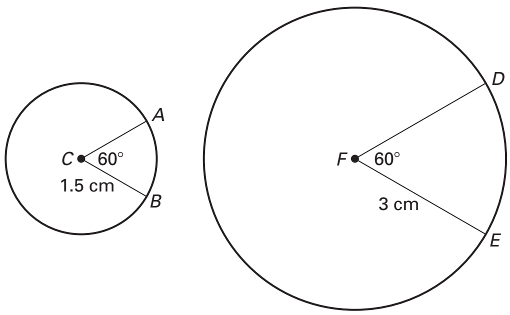
34. a. First count all the whole squares. There are about 64 whole squares. Next make groups of partially covered squares, so the combined area of each group is about 1 square unit. There are about 14 more squares, which makes a total of $64 + 14 = 78$ squares. Each side of each square is 2 kilometers, so the area of each square is $2^2 = 4$ square kilometers. 78 squares • 4 km^2/square $= 312$ square kilometers. The area of the planned landing region was about 312 square kilometers.

b. $P(\text{Beagle 2 landing in crater}) = \frac{\text{Area of crater}}{\text{Area of landing region}}$

$= \frac{\pi \cdot \left(\frac{1}{2}\right)^2}{312} \approx 0.0025$

The probability that Beagle 2 landed in the crater is about 0.0005 or 0.25%.

35.



Area of $\odot F = \pi r^2 = \pi \cdot 3^2 = 9\pi$

Area of sector $DEF = \frac{m\widehat{DE}}{360°} \cdot \pi r^2 = \frac{60°}{360°} \cdot \pi \cdot 3^2$

$= 1.5\pi$

Area of $\odot C = \pi r^2 = \pi \cdot (1.5)^2 = 2.25\pi$

Area of sector $ABC = \frac{m\widehat{AB}}{360°} \cdot \pi r^2$

$= \frac{60°}{360°} \cdot \pi(1.5)^2 = 0.375\pi$

Smaller circle: $P(\text{Point in sector}) = \frac{\text{Area of sector}}{\text{Area of circle}}$

$= \frac{0.375\pi}{2.25\pi}$

$= \frac{1}{6}$ or about 16.7%

Larger circle: $P(\text{Point in sector}) = \frac{\text{Area of sector}}{\text{Area of circle}}$

$= \frac{1.5\pi}{9\pi}$

$= \frac{1}{6}$ or about 16.7%

The probability of a randomly selected point being in the sector stays the same when the central angle stays the same and the radius of the circle doubles. As the radius doubles, the area of the sector and entire circle quadruples, but they are still proportional by the Areas of Similar Polygons Theorem. Because they are still proportional, the probabilities remain equal.

36. P(both pieces are at least 1 in.) is the same as $1 - P$(one piece is less than 1 inch) because both pieces may not be less than 1 inch. The probability of producing a piece less than 1 inch is $\frac{1}{3}$. So, $1 - \frac{1}{3} = \frac{2}{3} \approx 66.7\%$ is the probability we want to find.

37. a. From Exercise 30, the probability that one dart hits the yellow square is $\frac{1}{9}$. Because the throws are independent, you multiply the probabilities.

$[P(\text{Dart hits yellow square})]^2 = \left(\frac{1}{9}\right)^2 = \frac{1}{81}$

The probability that both darts hit the yellow square is $\frac{1}{81}$, or about 1.2%.

b. $P(\text{Dart hits outside the circle})$

$= \frac{\text{Area of outer square} - \text{Area of circle}}{\text{Area of outer square}}$

$= \frac{18^2 - \pi \cdot 9^2}{18^2} = \frac{324 - 81\pi}{324}$

Because the throws are independent, you multiply the probabilities.

$P(\text{Dart hits yellow square})$

$\cdot\ P(\text{Dart hits outside the circle}) = \frac{1}{9} \cdot \frac{324 - 81\pi}{324}$

≈ 0.024

The probability that the first dart hits the yellow square and the second hits outside the circle is about 2.4%.

c. From Exercise 30. The probability that one dart hits inside the circle but outside the yellow square is about 0.674. Because the throws are independent, you multiply the probabilities.

$[P(\text{Dart hits inside the circle but outside the yellow square})]^2 \approx (0.674)(0.674) \approx 0.454$

The probability that both darts hit inside the circle but outside the yellow square is about 45.4%.

38. $P(\text{Part of bird call erased}) = \text{Time of silence} + \text{Time of bird call} + \frac{\text{Time of erased data}}{\text{Total time of tape}}$

$= \frac{8 \text{ min} + 5 \text{ min} + 10 \text{ min}}{60 \text{ min}}$

$= \frac{23}{60}$ or about 38.3%

P(All of the bird call erased)

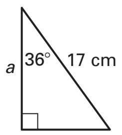
$= \dfrac{\text{Time of bird call} + \text{Time of erased data}}{\text{Total time of tape}}$

$= \dfrac{5 + 10}{60} = \dfrac{15}{60} = \dfrac{1}{4}$ or 25%

The probability that part of the bird call was erased is $\frac{23}{60}$, or about 38.3%. The probability that all of the bird call was erased was $\frac{1}{4}$, or 25%.

Quiz for the lessons "Areas of Regular Polygons" and "Use Geometric Probability"

1. The central angle of the regular pentagon measures $\frac{360°}{5} = 72°$ and the bisected angle measures $\frac{1}{2}(72°) = 36°$. Use a trigonometric ratio to find the apothem a.



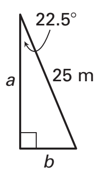
$\cos 36° = \dfrac{a}{17}$

$17 \cdot \cos 36° = a$

$A = \frac{1}{2}a \cdot ns = \frac{1}{2}(17 \cdot \cos 36°)(5)(20) \approx 687.66$

The area of the regular pentagon is about 687.7 square centimeters.

2. The central angle of the regular octagon measures $\frac{360°}{8} = 45°$ and the bisected angle measures $\frac{1}{2}(45°) = 22.5°$. Use a trigonometric ratio to find the apothem a and base b.



$\sin 22.5° = \dfrac{b}{25}$ $\qquad$ $\cos 22.5° = \dfrac{a}{25}$

$25 \cdot \sin 22.5° = b$ $\qquad$ $25 \cdot \cos 22.5° = a$

The side length of the octagon is $s = 2b = 50 \cdot \sin 22.5°$.

$A = \frac{1}{2}a \cdot ns = \frac{1}{2}(25 \cdot \cos 22.5)(8)(50 \cdot \sin 22.5)$

≈ 1767.8

The area of the regular octagon is about 1767.8 square meters.

3. $P(\text{point is in shaded region}) = \dfrac{\text{Area of shaded circle}}{\text{Area of large circle}}$

$= \dfrac{\pi r_s^2}{\pi r_\ell^2} = \dfrac{\pi\left(\frac{3}{2}\right)^2}{\pi\left(\frac{10}{2}\right)^2} = \dfrac{\frac{9}{4}\pi}{\frac{100}{4}\pi}$

$= \dfrac{9}{100}$

The probability that a randomly chosen point in the figure lies in the shaded region is $\frac{9}{100}$, or 9%.

4. $P(\text{Point is in shaded region}) = \dfrac{\text{Area of shaded region}}{\text{Area of rectangle}}$

$= \dfrac{\text{Area of rectangle} - \text{Area of trapezoid}}{\text{Area of rectangle}}$

$= \dfrac{bh - \frac{1}{2}h_t(b_1 + b_2)}{bh} = \dfrac{8(5) - \frac{1}{2}(2)(3 + 5)}{8(5)} = \dfrac{32}{40} = \dfrac{4}{5}$

The probability that a randomly chosen point in the figure lies in the shaded region is $\frac{4}{5}$, or 80%.

Lesson 11.5 Explore Solids

Investigating Geometry Activity for the lesson "Explore Solids"

1. a. 3 faces meet at each vertex.

b. 5 faces meet at each vertex.

2. The angle measures of all the angles in an equilateral triangle are 60°, so for 6 of these angles to meet means the total angle measure is $6 \cdot 60° = 360°$. A solid cannot have a vertex with 6 equilateral triangles because they would form a plane.

3. Three congruent regular hexagons meeting at a vertex would result in a plane being formed. The sum of the angles $(3 \cdot 120°)$ is 360° which will not form a convex vertex.

4. $F + V = E + 2$

Guided Practice for the lesson "Explore Solids"

1. The solid is formed by polygons, so it is a polyhedron. The base is a square, so it is a square pyramid. It has 5 faces, 5 vertices, and 8 edges.

2. The solid has a curved surface, so it is not a polyhedron.

3. The solid is formed by polygons, so it is a polyhedron. The two bases are congruent triangles, so it is a triangular prism. It has 5 faces, 6 vertices, and 9 edges.

4. The dodecahedron has 12 faces, 20 vertices, and 30 edges.

$F + V = E + 2$

$12 + 20 = 30 + 2$

$32 = 32$ ✓

5. The cross section is a triangle.

6. The cross section is a circle.

7. The cross section is a hexagon.

Exercises for the lesson "Explore Solids"

Skill Practice

1. The five Platonic solids and their faces are: regular tetrahedron, 4 faces; cube, 6 faces; regular octahedron, 8 faces; regular dodecahedron, 12 faces; regular icosahedron, 20 faces.

2. Euler's Theorem states the sum of the number of faces and vertices of a polyhedron is equal to the number of edges of the polyhedron plus two.

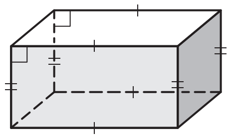
3. The solid is formed by polygons, so it is a polyhedron. The base is a pentagon, so it is a pentagonal pyramid.

4. The solid is formed by polygons, so it is a polyhedron. The two bases are congruent hexagons, so it is a hexagonal prism.

5. The solid has a curved surface, so it is not a polyhedron.

6. The bases of the prism are triangles, not rectangles. The solid is a triangular prism.

7.



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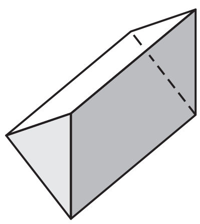

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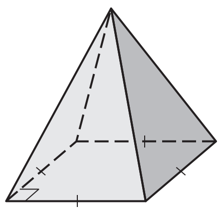
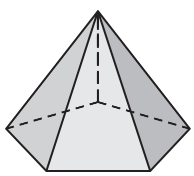


10.



11. $n + 12 = 18 + 2$

$n + 12 = 20$

$n = 8$

There are 8 faces.

12. $5 + n = 8 + 2$

$5 + n = 10$

$n = 5$

There are 5 vertices.

13. $10 + 16 = n + 2$

$26 = n + 2$

$24 = n$

There are 24 edges.

14. $n + 12 = 30 + 2$

$n + 12 = 32$

$n = 20$

There are 20 faces.

15. There are 4 faces, 4 vertices, and 6 edges.

$4 + 4 = 6 + 2$

$8 = 8$ ✓

16. There are 5 faces, 5 vertices, and 8 edges.

$5 + 5 = 8 + 2$

$10 = 10$ ✓

17. There are 5 faces, 6 vertices, and 9 edges.

$5 + 6 = 9 + 2$

$11 = 11$ ✓

18. There are 5 faces, 6 vertices, and 9 edges.

$5 + 6 = 9 + 2$

$11 = 11$ ✓

19. There are 8 faces, 12 vertices, and 18 edges.

$8 + 12 = 18 + 2$

$20 = 20$ ✓

20. There are 8 faces, 12 vertices, and 18 edges.

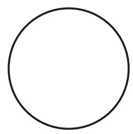
$8 + 12 = 18 + 2$

$20 = 20$ ✓

21. A cube is a solid formed by six congruent faces, so by name it is a regular hexahedron.

22. concave **23.** concave **24.** convex

25.

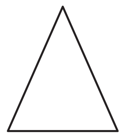


circle

26.

rectangle

27.



triangle

28. A; The cross section parallel to the square base is also a square.

29. Euler's Theorem proves this is incorrect.

$4 + 6 \stackrel{?}{=} 4 + 2$

$10 \neq 8$

Instead there should be 4 faces, 4 vertices, and 6 edges.

$4 + 4 = 6 + 2$

$8 = 8$ ✓

30. C; A triangular prism and a square pyramid both have 5 faces.

31. D; An octagonal prism has 10 faces, 16 vertices, and 24 edges.

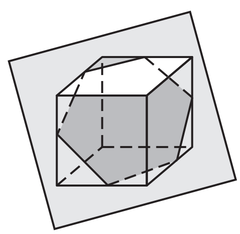
32. $32 + v = 90 + 2$

$32 + v = 92$

$v = 60$

Euler's Theorem can be used. The solid has 60 vertices.

33. The plane should intersect the cube at an angle so that it touches each of the six faces. The figure will look like:



Problem Solving

34. **a.** There are 10 vertices.

b. $7 + 10 = E + 2$

$17 = E + 2$

$15 = E$

There are 15 edges.

35. There are 18 edges and 12 vertices.

$8 + 12 = 18 + 2$

$20 = 20$ ✓

36. The cross section is a circle.

37. The cross section is a square.

38. The cross section is a rectangle.

39. A polyhedron with 4 vertices and 6 edges is a triangular pyramid. No, Euler's Theorem means that all polyhedrons with 4 vertices and 6 edges have the same number of faces.

40. **a.** The cross section is a rectangle.

b. The length of the rectangle is the hypotenuse of the right triangle formed by two sides of the cube. The length is $\sqrt{6^2 + 6^2} = \sqrt{72} = 6\sqrt{2}$.

$P = 6 + 6 + 6\sqrt{2} + 6\sqrt{2} = 12 + 12\sqrt{2} \approx 28.97$

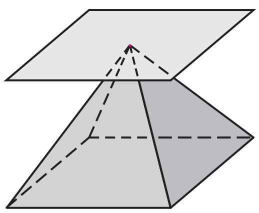
The perimeter is approximately 28.97 inches.

c. $A = \ell \cdot w = 6\sqrt{2} \cdot 6 = 36\sqrt{2}$

The area of the cross section is $36\sqrt{2}$ inches^2 or about 50.9 inches^2.

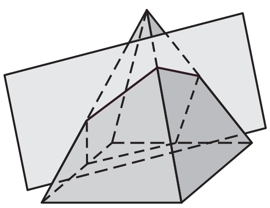
41. **a.** The cross section is a triangle.

b. Yes; *Sample answer:*



c. The cross section is a square.

d. Yes; *Sample answer:*

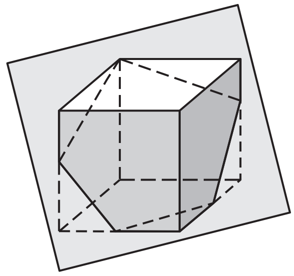


42.

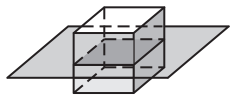
Platonic Solid	Faces	Vertices	Edges	$F + V = E + 2$
Tetrahedron	4	4	6	$4 + 4 = 6 + 2$
Cube	6	8	12	$6 + 8 = 12 + 2$
Octahedron	8	6	12	$8 + 6 = 12 + 2$
Dodecahedron	12	20	30	$12 + 20 = 30 + 2$
Icosahedron	20	12	30	$20 + 12 = 30 + 2$

43. No

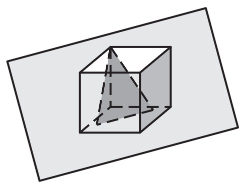
44. Yes;



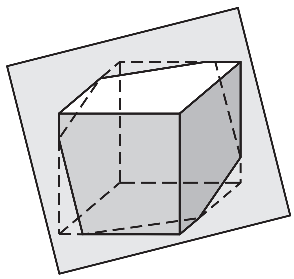
45. Yes;



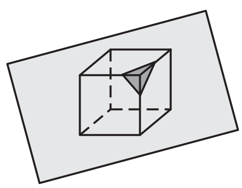
46. Yes;



47. Yes;



48. Yes;



49. **a.** There would be 7 faces, 10 vertices, and 15 edges.

b. There would be 7 faces, 10 vertices, and 15 edges.

c. The number of faces, vertices, and edges would remain the same.

d. There would be 9 faces, 14 vertices, and 21 edges.

50. a. There would be 5 faces, 6 vertices, and 9 edges.

b. There would be 5 faces, 6 vertices, and 9 edges.

c. The number of faces, vertices, and edges would remain the same.

d. Cutting two edges would make 6 faces, 7 vertices, and 11 edges.

51. The first solid has 8 vertices and each vertex has three 90° angles.

$D = 360° - 3(90°) = 90°$

$DV = 90(8) = 720°$

The second solid has 10 vertices and each vertex has two 90° angles and one 108° angle.

$D = 360° - 2(90°) - 108° = 72°$

$DV = 72(10) = 720°$

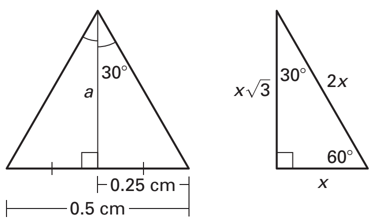
The third solid has 12 vertices and each vertex has two 90° angles and one 120° angle.

$D = 360° - 2(90°) - 120° = 60°$

$DV = 60(12) = 720°$

Mixed Review of Problem Solving for the lessons "Circumference and Arc Length", "Areas of Circles and Sectors", "Areas of Regular Polygons", "Use Geometric Probability", and "Explore Solids"

1. a. The central angle measures $\frac{360°}{6} = 60°$ and the bisected angle measures $\frac{1}{2}(60°) = 30°$. Use a special triangle to find the length of the apothem.



The apothem of a small mirror is $\frac{\sqrt{3}}{4}$ meters.

b. $A = \frac{1}{2}a \cdot ns = \frac{1}{2}\left(\frac{\sqrt{3}}{4}\right)(6)\left(\frac{1}{2}\right) \approx 0.65$

The area of one of the small mirrors is about 0.65 square meter.

c. $91(0.65) \approx 59.15$

The area of the primary mirror is about 59.2 square meters.

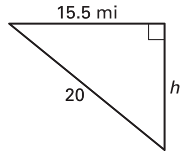
2. a. Area of illuminated water surface $= \frac{m\,\widehat{arc}}{360°} \cdot \pi r^2$

$$= \frac{216°}{360°} \cdot \pi \cdot 20^2$$

$$\approx 754$$

The area of the water's surface that is illuminated by the lighthouse is about 754 square miles.

b. The shortest distance between the lighthouse and the boat is the height of the triangle formed when the endpoints of the boat and illuminated water are connected to the lighthouse (which are the radii of the circle). The base of the original triangle is 31, so the base of the reference triangle is $\frac{1}{2}(31) = 15.5$. Use the Pythagorean Theorem to find the height of the triangle.



$20^2 = h^2 + 15.5^2$

$159.75 = h^2$

$12.64 \approx h$

The closest distance between the lighthouse and the boat is about 12.6 miles.

3. a. $P(\text{It will land it in the dish}) = \frac{\text{Area of red dish}}{\text{Area of jar}}$

$$= \frac{\pi rd^2}{\pi rj^2} = \frac{\pi\left(\frac{5}{2}\right)^2}{\pi\left(\frac{20}{2}\right)^2}$$

$$= \frac{\frac{25}{4}\pi}{\frac{400}{4}\pi} = \frac{1}{16}$$

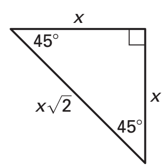
The probability that it will land in the dish is $\frac{1}{16}$, or 6.25%.

b. Because the probability is $\frac{1}{16}$, multiply 400 coins by $\frac{1}{16}$ to find out how man prizes you would expect people to win.

$400\left(\frac{1}{16}\right) = 25$

You would expect people to win 25 prizes if 400 coins are dropped.

4. Because the triangle is a right isosceles triangle, you can use a special triangle to find out the base lengths of the triangle, which are the diameters of the two smaller semi circles. From the diagram, $x = 4$. So, the diameter of the small semicircles is 4.



$P = \frac{1}{2}(\text{perimeter of large semicircle})$

$+ 2(\text{perimeter of small semicircle})$

$= \frac{1}{2}(\pi d) + 2\left(\frac{1}{2}\pi d_s\right) = \frac{1}{2}\pi(4\sqrt{2}) + \pi(4)$

$= 2\sqrt{2}\pi + 4\pi$

$A = \frac{1}{2}(\text{Area of large semicircle})$

$+ 2(\text{Area of small semicircle}) + \text{Area of triangle}$

$= \frac{1}{2}\pi\left(\frac{d_L}{2}\right)^2 + 2\left(\frac{1}{2}\pi\left(\frac{d_c}{2}\right)^2\right) + \frac{1}{2}x^2$

$= \frac{1}{2}\pi\left(\frac{4\sqrt{2}}{2}\right)^2 + \pi\left(\frac{4}{2}\right)^2 + \frac{1}{2}(4)^2 = 8\pi + 8$

The perimeter of the figure is $2\sqrt{2}\pi + 4\pi$ units and the area of the figure is $8\pi + 8$ square units.

5. *Sample answer:* The area of the given fan is

$A = \frac{120°}{360°} \cdot \pi \cdot 9^2 \approx 84.8 \text{ cm}^2$.

A fan with a radius of 8 cm and an arc of 160° would have an area of $A = \frac{160°}{360°} \cdot \pi \cdot 8^2 \approx 89.4 \text{ cm}^2$. Because the area of the new fan is greater than the area of the given fan, it does a better job of cooling you.

6. $F + V = E + 2$

$F + 6 = 7 + 2$

$F + 6 = 9$

$F = 3$

For a polyhedron to have 6 vertices and 7 edges it would need to have only 3 faces. Because it is not possible for a polyhedron to have only 3 faces, it is not possible for a polyhedron to have 6 vertices and 7 edges.

7. a. The cross section is a square.

b. $P = 4s = 4(10) = 40$

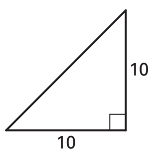
The perimeter is 40 feet.

$A = s^2 = 10^2 = 100$

The area is 100 square feet.

c. Isosceles right triangles are formed.

d.



Area $= \frac{1}{2}bh = \frac{1}{2}(10)(10) = 50$

The area is 50 square feet.

Lesson 11.6 Volume of Prisms and Cylinders

Guided Practice for the lesson "Volume of Prisms and Cylinders"

1. $V = (1)(1)(1) + (3)(1)(2) = 1 + 6 = 7$

The volume is 7 units^3.

2. $V = Bh = (5)^2(12) = 300$

The volume is 300 ft^3.

3. $V = \pi r^2 h$

$684\pi = \pi r^2(18)$

$38 = r^2$

$\sqrt{38} = r$

The radius is $\sqrt{38}$ inches.

4. $V = Bh = \frac{1}{2}(9 \cdot 5)(8) = 180$

The volume is 180 m^3.

5. B = Area of triangle − area of square

$= \frac{1}{2}(10)(5\sqrt{3}) - 3^2 = 25\sqrt{3} - 9$

$V = Bh = (25\sqrt{3} - 9)(6) \approx 205.81$

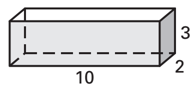
The volume is about 205.81 ft^3.

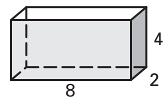
Exercises for the lesson "Volume of Prisms and Cylinders"

Skill Practice

1. The volume of a solid is measured in cubic units.

2. Not necessarily. For example:





$S = 2B + Ph$

$= 2(20) + (24)(3)$

$= 112$ square units

$V = Bh$

$= 20(3)$

$= 60$ cubic units

$S = 2B + Ph$

$= 2(16) + (20)(4)$

$= 112$ square units

$V = Bh$

$= 16(4)$

$= 64$ cubic units

Two solids can have the same surface area and not have the same volume.

3. A; Volume of box $= Bh = (15 \cdot 9)(3) = 405$ cubic inches

Volume of cube $= s^3 = 3^3 = 27$ cubic inches

Number of cubes $= \frac{405}{27} = 15$

15 three inch cubes can fit in the box.

4. $V = 2(3)(1) + 7(2)(1) = 6 + 14 = 20$ unit cubes

The volume is 20 cubic units.

5. $V = 1(4)(2) + 5(1)(2) = 8 + 10 = 18$ unit cubes

The volume is 18 cubic units.

6. $V = 3(1)(4) + 5(3)(4) = 12 + 60 = 72$ unit cubes

The volume is 72 cubic units.

7. $V = Bh = \left(\frac{1}{2}(7 \cdot 10)\right) \cdot 5 = 175$

The volume is 175 in.^{3}.

8. $V = Bh = (4 \cdot 2)(1.5) = 12$

The volume is 12 m^3.

9. $V = Bh = 6\left(\frac{1}{2}(7.5)(3.75\sqrt{3})\right)(18) = 1518.75\sqrt{3} \approx 2630.55$

The volume is about 2630.55 cm^3.

10. $V = \pi r^2 h = \pi(7)^2(12) = 588\pi \approx 1847.26$

The volume is about 1847.26 ft^3.

11. $V = \pi r^2 h = \pi\left(\frac{10}{2}\right)^2(16) = 400\pi \approx 1256.64$

The volume is about 1256.64 in.^{3}.

12. $V = \pi r^2 h = \pi\left(\frac{26.8}{2}\right)^2(9.8) = 1759.688\pi \approx 5528.22$

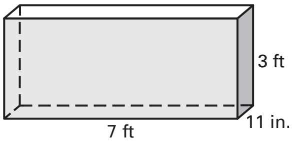
The volume is about 5528.22 cm^3.

13. The volume of a right cylinder is $\pi r^2 h$ not $2\pi rh$.

$V = \pi r^2 h = \pi(4)^2(3) = 48\pi$

The volume is 48π ft^3.

14.



In cubic inches,

$V = Bh = (7 \cdot 12)(3 \cdot 12)(11) = 33{,}264.$

The volume is 33,264 in.^{3} = 19.25 ft^3.

15. $V = Bh$

$1000 = (x \cdot x) \cdot x$

$1000 = x^3$

$10 = x$

The value of x is 10 inches.

16. $B = \frac{1}{2}bh = \frac{1}{2}(5)\left(\sqrt{9^2 - 2.5^2}\right) \approx 21.615$

$V = Bh$

$45 \approx 21.615x$

$x \approx 2.08$

The value of x is about 2.08 cm.

17. $V = \pi r^2 h$

$128\pi = \pi\left(\frac{x}{2}\right)^2(8)$

$128\pi = 2\pi x^2$

$64 = x^2$

$8 = x$

The value of x is 8 inches.

18. $V = \pi R^2 h - \pi r^2 h$

$= \pi(3)^2(7) - \pi(1)(7)$

$= 63\pi - 7\pi = 56\pi \approx 175.93$

The volume is about 175.93 m^3.

19. $V = Bh = [(7.8)(12.4) - (1.8)(3)] \cdot 9$

$= (96.72 - 5.4) \cdot 9$

$= (91.32)9 = 821.88$

The volume is 821.88 ft^3.

20. $V = Bh + \frac{1}{2}\left(\pi r^2 h\right)$

$= (4 \cdot 4)(4) + \frac{1}{2}\left(\pi(2)^2(4)\right)$

$= 64 + 8\pi$

≈ 89.13

The volume is about 89.13 in.^{3}

21. A; $V = \pi r^2 h$

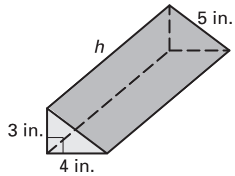
$64\pi = \pi(4)^2 h$

$64\pi = 16\pi h$

$4 = h$

The height of the cylinder is 4 feet.

22.



$V = Bh$

$96 = \frac{1}{2}(3 \cdot 4)h$

$96 = 6h$

$16 = h$

The height of the prism is 16 inches.

23. $V = \pi r^2 h$

$1005.5 = \pi r^2(8)$

$40 \approx r^2$

$6.33 \approx r$

$d = 2r \approx 2(6.33) \approx 12.65$

The diameter is about 12.65 cm.

24. $V = Bh = (4 \cdot 7)(6) = 168$

The volume is 168 in.^{3}

25. $V = \pi r^2 h = \pi(8)^2(14) = 896\pi \approx 2814.87$

The volume is about 2814.87 ft^3.

26. $\sin 60° = \frac{h}{18}$

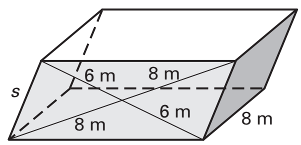
$18 \sin 60° = h$

$9\sqrt{3} = h$

$V = \pi r^2 h = \pi\left(\frac{12}{2}\right)^2\left(9\sqrt{3}\right) = 324\pi\sqrt{3} \approx 1763.01$

The volume is about 1763.01 m^3.

27.



$s = \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10$

Lateral area = $P \cdot h = 4(10) \cdot 8 = 320$

The lateral area is 320 m^2.

B = area of a rhombus = $\frac{1}{2}d_1 d_2 = \frac{1}{2}(12)(16) = 96$

Surface area = $2B + Ph = 2(96) + 320 = 512$

The surface area is 512 m^2.

Volume = $Bh = 96 \cdot 8 = 768$

The volume is 768 m^3.

Problem Solving

28. B = Area of rectangular face − Area of cylindrical end

$= 9.5 - \pi\left(\frac{3}{2}\right)^2$

$= 45 - \frac{9}{4}\pi$

$V = Bh = \left(45 - \frac{9}{4}\pi\right)(17) \approx 644.83$

The volume is about 644.83 mm^3.

29. a. V = Volume of large prism − Volume of two smaller prisms

$V = 15.75(8)(8) - (4.5)(4)(8) - (4.5)(4)(8)$

$= 1008 - 144 - 144$

$= 720$

The volume is 720 $in.^3$.

b. B = Area of large rectangle − 2 • Area of small rectangle

$= 15.75(8) - 2(4.5 \cdot 4)$

$= 90$

$V = Bh = 90(8) = 720$

The volume is 720 $in.^3$.

c. The answers are the same.

30. a. $V = \pi r^2 h$

$= \pi\left(\frac{1000}{2}\right)^2(400)$

$= 100{,}000{,}000\pi$

$\approx 314{,}159{,}265$

The volume of Blue Hole is about 314,159,265 ft^3.

b. Gallons of water $= 314{,}159{,}265 \text{ ft}^3 \times \frac{7.48}{1 \text{ ft}^3} \approx 2{,}349{,}911{,}302$

There are about 2,349,911,302 gallons of water in the Blue Hole.

31. $C = 2\pi r$

$10 = 2\pi r$

$\frac{5}{\pi} = r$

$V = \pi r^2 h = \pi\left(\frac{5}{\pi}\right)^2(20) \approx 159.15$

The volume of the column is about 159.15 ft^3.

32. The first cylinder has volume

$V = \pi r^2 h = \pi(3)^2(5) = 45\pi$

The second cylinder has volume

$V = \pi r^2 h = \pi(5)^2(3) = 75\pi$

The second cylinder has a greater volume because 75π is greater than 45π.

33. a. $V = \frac{3}{4}Bh = \frac{3}{4}(30 \cdot 10)(20) = 4500$

The volume of the water is 4500 $in.^3$.

b. The volume of the rock is equal to the volume the water rose.

$V = Bh = (30 \cdot 10)(0.25) = 75$

The volume of the rock is 75 $in.^3$.

c. The capacity of the tank is $(30)(10)(20) = 6000$ $in.^3$.

Let n represent the number of rocks that can be placed in tank.

Then, $6000 = 4500 + 75n$

$1500 = 75n$

$20 = n$

So, 20 rocks can be placed in the aquarium before the water spills out.

34. $V = Bh$ = (Area of rectangular bottom + Area of triangular top) h

$9072 = \left[(8)(18) + \frac{1}{2}(18)\left(\sqrt{x^2 - 9^2}\right)\right](36)$

$252 = 144 + 9\sqrt{x^2 - 81}$

$108 = 9\sqrt{x^2 - 81}$

$12 = \sqrt{x^2 - 81}$

$12^2 = \left(\sqrt{x^2 - 81}\right)^2$

$144 = x^2 - 81$

$225 = x^2$

$\sqrt{225} = x$

$15 = x$

Each half of the roof is 15 ft by 36 ft.

Problem Solving Workshop for the lesson "Volume of Prisms and Cylinders"

1. a. Volume V of pencil holder = Volume of large prism − Volume of 2 cylinders

$= 7.5 \cdot 4 \cdot 4 - 2\left(\pi(1.5)^2(4)\right)$

$= 120 - 18\pi$

≈ 63.45

The volume of the pencil holder is about 63.45 $in.^3$.

b. B = Area of rectangle − Area of two circles

$= (7.5)(4) - 2(\pi)(1.5)^2$

$= 30 - 4.5\pi$

$V = Bh = (30 - 4.5\pi)4 \approx 63.45$

The volume of the pencil holder is about 63.45 $in.^3$.

2. The student was correct that 4 times the base area of the cylinders needs to be subtracted, but they forgot that the lateral area of each cylinder needs to be added.

$S = 2(7.5 \cdot 4) + 2(7.5 \cdot 4) + 2(4 \cdot 4) - 4\left(\pi(1.5)^2\right)$
$\quad + 2\left(2\pi(1.5)4\right)$

$S = 152 + 15\pi$

$S \approx 199.12$

The surface area is about 199.12 square inches.

3. Volume of cylinder $= \pi R^2 h$

Volume of hole $= \frac{1}{2}\pi R^2 h = \pi r^2 h$

$\pi r^2 h = \frac{1}{2}\pi R^2 h$

$r^2 = \frac{1}{2}R^2$

$r = \sqrt{\frac{1}{2}R^2}$

$r = \frac{\sqrt{2}}{2}R$

For the hole to have half the volume of the cylinder, r should be $\frac{\sqrt{2}}{2}R$.

4. $\begin{matrix}\text{Volume} \\ \text{of solid}\end{matrix} = \begin{matrix}\text{Volume of} \\ \text{large prism}\end{matrix} - \begin{matrix}\text{Volume of} \\ \text{small prism}\end{matrix}$

$V = (4)(2)(5) - (1)(1)(5) = 40 - 5 = 35$

The volume is 35 ft^3.

5. $V =$ Area of sector $\cdot\, h$

$= \frac{60°}{360°} \cdot \pi(2)^2 \cdot 3.5$

$= \frac{1}{6}(4\pi) \cdot 3.5$

$= \frac{7}{3}\pi$

≈ 7.33

The volume is about 7.33 in.^3.

6. **a.** The surface area can be found by finding the sum of the areas of each exposed face.

b. Not all of the faces of the beams are exposed, so adding the individual surface areas will result in a total that is greater than the actual surface area.

Extension for the lesson "Volume of Prisms and Cylinders"

1. volume $= 8 \cdot 8 \cdot 4.8 = 307.2 \text{ cm}^3$

density $= \frac{\text{mass}}{\text{volume}} = \frac{450\text{g}}{307.2 \text{ cm}^3} \approx 1.46 \text{ g/cm}^3$ or about 1.5 g/cm^3

2. area $= 369 \cdot 281 = 103{,}689 \text{ mi}^2$

population density $= \frac{\text{number of people}}{\text{area of land}} = \frac{5{,}024{,}728 \text{ people}}{103{,}689 \text{ mi}^2}$

≈ 48.46

In 2009, the population density of Colorado was about 48 people per square mile.

3. 2000: population density $=$

$\frac{\text{number of people}}{\text{area of land}} = \frac{2.74(7{,}393{,}354)}{261{,}797 \text{ mi}^2} = \frac{20{,}257{,}789.96}{261{,}797 \text{ mi}^2}$

≈ 77.38

2009: population density $=$

$\frac{\text{number of people}}{\text{area of land}} = \frac{24{,}782{,}302}{261{,}797 \text{ mi}^2} \approx 94.66$

About 77 people per square mile in 2000; in 2009, the population density was about 95 people per square mile, or 18 people per square mile greater than in 2000.

4. daily cost per cubic foot of first house:

$\frac{\text{cost}}{\text{cubic feet}} = \frac{\$7}{30{,}000 \text{ ft}^3} \approx \$0.00023/\text{ft}^3$

daily cost per cubic foot of second house:

$\frac{\text{cost}}{\text{cubic feet}} = \frac{\$6.50}{25{,}000 \text{ ft}^3} \approx \$0.00026/\text{ft}^3$

The 30,000 cubic foot house costs less to cool.

5. The heavier object. *Sample answer:* The weight of an object is the product of its mass and the gravity on Earth. Because the effect of gravity is the same for both objects, the object that is heavier has a greater mass.

Lesson 11.7 Volume of Pyramids and Cones

Investigating Geometry Activity for the lesson "Volume of Pyramids and Cones"

Step 3

It takes 3 times to fill the prism. So, the volume of the prism is 3 times the volume of the pyramid.

1. The areas of the bases are the same.

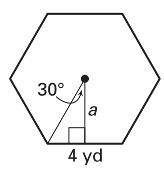
2. The heights are approximately the same.

3. The ratio of the volume of the pyramid to the volume of the prism is 1 to 3 .

4. $V = \frac{1}{3}Bh$, where B is the area of the base and h is the height.

Guided Practice for the lesson "Volume of Pyramids and Cones"

1. $\frac{360°}{6} = 60°$



The apothem bisects the central angle, so a 30°-60°-90° triangle is formed.

$a = \frac{2}{\tan 30°} = 2\sqrt{3}$

$B = \frac{1}{2}aP$

$= \frac{1}{2}\left(2\sqrt{3}\right)(6 \cdot 4)$

$= 24\sqrt{3}$

$V = \frac{1}{3}Bh$

$= \frac{1}{3}\left(24\sqrt{3}\right)(11)$

$= 88\sqrt{3} \approx 152.42$

The volume is about 152.42 yd^3.

2. $h^2 = \ell^2 - r^2$

$h^2 = 8^2 - 5^2$

$h^2 = 39$

$h = \sqrt{39}$

$V = \frac{1}{3}Bh$

$= \frac{1}{3}\left(\pi(5)^2\right)\left(\sqrt{39}\right)$

$= \frac{25}{3}\pi\sqrt{39}$

≈ 163.49

The volume is about 163.49 m^3.

3. $V = \frac{1}{3}Bh$

$1350\pi = \frac{1}{3}(\pi)(18)^2h$

$1350\pi = 108\pi h$

$12.5 = h$

The height is 12.5 meters.

4. $\tan 40° = \frac{r}{5.8}$

$r = 5.8 \tan 40°$

$V = \frac{1}{3}\left(\pi r^2\right)h$

$= \frac{1}{3}(\pi)(5.8 \tan 40°)^2(5.8)$

≈ 143.86

The volume is about 143.86 in.^3.

5. Volume = Volume of cylinder + Volume of cone

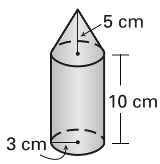
$= \pi r^2h + \frac{1}{3}\left(\pi r^2\right)h$

$= \pi(3)^2(10) + \frac{1}{3}\left(\pi \cdot 3^2\right)5$

$= 90\pi + 15\pi$

$= 105\pi$

≈ 329.87



The volume of the solid is about 329.87 cubic centimeters.

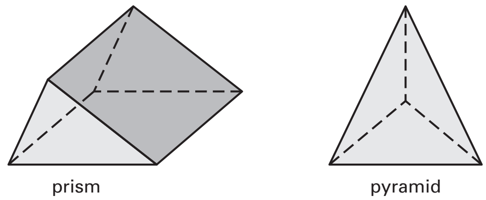
6. Flow rate $= \frac{\text{volume}}{\text{time}} = \frac{101\text{mL}}{3.2\text{ s}} \approx 31.56$ mL/s

The flow rate of the sand is about 31.56 mL/s.

Exercises for the lesson "Volume of Pyramids and Cones"

Skill Practice

1. A triangular prism has two congruent triangular bases that are parallel to each other. The lateral faces are all parallelograms. A triangular pyramid has a single triangular base with lateral faces are triangles with a common vertex.



Triangular prism Triangular pyramid

2. The volume of a square pyramid is one third the volume of a square prism with the same base and height.

3. $V = \frac{1}{3}Bh = \frac{1}{3}(5^2)(6) = 50$

The volume is 50 cm^3.

4. $V = \frac{1}{3}Bh = \frac{1}{3}(\pi r^2)h = \frac{1}{3}(\pi \cdot 10^2)(13) = \frac{1300}{3}\pi \approx 1361.36$

The volume is about 1361.36 mm^3.

5. $V = \frac{1}{3}Bh = \frac{1}{3}(2.5)(4) = \frac{40}{3} \approx 13.33$

The volume is about 13.33 in.^3.

6. $V = \frac{1}{3}Bh = \frac{1}{3}(\pi r^2)h = \frac{1}{3}(\pi \cdot 1^2)(2) = \frac{2}{3}\pi \approx 2.09$

The volume is about 2.09 m^3.

7. $V = \frac{1}{3}Bh = \frac{1}{3}\left(\frac{1}{2} \cdot 3 \cdot 4\right)(3) = 6$

The volume is 6 in.^3.

8. $V = \frac{1}{3}Bh = \frac{1}{3}\left(6 \cdot \frac{1}{2} \cdot 6\sqrt{3} \cdot 12\right)(17) = 1224\sqrt{3} \approx 2120.03$

The volume is about 2120.03 ft^3.

9. The slant height was used instead of the actual height.

$h = \sqrt{15^2 - 9^2} = \sqrt{144} = 12$

$V = \frac{1}{3}\pi(9^2)(12) = 324\pi \approx 1018$

The volume is about 1018 ft^3.

10. The volume of the pyramid is $\frac{1}{3}Bh$ not $\frac{1}{2}Bh$.

$V = \frac{1}{3}(7^2)(10) = \frac{490}{3} \approx 163.3$

The volume is about 163.3 ft^3.

11. D; $V = \frac{1}{3}Bh$

$45 = \frac{1}{3}B(9)$

$45 = 3B$

$15 = B$

The area of the base is 15 ft^2.

12. $V = \frac{1}{3}Bh$

$200 = \frac{1}{3}(10^2)x$

$200 = \frac{100}{3}x$

$6 = x$

The value of x is 6 cm.

13. $V = \frac{1}{3}Bh$

$216\pi = \frac{1}{3}(\pi r^2)h$

$216\pi = \frac{1}{3}(\pi \cdot x^2)(18)$

$216\pi = 6\pi x^2$

$36 = x^2$

$6 = x$

The value of x is 6 inches.

14. $V = \frac{1}{3}Bh$

$7\sqrt{3} = \frac{1}{3}\left(\frac{1}{2}(2\sqrt{3})(3)\right)(x)$

$7\sqrt{3} = \sqrt{3}x$

$7 = x$

The value of x is 7 feet.

15. $\tan 60° = \frac{\text{opp.}}{\text{adj.}}$

$\sqrt{3} = \frac{22}{r}$

$r = \frac{22}{\sqrt{3}}$

$V = \frac{1}{3}(\pi r^2)h \approx \frac{1}{3}\pi\left(\frac{22}{\sqrt{3}}\right)^2(22) \approx 3716.85$

The volume is about 3716.85 ft^3.

16. $\tan 32° = \frac{\text{opp.}}{\text{adj.}}$

$\tan 32° = \frac{7}{h}$

$h = \frac{7}{\tan 32°}$

$V = \frac{1}{3}(\pi r^2)h \approx \frac{1}{3}\pi(7)^2\left(\frac{7}{\tan 32°}\right) \approx 574.82$

The volume is about 574.82 yd^3.

17. $\cos 54° = \frac{\text{adj.}}{\text{hyp.}}$

$\cos 54° = \frac{r}{15}$

$r = 15 \cos 54°$

$\sin 54° = \frac{\text{opp.}}{\text{hyp.}}$

$\sin 54° = \frac{h}{15}$

$h = 15 \sin 54°$

$V = \frac{1}{3}(\pi r^2)h$

$= \frac{1}{3}\pi(15 \cos 54°)^2(15 \sin 54°)$

≈ 987.86

The volume is about 987.86 cm^3.

18. B; $\tan 29° = \frac{\text{opp.}}{\text{adj.}}$

$\tan 29° = \frac{5}{h}$

$h = \frac{5}{\tan 29°}$

$V = \frac{1}{3}(\pi r^2)h \approx \frac{1}{3}(\pi \cdot 5^2)\left(\frac{5}{\tan 29°}\right) \approx 236.15$

The approximate volume of the cone is 236.15 ft^3.

19. $V = \frac{1}{3}(\pi r^2)h$

$143.6 = \frac{1}{3}(\pi \cdot 4^2)h$

$143.6 = \left(\frac{16\pi}{3}\right)h$

$8.57 \approx h$

The height of the cone is about 8.57 cm.

20. $V =$ Volume of cylinder + Volume of cone

$= \pi r^2h + \frac{1}{3}(\pi r^2)h = \pi \cdot 3^2 \cdot 7 + \frac{1}{3}(\pi \cdot 3^2)(3)$

$= 63\pi + 9\pi = 72\pi \approx 226.19$

The volume is about 226.19 cm^3.

21. $V =$ Volume of cube − Volume of pyramid

$= s^3 - \frac{1}{3}Bh = 10^3 - \frac{1}{3}(5\sqrt{2})^2(10) \approx 833.33$

The volume is about 833.33 in.^3.

22. $V =$ Volume of large cone + Volume of small cone

$= \frac{1}{3}(\pi R^2)H + \frac{1}{3}(\pi r^2)h = \frac{1}{3}(\pi \cdot 1^2)(2) + \frac{1}{3}\left(\pi\left(\frac{1}{2}\right)^2\right)(1)$

$= \frac{2}{3}\pi + \frac{1}{12}\pi = \frac{3}{4}\pi \approx 2.36$

The volume is about 2.36 ft^3.

23. $V = 2 \cdot$ Volume of pyramid $= 2\left(\frac{1}{3}Bh\right) = \frac{2}{3}(3.3^2)(2.3)$

≈ 16.70

The volume is about 16.70 cm^3.

24. $V =$ Volume of cube − Volume of cone

$= S^3 - \frac{1}{3}(\pi r^2)h = 5.1^3 - \frac{1}{3}(\pi \cdot 2.55^2)(5.1)$

≈ 97.92

The volume is about 97.92 m^3.

25. V = Volume of cylinder − Volume of pyramid

$= (\pi r^2)h - \frac{1}{3}Bh$

$B = \frac{1}{2}aP$

$a = 2 \cos 22.5°$

$P = 8(4 \sin 22.5°)$

$V = (\pi \cdot 2^2)(3) - \frac{1}{3}\left(\frac{1}{2}(2 \cos 22.5°)(8)(4 \sin 22.5°)\right)$

≈ 26.39

The volume is about 26.39 yd^3.

26. The volume of a cone is proportional to the height, so to double the volume, change the height from h to $2h$.

The volume of a cone is proportional to the square of the radius of the base, so to double the volume, change the radius from r to $r\sqrt{2}$.

27. Drawing a diagonal in the square creates two 45°-45°-90° triangles. The diagonal has a length of $5\sqrt{2}$ meters, which is also the diameter of the base of the cone.

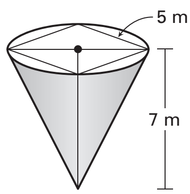
So, the radius is $\frac{5\sqrt{2}}{2}$.

$V = \frac{1}{3}(\pi r^2)h$

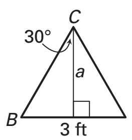
$= \frac{1}{3}\left(\pi\left(\frac{5\sqrt{2}}{2}\right)^2\right)(7)$

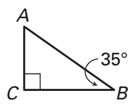
≈ 91.63

The volume is about 91.63 m^3.



28.





$\sin 30° = \frac{\text{opp.}}{\text{hyp.}}$

$0.5 = \frac{1.5}{BC}$

$BC = 3$

$\tan 30° = \frac{\text{opp.}}{\text{adj.}}$

$\frac{\sqrt{3}}{3} = \frac{1.5}{a}$

$a = 1.5\sqrt{3}$

$\tan 35° = \frac{\text{opp.}}{\text{adj.}}$

$\tan 35° = \frac{AC}{3}$

$AC = 3 \tan 35° \approx 2.10$

$V = \frac{1}{3}Bh$

$\approx \frac{1}{3}\left(6 \cdot \frac{1}{2} \cdot 1.5\sqrt{3} \cdot 3\right)(2.10)$

≈ 16.37

The volume is about 16.37 ft^3.

Problem Solving

29. a. $V = \frac{1}{3}(\pi r^2)h = \frac{1}{3}(\pi \cdot 4^2)(12) = 64\pi \approx 201$

There are about 201 cubic inches of frosting in the bag.

b. $\frac{201 \text{ in.}^3}{15 \text{ flowers}} = 13.4 \text{ in.}^3$ per flower

About 13.4 cubic inches of frosting are used per flower.

30. $V = \frac{1}{3}(\pi r^2)h = \frac{1}{3}(\pi \cdot 3^2)(8) = 24\pi \approx 75.40$

The volume of the small cup is about 75.4 in.^3.

31. The cone and the cylinder have the same base and height, so the volume of the cone is $\frac{1}{3}$ • Volume of cylinder.

This means you have to buy 3 small cups of popcorn to have the same amount as the large cup.

32. The cost of the large cup is twice the cost of the small cup but the volume is three times as great. So, the large cylindrical cup gives you more popcorn for your money.

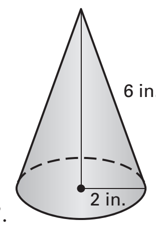
33. $h = \sqrt{6^2 - 2^2} = \sqrt{32} = 4\sqrt{2}$

$V = \frac{1}{3}(\pi r^2)h$

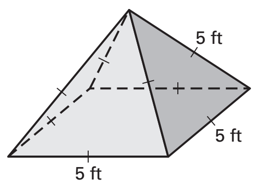
$= \frac{1}{3}(\pi \cdot 2^2)(4\sqrt{2})$

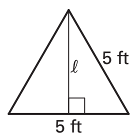
≈ 23.70

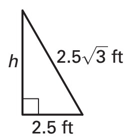
The volume is about 23.70 in.^3.



34.







$\ell = \sqrt{5^2 - 2.5^2} = 2.5\sqrt{3}$

$h = \sqrt{(2.5\sqrt{3})^2 - 2.5^2} = 2.5\sqrt{2}$

$V = \frac{1}{3}Bh = \frac{1}{3}(5^2)(2.5\sqrt{2}) \approx 29.46$

The volume is about 29.46 ft^3.

35. a. Original volume $= \frac{1}{3}Bh = \frac{1}{3}(7^2)(10) = \frac{490}{3}$

Let $h = 2(10)$: $V = \frac{1}{3}Bh = \frac{1}{3}(7^2)(20) = 2\left(\frac{490}{3}\right)$

The volume is doubled.

b. Let $S =$ side length $= 2(7)$

$V = \frac{1}{3}Bh = \frac{1}{3}(2(7))^2(10) = 4\left(\frac{490}{3}\right)$

The original volume is multiplied by four.

c. If the height of a pyramid is multiplied by x, the volume is multiplied by a factor of x. If the side length is multiplied by x, the volume is multiplied by a factor of x^2.

36. a. Volume = Volume of cylinder + Volume of cone

$$= \pi r^2 h + \frac{1}{3}(\pi r^2)h$$

$$= \pi(2.5^2)(7.5) + \frac{1}{3}(\pi \cdot 2.5^2)(4)$$

$$\approx 173.44 \text{ in.}^3$$

$$\frac{173.44 \text{ in.}^3}{14.4 \text{ in.}^3} \approx 12.04$$

The container can hold about 12 cups.

b. $2 \cdot \frac{1}{3} = \frac{2}{3}$ cups a day.

days $= \frac{2}{\frac{2}{3}} = 12 \cdot \frac{3}{2} = 18$

The feeder can go 18 days before it needs to be refilled.

37. Volume of big prism $= 6\left(\frac{\sqrt{3}}{4}S^2\right)h$

$$= 6\left(\frac{\sqrt{3}}{4}(3.5)^2\right)(1.5)$$

$$\approx 47.74$$

Volume of small prism $= 6\left(\frac{\sqrt{3}}{4}S^2\right)h$

$$= 6\left(\frac{\sqrt{3}}{4}(3.25)^2\right)(0.25)$$

$$\approx 6.86$$

Volume of pyramid $= \frac{1}{3} \cdot 6\left(\frac{\sqrt{3}}{4}S^2\right)h$

$$= \frac{1}{3} \cdot 6\left(\frac{\sqrt{3}}{4}(3^2)\right)(3)$$

$$\approx 23.38$$

Volume of solid $= 47.74 + 6.86 + 23.38 = 77.98$

The volume of the deck prism is about 77.98 in.^{3}.

38. a. Average area $= \pi \cdot$ (average value of R^2) $= \pi\left(\frac{R^2}{3}\right)$

b. $V_{\text{cone}} = \frac{1}{3}Bh = \frac{1}{3}\pi R^2 h = \frac{\pi R^2}{3} \cdot h,$

where B is the area of the base of the cone, $\frac{\pi R^2}{3}$ is the average area of a circular cross section, and h is the height.

39. a. $h = 2r \rightarrow r = \frac{h}{2}$

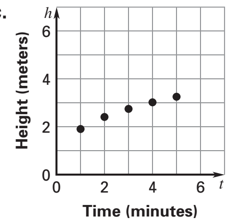
$V = \frac{1}{3}Bh = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2(h) = \frac{1}{3}\pi\left(\frac{h^2}{4}\right)(h) = \frac{\pi h^3}{12},$

where B is the area of the base of the cone, r is the radius, and h is the height.

b.

Time (minutes)	Height (meters)
1	1.90
2	2.40
3	2.74
4	3.02
5	3.25

c.



There is not a linear relationship between the height of the water and time. The data points on the graph do not form a straight line, so there cannot be a linear relationship.

40. $V = [\text{area of base}_1 + \text{area of base}_2 + \text{geometric mean}]\frac{1}{3}h$

$$= \left[\pi(3)^2 + \pi(9)^2 + \sqrt{\pi(3)^2 \cdot \pi(9)^2}\right]\frac{1}{3}(10)$$

$$\approx [9\pi + 81\pi + 84.82]\frac{10}{3}$$

$$\approx [367.57]\frac{10}{3}$$

$$\approx 1225.22$$

The volume of the frustum is about 1225.22 cm^3.

41. a.

$$\frac{h_1 + h_2}{r_2} = \frac{h_1}{r_1}$$

$$h_1 r_2 = r_1(h_1 + h_2)$$

$$h_1 r_2 = h_1 r_1 + h_2 r_1$$

$$h_1 r_2 - h_1 r_1 = h_2 r_1$$

$$h_1(r_2 - r_1) = h_2 r_1$$

$$h_1 = \frac{h_2 r_1}{r_2 - r_1}$$

b. $V = \frac{\pi r_2^2(h_1 + h_2)}{3} - \frac{\pi r_1^2 h_1}{3}$

$$= \pi r_2^2 \frac{\left(\frac{r_1 h_2}{r_2 - r_1} + h_2\right)}{3} - \frac{\pi r_1^2\left(\frac{r_1 h_2}{r_2 - r_1}\right)}{3}$$

c. $V = \pi r_2^2 \frac{\left(\frac{r_1 h_2}{r_2 - r_1} + h_2\right)}{3} - \frac{\pi r_1^2\left(\frac{r_1 h_2}{r_2 - r_1}\right)}{3}$

$$= \frac{1}{3}\left(\pi r_2^{\,2}\left(\frac{r_1h_2 + r_2h_2 - r_1h_2}{r_2 - r_1}\right) - \pi\left(\frac{r_1^{\,3}h_2}{r_2 - r_1}\right)\right)$$

$$= \frac{\pi}{3}\left(\frac{r_2^{\,3}h_2}{r_2 - r_1} - \frac{r_1^{\,3}h_2}{r_2 - r_1}\right)$$

$$= \frac{\pi h_2}{3}\left(\frac{r_2^{\,3} - r_1^{\,3}}{r_2 - r_1}\right)$$

$$= \frac{\pi h_2}{3}\left(\frac{(r_2 - r_1)(r_2^{\,2} + r_1r_2 + r_1^{\,2})}{r_2 - r_1}\right)$$

$$= \frac{h_2}{3}(\pi r_2^{\,2} + \pi r_1r_2 + \pi r_1^{\,2})$$

The volume is equal to one-third of the height times the area of the lower base plus the geometric mean of the two bases plus the area of the upper base.

42. If you rotate around the side of length 15, the solid of rotation is a cone with height 15 and base radius 20.

$V_1 = \frac{1}{3}\pi r^2h = \frac{1}{3}\pi(20)^2(15) = 2000\pi$

If you rotate around the side of length 20, the solid of rotation is a cone with height 20 and base radius 15.

$V_2 = \frac{1}{3}\pi r^2h = \frac{1}{3}\pi(15)^2(20) = 1500\pi$

Use the Geometric Mean Theorems 7.6 and 7.7 to show that the altitude drawn to the hypotenuse creates 9-12-15 and 12-16-20 right triangles.

If you rotate around the hypotenuse, the solid of rotation is two cones with a common base radius 12. One cone has height 9 and the other has height 16.

$V_3 = \frac{1}{3}\pi r^2h_1 + \frac{1}{3}\pi r^2h_2 = \frac{1}{3}\pi(12)^2(9) + \frac{1}{3}\pi(12)^2(16) =$
$432\pi + 768\pi = 1200\pi$

Quiz for the lessons "Explore Solids", "Volume of Prisms and Cylinders" and "Volume of Pyramids and Cones"

1. $F + V = E + 2$

$F + 8 = 12 + 2$

$F + 8 = 14$

$F = 6$

The polyhedron has 6 faces.

2. $V = Bh = \frac{1}{2}(10)(15)(7) = 525$

The volume is 525 cm^3.

3. $V = \pi r^2h = \pi(6)^2(10) = 360\pi \approx 1130.97$

The volume is about 1130.97 $in.^3$.

4. $V = \pi r^2h = \pi(4.5)^2(16) = 324\pi \approx 1017.88$

The volume is about 1017.88 m^3.

5. $V = \frac{1}{3}Bh = \frac{1}{3}(3^2)(2) = 6$

The volume is 6 cm^3.

6. $V = \frac{1}{3}(\pi r^2)h = \frac{1}{3}(\pi \cdot 30^2)(40) \approx 37{,}699.11$

The volume is about 37,699.11 ft^3.

7. $V = \frac{1}{3}(\pi r^2)h = \frac{1}{3}(\pi \cdot 8^2)(15) = 320\pi \approx 1005.31$

The volume is about 1005.31 yd^3.

8. The cups have the same radius and height, so the volume of the cone is one-third the volume of the cylinder. When one conical cup is poured into the cylindrical cup, one third of the cylinder will be full. The full level will be at a height that is one-third the cylinder's height or $\frac{1}{3}(6) = 2$ inches.

Spreadsheet Activity for the lesson "Volume of Pyramids and Cones"

1. The radius should be 2.25 cm and the height should be approximately 4.53 cm. The value in the surface area column decreases as you approach 95.81 and increases after.

2. The radius should be about 4.64 cm and the height should be about 9.29 cm.

Extension for the lesson "Volume of Prisms and Cylinders"

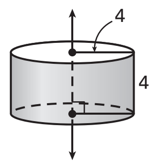
1–3. Check students' sketches.

1. Cylinder with height 8 and base radius 6

2. Cone with height 4 and base radius 3

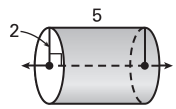
3. Sphere with radius 7

4.



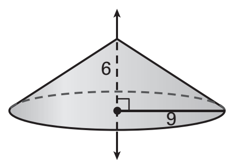
Cylinder with height 4 and base radius 4

5.



Cylinder with height 5 and base radius 2

6.



Cone with height 6 and base radius 9

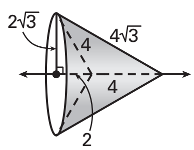
7–9. Check students' sketches.

7. $V = \pi r^2h = \pi(8^2)(8) = 512\pi$

8. $V = \frac{1}{3}\pi r^2h = \frac{1}{3}\pi(6)^2(6) = 72\pi$

9. $V = 2 \cdot \frac{1}{3}\pi r^2h = 2 \cdot \frac{1}{3}\pi(2^2)(2) = 5\frac{1}{3}\pi$

10. $V = \frac{1}{3}\pi r^2h = \frac{1}{3}\pi(2\sqrt{3})^2(6) - \frac{1}{3}\pi(2\sqrt{3})^2(2)$
$= 24\pi - 8\pi = 16\pi$



Lesson 11.8 Surface Area and Volume of Spheres

Guided Practice for the lesson "Surface Area and Volume of Spheres"

1. $S = 4\pi r^2 = 4\pi(20)^2 = 1600\pi \approx 5026.55$

The surface area is about 5026.55 ft^2.

2. $S = 4\pi r^2$

$30\pi = 4\pi r^2$

$7.5 = r^2$

$2.74 \approx r$

The radius is about 2.74 m.

3. $C = 2\pi r$

$2\pi r = 6\pi$

$r = 3$

$S = 4\pi r^2 = 4\pi(3)^2 = 36\pi$

The surface area of the inner ball is 36π, or about 113.10 ft^2.

4. $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(5)^3 \approx 523.60$

The volume is about 523.60 yd^3.

5. V = Volume of cone + Volume of hemisphere

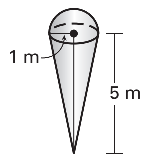
$= \frac{1}{3}\pi r^2 h + \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$

$= \frac{1}{3}\pi(1)^2(5) + \frac{2}{3}\pi(1)^3$

$= \frac{5}{3}\pi + \frac{2}{3}\pi$

$= \frac{7}{3}\pi$

≈ 7.33



The volume is about 7.33 m^3.

Exercises for the lesson "Surface Area and Volume of Spheres"

Skill Practice

1. The formula for the surface area of a sphere is $4\pi r^2$ and the formula for the volume of a sphere is $\frac{4}{3}\pi r^3$, where r is the radius of the sphere.

2. The center of the sphere must be contained for the intersection to be a great circle.

3. $S = 4\pi r^2 = 4\pi(4)^2 = 64\pi \approx 201.06$

The surface area is about 201.06 ft^2.

4. $S = 4\pi r^2 = 4\pi(7.5)^2 = 225\pi \approx 706.86$

The surface area is about 706.86 cm^2.

5. $S = 4\pi r^2 = 4\pi(9.15)^2 = 334.89\pi \approx 1052.09$

The surface area is about 1052.09 m^2.

6. B; $S = 4\pi r^2$

$32\pi = 4\pi r^2$

$8 = r^2$

$\sqrt{8} = r$

The approximate radius is 2.83 m.

7. $C = 2\pi r$

$9.6\pi = 2\pi r$

$4.8 = r$

The radius is 4.8 inches.

8. $d = 2r = 2(4.8) = 9.6$

The diameter is 9.6 inches.

9. $S = \frac{1}{2}(4\pi r^2) = \frac{1}{2}(4\pi(4.8)^2) = 46.08\pi \approx 144.76$

The surface area of the hemisphere is about 144.76 $in.^2$.

10. The surface area of a hemisphere is half the surface area of the sphere.

$S = \frac{1}{2}(4\pi r^2) = \frac{1}{2}(4\pi(5)^2) = 50\pi \approx 157.08$

The surface area of the hemisphere is about 157.08 ft^2.

11. $C = 2\pi r$

$48.4\pi = 2\pi r$

$24.2 = r$

$S = 4\pi r^2 = 4\pi(24.2)^2 \approx 7359.37$

The surface area is about 7359.37 cm^2.

12. $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(6)^3 = 288\pi \approx 904.78$

The volume is about 904.78 $in.^3$.

13. $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(40)^3 \approx 268{,}082.57$

The volume is about 268,082.57 mm^3.

14. $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(2.5)^3 \approx 65.45$

The volume is about 65.45 cm^3.

15. The formula for the volume of a sphere is $\frac{4}{3}\pi r^3$ not $\frac{4}{3}\pi r^2$.

$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(8)^3 = 682.66\pi \approx 2144.66$

The volume is about 2144.66 ft^3.

16. $V = \frac{4}{3}\pi r^3$

$1436.76 = \frac{4}{3}\pi r^3$

$343 \approx r^3$

$7 \approx r$

The radius is about 7.00 m.

17. $V = \frac{4}{3}\pi r^3$

$91.95 = \frac{4}{3}\pi r^3$

$21.95 \approx r^3$

$2.80 \approx r$

The radius is about 2.80 cm.

18. $V = \frac{4}{3}\pi r^3$

$20{,}814.37 = \frac{4}{3}\pi r^3$

$4969.06 \approx r^3$

$17.06 \approx r$

The radius is about 17.06 in.

19. $V = \frac{4}{3}\pi r^3$

$36\pi = \frac{4}{3}\pi r^3$

$27 = r^3$

$3 = r$

The radius is 3 feet, so the diameter is $2(3) = 6$ feet.

20. A; $S = 4\pi r^2$

$V = \frac{4}{3}\pi r^3 = \frac{r}{3}(4\pi r^2) = \frac{rS}{3}$

The relationship is $V = \frac{rS}{3}$.

21. $S = \pi r^2 + 2\pi rh + \frac{1}{2}(4\pi r^2)$

$= \pi(3.3)^2 + 2\pi(3.3)(7) + \frac{1}{2}(4)(\pi)(3.3)^2$

≈ 247.78

The surface area is about 247.78 in.^{2}.

$V = \pi r^2 h - \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$

$= \pi(3.3)^2(7) - \frac{1}{2}\left(\frac{4}{3}\right)(\pi)(3.3)^3$

≈ 164.22

The volume is about 164.22 in.^{3}.

22. $S = \pi r^2 + 2\pi rh + \frac{1}{2}(4\pi r^2)$

$= \pi(5.8)^2 + 2\pi(5.8)(14) + \frac{1}{2}(4)(\pi)(5.8)^2$

≈ 827.24

The surface area is about 827.24 ft^2.

$V = \pi r^2 h + \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$

$= \pi(5.8)^2(14) + \frac{1}{2}\left(\frac{4}{3}\right)(\pi)(5.8)^3$

≈ 1888.21

The volume is about 1888.21 ft^3.

23. $S = \pi r\ell + \frac{1}{2}(4\pi r^2)$

$= \pi(4.9)^2(13.52) + \frac{1}{2}(4)(\pi)(4.9)^2$

≈ 358.97

The surface area is about 358.97 cm^2.

$V = \frac{1}{3}\pi r^2 h + \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$

$= \frac{1}{3}\pi(4.9)^2(12.6) + \frac{1}{2}\left(\frac{4}{3}\right)(\pi)(4.9)^3$

≈ 563.21

The volume is about 563.21 cm^3.

24–27.

	Radius of sphere	Circumference of great circle
24.	10 ft	20π ft
25.	13 in.	26π in.
26.	25 cm	50π cm
27.	21 m	42π m

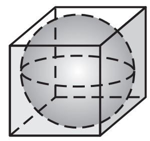
	Surface area of sphere	Volume of sphere
24.	400π ft^2	$\left(\frac{4000}{3}\right)\pi$ ft^3
25.	676π in.^{2}	$\left(\frac{8788}{3}\right)\pi$ in.^{3}
26.	2500π cm^2	$\left(\frac{62{,}500}{3}\right)\pi$ cm^3
27.	1764π m^2	$12{,}348\pi$ m^3

28. C; Volume of cube $= s^3$

$s^3 = 64$

$s = 4$

$r = \frac{s}{2} = 2$



Surface area of sphere $= 4\pi r^2$

$= 4\pi(2)^2$

$= 16\pi$

The surface area is 16π cm^2.

29. a. $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(1)^3 = \frac{4}{3}\pi$ in.^{3}

Sample answers: $r = 1$ in., $h = \frac{4}{3}$ in.

$r = 0.5$ in., $h = \frac{16}{3}$ in.

$r = 2$ in., $h = \frac{1}{3}$ in.

b. The surface area of the cylinder will be greater than the surface area of the sphere when $2\pi rh + 2\pi r^2 > 4\pi$. Solve to get $r(h + r) > 2$. The surface area of the cylinder will be greater than the surface area of the sphere when the product of the radius and the sum of the height and the radius of the cylinder is greater than 2.

Problem Solving

30. $V = \pi r^2 h + \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$

$= \pi(10)^2(60) + \frac{1}{2}\left(\frac{4}{3}\right)(\pi)(10)^3$

$\approx 20{,}943.95$

The volume of the grain silo is about 20,944 ft^3.

31. $C = 2\pi r$

$24{,}855 = 2\pi r$

$\frac{24855}{2\pi} = r$

$S = \frac{1}{2}(4\pi r^2) = \frac{1}{2}(4)(\pi)\left(\frac{24855}{2\pi}\right)^2 \approx 98{,}321{,}312.33$

The surface area of the western hemisphere is approximately 98,321,312 square miles.

32. a. $V = \frac{4}{3}\pi r^3$

$1427.54 = \frac{4}{3}\pi r^3$

$340.8 \approx r^3$

$6.99 \approx r$

The radius is about 6.99 cm.

b. $S = 4\pi r^2 = 4\pi(6.99)^2 \approx 613.99$

The surface area is about 613.99 cm^2.

33. a. $C = 2\pi r$

$8 = 2\pi r$

$\frac{4}{\pi} = r$

$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{4}{\pi}\right)^3 \approx 8.65$

The volume of a tennis ball is about 8.65 $in.^3$.

b. $V =$ Volume of cylinder $-$ 3 • volume of one ball

$= \pi r^2 h - 3\left(\frac{4}{3}\pi r^3\right)$

$= \pi(1.43)^2(8.625) - 3\left(\frac{4}{3}\pi\left(\frac{4}{\pi}\right)^3\right)$

≈ 29.47

There is about 29.47 $in.^3$ of space not taken up.

34. a. $C = 2\pi r$

$27\pi = 2\pi r$

$13.5 = r$

$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(13.5)^3 \approx 10{,}306$

The volume of the balloon is about 10,306 cm^3.

b. The radius is cubed, so $(2r)^3$ would give $8r^3$. This means the volume should be eight times the original volume.

c. $V = \frac{4}{3}\pi(2r)^3 = \frac{4}{3}\pi(2.7)^3 \approx 82447.96 \approx 8(10{,}306)$

The volume is about 82,447.96 cm^3. The prediction from part (b) was correct. The ratio of this volume to the original is 8 to 1 or 8 : 1 .

35. a. Surface area of Torrid Zone $= 2\pi rh$

$= 2\pi(3963)(3250)$

$\approx 80{,}925{,}856 \text{ mi}^2$

Surface area of Earth $= 4\pi r^2$

$= 4\pi(3963)^2$

$\approx 197{,}359{,}488 \text{ mi}^2$

b. Probability of Torrid Zone $= \frac{\text{Surface area of Torrid Zone}}{\text{Surface area of the Earth}}$

$= \frac{80{,}925{,}856 \text{ mi}^2}{197{,}359{,}488 \text{ mi}^2}$

≈ 0.41

The probability that a meterorite will land in the Torrid Zone is about 41%.

36. a. Solid I: $S = 4\pi r^2$

Solid II: $S = 2\pi r^2 + 2\pi r(2r) = 6\pi r^2$

Solid III: $S = 2\left[\pi r\left(r\sqrt{2}\right) + \pi r^2\right] \approx 4.83\pi r^2$

Solid I, Solid III, Solid II

b. Solid I: $V = \frac{4}{3}\pi r^3$

Solid II: $V = \pi r^2(2r) = 2\pi r^3$

Solid III: $V = 2\left[\frac{1}{3}(\pi r^2)(r)\right] = \frac{2}{3}\pi r^3$

Solid III, Solid I, Solid II

37. A sphere is formed.

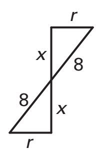
$S = 4\pi r^2 = 4\pi(9)^2 \approx 1017.88$

The surface area is about 1018 $in.^2$.

$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(9)^3 \approx 3053.63$

The volume is about 3054 $in.^3$.

38. a. The diagonal of the cylinder's height and diameter is equal to the diameter of the sphere.



Using the Pythagorean Theorem:

$r^2 = 8^2 - x^2$

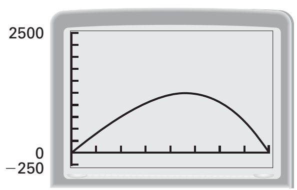
$r^2 = 64 - x^2$

Volume of the cylinder $= \pi r^2 h$

$= \pi(64 - x^2)(2x)$

$= 2\pi x(64 - x^2)$

b.



The maximum value occurs when x is about 4.62 m.

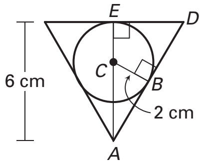
c. $V = 2\pi x(64 - x^2)$

$= 2\pi(4.62)(64 - 4.62^2)$

≈ 1238.22

The maximum volume is about 1238.22 m^3.

39. The lateral edge of the cone is tangent to the sphere, so a radius drawn from the sphere is perpendicular to the tangent lateral side. By the AA Similarity Postulate, two similar right triangles are formed by this radius. $\overline{AC}$ has a length of 4 cm because $\overline{EC}$ is a radius of the sphere and has length 2 cm. By the Pythagorean Theorem, $\overline{AB}$ has length $\sqrt{4^2 - 2^2} = \sqrt{12} = 2\sqrt{3}$ cm.



$\frac{BC}{DE} = \frac{AB}{EA}$ $\qquad$ $\frac{BC}{DE} = \frac{AC}{AD}$

$\frac{2}{DE} = \frac{2\sqrt{3}}{6}$ $\qquad$ $\frac{2}{2\sqrt{3}} = \frac{4}{AD}$

$DE(2\sqrt{3}) = 12$ $\qquad$ $2(AD) = 8\sqrt{3}$

$DE = 2\sqrt{3}$ $\qquad$ $AD = 4\sqrt{3}$

The radius of the cone is $2\sqrt{3}$ cm and the lateral edge is $4\sqrt{3}$ cm.

$S = \pi r\ell + \pi r^2$

$= \pi(2\sqrt{3})(4\sqrt{3}) + \pi(2\sqrt{3})^2$

$= 36\pi$

≈ 113.10

The surface area is about 113.10 cm^2.

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(2\sqrt{3})^2(6) = 24\pi \approx 75.40$

The volume is about 75.40 cm^3.

Lesson 11.9 Explore Similar Solids

Investigating Geometry Activity for the lesson "Explore Similar Solids"

Step 1 and Step 2:

	Scale factor of Solid A to Solid B	Surface area of Solid A, S_A	Surface area of Solid B, S_B
Pair 1	$\frac{1}{2}$	72	288
Pair 2	$\frac{2}{3}$	28π	63π
Pair 3	$\frac{3}{1}$	675	75

	$\frac{S_A}{S_B}$	V_A	V_B	$\frac{V_A}{V_B}$
Pair 1	$\frac{1}{4}$	36	288	$\frac{1}{8}$
Pair 2	$\frac{4}{9}$	20π	67.5π	$\frac{8}{27}$
Pair 3	$\frac{9}{1}$	974.28	36.08	$\frac{27}{1}$

Step 3:
$\frac{S_A}{S_B}$ is the square of the scale factor while $\frac{V_A}{V_B}$ is the cube of the scale factor.

1. The ratio of the surface areas of similar solids is the square of the scale factor of the solids. The ratio of the volumes of similar solids is the cube of the scale factor of the solids.

2. ratio of surface areas $= \frac{1}{k^2}$

ratio of volumes $= \frac{1}{k^3}$

Guided Practice for the lesson "Explore Similar Solids"

1. Lengths $= \frac{12}{9} = \frac{4}{3}$

Widths $= \frac{16}{12} = \frac{4}{3}$

Heights $= \frac{4}{3}$

The prisms are similar because the ratios of corresponding linear measures are equal.

2. Heights $= \frac{15}{10} = \frac{3}{2}$

Radii $= \frac{10}{5} = 2$

The cones are not similar because the ratios of corresponding linear measures are not equal.

3. $\frac{C^2}{D^2} = \frac{54}{150}$

$\frac{C}{D} = \frac{3\sqrt{6}}{5\sqrt{6}}$

$\frac{C}{D} = \frac{3}{5}$

The scale factor of C to D is $\frac{3}{5}$.

The edge length of C is $\sqrt{\frac{54}{6}} = 3$ units.

$\frac{3^3}{5^3} = \frac{C^3}{D^3}$

$\frac{27}{125} = \frac{27}{D^3}$

$D^3 = 125$

The volume of D is 125 cubic units.

4. $\frac{2^3}{1^3} = \frac{\text{Price}}{\$1.50}$

Price = 8($1.50) = $12

The price of the large ball should be $12 for neither ball to be a better buy.

Exercises for the lesson "Explore Similar Solids"

Skill Practice

1. Two solids are similar if they are the same type of solid with equal ratios of all corresponding linear measures.

2. If the corresponding linear measures have ratio $a:b$, then the volumes are in the ratio $a^3:b^3$.

3. Radii $= \frac{7}{4}$

Heights $= \frac{16}{10} = \frac{8}{5}$

The solids are not similar because the ratios of corresponding linear measures are not equal.

4. Lengths $= \frac{5}{9}$

Widths $= \frac{7}{12.6} = \frac{5}{9}$

Heights $= \frac{11}{14.8} = \frac{55}{74}$

The solids are not similar because the ratios of corresponding linear measures are not equal.

5. Lengths $= \frac{6}{8} = \frac{3}{4}$

Widths $= \frac{13.5}{18} = \frac{3}{4}$

Heights $= \frac{4.5}{6} = \frac{3}{4}$

The solids are similar because the ratios of corresponding linear measures are equal.

6. Radii $= \frac{8}{12} = \frac{2}{3}$

Heights $= \frac{18}{27} = \frac{2}{3}$

The solids are similar because the ratios of corresponding linear measures are equal.

7. D; Lengths $= \frac{6}{15} = \frac{2}{5}$

Widths $= \frac{4}{10} = \frac{2}{5}$

Heights $= \frac{10}{25} = \frac{2}{5}$

15 feet by 10 feet by 25 feet

8. $\frac{\text{Surface area of A}}{\text{Surface area of B}} = \frac{a^2}{b^2}$

$\frac{150\pi}{\text{Surface area of B}} = \frac{1^2}{2^2}$

Surface area of B $= 600\pi$ in.^{2}

$\frac{\text{Volume of A}}{\text{Volume of B}} = \frac{a^3}{b^3}$

$\frac{250\pi}{\text{Volume of B}} = \frac{1^3}{2^3}$

Volume of B $= 2000\pi$ in.^{3}

9. $\frac{\text{Surface area of A}}{\text{Surface area of B}} = \frac{a^2}{b^2}$

$\frac{1500}{\text{Surface area of B}} = \frac{3^2}{1^2}$

Surface area of B ≈ 166.67 m^2

$\frac{\text{Volume of A}}{\text{Volume of B}} = \frac{a^3}{b^3}$

$\frac{3434.6}{\text{Volume of B}} = \frac{3^3}{1^3}$

Volume of B $= 127.21$ m^3

10. $\frac{\text{Surface area of A}}{\text{Surface area of B}} = \frac{a^2}{b^2}$

$\frac{2356.2}{\text{Surface area of B}} = \frac{5^2}{2^2}$

Surface area of B ≈ 377 cm^2

$\frac{\text{Volume of A}}{\text{Volume of B}} = \frac{a^3}{b^3}$

$\frac{7450.9}{\text{Volume of B}} = \frac{5^3}{2^3}$

Volume of B ≈ 476.86 cm^3

11. The ratio of the volumes is $\frac{a^3}{b^3}$, not $\frac{a^2}{b^2}$.

$\frac{500\pi}{\text{Volume of B}} = \frac{1^3}{4^3}$

12. $\frac{a^3}{b^3} = \frac{8\pi}{125\pi}$

$\frac{a^3}{b^3} = \frac{8}{125}$

$\frac{a}{b} = \frac{2}{5}$

The scale factor of Solid I to Solid II is 2 : 5 .

13. $\frac{a^3}{b^3} = \frac{27}{729}$

$\frac{a}{b} = \frac{3}{9}$

$\frac{a}{b} = \frac{1}{3}$

The scale factor of Solid I to Solid II is 1 : 3 .

14. $\frac{a^2}{b^2} = \frac{288}{128}$

$\frac{a^2}{b^2} = \frac{144}{64}$

$\frac{a}{b} = \frac{12}{8}$

$\frac{a}{b} = \frac{3}{2}$

The scale factor of Solid I to Solid II is 3 : 2 .

15. $\frac{a^2}{b^2} = \frac{192}{108}$

$\frac{a^2}{b^2} = \frac{16}{9}$

$\frac{a}{b} = \frac{4}{3}$

The scale factor of Solid I to Solid II is 4 : 3 .

16. C; $\frac{a^3}{b^3} = \frac{8\pi}{27\pi}$

$\frac{a^3}{b^3} = \frac{8}{27}$

$\frac{a}{b} = \frac{2}{3}$

$\frac{a^2}{b^2} = \frac{2^2}{3^2}$

$= \frac{4}{9}$

17. $\frac{a^3}{b^3} = \frac{2\pi}{16\pi}$

$\frac{a^3}{b^3} = \frac{1}{8}$

$\frac{a}{b} = \frac{1}{2}$

$\frac{a^2}{b^2} = \frac{1}{4}$

The ratio of the surface area of the smaller sphere to the surface area of the larger sphere is 1 : 4.

18. $\frac{\text{Surface area of A}}{\text{Surface area of B}} = \frac{a^2}{b^2}$

$\frac{78\pi}{\text{Surface area of B}} = \frac{2^2}{3^2}$

The surface area of B or the large cylinder is $175.5\pi \approx 551.35 \text{ m}^2$.

19. Scale factor $= \frac{4}{8} = \frac{1}{2}$

Surface area of Solid I $= \pi r^2 + 2\pi rh + \pi r\ell$

$= \pi(2)^2 + 2\pi(2)(4) + \pi(2)(\sqrt{13})$

$\approx 85.49 \text{ ft}^2$

Volume of Solid I $= \pi r^2 h + \frac{1}{3}\pi r^2 h$

$= \pi(2)^2(4) + \frac{1}{3}\pi(2)^2(3)$

$\approx 62.83 \text{ ft}^3$

$\frac{\text{Surface area of Solid I}}{\text{Surface area of Solid II}} = \frac{a^2}{b^2}$

$\frac{85.49}{\text{Surface area of Solid II}} = \frac{1^2}{2^2}$

Surface area of Solid II $\approx 341.96 \text{ ft}^2$

$\frac{\text{Volume of Solid I}}{\text{Volume of Solid II}} = \frac{a^3}{b^3}$

$\frac{62.83}{\text{Volume of Solid II}} = \frac{1^3}{2^3}$

Volume of Solid II $\approx 502.64 \text{ ft}^3$

20. Scale factor $= \frac{8}{3}$

Surface area of Solid I $= B + P\ell + 4\left(\frac{1}{4} \cdot \sqrt{3} \cdot 3^2\right)$

$= 3^2 + 12(8) + 4\left(\frac{1}{4} \cdot \sqrt{3} \cdot 3^2\right)$

$\approx 120.59 \text{ cm}^2$

Volume of Solid II $= Bh + \frac{1}{3}Bh$

$= (3^2)(8) + \frac{1}{3}(3^2)(1.5\sqrt{2})$

$\approx 78.36 \text{ cm}^3$

$\frac{\text{Surface area of Solid I}}{\text{Surface area of Solid II}} = \frac{a^2}{b^2}$

$\frac{120.59}{\text{Surface area of Solid II}} = \frac{8^2}{3^2}$

Surface area of Solid II $\approx 16.96 \text{ cm}^2$

$\frac{\text{Volume of Solid I}}{\text{Volume of Solid II}} = \frac{a^3}{b^3}$

$\frac{78.36}{\text{Volume of Solid II}} = \frac{8^3}{3^3}$

Volume of Solid II $\approx 4.13 \text{ cm}^3$

21. Scale factor $= \frac{4}{7}$

Surface area of Solid I $= 2\ell h + 2\ell w - 2\pi r^2 + 2\pi rh$

$= 2(4)(4) + 2(4)(4) - 2\pi(2^2) + 2\pi(2)(4)$

$\approx 89.13 \text{ in.}^2$

Volume of Solid I $= Bh$

$= (4^2 - \pi(2^2))(4)$

$\approx 13.7345 \text{ in.}^3$

$\frac{\text{Surface area of Solid I}}{\text{Surface area of Solid II}} = \frac{a^2}{b^2}$

$\frac{89.13}{\text{Surface area of Solid II}} = \frac{4^2}{7^2}$

Surface area of Solid II $\approx 272.96 \text{ in.}^2$

$\frac{\text{Volume of Solid I}}{\text{Volume of Solid II}} = \frac{a^3}{b^3}$

$\frac{13.7345}{\text{Volume of Solid II}} = \frac{4^3}{7^3}$

Volume of Solid II $\approx 73.61 \text{ in.}^3$

22. Scale factor $= \frac{8}{5}$

Surface area of Solid I $= 5(8) + 26(5) + 2\pi\left(\frac{1}{2}\right)(8)^3 + 3(1 \cdot 8)$

$\approx 219.13 \text{ m}^2$

Volume of Solid I $= (5^2)(8)$

$= 200 \text{ m}^3$

$\frac{\text{Surface area of Solid I}}{\text{Surface area of Solid II}} = \frac{a^2}{b^2}$

$\frac{219.13}{\text{Surface area of Solid II}} = \frac{8^2}{5^2}$

Surface area of Solid II $\approx 85.60 \text{ m}^2$

$\frac{\text{Volume of Solid I}}{\text{Volume of Solid II}} = \frac{a^3}{b^3}$

$\frac{200}{\text{Volume of Solid II}} = \frac{8^3}{5^3}$

Volume of Solid II $\approx 48.83 \text{ m}^3$

23. $h = 2r$

$S = 2\pi r^2 + 2\pi rh$

$54\pi = 2\pi r^2 + 2\pi r(2r)$

$54\pi = 6\pi r^2$

$9 = r^2$

$r = 3$

$h = 2r = 6$

$\frac{(\text{radius of small cylinder})^2}{(\text{radius of large cylinder})^2} = \frac{54\pi}{384\pi}$

$\frac{3^2}{(\text{radius of large cylinder})^2} = \frac{9}{64}$

radius of large cylinder = 8

height of large cylinder = 8(2) = 16

The radius and height of the small cylinder are 3 ft and 6 ft.

The radius and height of the large cylinder are 8 ft and 16 ft.

24. a. The ratio from the small cone to big cone is 8 : 10 or 4 : 5.

The ratio of the area is then $4^2 : 5^2$ or 16 : 25.

b. 4:5

c. $4^2 : 5^2$ or 16 : 25

d. $4^3 : 5^3$ or 64 : 125

e. The volume of the top part of the cone is $\frac{64}{125}$.

If 1 represents the volume of the entire cone, $1 - \frac{64}{125}$ represents the volume of the bottom part of the cone. $1 - \frac{64}{125} = \frac{61}{125}$. The ratio of the pieces is $\frac{64}{125} : \frac{61}{125}$ or 64 : 61.

Problem Solving

25. $\frac{\text{Volume of smaller mug}}{\text{Volume of larger mug}} = \frac{a^3}{b^3}$

$\frac{\text{Volume of smaller mug}}{12} = \frac{3.5^3}{4^3}$

Volume of smaller mug ≈ 8.04. So, the capacity of the smaller mug is about 8.04 fluid ounces.

26. The units should be the same for each part of the ratio.

$$\frac{\text{Volume of small binoculars}}{\text{Volume of large binoculars}} = \frac{a^3}{b^3} = \frac{(0.5\text{ feet})^3}{(45\text{ feet})^3} = \frac{0.125}{91125} = \frac{1}{729{,}000}$$

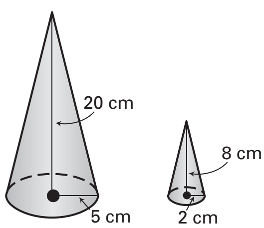
The ratio of the volumes is 1 : 729,000.

27. $\frac{\text{Volume of small bowl}}{\text{Volume of large bowl}} = \frac{a^3}{b^3}$

$\frac{\text{Volume of small bowl}}{64} = \frac{3^3}{4^3}$

Volume of small bowl = 27. So, the small bowl requires 27 fluid ounces of lemonade.

28. *Sample answer:*



29. a. Volume of smaller orange $= \frac{4}{3}\pi(1.6)^3 \approx 17.16\text{ in.}^3$

Volume of larger orange $= \frac{4}{3}\pi(2)^3 \approx 33.51\text{ in.}^3$

b. Ratio of diameters $= \frac{3.2}{4} = 0.8$

Ratio of volumes $\approx \frac{17.16}{33.51} \approx 0.51 \approx (0.8)^3$

The ratio of the volumes is the ratio of the diameters cubed.

c. Small orange: diameter $= 3.2 - 2\left(\frac{1}{8}\right) = 2.95$ in.

Large orange: diameter $= 4 - 2\left(\frac{1}{8}\right) = 3.75$ in.

d. $S = 4\pi(1.475)^2 \approx 27.34\text{ in.}^2$

$V = \frac{4}{3}\pi(1.475)^3 \approx 13.44\text{ in.}^3$

$S = 4\pi(1.875)^2 \approx 44.18\text{ in.}^2$

$V = \frac{4}{3}\pi(1.875)^3 \approx 27.61\text{ in.}^3$

ratio of surface areas $\approx \frac{27.34}{44.18} \approx 0.62$

ratio of volumes $\approx \frac{13.44}{27.61} \approx 0.49 \approx 0.62(0.8)$

The ratio of the volumes is about 0.8 times the ratio of the surface areas. The original ratio was $\frac{3.2}{4} = 0.8$.

30. a. $\frac{a}{b}; \frac{a^3}{b^3}$

b. Cone I: $V = \frac{1}{3}(\pi a^2)(2a) = \frac{2}{3}\pi a^3$

Cone II: $V = \frac{1}{3}(\pi b^2)(2b) = \frac{2}{3}\pi b^3$

c. $\frac{\text{Volume of Cone I}}{\text{Volume of Cone II}} = \frac{\frac{2}{3}\pi a^3}{\frac{2}{3}\pi b^3} = \frac{a^3}{b^3}$

The answer is equivalent to the answer from part (a).

31. a. $\frac{\text{model length}}{\text{actual length}} = \frac{1}{18}$

$\frac{8\text{ in.}}{\text{actual length}} = \frac{1}{18}$

actual length = 144 in.

The length of the actual car is 144 in. or 12 ft.

b. $\frac{\text{Surface area of model tire}}{\text{Surface area of actual tire}} = \frac{a^2}{b^2}$

$\frac{121\text{ square inches}}{\text{Surface area of actual tire}} = \frac{1^2}{18^2}$

Surface area = 3920.4

The surface area of each tire of the actual car is 3920.4 in.^{2}

c. $\frac{\text{Volume of model engine}}{\text{Volume of actual engine}} = \frac{a^3}{b^3}$

$\frac{\text{Volume of model engine}}{8748} = \frac{1^3}{18^3}$

Volume = 1.5

The volume of the model car's engine is 1.5 cubic inches.

32. $\dfrac{\text{Volume of small cylinder}}{\text{Volume of large cylinder}} = \dfrac{a^3}{b^3}$

$$\frac{16\pi}{432\pi} = \frac{1}{27} = \frac{1^3}{3^3}$$

The common ratio between cylinders is $\frac{1}{3}$ or 1 : 3.

$\dfrac{\text{Lateral area of small cylinder}}{\text{Lateral area of large cylinder}} = \dfrac{a^2}{b^2}$

$\dfrac{\text{Lateral area of small cylinder}}{72\pi} = \dfrac{1^2}{3^2}$

Lateral area $= 8\pi$

The lateral area of the small cylinder is 8π square units.

33. The ratio of the small snowball to the medium snowball is 5 : 7. So, the ratio of their volumes is $5^3 : 7^3$.

$\dfrac{5^3}{7^3} = \dfrac{1.2}{x} \rightarrow x = 3.2928$

The weight of the medium snowball is 3.2928 kilograms.

The ratio of the small snowball to the large snowball is 5 : 9 . So, the ratio of their volumes is $5^3 : 9^3$.

$\dfrac{5^3}{9^3} = \dfrac{1.2}{x} \rightarrow x = 6.9984$

The weight of the large snowball is 6.9984 kilograms.

$1.2 + 3.2928 + 6.9984 = 11.4912$

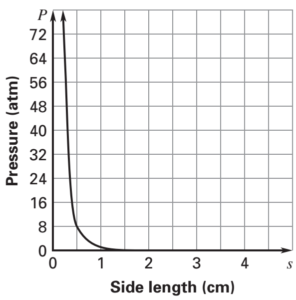
The total weight of the snow used is about 11.5 kilograms.

34. a. $P = \dfrac{k}{s^3}$, where k is the constant of variation and s is the side length.

b.

Side length *s* (cm)	$\frac{1}{4}$	$\frac{1}{2}$	1	2	3
Pressure *P* (atm)	$64k$	$8k$	k	$\frac{k}{8}$	$\frac{k}{64}$

c.



The graph does not show a linear relationship. The points on the graph cannot be connected with a straight line, so there is not a linear relationship.

35. The top of the pyramid is similar to the entire pyramid.

Let $V =$ Volume of top pyramid

$h =$ height of top pyramid

$$\frac{V}{2V} = \frac{h^3}{12^3}$$

$$\frac{1}{2} = \frac{h^3}{1728}$$

$$2h^3 = 1728$$

$$h^3 = 864$$

$$h = (864)^{1/3}$$

$$= 6 \cdot 2^{2/3}$$

$$\approx 9.52$$

The height of the top of the pyramid is about 9.52 feet.

Quiz for the lessons "Surface Area and Volume of Spheres" and "Explore Similar Solids"

1. $S = 4\pi r^2 = 4\pi(7)^2 \approx 615.75$

The surface area is about 615.75 cm^2.

$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(7)^3 \approx 1436.76$

The volume is about 1436.76 cm^3.

2. $S = 4\pi r^2 = 4\pi(11.5)^2 \approx 1661.90$

The surface area is about 1661.90 m^2.

$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(11.5)^3 \approx 6370.63$

The volume is about 6370.63 m^3.

3. $S = 4\pi r^2 = 4\pi(10.7)^2 \approx 1438.72$

The surface area is about 1438.72 ft^2.

$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(10.7)^3 \approx 5131.45$

The volume is about 5131.45 ft^3.

4. $\dfrac{\text{Surface area of A}}{\text{Surface area of B}} = \dfrac{a^2}{b^2}$

$\dfrac{114}{\text{Surface area of B}} = \dfrac{1^2}{3^2}$

$S = 1026$

$\dfrac{\text{Volume of A}}{\text{Volume of B}} = \dfrac{a^3}{b^3}$

$\dfrac{72}{\text{Volume of B}} = \dfrac{1^3}{3^3}$

$V = 1944$

The surface area of B is 1026 in.^2 and the volume is 1944 in.^3.

5. $\frac{\text{Surface area of A}}{\text{Surface area of B}} = \frac{a^2}{b^2}$

$\frac{170\pi}{\text{Surface area of B}} = \frac{2^2}{3^2}$

$S = 382.5\pi$

$\frac{\text{Volume of A}}{\text{Volume of B}} = \frac{a^3}{b^3}$

$\frac{300\pi}{\text{Volume of B}} = \frac{2^3}{3^3}$

$V = 1012.5\pi$

The surface area of B is 382.5π m^2 and the volume is 1012.5π m^3.

6. $\frac{\text{Surface area of A}}{\text{Surface area of B}} = \frac{a^2}{b^2}$

$\frac{383}{\text{Surface area of B}} = \frac{5^2}{4^2}$

$S = 245.12$

$\frac{\text{Volume of A}}{\text{Volume of B}} = \frac{a^3}{b^3}$

$\frac{440}{\text{Volume of B}} = \frac{5^3}{4^3}$

$V = 225.28$

The surface area of B is 245.12 cm^2 and the volume is 225.28 cm^3.

7. $\frac{\text{Volume of larger cone}}{\text{Volume of smaller cone}} = \frac{a^3}{b^3}$

$\frac{729\pi}{343\pi} = \frac{729}{343} = \frac{9^3}{7^3}$

The scale factor is 9 : 7 or $\frac{9}{7}$.

Extension for the lesson "Volume of Pyramids and Cones"

1–7. Sample answers are given.

1. A plane parallel to lateral sides and passing through opposite vertices of the hexagonal base.
2. A plane that contains the vertex of the pyramid and passes through opposite corners of the base.
3. A plane that passes through the center of the sphere.
4. A line that contains the centers of the bases is an axis. Rotation of 60° maps the prism onto itself.
5. The axis contains the vertex and the center of the base (rotations of 60°, 120°, or 180°).
6. A line that passes through any two opposite vertices is an axis (rotations of 90° or 180°).
7. Yes; a solid such as a coffee mug with a handle has a plane of symmetry but not rotational symmetry.
8. Check students' examples.
9. 6 lines that pass through midpoints of opposite edges (180°), and 4 lines that pass through opposite vertices (120°)

Mixed Review of Problem Solving for the lessons "Volume of Prisms and Cylinders", "Volume of Pyramids and Cones", "Surface Area and Volume of Spheres", and "Explore Similar Solids"

1. a. $V = Bh = (24 \cdot 16)(20) = 7680$

The volume is 7680 in.^{3}.

b. $V = Bh = (8 \cdot 2)(3) = 48$

The volume is 48 in.^{3}.

c. Number of boxes $= \frac{\text{Volume of container}}{\text{Volume of one box}} = \frac{7680}{48} = 160$

So, 160 boxes of cookies can fit in the container.

2. a. $V = \pi r^2 h = \pi(1.25)^2(7) \approx 10.94\pi \approx 34.36$

The volume is about 34.36 in.^{3}.

b. 18 fluid ounces $\cdot \left(\frac{1 \text{ in.}^3}{0.554 \text{ fluid ounces}}\right) \approx 32.49$ in.^{3}

The volume of the 18 fluid ounces is about 32.49 cubic inches. Since this volume is less than the volume of the cylinder, all of the juice will fit in the cup.

3. a. $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(6)^2(10) \approx 376.99$

The volume of the funnel is about 377 cm^3.

b. 377 cm^3 = 377 mL

$\frac{45 \text{ mL}}{1 \text{ sec}} = \frac{377 \text{ mL}}{\text{time}}$

time ≈ 8.38

It takes about 8.38 seconds to empty the funnel.

c. $V = \frac{1}{3}\pi(10)^2(6) \approx 628.32$

The volume of the new funnel is about 628.32 cm^3, which means it can hold about 628.32 mL.

$\frac{45 \text{ mL}}{1 \text{ sec}} = \frac{628.32 \text{ mL}}{\text{time}}$

time ≈ 13.96

It would take about 13.96 seconds to empty the funnel.

d. The volume of the funnel in part (c) is greater then the volume of the funnel in part (a). A greater volume means the funnel can hold more oil. If the rate at which the oil flows stays constant, it would have to take the funnel in part (c) longer because it has a greater volume.

4. a. $C = 2\pi r$

$29.5 = 2\pi r$

$\frac{29.5}{2\pi} = r$

$S = 4\pi r^2 = 4\pi\left(\frac{29.5}{2\pi}\right)^2 \approx 277$

The surface area of a men's basketball is about 277 in.^{2}.

$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{29.5}{2\pi}\right)^3 \approx 433.53$

The volume of a men's basketball is about 433.53 in.^{3}.

b. $C = 2\pi r$

$28.5 = 2\pi r$

$\frac{28.5}{2\pi} = r$

$S = 4\pi r^2 = 4\pi\left(\frac{28.5}{2\pi}\right)^2 \approx 258.55$

The surface area of a women's basketball is about 258.55 in.^{2}.

$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{28.5}{2\pi}\right)^3 \approx 390.92$

The volume of a women's basketball is about 390.92 in.^{3}.

c. $\frac{\text{Surface area of women's ball}}{\text{Surface area of men's ball}} = \frac{a^2}{b^2}$

$\frac{\text{Surface area of women's ball}}{277} = \frac{28.5^2}{29.5^2}$

Surface area of women's ball ≈ 258.54 in.^{2}

$\frac{\text{Volume of women's ball}}{\text{Volume of men's ball}} = \frac{a^3}{b^3}$

$\frac{\text{Volume of women's ball}}{433.53} = \frac{28.5^3}{29.5^3}$

Volume of women's ball ≈ 390.92 in.^{3}

The answers are approximately equivalent. So, the answers agree with the answers in part (b).

5. Volume of rock = volume the water rises

$\frac{4}{3}\pi r^3 = \pi(2)^2\left(\frac{9}{64}\right)$

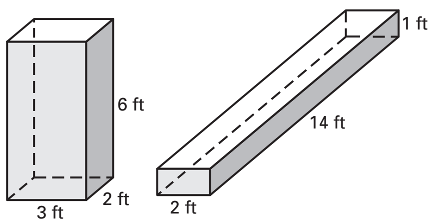
$\frac{4}{3}\pi r^3 = \frac{9}{16}\pi$

$r^3 = \frac{27}{64}$

$r = \frac{3}{4}$

The radius of the rock is $\frac{3}{4}$ inch.

6. *Sample answer:*



Original $S = 72$ ft^2 New $S = 88$ ft^2

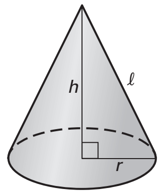
Original $V = 36$ ft^3 New $V = 28$ ft^3

7. Volume of cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(r)^2(r) = \frac{1}{3}\pi r^3$

Volume of hemisphere $= \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = 2\left(\frac{1}{3}\pi r^3\right)$

The volume of the hemisphere is twice the volume of the cone.

8. *Sample answer:*



The slant height of the cone is the hypotenuse of the triangle formed by the height, radius, and slant height. The hypotenuse of a triangle must be longer than each of the legs, so the height will always be less than the slant height.

Chapter Review for the chapter "Measurement of Figures and Solids"

1. A sphere is the set of all points in space equidistant from a given point.

2. Either pair of parallel sides can be used as the bases of a parallelogram and the height is the perpendicular distance between them.

3. An apothem of the square is $\overline{XZ}$.

4. A radius of the square is $\overline{XY}$.

5. $C = \pi d$

$94.24 = \pi d$

$\frac{94.24}{\pi} = d$

$30 \approx d$

The diameter of $\odot F$ is about 30 feet.

6. $\frac{\text{Arc length of } \widehat{GH}}{C} = \frac{m\widehat{GH}}{360^\circ}$

$\frac{5.5}{C} = \frac{35^\circ}{360^\circ}$

$5.5(360^\circ) = 35^\circ(C)$

$56.57 = C$

The circumference of $\odot F$ is about 56.57 centimeters.

7. $\frac{\text{Arc length of } \widehat{GH}}{2\pi r} = \frac{m\widehat{GH}}{360^\circ}$

$\frac{\text{Arc length of } \widehat{GH}}{2\pi(13)} = \frac{115^\circ}{360^\circ}$

Arc length of $\widehat{GH} = \frac{115^\circ(26\pi)}{360^\circ}$

Arc length of $\widehat{GH} \approx 26.09$

The length of $\widehat{GH}$ is about 26.09 inches.

8. Area of sector $TWU = \frac{m\widehat{TWU}}{360^\circ} \cdot \pi r^2$

$= \frac{240^\circ}{36^\circ} \cdot \pi \cdot 9^2 \approx 169.65$

The area of the blue shaded region is about 169.65 square inches.

9. Area of blue shaded region = Area of rectangle − Area of semicircle

$$= bh - \frac{1}{2}\pi r^2$$

$$= 6(4) - \frac{1}{2}\pi\left(\frac{4}{2}\right)^2$$

$$= 24 - 2\pi \approx 17.72$$

The area of the blue shaded region is about 17.72 square inches.

10. Area of red sector $= \frac{m\widehat{RQ}}{360°} \cdot \pi r^2$

$$27.93 = \frac{50°}{360°} \cdot \pi \cdot r^2$$

$$8 \approx r$$

Find the measure of the major arc.

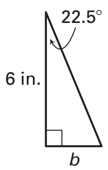
$m\widehat{RQ} = 50$, so $m\widehat{RTQ} = 360° - 50° = 310°$.

Area of blue sector $= \frac{m\widehat{RTQ}}{360°} \cdot \pi r^2$

$$\approx \frac{310°}{360°} \cdot \pi(8)^2 \approx 173.14$$

The area of the blue shaded region is about 173.14 square feet.

11. The central angle is $\frac{360°}{8} = 45°$ and the bisected angle is $\frac{1}{2}(45°) = 22.5°$. Use a trigonometric ratio to find the side length.



$$\tan 22.5° = \frac{b}{6}$$

$$6 \cdot \tan 22.5° = b$$

The side length $s = 2b = 12 \cdot \tan 22.5°$

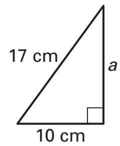
An octagon has 8 sides, so the perimeter is

$P = 8(12 \cdot \tan 22.5°) = 96 \cdot \tan 22.5° \approx 39.8$.

$$A = \frac{1}{2}a \cdot ns = \frac{1}{2}(6) \cdot 8(12 \tan 22.5°) \approx 119.3$$

The perimeter of the platter is about 39.8 inches and the area is about 119.3 square inches.

12. The pentagon's side length is 20 centimeters, so the length of the base of the triangle is $\frac{1}{2}(20) = 10$ centimeters. Use the Pythagorean Theorem to find the apothem.



$$10^2 + a^2 = 17^2$$

$$a^2 = 189$$

$$a = \sqrt{189}$$

$$A = \frac{1}{2}a \cdot ns = \frac{1}{2}(\sqrt{189})(5)(20) \approx 687.39$$

The area of the jigsaw puzzle is about 687.4 square centimeters.

13. $P(k \text{ is on } \overline{AB}) = \frac{\text{length of } \overline{AB}}{\text{length of } \overline{AC}} = \frac{|2 - (-2)|}{|5 - (-2)|} = \frac{4}{7}$

The probability that k is on $\overline{AB}$ is $\frac{4}{7}$.

14. $P(\text{Point is in shaded region}) = \frac{\text{Area of shaded region}}{\text{Area of semicircle}}$

$$= \frac{\frac{25°}{360°} \cdot \pi \cdot \left(\frac{12}{2}\right)^2}{\frac{1}{2} \cdot \pi \cdot \left(\frac{12}{2}\right)^2} = \frac{\frac{25°}{360°}}{\frac{1}{2}} \approx 0.139$$

The probability that a randomly chosen point in the figure lies in the shaded region is about 13.9%.

15. $P(\text{Point is in shaded area})$

$$= \frac{\text{Area of triangle}}{\text{Area of semicircle} + \text{Area of triangle}}$$

$$= \frac{\frac{1}{2}(6)(15)}{\frac{1}{2}\pi \cdot \left(\frac{6}{2}\right)^2 + \frac{1}{2}(6)(15)} = \frac{45}{4.5\pi + 45} \approx 0.761$$

The probability that a randomly chosen point in the figure lies in the shaded region is about 76.1%.

16. $P(\text{Point is in shaded area})$

$$= \frac{\text{Area of rectangle} + \text{Area of semicircle}}{\text{Area of square}}$$

$$= \frac{4\left(\frac{4}{2}\right) + \frac{1}{2} \cdot \pi \cdot \left(\frac{4}{2}\right)^2}{4^2} = \frac{8 + 2\pi}{16} \approx 0.893$$

The probability that a randomly chosen point in the figure lies in the shaded region is about 89.3%

17. $20 + n = 30 + 2$

$n = 12$

18. $n + 6 = 12 + 2$

$n = 8$

19. $14 + 24 = n + 2$

$36 = n$

20. $V = Bh = (1.5)(2.1)(3.6) = 11.34$

The volume is 11.34 cubic meters.

21. $V = Bh = (\pi r^2)h = \pi(2)^2(8) = 32\pi \approx 100.53$

The volume is about 100.53 cubic millimeters.

22. $V = Bh \approx \left(5\left(\frac{1}{2}\right)(2)(1.37638)\right)(4) \approx 27.53$

The volume is about 27.53 cubic yards.

23. $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(8)^2(15) = 320\pi \approx 1005.31$

The volume is about 1005.31 cubic centimeters.

24. $V = \frac{1}{3}Bh$

$60 = \frac{1}{3}B(15)$

$60 = 5B$

$12 = B$

The area of the base is 12 square inches.

25. $S = 4\pi r^2 = 4\pi(1195)^2 \approx 17{,}945{,}091.40$

$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(1195)^3 \approx 7{,}148{,}128{,}072.97$

The surface area and volume of Pluto are about 17,945,091.4 square kilometers and 7,148,128,072.97 cubic kilometers.

26. $V = S^3 + \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = 6^3 + \frac{2}{3}\pi(3)^3$

$= 216 + 18\pi \approx 272.55$

The volume is about 272.55 cubic meters.

27. $\frac{\text{Surface area of A}}{\text{Surface area of B}} = \frac{a^2}{b^2}$

$\frac{62}{\text{Surface area of B}} = \frac{1^2}{4^2}$

Surface area of B $= 992$

$\frac{\text{Volume of A}}{\text{Volume of B}} = \frac{a^3}{b^3}$

$\frac{30}{\text{Volume of B}} = \frac{1^3}{4^3}$

Volume of B $= 1920$

The surface area of Solid B is 992 square centimeters and the volume of Solid B is 1920 cubic centimeters.

28. $\frac{\text{Surface area of A}}{\text{Surface area of B}} = \frac{a^2}{b^2}$

$\frac{112\pi}{\text{Surface area of B}} = \frac{1^2}{3^2}$

Surface area of B $= 1008\pi$

$\frac{\text{Volume of A}}{\text{Volume of B}} = \frac{a^3}{b^3}$

$\frac{160\pi}{\text{Volume of B}} = \frac{1^3}{3^3}$

Volume of B $= 4320\pi$

The surface area of Solid B is $1008\pi \approx 3166.73$ square meters and the volume of Solid B is $4320\pi \approx 13{,}571.68$ cubic meters.

29. $\frac{\text{Surface area of A}}{\text{Surface area of B}} = \frac{a^2}{b^2}$

$\frac{144\pi}{\text{Surface area of B}} = \frac{2^2}{5^2}$

Surface area of B $= 900\pi$

$\frac{\text{Volume of A}}{\text{Volume of B}} = \frac{a^3}{b^3}$

$\frac{288\pi}{\text{Volume of B}} = \frac{2^3}{5^3}$

Volume of B $= 4500\pi$

The surface area of Solid B is $900\pi \approx 2827.43$ square yards and the volume of Solid B is $4500\pi \approx 14{,}137.17$ cubic yards.

Chapter Test for the chapter "Measurement of Figures and Solids"

1. $\frac{\text{Arc length of } \widehat{EHD}}{C} = \frac{m\widehat{EHD}}{360°}$

$\frac{64}{C} = \frac{210°}{360°}$

$64(360°) = C(210°)$

$109.71 \approx C$

The circumference of $\odot F$ is about 109.71 inches.

2. $\frac{\text{Arc length of } \widehat{GH}}{2\pi r} = \frac{m\widehat{GH}}{360°}$

$\frac{35}{2\pi(27)} = \frac{m\widehat{GH}}{360°}$

$360° \cdot \frac{35}{2\pi(27)} = m\widehat{GH}$

$74.27° \approx m\widehat{GH}$

The measure of $\widehat{GH}$ is about 74.3°.

3. Find the measure of the major arc.

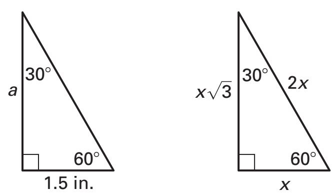
$m\,\widehat{QTR} = 360° - 105° = 255°$

Area of sector $QTR = \frac{m\,\widehat{QTR}}{360°} \cdot \pi r^2$

$= \frac{255°}{360°} \cdot \pi \cdot 8^2 \approx 142.42$

The area of the shaded sector is about 142.42 square inches.

4. A hexagon has 6 sides. The perimeter is 18 inches, so the side length is $18 \div 6 = 3$ inches. The central angle is $\frac{360°}{6} = 60°$ and the bisected angle is $\frac{1}{2}(60°) = 30°$. Use a special triangle to find the apothem.



The apothemis about $1.5\sqrt{3} \approx 2.6$ inches.

$A = \frac{1}{2}a \cdot ns \approx \frac{1}{2}(1.5\sqrt{3})(6)(3) \approx 23.38$

The area of the tile is about 23.4 square inches.

5. $P(\text{Point is in red region})$

$$= \frac{2(\text{area of red semicircle})}{\text{Area of square} + 2(\text{Area of seimicircle})}$$

$$= \frac{2\left(\frac{1}{2} \cdot \pi \cdot \left(\frac{10}{2}\right)^2\right)}{10^2 + 2\left(\frac{1}{2} \cdot \pi \cdot \left(\frac{10}{2}\right)^2\right)} = \frac{25\pi}{100 + 25\pi} \approx 0.44$$

The probability that a randomly chosen point in the figure lies in the red region is about 44%.

6. $P(\text{Point is in blue region})$

$$= \frac{\text{Area of square} - 2(\text{Area of red semicircle})}{\text{Area of square} + 2(\text{Area of semicircle})}$$

$$= \frac{10^2 - 2\left(\frac{1}{2} \cdot \pi \cdot \left(\frac{10}{2}\right)^2\right)}{10^2 + 2\left(\frac{1}{2} \cdot \pi \cdot \left(\frac{10}{2}\right)^2\right)} = \frac{100 - 25\pi}{100 + 25\pi} \approx 0.12$$

The probability that a randomly chosen point in the figure lies in the blue region is about 12%.

7. Faces = 9

Vertices = 9

Edges = 16

$F + V = E + 2$

$9 + 9 = 16 + 2$ ✓

8. Faces = 8

Vertices = 12

Edges = 18

$F + V = E + 2$

$8 + 12 = 18 + 2$ ✓

9. Faces = 10

Vertices = 16

Edges = 24

$F + V = E + 2$

$10 + 16 = 24 + 2$ ✓

10. $V = Bh = (12.7)(4) = 336$

The volume is 336 cubic centimeters.

11. $V = Bh = 5\left(\frac{1}{2} \cdot 8 \cdot 5.5055\right)(15.5) \approx 1706.71$

The volume is about 1706.71 cubic meters.

12. $V = \pi r^2 h = \pi(10.95)^2(10.3) \approx 3879.85$

The volume is about 3879.85 cubic feet.

13. $S = 4\pi r^2 = 4\pi(17.5)^2 \approx 3848.45$

$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(17.5)^3 \approx 22{,}449.30$

The surface area and volume of the marble are about 3848.45 square millimeters and 22,449.30 cubic millimeters.

14. $\frac{\text{Surface area of smaller can}}{\text{Surface area of larger can}} = \frac{a^2}{b^2}$

$\frac{308\pi}{\text{Surface area of larger can}} = \frac{2^2}{3^2}$

The surface area of the larger can is 693π square inches.

$\frac{\text{Volume of smaller can}}{\text{Volume of larger can}} = \frac{a^3}{b^3}$

$\frac{735\pi}{\text{Volume of larger can}} = \frac{2^3}{3^3}$

The volume of the larger can is about 2480.63π cubic inches.

Extra Practice

For the chapter "Measurement of Figures and Solids"

1. Arc length of $\widehat{AB} = \frac{90°}{360°} \cdot 2\pi(3) \approx 4.71$ meters

2. Arc length of $\widehat{AB} = \frac{120°}{360°} \cdot 2\pi(10) \approx 20.94$ feet

3. Arc length of $\widehat{AB} = \frac{30°}{360°} \cdot 2\pi\left(\frac{8}{2}\right) \approx 2.09$ inches

4. Arc length of $\widehat{AB} = \frac{150°}{360°} \cdot 2\pi\left(\frac{20}{2}\right) \approx 26.18$ centimeters

5. $A = \pi r^2 = \pi(3^2) = 9\pi$ in.^{2}

The area of a circle with a 3 inch radius is 9π square inches. So, the area is about 28.27 square inches.

6. $A = \pi r^2 = \pi(2.5)^2 = 6.25\pi$

The area of a circle with a 2.5 centimeter radius is 6.25π square centimeters. So, the area is about 19.63 square centimeters.

7. $A = \pi r^2 = \pi\left(\frac{20}{2}\right)^2 = 100\pi$

The area of a circle with a 20 foot diameter is 100π square feet. So, the area is about 314.16 square feet.

8. $A = \pi r^2 = \pi\left(\frac{13}{2}\right)^2 = 42.25\pi$

The area of a circle with a 13 meter diameter is 42.25π square meters. So, the area is about 132.73 square meters.

9. $m\widehat{DE} = 45°$

$m\widehat{DGE} = 360° - 45° = 315°$

Area of small sector $= \frac{m\widehat{DE}}{360°} \cdot \pi r^2$

$= \frac{45°}{360°} \cdot \pi(5^2) \approx 9.82$ in.^{2}

Area of large sector $= \frac{m\widehat{DGE}}{360°} \cdot \pi r^2$

$= \frac{315°}{360°} \cdot \pi(5^2) \approx 68.72$ in.^{2}

10. Because $\angle DFE$ is a straight angle, both sectors have the same area.

Area of each sector $= \frac{m\widehat{DHE}}{360°} \cdot \pi r^2$

$= \frac{180°}{360°} \cdot \pi\left(\frac{22}{2}\right)^2 \approx 190.07$ cm^2

11. $m\widehat{DE} = 100°$

$m\widehat{DGE} = 360° - 100° = 260°$

$$\text{Area of small sector} = \frac{m\widehat{DE}}{360°} \cdot \pi r^2$$
$$= \frac{100°}{360°} \cdot \pi(7^2) \approx 42.76 \text{ ft}^2$$
$$\text{Area of large sector} = \frac{m\widehat{DGE}}{360°} \cdot \pi r^2$$
$$= \frac{260°}{360°} \cdot \pi(7^2) \approx 111.18 \text{ ft}^2$$

12. $m\widehat{DGE} = 240°$

$m\widehat{DE} = 360° - 240° = 120°$

$$\text{Area of small sector} = \frac{m\widehat{DE}}{360°} \cdot \pi r^2$$
$$= \frac{120°}{360°} \cdot \pi(2^2) \approx 4.19 \text{ yd}^2$$
$$\text{Area of large sector} = \frac{m\widehat{DGE}}{360°} \cdot \pi r^2$$
$$= \frac{240°}{360°} \cdot \pi(2^2) \approx 8.38 \text{ yd}^2$$

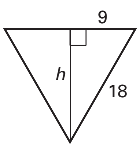
13. The measure of a central angle of a regular octagon is $\frac{360°}{8} = 45°$.

14. The measure of a central angle of a regular dodecagon is $\frac{360°}{12} = 30°$.

15. The measure of a central angle of a regular 20-gon is $\frac{360°}{20} = 18°$.

16. The measure of a central angle of a regular 25-gon is $\frac{360°}{25} = 14.4°$.

17. $P = 3(18) = 54$ units



$h^2 + 9^2 = 18^2$

$h^2 + 81 = 324$

$h^2 = 243$

$h = \sqrt{243} = 9\sqrt{3}$

$A = \frac{1}{2}bh$

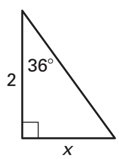
$= \frac{1}{2}(18)9\sqrt{3}$

$= 81\sqrt{3}$

≈ 140.30 square units

18. Measure of a central angle: $\frac{360°}{5} = 72°$

The apothem bisects a 72° central angle to form a right triangle with a 36° angle and a longer leg that is 2 units long.



$\tan 36° = \frac{x}{2}$

$2 \tan 36° = x$

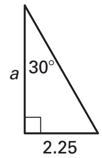
The length of a side is $2x$.

Perimeter: $P = 5 \cdot 2x = 10(2 \tan 36°) \approx 14.53$ units

Area: $A = \frac{1}{2}aP \approx \frac{1}{2}(2)(14.53) = 14.53$ square units

19. Measure of a central angle: $\frac{360°}{6} = 60°$

The apothem bisects a 60° central angle to form a right triangle with a 30° angle and a shorter leg that is $\frac{4.5}{2} = 2.25$ units long.



$\tan 30° = \frac{2.25}{a}$

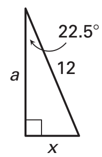
$a = \frac{2.25}{\tan 30°}$

Perimeter: $P = 6(4.5) = 27$ units

Area: $A = \frac{1}{2}aP = \frac{1}{2}\left(\frac{2.25}{\tan 30°}\right)(27) \approx 52.61$ square units

20. Measure of a central angle: $\frac{360°}{8} = 45°$

The apothem bisects a 45° central angle to form a right triangle with a 22.5° angle and a hypotenuse that is 12 units long.



$\cos 22.5° = \frac{a}{12}$ $\quad$ $\sin 22.5° = \frac{x}{12}$

$12 \cos 22.5° = a$ $\quad$ $12 \sin 22.5° = x$

Side length: $2x = 24 \sin 22.5°$

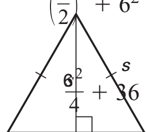
Perimeter: $P = 8(24 \sin 22.5°) \approx 73.48$ units

Area: $A = \frac{1}{2}aP$

$\approx \frac{1}{2}(12 \cos 22.5°)(73.48)$

≈ 407.29 square units

21. The center of the circle is the point of concurrency of the medians of the regular triangle, so the radius of the circle is $\frac{2}{3}$ the height of the triangle. Then the height of the triangle is 6. Using a side length of s:



$\left(\frac{s}{2}\right)^2 + 6^2 = s^2$

$\frac{s^2}{4} + 36 = s^2$

$36 = \frac{3}{4}s^2$

$48 = s^2$

$4\sqrt{3} = s$

$$P = \frac{\text{Area of circle} - \text{Area of triangle}}{\text{Area of circle}}$$
$$= \frac{\pi(4^2) - \frac{1}{2}(4\sqrt{3})6}{\pi(4^2)} \approx 0.587$$

There is about a 58.7% probability that a randomly chosen point in the circle lies in the shaded region.

22. $P = \dfrac{\text{Area of sector} - \text{Area of triangle}}{\text{Area of circle}}$

$= \dfrac{\frac{90°}{360°} \cdot \pi(15^2) - \frac{1}{2}(15)(15)}{\pi(15^2)}$

≈ 0.091

There is about a 9.1% probability that a randomly chosen point in the circle lies in the shaded region.

23. In the large triangle, the base = 10 = the height.

The small triangle at the top is similar to the large triangle and its height is 3. So its base must be equal to 3.

$P = \dfrac{\text{Sum of areas of shaded triangles}}{\text{Area of large triangle}}$

$= \dfrac{\frac{1}{2}(3)(3) + \frac{1}{2}(3)(7)}{\frac{1}{2}(10)(10)}$

$= 0.3$

There is a 30% probability that a randomly chosen point in the large triangle lies in the shaded region.

24. Base of rectangle: $4(6) = 24$

Height of rectangle: $2(6) = 12$

$P = \dfrac{\text{Area of rectangle} - \text{Sum of areas of circles}}{\text{Area of rectangle}}$

$= \dfrac{24(12) - [\pi(6^2) + \pi(6^2)]}{24(12)}$

≈ 0.215

There is about a 21.5% probability that a randomly chosen point in the rectangle lies in the shaded region.

25. $P = \dfrac{4.5}{120} = 0.0375$

There is a 3.75% chance that your favorite song will be playing when you randomly turn on the radio.

26. No, the solid is not a polyhedron; it is not bounded by polygons.

27. Yes, the solid is a polyhedron; the solid is a pentagonal prism; the solid is bounded by polygons and has pentagons for bases.

28. No, the solid is not a polyhedron; it is not bounded by polygons and the bases are not polygons.

29. Yes, the solid is a polyhedron; the solid is a triangular pyramid; the solid is bounded by polygons and has a triangle for a base.

30. $B = (4)(3.5) = 14$

$V = Bh$

$V = (14)(2)$

$V = 28 \text{ ft}^2$

31. $B = \frac{1}{2}(14)(14) = 98$

$V = Bh$

$V = (98)(20)$

$V = 1960 \text{ cm}^3$

32. $V = \pi r^2 h$

$V = \pi(1.15)^2(7.2)$

$V = \pi(1.3225)(7.2)$

$V = 9.522\pi$

$V \approx 29.91 \text{ mm}^3$

33. $B = x \cdot x = x^2$

$V = Bh$

$8 = (x^2)(x)$

$8 = x^3$

$2 \text{ cm} = x$

34. $B = \frac{1}{2}(3)(6) = 9$

$V = Bh$

$72 = 9x$

$8 \text{ ft} = x$

35. $V = \pi r^2 h$

$628 = \pi x^2(8)$

$\frac{628}{8\pi} = x^2$

$24.99 \approx x^2$

$5.00 \text{ in.} \approx x$

36. $B = (12)(12) = 144$

$V = \frac{1}{3}Bh$

$V = \frac{1}{3}(144)(15)$

$V = 720 \text{ in.}^3$

37. Find the apothem and perimeter of the hexagonal base:

$a = (2.5)(\tan 60°) \approx 4.33013$

$P = (5)(6) = 30$

$B = \frac{1}{2}aP \approx \frac{1}{2}(4.33013)(30) \approx 64.95195$

$V = \frac{1}{3}Bh$

$V \approx \frac{1}{3}(64.95195)(8)$

$V \approx 173.21 \text{ ft}^3$

38. $V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi(7.3)^2(11.4)$

$V = \frac{1}{3}\pi(53.29)(11.4)$

$V = 202.502\pi$

$V \approx 636.18 \text{ m}^3$

39. Find the radius of the base:

$\tan 45° = \frac{18}{r}$

$(r)(\tan 45°) = 18$

$r = \frac{18}{\tan 45°}$

$r = 18$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi(18)^2(18)$

$V = \frac{1}{3}\pi(324)(18)$

$V = 1944\pi$

$V \approx 6107.26 \text{ in.}^3$

40. Find the height:

$\tan 30° = \frac{h}{5}$

$5 \cdot \tan 30° = h$

$2.886751 \approx h$

$V = \frac{1}{3}\pi r^2 h$

$V \approx \frac{1}{3}\pi(5)^2(2.886751)$

$V \approx \frac{1}{3}\pi(25)(2.886751)$

$V \approx 24.056258\pi$

$V \approx 75.57 \text{ m}^3$

41. Find the radius of the base:

$\tan 68° = \frac{4.2}{r}$

$(r)(\tan 68°) = 4.2$

$r = \frac{4.2}{\tan 68°}$

$r \approx 1.696910$

$V = \frac{1}{3}\pi r^2 h$

$V \approx \frac{1}{3}\pi(1.696910)^2(4.2)$

$V \approx 4.031305\pi$

$V \approx 12.66 \text{ ft}^3$

42. $S = 4\pi r^2$

$S = 4\pi(13)^2$

$S = 4\pi(169)$

$S = 676\pi$

$S \approx 2123.72 \text{ m}^2$

$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi(13)^3$

$V = \frac{4}{3}\pi(2197)$

$V \approx 2929.333\pi$

$V \approx 9202.77 \text{ m}^3$

43. $S = 4\pi r^2$

$S = 4\pi(1.8)^2$

$S = 4\pi(3.24)$

$S = 12.96\pi$

$S \approx 40.72 \text{ in.}^2$

$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi(1.8)^3$

$V = \frac{4}{3}\pi(5.832)$

$V = 7.776\pi$

$V \approx 24.43 \text{ in.}^3$

44. $S = 4\pi r^2$

$S = 4\pi(14)^2$

$S = 4\pi(196)$

$S = 784\pi$

$S \approx 2463.01 \text{ yd}^2$

$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi(14)^3$

$V = \frac{4}{3}\pi(2744)$

$V \approx 3658.667\pi$

$V \approx 11{,}494.04 \text{ yd}^3$

45. $S = 4\pi r^2$

$S = 4\pi(6.85)^2$

$S = 4\pi(46.9225)$

$S = 187.69\pi$

$S \approx 589.65 \text{ cm}^2$

$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi(6.85)^3$

$V = \frac{4}{3}\pi(321.419125)$

$V \approx 428.558833\pi$

$V \approx 1346.36 \text{ cm}^3$

46. $S = 4\pi r^2$

$S = 4\pi(20)^2$

$S = 4\pi(400)$

$S = 1600\pi$

$S \approx 5026.55 \text{ in.}^2$

$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi(20)^3$

$V = \frac{4}{3}\pi(8000)$

$V \approx 10666.667\pi$

$V \approx 33{,}510.32 \text{ in.}^3$

47. $S = 4\pi r^2$

$S = 4\pi(17.5)^2$

$S = 4\pi(306.25)$

$S = 1225\pi$

$S \approx 3848.45 \text{ mm}^2$

$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi(17.5)^3$

$V = \frac{4}{3}\pi(5359.375)$

$V \approx 7145.833\pi$

$V \approx 22{,}449.30 \text{ mm}^3$

48. $S = 4\pi r^2$

$S = 4\pi(7.6)^2$

$S = 4\pi(57.76)$

$S = 231.04\pi$

$S \approx 725.83 \text{ m}^2$

$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi(7.6)^3$

$V = \frac{4}{3}\pi(438.976)$

$V \approx 585.301333\pi$

$V \approx 1838.78 \text{ m}^3$

49. $S = 4\pi r^2$

$S = 4\pi(11.5)^2$

$S = 4\pi(132.25)$

$S = 529\pi$

$S \approx 1661.90 \text{ ft}^2$

$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi(11.5)^3$

$V = \frac{4}{3}\pi(1520.875)$

$V \approx 2027.833\pi$

$V \approx 6370.63 \text{ ft}^3$

50. $\frac{\text{Surface area of A}}{\text{Surface area of B}} = \frac{a^2}{b^2}$

$\frac{324\pi}{\text{Surface area of B}} = \frac{3^2}{2^2}$

$\frac{324\pi}{\text{Surface area of B}} = \frac{9}{4}$

$(9)(\text{Surface area of B}) = 4(324\pi)$

$(9)(\text{Surface area of B}) = 1296\pi$

$\text{Surface area of B} = 144\pi \text{in.}^2$

$\frac{\text{Volume of A}}{\text{Volume of B}} = \frac{a^3}{b^3}$

$\frac{972\pi}{\text{Volume of B}} = \frac{3^3}{2^3}$

$\frac{972\pi}{\text{Volume of B}} = \frac{27}{8}$

$(27)(\text{Volume of B}) = 8(972\pi)$

$(27)(\text{Volume of B}) = 7776\pi$

$\text{Volume of B} = 288\pi \text{ in.}^3$

51. $\frac{\text{Surface area of A}}{\text{Surface area of B}} = \frac{a^2}{b^2}$

$\frac{864}{\text{Surface area of B}} = \frac{2^2}{1^2}$

$\frac{864}{\text{Surface area of B}} = \frac{4}{1}$

$(4)(\text{Surface area of B}) = 864$

$\text{Surface area of B} = 216 \text{ ft}^2$

$\frac{\text{Volume of A}}{\text{Volume of B}} = \frac{a^3}{b^3}$

$\frac{1728}{\text{Volume of B}} = \frac{2^3}{1^3}$

$\frac{1728}{\text{Volume of B}} = \frac{8}{1}$

$(8)(\text{Volume of B}) = 1728$

$\text{Volume of B} = 216 \text{ ft}^3$

52. $\frac{\text{Surface area of A}}{\text{Surface area of B}} = \frac{a^2}{b^2}$

$\frac{64\pi}{\text{Surface area of B}} = \frac{4^2}{7^2}$

$\frac{64\pi}{\text{Surface area of B}} = \frac{16}{49}$

$(16)(\text{Surface area of B}) = 49(64\pi)$

$(16)(\text{Surface area of B}) = 3136\pi$

$\text{Surface area of B} = 196\pi \text{ cm}^2$

$\frac{\text{Volume of A}}{\text{Volume of B}} = \frac{a^3}{b^3}$

$\frac{64\pi}{\text{Volume of B}} = \frac{4^3}{7^3}$

$\frac{64\pi}{\text{Volume of B}} = \frac{64}{343}$

$(64)(\text{Volume of B}) = 343(64\pi)$

$\text{Volume of B} = 343\pi \text{ cm}^3$