Chapter 6 Systems of Equations and Inequalities

Prerequisite Skills for the chapter "Systems of Equations and Inequalities"

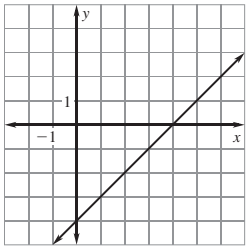
1. The least common multiple of 10 and 15 is 30.

2. Two lines in the same plane are *parallel* if they do not intersect.

3. $x - y = 4$

$-y = -x + 4$

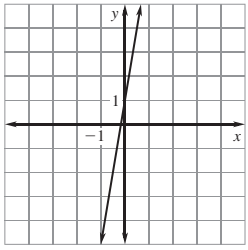
$y = x - 4$



4. $6x - y = -1$

$-y = -6x - 1$

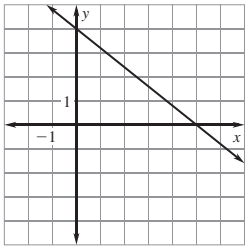
$y = 6x + 1$



5. $4x + 5y = 20$

$5y = -4x + 20$

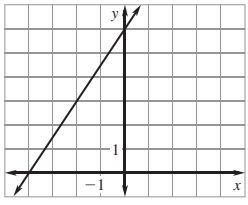
$y = -\frac{4}{5}x + 4$



6. $3x - 2y = -12$

$-2y = -3x - 12$

$y = \frac{3}{2}x + 6$



7. $5m + 4 - m = 20$

$4m + 4 = 20$

$4m = 16$

$m = 4$

8. $10(z + 5) + z = 6$

$10z + 50 + z = 6$

$11z + 50 = 6$

$11z = -44$

$z = -4$

9. $y = 2x - 3$

$y + 2x = -3$

$y = -2x - 3$

The lines are not parallel because their slopes are not the same.

10. $y - 5x = -1$

$y = 5x - 1$

$y - 5x = 1$

$y = 5x + 1$

The lines are parallel because their slopes are both 5.

11. $y = x + 10$

$x - y = -9$

$-y = -x - 9$

$y = x + 9$

The lines are parallel because their slopes are both 1.

12. $6x - y = 4$

$-y = -6x + 4$

$y = 6x - 4$

$4x - y = 6$

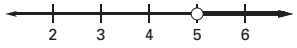
$-y = -4x + 6$

$y = 4x - 6$

The lines are not parallel because their slopes are not the same.

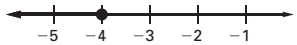
13. $m + 4 > 9$

$m > 9$



14. $-6t \geq 24$

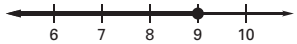
$t \leq 4$



15. $2x - 5 \leq 13$

$2x \leq 18$

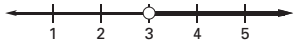
$x \leq 9$



16. $-5y + 1 < -14$

$-5y < -15$

$y > 3$



Lesson 6.1 Solve Linear Systems by Graphing

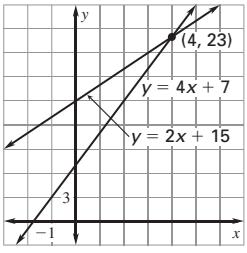
Investigating Algebra Activity for the lesson "Solve Linear Systems by Graphing"

x	$y = 2x + 15$	$y = 4x + 7$
0	15	7
1	17	11
2	19	15
3	21	19
4	23	23
5	25	27

The solution is $x = 4$.

1. Bill and his brother will have the same number of books in 4 months. They will each have 23 books.

2.



The graphs intersect at the point with the x-value that is the solution to the two equations.

3.

x	$y = 2x + 3$	$y = -3x + 18$
0	3	18
1	5	15
2	7	12
3	9	9
4	11	6

The solution is $x = 3, y = 9$.

4.

x	$y = -x + 1$	$y = 2x - 5$
0	1	-5
1	0	-3
2	-1	-1
3	-2	1
4	-3	3

The solution is $x = 2, y = -1$.

5.

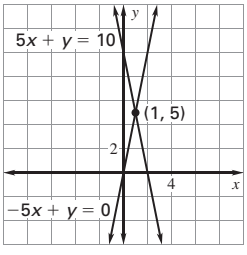
x	$y = -3x + 1$	$y = 5x - 31$
0	1	-31
1	-2	-26
2	-5	-21
3	-8	-16
4	-11	-11

The solution is $x = 4, y = -11$.

Lesson 6.1 Solve Linear Systems by Graphing

Guided Practice for the lesson "Solve Linear Systems by Graphing"

1.



(1, 5)

$-5x + y = 0$ $\qquad$ $5x + y = 10$

$-5(1) + 5 \stackrel{?}{=} 0$ $\qquad$ $5(1) + 5 \stackrel{?}{=} 10$

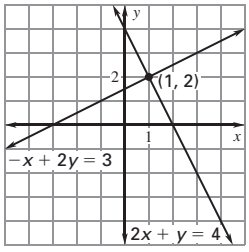
$0 = 0$ ✓ $\qquad$ $10 = 10$ ✓

Because the ordered pair (1, 5) is a solution of each equation, it is a solution of the system.

2. $-x + 2y = 3$ $\qquad$ $2x + y = 4$

$2y = x + 3$ $\qquad$ $y = -2x + 4$

$y = \frac{1}{2}x + \frac{3}{2}$



(1, 2)

$-x + 2y = 3$ $\qquad$ $2x + y = 4$

$-1 + 2(2) \stackrel{?}{=} 3$ $\qquad$ $2(1) + 2 \stackrel{?}{=} 4$

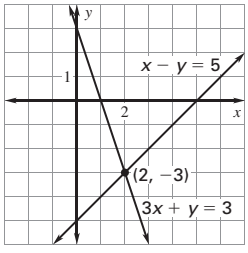
$3 = 3$ ✓ $\qquad$ $4 = 4$ ✓

Because the ordered pair (1, 2) is a solution of each equation, it is a solution of the system.

3. $x - y = 5$ $\qquad$ $3x + y = 3$

$-y = -x + 5$ $\qquad$ $y = -3x + 3$

$y = x - 5$



(2, −3)

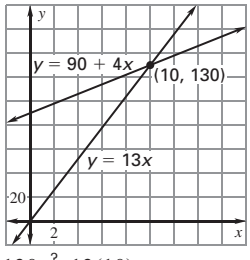
$x - y = 5$ $\qquad$ $3x + y = 3$

$2 - (-3) \stackrel{?}{=} 5$ $\qquad$ $3(2) + (-3) \stackrel{?}{=} 3$

$5 = 5$ ✓ $\qquad$ $3 = 3$ ✓

Because the ordered pair (2, −3) is a solution of each equation, it is a solution of the system.

4. $y = 13x$ $\qquad$ $y = 90 + 4x$

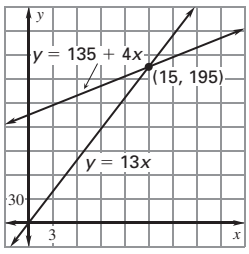


$130 \stackrel{?}{=} 13(10)$ $\qquad$ $130 \stackrel{?}{=} 90 + 4(10)$

$130 = 130$ ✓ $\qquad$ $130 = 130$ ✓

The cost is the same at 10 sessions.

5. $y = 13x$ $\qquad$ $y = 135 + 4x$



$195 \stackrel{?}{=} 13(15)$ $\qquad$ $195 \stackrel{?}{=} 135 + 4(15)$

$195 = 195$ ✓ $\qquad$ $195 = 195$ ✓

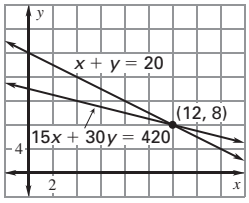
The cost is the same at 15 sessions.

6. $x + y = 20$

$15x + 30y = 420$

$30y = -15x + 420$

$y = -\frac{1}{2}x + 14$



$12 + 8 \stackrel{?}{=} 20$ $\qquad$ $15(12) + 30(8) \stackrel{?}{=} 420$

$20 = 20$ ✓ $\qquad$ $420 = 420$ ✓

8 bicycles were rented.

Exercises for the lesson "Solve Linear Systems by Graphing"

Skill Practice

1. A *solution* of a system of linear equations in two variables is an ordered pair that satisfies each equation in the system.

2. Graph both equations on the same coordinate plane. Find the point where the lines appear to intersect, substitute the x and y values from this point into each equation. If the ordered pair is a solution to each equation, it is a solution of the system.

3. $x + y = -2$ $\qquad$ $x + 5y = 2$

$-3 + 1 \stackrel{?}{=} -2$ $\qquad$ $-3 + 5(1) \stackrel{?}{=} 2$

$-2 = -2$ ✓ $\qquad$ $2 = 2$ ✓

$(-3, 1)$ is a solution.

4. $2x - 3y = 4$ $\qquad$ $2x + 8y = 11$

$2(5) - 3(2) \stackrel{?}{=} 4$ $\qquad$ $2(5) + 8(2) \stackrel{?}{=} 11$

$4 = 4$ ✓ $\qquad$ $26 \neq 11$

$(5, 2)$ is not a solution.

5. $6x + 5y = -7$ $\qquad$ $x - 2y = 0$

$6(-2) + 5(1) \stackrel{?}{=} -7$ $\qquad$ $-2 - 2(1) \stackrel{?}{=} 0$

$-7 \neq -7$ ✓ $\qquad$ $-4 \neq 0$

$(-2, 1)$ is not a solution.

6. B; $(0, -2)$

$x + y = -2$ $\qquad$ $7x - 4y = 8$

$-2 + 0 \stackrel{?}{=} -2$ $\qquad$ $7(-2) - 4(0) \stackrel{?}{=} 8$

$-2 = -2$ ✓ $\qquad$ $-14 \neq 8$

$0 + (-2) \stackrel{?}{=} -2$ $\qquad$ $7(0) - 4(-2) \stackrel{?}{=} 8$

$-2 = -2$ ✓ $\qquad$ $8 = 8$ ✓

7. B; $(-3, 6)$

$2x + 3y = 12$ $\qquad$ $10x + 3y = -12$

$2(-3) + 3(3) \stackrel{?}{=} 12$ $\qquad$ $10(-3) + 3(3) \stackrel{?}{=} -12$

$3 \neq 12$ $\qquad$ $-21 \neq -12$

$2(-3) + 3(6) \stackrel{?}{=} -12$ $\qquad$ $10(-3) + 3(6) \stackrel{?}{=} -12$

$12 = 12$ ✓ $\qquad$ $-12 = -12$ ✓

8. $(1, -3)$

$x - y = 4$ $\qquad$ $4x + y = 1$

$1 - (-3) \stackrel{?}{=} 4$ $\qquad$ $4(1) + (-3) \stackrel{?}{=} -1$

$4 = 4$ ✓ $\qquad$ $1 = 1$ ✓

$(1, -3)$ is a solution.

9. $(4, 2)$

$-x + y = -2$ $\qquad$ $2x - y = 6$

$-4 + 2 \stackrel{?}{=} -2$ $\qquad$ $2(4) - 2 \stackrel{?}{=} 6$

$-2 = -2$ ✓ $\qquad$ $6 = 6$ ✓

$(4, 2)$ is a solution.

10. $(3, 2)$

$x + y = 5$ $\qquad$ $-2x + y = -4$

$3 + 2 \stackrel{?}{=} 5$ $\qquad$ $-2(3) + 2 \stackrel{?}{=} -4$

$5 = 5$ ✓ $\qquad$ $-4 = -4$ ✓

$(3, 2)$ is a solution.

11. The error is that the y-intercept of equation 2 should be -1, not -3. When graphed correctly, the lines intersect at $(-3, -3)$.

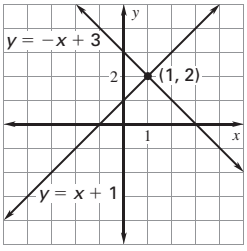
$x - 3y = 6$ $\qquad$ $2x - 3y = 3$

$-3 - 3(-3) \stackrel{?}{=} 6$ $\qquad$ $2(-3) - 3(-3) \stackrel{?}{=} 3$

$6 = 6$ ✓ $\qquad$ $3 = 3$ ✓

$(-3, -3)$ is a solution to the linear system.

12. $y = -x + 3$ $\qquad$ $y = x + 1$



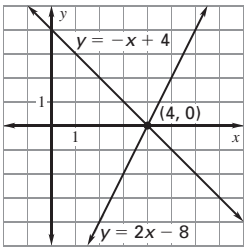
Test $(1, 2)$.

$2 \stackrel{?}{=} -1 + 3$ $\qquad$ $2 \stackrel{?}{=} 1 + 1$

$2 = 2$ ✓ $\qquad$ $2 = 2$ ✓

$(1, 2)$ is a solution.

13. $y = -x + 4$ $\qquad$ $y = 2x - 8$



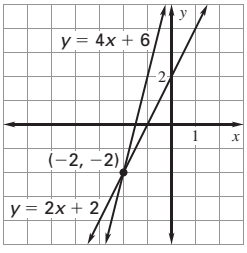
Test (4, 0).

$0 \stackrel{?}{=} -4 + 4$ $\qquad$ $0 \stackrel{?}{=} 2(4) - 8$

$0 = 0$ ✓ $\qquad$ $0 = 0$ ✓

(4, 0) is a solution.

14. $y = 2x + 2$ $\qquad$ $y = 4x + 6$



Test $(-2, -2)$.

$-2 \stackrel{?}{=} 2(-2) + 2$ $\qquad$ $-2 \stackrel{?}{=} 4(-2) + 6$

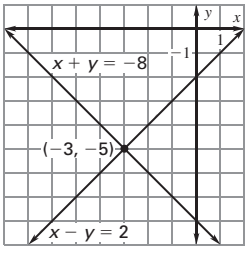
$-2 = -2$ ✓ $\qquad$ $-2 = -2$ ✓

$(-2, -2)$ is a solution.

15. $x - y = 2$ $\qquad$ $x + y = -8$

$-y = -x + 2$ $\qquad$ $y = -x - 8$

$y = x - 2$



Test $(-3, -5)$.

$-3 - (-5) \stackrel{?}{=} 2$ $\qquad$ $-3 + (-5) \stackrel{?}{=} -8$

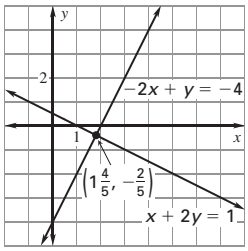
$2 = 2$ ✓ $\qquad$ $-8 = -8$ ✓

$(-3, -5)$ is a solution.

16. $x + 2y = 1$ $\qquad$ $-2x + y = -4$

$2y = -x + 1$ $\qquad$ $y = 2x - 4$

$y = -\frac{1}{2}x + \frac{1}{2}$



Test $\left(1\frac{4}{5}, -\frac{2}{5}\right)$.

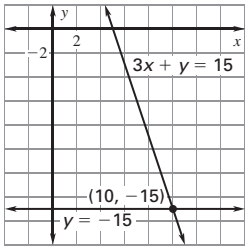
$1\frac{4}{5} + 2\left(-\frac{2}{5}\right) \stackrel{?}{=} 1$ $\qquad$ $-2\left(1\frac{4}{5}\right) + \left(-\frac{2}{5}\right) \stackrel{?}{=} -4$

$1 = 1$ ✓ $\qquad$ $-4 = -4$ ✓

$\left(1\frac{4}{5}, -\frac{2}{5}\right)$ is a solution.

17. $3x + y = 15$ $\qquad$ $y = -15$

$y = -3x + 15$



Test $(10, -15)$.

$3(10) + (-15) \stackrel{?}{=} 15$ $\qquad$ $-15 = -15$ ✓

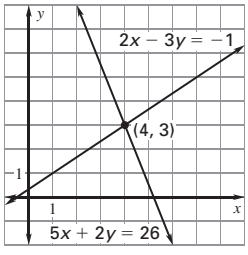
$15 = 15$ ✓

$(10, -15)$ is a solution.

18. $2x - 3y = -1$ $\qquad$ $5x + 2y = 26$

$-3y = -2x - 1$ $\qquad$ $2y = -5x + 26$

$y = \frac{2}{3}x + \frac{1}{3}$ $\qquad$ $y = -\frac{5}{2}x + 13$



Test (4, 3).

$2(4) - 3(3) \stackrel{?}{=} -1$ $\qquad$ $5(4) + 2(3) \stackrel{?}{=} 26$

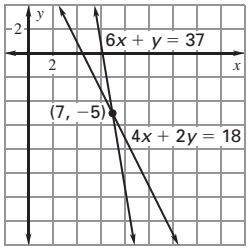
$-1 = -1$ ✓ $\qquad$ $26 = 26$ ✓

(4, 3) is a solution.

19. $6x + y = 37$ $\qquad 4x + 2y = 18$

$y = -6x + 37$ $\qquad 2y = -4x + 18$

$y = -2x + 9$



Test (7, −5).

$6(7) + (-5) \stackrel{?}{=} 37$ $\qquad 4(7) + 2(-5) \stackrel{?}{=} 18$

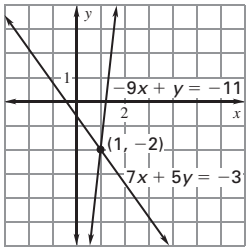
$37 = 37$ ✓ $\qquad 18 = 18$ ✓

(7, −5) is a solution.

20. $7x + 5y = -3$ $\qquad -9y + y = -11$

$5y = -7x - 3$ $\qquad y = 9x - 11$

$y = -\frac{7}{5}x - \frac{3}{5}$



Test (1, −2).

$7(1) + 5(-2) \stackrel{?}{=} -3$ $\qquad -9(1) + (-2) \stackrel{?}{=} -11$

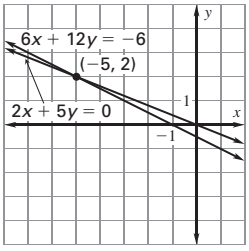
$-3 = -3$ ✓ $\qquad -11 = -11$ ✓

(1, −2) is a solution.

21. $6x + 12y = -6$ $\qquad 2x + 5y = 0$

$12y = -6x - 6$ $\qquad 5y = -2x$

$y = -\frac{1}{2}x - \frac{1}{2}$ $\qquad y = -\frac{2}{5}x$



Test (−5, 2).

$6(-5) + 12(2) \stackrel{?}{=} -6$ $\qquad 2(-5) + 5(2) \stackrel{?}{=} 0$

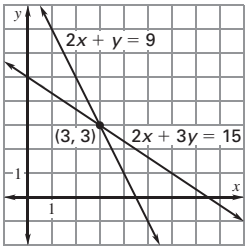
$-6 = -6$ ✓ $\qquad 0 = 0$ ✓

(−5, 2) is a solution.

22. $2x + y = 9$ $\qquad 2x + 3y = 15$

$y = -2x + 9$ $\qquad 3y = -2x + 15$

$y = -\frac{2}{3}x + 5$



Test (3, 3).

$2(3) + 3 \stackrel{?}{=} 9$ $\qquad 2(3) + 3(3) \stackrel{?}{=} 15$

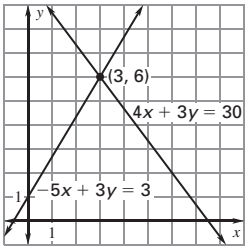
$9 = 9$ ✓ $\qquad 15 = 15$ ✓

(3, 3) is a solution.

23. $-5x + 3y = 3$ $\qquad 4x + 3y = 30$

$3y = 5x + 3$ $\qquad 3y = -4x + 30$

$y = \frac{5}{3}x + 1$ $\qquad y = -\frac{4}{3}x + 10$



Test (3, 6).

$-5(3) + 3(6) \stackrel{?}{=} 3$ $\qquad 4(3) + 3(6) \stackrel{?}{=} 30$

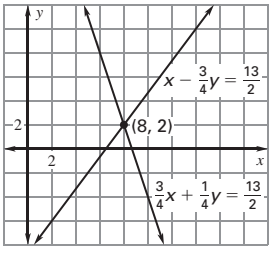
$3 = 3$ ✓ $\qquad 30 = 30$ ✓

(3, 6) is a solution.

24. $\frac{3}{4}x + \frac{1}{4}y = \frac{13}{2}$ $\qquad x - \frac{3}{4}y = \frac{13}{2}$

$\frac{1}{4}y = -\frac{3}{4}x + \frac{13}{2}$ $\qquad -\frac{3}{4}y = -x + \frac{13}{2}$

$y = -3x + 26$ $\qquad y = \frac{4}{3}x - 8\frac{2}{3}$



Test (8, 2).

$\frac{3}{4}(8) + \frac{1}{4}(2) \stackrel{?}{=} \frac{13}{2}$ $\qquad 8 - \frac{3}{4}(2) \stackrel{?}{=} \frac{13}{2}$

$\frac{13}{2} = \frac{13}{2}$ ✓ $\qquad \frac{13}{2} = \frac{13}{2}$ ✓

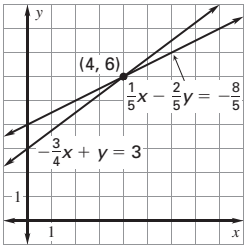
(8, 2) is a solution.

25. $\frac{1}{5}x - \frac{2}{5}y = -\frac{8}{5}$ $\quad -\frac{3}{4}x + y = 3$

$x - 2y = -8$ $\quad y = \frac{3}{4}x + 3$

$-2y = -x - 8$

$y = \frac{1}{2}x + 4$



Test (4, 6).

$\frac{1}{5}(4) - \frac{2}{5}(6) \stackrel{?}{=} -\frac{8}{5}$ $\quad -\frac{3}{4}(4) + 6 \stackrel{?}{=} 3$

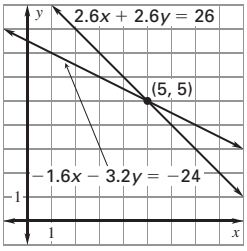
$-\frac{8}{5} = -\frac{8}{5}$ ✓ $\quad 3 = 3$ ✓

(4, 6) is a solution.

26. $-1.6x - 3.2y = -24$ $\quad 2.6x + 2.6y = 26$

$-3.2y = 1.6x - 24$ $\quad x + y = 10$

$y = -0.5x + 7.5$ $\quad y = -x + 10$



Test (5, 5).

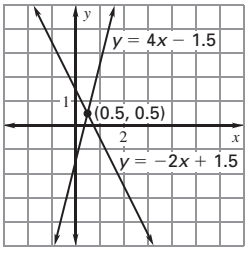
$-1.6(5) - 3.2(5) \stackrel{?}{=} -24$ $\quad 2.6(5) + 2.6(5) \stackrel{?}{=} 26$

$-24 = -24$ ✓ $\quad 26 = 26$ ✓

(5, 5) is a solution.

27. Answers will vary.

28. $y = 4x - 1.5$ $\quad y = -2x + 1.5$



Check (0.5, 0.5).

$0.5 \stackrel{?}{=} 4(0.5) - 1.5$ $\quad 0.5 \stackrel{?}{=} -2(0.5) + 1.5$

$0.5 = 0.5$ ✓ $\quad 0.5 = 0.5$ ✓

It is important to check your solution because the coordinates of the solution may not be obvious by looking at the graph.

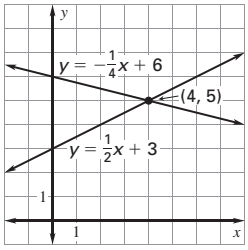
29. a. $-\frac{1}{4}x + 6 = \frac{1}{2}x + 3$

$6 = \frac{3}{4}x + 3$

$3 = \frac{3}{4}x$

$4 = x$

b. $y = -\frac{1}{4}x + 6$ $\quad y = \frac{1}{2}x + 3$



Test (4, 5).

$5 \stackrel{?}{=} -\frac{1}{4}(4) + 6$ $\quad 5 \stackrel{?}{=} \frac{1}{2}(4) + 3$

$5 = 5$ ✓ $\quad 5 = 5$ ✓

The solution is (4, 5).

c. The two equations from the system in part (b) were set equal to each other for the equation in part (a).

d. Set each side of the equation equal to y.

$y = -\frac{2}{5}x + 5$ $\quad y = \frac{1}{5}x + 2$

Graph both equations on the same coordinate plane. The point where the graphs intersect is the solution. The x-coordinate of the intersection point is the value of x in the given equation.

30. $-3x + 2y = 1$

$2y = 3x + 1$

$y = \frac{3}{2}x + \frac{1}{2}$

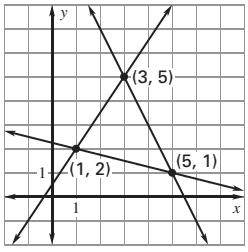
$2x + y = 11$

$y = -2x + 11$

$x + 4y = 9$

$4y = -x + 9$

$y = -\frac{1}{4}x + \frac{9}{4}$



Line 1 & 2: Test (3, 5).

$-3(3) + 2(5) \stackrel{?}{=} 1$ $\quad 2(3) + 5 \stackrel{?}{=} 11$

$1 = 1$ ✓ $\quad 11 = 11$ ✓

Line 2 & 3: Test (5, 1).

$2(5) + 1 \stackrel{?}{=} 11$ $\quad 5 + 4(1) \stackrel{?}{=} 9$

$11 = 11$ ✓ $\quad 9 = 9$ ✓

Line 3 & 1: Test (1, 2).

$1 + 2(4) \stackrel{?}{=} 9$ $\quad -3(1) + 2(2) \stackrel{?}{=} 1$

$9 = 9$ ✓ $\quad 1 = 1$ ✓

The vertices of the triangles are (3, 5), (5, 1), (1, 2).

Problem Solving

31. The percent who watch 1 hour or less will equal the percent who watch more than 1 hour 50 years after 1990, or in 2040.

32. B; $y = -484x + 17{,}424$

$y = -330x + 15{,}840$

33. Let x = number of small cards.

Let y = number of large cards.

$x + y = 25$

$3x + 5y = 95$

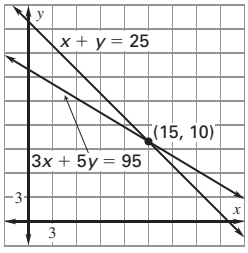
$3x + 5y = 95$

$5y = -3x + 95$

$y = -\frac{3}{5}x + 19$

$x + y = 25$

$y = -x + 25$



Test (15, 10)

$15 + 10 \stackrel{?}{=} 25$

$25 = 25$ ✓

$3(15) + 5(10) \stackrel{?}{=} 95$

$95 = 95$ ✓

She sold 15 small cards and 10 large cards.

34. a. Let x = minutes on stair machine.

Let y = minutes on stationary bike.

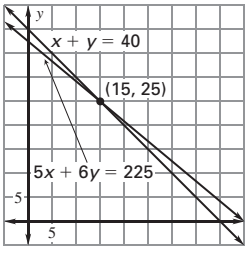
$x + y = 40$

$y = -x + 40$

$5x + 6y = 225$

$6y = -5x + 225$

$y = -\frac{5}{6}x + 37.5$



You should spend 15 minutes on the stair machine and 25 minutes on the stationary bike.

b. Let x = minutes on elliptical trainer.

Let y = minutes on stair machine.

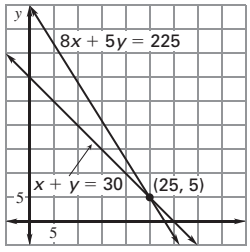
$x + y = 30$

$y = -x + 30$

$8x + 5y = 225$

$5y = -8x + 225$

$y = -\frac{8}{5}x + 45$



You should spend 25 minutes on the elliptical trainer and 5 minutes on the stair machine.

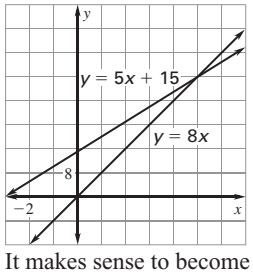
35. a. $y = 5x + 15$

$y = 8x$

b.

x	$y = 5x + 15$	$y = 8x$
1	20	8
2	25	16
3	30	24
4	35	32
5	40	40

c.



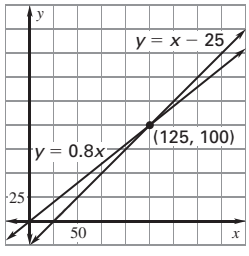
It makes sense to become a club member if you attend more than 5 movies a year. The graph shows that the y-values (total cost) are higher on the line representing a non-member when x (number of movies viewed) is greater than 5.

36. Let x = purchase price.

Let y = amount paid.

$y = x - 25$

$y = 0.8x$



You should choose \$25 off if your purchase is less than \$125, and you should choose 20% off if your purchase is more than \$125, because 20% of amounts greater than \$125 is more than \$25.

Graphing Calculator Activity for the lesson "Solve Linear Systems by Graphing"

1. $Y_1 = x + 4$

$Y_2 = -3x - 2$

The solution is about $(-1.5, 2.5)$.

2. $5x + y = -4$

$Y_1 = -5x - 4$

$x - y = -2$

$-y = -x - 2$

$Y_2 = x + 2$

The solution is about $(-1, 1)$.

3. $-0.45x - y = 1.35$

$-y = 0.45x + 1.35$

$Y_1 = -0.45x - 1.35$

$-1.8x + y = -1.8$

$Y_2 = 1.8x - 1.8$

The solution is about $(0.2, -1.44)$

4. $-0.4x + 0.8y = -16$

$0.8y = 0.4x - 16$

$Y_1 = 0.5x - 20$

$1.2x + 0.4y = 1$

$0.4y = -1.2x + 1$

$Y_2 = -3x + 2.5$

The solution is about $(6.43, -16.79)$

Lesson 6.2 Solve Linear Systems by Substitution

Guided Practice for the lesson "Solve Linear Systems by Substitution"

1. $y = 2x + 5$

$3x + y = 10$

$3x + 2x + 5 = 10$

$5x = 5$

$x = 1$

$y = 2(1) + 5$

$y = 7$

The solution is $(1, 7)$.

Check:

$7 \stackrel{?}{=} 2(1) + 5$ $\qquad$ $3(1) + 7 \stackrel{?}{=} 10$

$7 = 7$ ✓ $\qquad$ $10 = 10$ ✓

2. $x - y = 3 \rightarrow x = y + 3$

$x + 2y = -6$

$y + 3 + 2y = -6$

$3y = -9$

$y = -3$

$x = -3 + 3$

$x = 0$

The solution is $(0, -3)$.

Check:

$0 - (-3) \stackrel{?}{=} 3$ $\qquad$ $0 + 2(-3) \stackrel{?}{=} -6$

$3 = 3$ ✓ $\qquad$ $-6 = -6$ ✓

3. $3x + y = -7$

$y = -3x - 7$

$-2x + 4y = 0$

$-2x + 4(-3x - 7) = 0$

$-2x + (-12x) - 28 = 0$

$-14x = 28$

$x = -2$

$y = -3(-2) - 7$

$y = -1$

The solution is $(-2, -1)$.

Check:

$3(-2) + (-1) \stackrel{?}{=} -7$ $\qquad$ $-2(-2) + 4(-1) \stackrel{?}{=} 0$

$-7 = -7$ ✓ $\qquad$ $0 = 0$ ✓

4. $y = 10 + 21.95x$

$y = 10 + 21.95(20)$

$y = 449$

The cost for the internet service provider is $449 after 20 months.

$y = 22.45x$

$y = 22.45(20)$

$y = 449$

The cost for the website hosting company is also $449 after 20 months.

5. $y = 5 + 21.95x$

$y = 22.45x$

$22.45x = 5 + 21.95x$

$0.5x = 5$

$x = 10$

The total cost will be the same for both companies after 10 months.

6. $x + y = 16$

$x + 0.5y = 0.7(16)$

$16 - y + 0.5y = 11.2$

$-0.5y = -4.8$

$y = 9.6$

$x + 9.6 = 16$

$x = 6.4$

Mix 6.4 quarts of 100% antifreeze and 9.6 quarts of 50% antifreeze and 50% water mix to get 16 quarts of 70% antifreeze and 30% water mix.

Exercises for the lesson "Solve Linear Systems by Substitution"

Skill Practice

1. Answers will vary.

2. Solve equation 2 for y, because y doesn't have a coefficient in this equation. Then substitute that value into equation 1, and solve equation 1 for x.

3. $x = 17 - 4y$
$y = x - 2$
$y = 17 - 4y - 2$
$5y = 15$
$y = 3$
$x = 17 - 4(3)$
$x = 5$
The solution is (5, 3).

4. $y = 2x - 1$
$2x + y = 3$
$2x + 2x - 1 = 3$
$4x = 4$
$x = 1$
$y = 2(1) - 1$
$y = 1$
The solution is (1, 1).

5. $x = y + 3$
$2x - y = 5$
$2(y + 3) - y = 5$
$2y + 6 - y = 5$
$y = -1$
$x = -1 + 3$
$x = 2$
The solution is (2, −1).

6. $4x - 7y = 10$
$y = x - 7$
$4x - 7(x - 7) = 10$
$4x - 7x + 49 = 10$
$-3x = -39$
$x = 13$
$y = 13 - 7$
$y = 6$
The solution is (13, 6).

7. $x = 16 - 4y$
$3x + 4y = 8$
$3(16 - 4y) + 4y = 8$
$48 - 12y + 4y = 8$
$-8y = -40$
$y = 5$
$x = 16 - 4(5)$
$x = -4$
The solution is (−4, 5).

8. $-5x + 3y = 51$
$y = 10x - 8$
$-5x + 3(10x - 8) = 51$
$-5x + 30x - 24 = 51$
$25x = 75$
$x = 3$
$y = 10(3) - 8$
$y = 22$
The solution is (3, 22).

9. $2x = 12 \rightarrow x = 6$
$x - 5y = -29$
$6 - 5y = -29$
$y = 7$
The solution is (6, 7).

10. $2x - y = 23$
$x - 9 = -1$
$x = 8$
$2(8) - y = 23$
$-y = 7$
$y = -7$
The solution is (8, −7).

11. $x + y = 0 \rightarrow x = -y$
$x - 2y = 6$
$-y - 2y = 6$
$-3y = 6$
$y = -2$
$x = 2$
The solution is (2, −2).

12. $2x + y = 9$
$y = -2x + 9$
$4x - y = -15$
$4x - (-2x + 9) = -15$
$4x + 2x - 9 = -15$
$6x = -6$
$x = -1$
$y = -2(-1) + 9$
$y = 11$
The solution is (−1, 11).

13. $5x + 2y = 9$
$x + y = -3$
$y = -3 - x$
$5x + 2(-3 - x) = 9$
$5x - 6 - 2x = 9$
$3x = 15$
$x = 5$
$y = -3 - 5$
$y = -8$
The solution is (5, −8).

14. $5x + 4y = 32$

$9x - y = 33$

$-y = -9x + 33$

$y = 9x - 33$

$5x + 4(9x - 33) = 32$

$5x + 36x - 132 = 32$

$41x = 164$

$x = 4$

$y = 9(4) - 33$

$y = 3$

The solution is (4, 3).

15. $11x - 7y = -14$

$x - 2y = -4$

$x = 2y - 4$

$11(2y - 4) - 7y = -14$

$22y - 44 - 7y = -14$

$15y = 30$

$y = 2$

$x = 2(2) - 4$

$x = 0$

The solution is (0, 2).

16. $20x - 30y = -50$

$x + 2y = 1$

$x = -2y + 1$

$20(-2y + 1) - 30y = -50$

$-40y + 20 - 30y = -50$

$-70y + 20 = -50$

$-70y = -70$

$y = 1$

$x = -2(1) + 1$

$x = -1$

The solution is $(-1, 1)$.

17. $6x + y = 4$

$x - 4y = 19$

$x = 4y + 19$

$6(4y + 19) + y = 4$

$24y + 114 + y = 4$

$25y = -110$

$y = -4.4$

$x = 4(-4.4) + 19$

$x = 1.4$

The solution is $(1.4, -4.4)$.

18. A; (6, 7)

$4x - y = 17$

$y = 4x - 17$

$-9x + 8y = 2$

$-9x + 8(4x - 17) = 2$

$-9x + 32x - 136 = 2$

$23x = 138$

$x = 6$

$y = 4(6) - 17$

$y = 7$

19. The error is that, in step 3, 6 should have been substituted for x in the equation, not for y.

$y = 9 - 3x$

$y = 9 - 3(6)$

$y = -9$

20. $4.5x + 1.5y = 24$

$x - y = 4$

$x = 4 + y$

$4.5(4 + y) + 1.5y = 24$

$18 + 4.5y + 1.5y = 24$

$18 + 6y = 24$

$6y = 6$

$y = 1$

$x = 4 + 1 = 5$

The solution is (5, 1).

21. $35x + y = 20$

$y = 20 - 35x$

$1.5x - 0.1y = 18$

$1.5x - 0.1(20 - 35x) = 18$

$1.5x - 2 + 3.5x = 18$

$5x = 20$

$x = 4$

$y = 20 - 35(4)$

$y = -120$

The solution is $(4, -120)$.

22. $3x - 2y = 8$

$0.5x + y = 17$

$y = 17 - 0.5x$

$3x - 2(17 - 0.5x) = 8$

$3x - 34 + x = 8$

$4x = 42$

$x = 10.5$

$y = 17 - 0.5(10.5)$

$y = 11.75$

The solution is (10.5, 11.75).

23. $0.5x + 0.6y = 5.7$

$2x - y = -1$

$-y = -2x - 1$

$y = 2x + 1$

$0.5x + 0.6(2x + 1) = 5.7$

$0.5x + 1.2x + 0.6 = 5.7$

$1.7x = 5.1$

$x = 3$

$y = 2(3) + 1$

$y = 7$

The solution is (3, 7).

24. $x - 9 = 0.5y$

$x = 0.5y + 9$

$2.2x - 3.1y = -0.2$

$2.2(0.5y + 9) - 3.1y = -0.2$

$1.1y + 19.8 - 3.1y = -0.2$

$-2y = -20$

$y = 10$

$x = 0.5(10) + 9$

$x = 14$

The solution is (14, 10).

25. $0.2x + y = -1.8$

$y = -1.8 - 0.2x$

$1.8y + 5.5x = 27.6$

$1.8(-1.8 - 0.2x) + 5.5x = 27.6$

$-3.24 - 0.36x + 5.5x = 27.6$

$5.14x = 30.84$

$x = 6$

$y = -1 \cdot 8 - 0.2(6)$

$y = -3$

The solution is (6, −3).

26. $\frac{1}{2}x + \frac{1}{4}y = 5$

$x - \frac{1}{2}y = 1$

$x = \frac{1}{2}y + 1$

$\frac{1}{2}\left(\frac{1}{2}y + 1\right) + \frac{1}{4}y = 5$

$\frac{1}{4}y + \frac{1}{2} + \frac{1}{4}y = 5$

$\frac{1}{2}y = 4\frac{1}{2}$

$y = 9$

$x = \frac{1}{2}(9) + 1$

$x = 5\frac{1}{2}$

The solution is $\left(5\frac{1}{2}, 9\right)$.

27. $x + \frac{1}{3}y = -2$

$x = -\frac{1}{3}y - 2$

$-8x - \frac{2}{3}y = 4$

$-8\left(-\frac{1}{3}y - 2\right) - \frac{2}{3}y = 4$

$\frac{8}{3}y + 16 - \frac{2}{3}y = 4$

$2y + 16 = 4$

$2y = -12$

$y = -6$

$x = -\frac{1}{3}(-6) - 2$

$x = 0$

The solution is (0, −6).

28. $\frac{3}{8}x + \frac{3}{4}y = 12$

$\frac{2}{3}x + \frac{1}{2}y = 13$

$\frac{1}{2}y = 13 - \frac{2}{3}x$

$y = 26 - \frac{4}{3}x$

$\frac{3}{8}x + \frac{3}{4}\left(26 - \frac{4}{3}x\right) = 12$

$\frac{3}{8}x + \frac{39}{2} - x = 12$

$-\frac{5}{8}x = -\frac{15}{2}$

$x = 12$

$y = 26 - \frac{4}{3}(12)$

$y = 10$

The solution is (12, 10).

29. Once you have obtained a solution using substitution, graph both equations on the same coordinate plane. The point where the lines intersect should be the same as the solution you got using substitution.

30. $ax + by = -16$

$-9a + 4b = -16$

$ax - by = -56$

$-9a - 4b = -56$

$-4b = -56 + 9a$

$b = 14 - \frac{9}{4}a$

$-9a + 4\left(14 - \frac{9}{4}a\right) = -16$

$-9a + 56 - 9a = -16$

$-18a = -72$

$a = 4$

$b = 14 - \frac{9}{4}(4)$

$b = 5$

The solution is $a = 4$, $b = 5$.

Problem Solving

31. Let x = bags of popcorn sold.

Let y = pretzels sold.

$x = 2y$

$2.50x + 2y = 336$

$2.50(2y) + 2y = 336$

$5y + 2y = 336$

$7y = 336$

$y = 48$

$x = 2(48)$

$x = 96$

They sold 96 bags of popcorn and 48 pretzels.

32. Let x = number of person tubes.

Let y = number of cooler tubes.

$x + y = 26 \rightarrow x = 26 - y$

$15x + 7.50y = 360$

$15(26 - y) + 7.50y = 360$

$390 - 15y + 7.50y = 360$

$-7.50y = -30$

$y = 4$

$x = 26 - 4 = 22$

They rented 22 tubes for people and 4 "cooler" tubes.

33. $x + y = 9 \rightarrow x = 9 - y$

$1.5x = 1.2y$

$1.5(9 - y) = 1.2y$

$13.5 - 1.5y = 1.2y$

$13.5 = 2.7y$

$5 = y$

$x = 9 - 5 = 4$

The length from A to the string is represented by x, which is 4 inches. The length from the string to B is represented by y, which is 5 inches. The string should be placed 4 inches from point A.

34. a. $d = rt$

$d = 1.9t \leftarrow$ Lane 2 swimmer

$d = 1.8(t + 1.2)$

$1.9t = 1.8(t + 1.2)$

$1.9t = 1.8t + 2.16$

$0.1t = 2.16$

$t = 21.6$

The swimmer in lane 2 will catch up after 21.6 seconds.

b. The race will end when the swimmers have gone 400 meters. If d is less than 400 when $t = 21.6$ seconds, then the swimmer in lane 2 will catch up before the race ends.

$d = 1.9t$

$d = 1.9(21.6)$

$d = 41.04$ meters

The swimmer in lane 2 will catch up to the swimmer in lane 1 before the race ends.

35. Let x = mL of 1% hydrochloric acid solution.

Let y = mL of 5% hydrochloric acid solution.

$x + y = 100 \rightarrow x = 100 - y$

$0.01x + 0.05y = 0.03(100)$

$0.01(100 - y) + 0.05y = 3$

$1 - 0.01y + 0.05y = 3$

$0.04y = 2$

$y = 50$

$x = 100 - 50 = 50$

You need to mix 50 mL of the 1% solution and 50 mL of the 5% solution.

36. Let x = number of dimes.

Let y = number of quarters.

$x = y + 3$

$0.1x + 0.25y = 4.50$

$0.1(y + 3) + 0.25y = 4.50$

$0.1y + 0.3 + 0.25y = 450$

$0.35y = 4.2$

$y = 12$

$x = 12 + 3 = 15$

She has 12 quarters.

37. Let x = time cheetah runs.

Let y = time gazelle runs.

$x = y$

$88x - 73y = 350$

$88x - 73x = 350$

$15x = 350$

$x = 23.3$

The cheetah would catch up to the gazelle in 23.3 seconds, but since the cheetah can only sustain this speed for 20 seconds, the gazelle can stay ahead of the cheetah.

38. Let x = bushels of 100% vermiculite.

Let y = bushels of 60% vermiculite.

$x + 5 = y$

$1x + 0.5(5) = 0.6y$

$x + 2.5 = 0.6(x + 5)$

$x + 2.5 = 0.6x + 3$

$0.4x = 0.5$

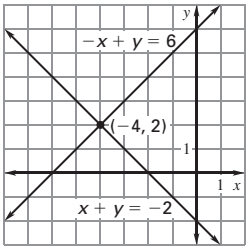
$x = 1.25$

In order to make a mix that is 60% vermiculite and 40% peat moss, he would have to add 1.25 bushels of the 100% vermiculite which would give him 6.25 bushels total of the mixture. Since he only needs 6 bushels, he does have enough of the 50%/50% mix.

Quiz for the lessons "Solve Linear Systems by Graphing" and "Solve Linear Systems by Substitution"

1. $x + y = -2$ $\qquad -x + y = 6$

$y = -x - 2$ $\qquad y = x + 6$



Test $(-4, 2)$.

$-4 + 2 \stackrel{?}{=} -2$ $\qquad -(-4) + 2 \stackrel{?}{=} 6$

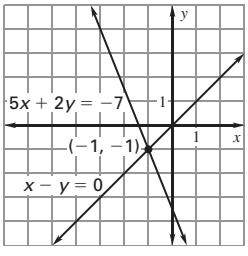
$-2 = -2$ ✓ $\qquad 6 = 6$ ✓

$(-4, 2)$ is a solution.

2. $x - y = 0$ $\qquad 5x + 2y = -7$

$y = x$ $\qquad 2y = -5x - 7$

$y = -\frac{5}{2}x - \frac{7}{2}$



Test $(-1, -1)$. $\qquad 5(-1) + 2(-1) \stackrel{?}{=} -7$

$-1 - (-1) \stackrel{?}{=} 0$ $\qquad -7 = -7$ ✓

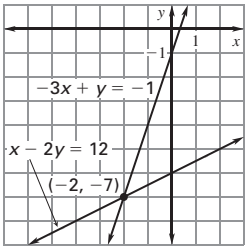
$0 = 0$ ✓

$(-1, -1)$ is a solution.

3. $x - 2y = 12$ $\qquad -3x + y = -1$

$-2y = -x + 12$ $\qquad y = 3x - 1$

$y = \frac{1}{2}x - 6$



Test $(-2, -7)$.

$-2 - 2(-7) \stackrel{?}{=} 12$ $\qquad -3(-2) + (-7) \stackrel{?}{=} -1$

$12 = 12$ ✓ $\qquad -1 = -1$ ✓

$(-2, -7)$ is a solution.

4. $y = x - 4$

$-2x + y = 18$

$-2x + x - 4 = 18$

$-x = 22$

$x = -22$

$y = -22 - 4 = -26$

The solution is $(-22, -26)$.

5. $y = 4 - 3x$

$5x - y = 22$

$5x - (4 - 3x) = 22$

$5x - 4 + 3x = 22$

$8x = 26$

$x = 3.25$

$y = 4 - 3(3.25)$

$y = -5.75$

The solution is $(3.25, -5.75)$.

6. $x = y + 9$

$5x - 3y = 7$

$5(y + 9) - 3y = 7$

$5y + 45 - 3y = 7$

$2y = -38$

$y = -19$

$x = -19 + 9 = -10$

The solution is $(-10, -19)$.

7. $2y + x = -4$

$x = -2y - 4$

$y - x = -5$

$y - (-2y - 4) = -5$

$y + 2y + 4 = -5$

$3y = -9$

$y = -3$

$x = -2(-3) - 4 = 2$

The solution is $(2, -3)$.

8. $5x - 4y = 27$

$-2x + y = 3$

$y = 2x + 3$

$5x - 4(2x + 3) = 27$

$5x - 8x - 12 = 27$

$-3x = 39$

$x = -13$

$y = 2(-13) + 3 = -23$

The solution is $(-13, -23)$.

9. $3x - 5y = 13$

$x + 4y = 10$

$x = 10 - 4y$

$3(10 - 4y) - 5y = 13$

$30 - 12y - 5y = 13$

$-17y = -17$

$y = 1$

$x = 10 - 4(1) = 6$

The solution is (6, 1).

Problem Solving Workshop for the lesson "Solve Linear Systems by Substitution"

1.

Miles	Company 1	Company 2
1	2.80	3.20
2	4.40	4.70
3	6.00	6.20
4	7.60	7.70
5	9.20	9.20

Each taxi will cost the same after 5 miles.

2. Let x = number of adult tickets.

Let y = number of student tickets.

$x + y = 120 \rightarrow x = 120 - y$

$5x + 3y = 460$

$5(120 - y) + 3y = 460$

$600 - 5y + 3y = 460$

$-2y = -140$

$y = 70$

$x = 120 - 70 = 50$

50 adult tickets were purchased, and 70 student tickets were purchased.

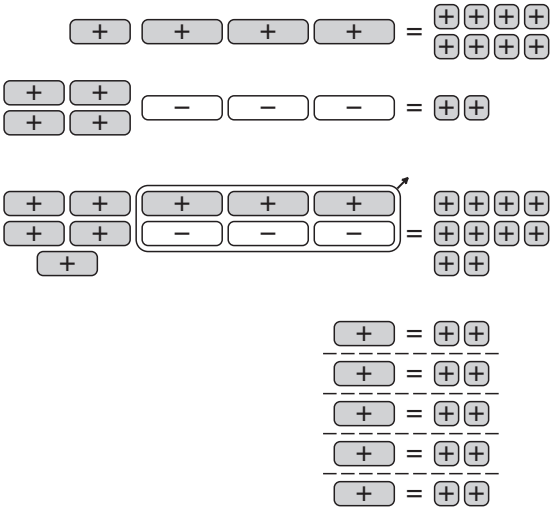
Student Tickets	Adult Tickets	Total Cost ($)
100	20	400
90	30	420
80	40	440
70	**50**	**460**
60	60	480
50	70	500

Lesson 6.3 Solve Linear Systems by Adding or Subtracting

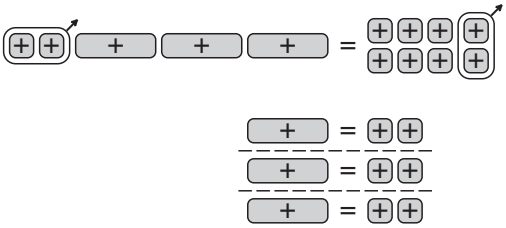
Investigating Algebra Activity for the lesson "Solve Linear Systems by Adding or Subtracting"

1. $x + 3y = 8$

$4x - 3y = 2$



So, $x = 2$.

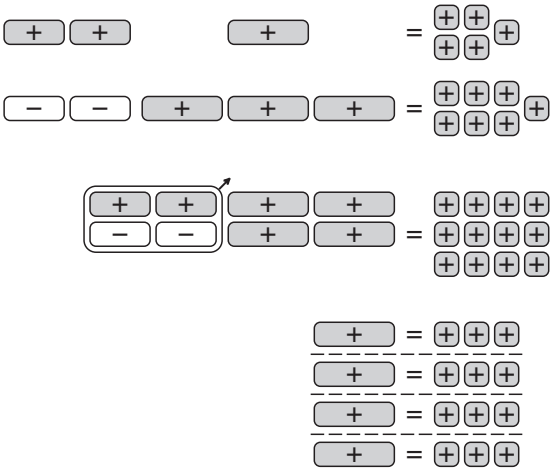


So, $y = 2$.

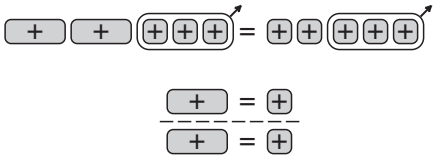
The solution to the system is (2, 2).

2. $2x + y = 5$

$-2x + 3y = 7$



So, $y = 3$.

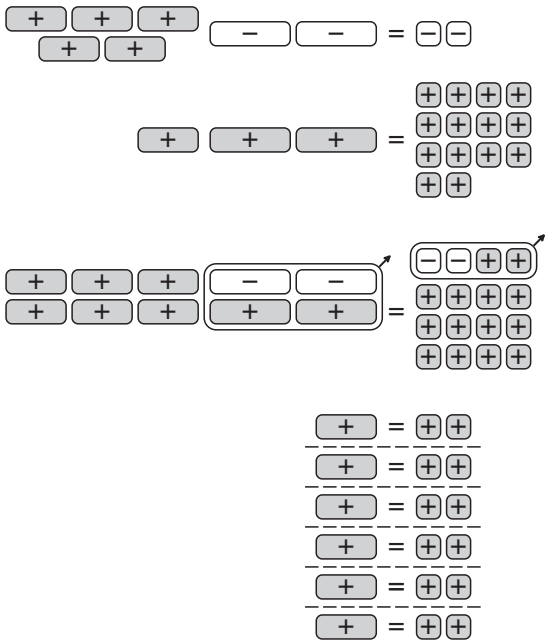


So, $x = 1$.

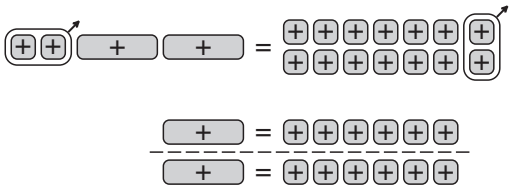
The solution to the system is (1, 3).

3. $5x - 2y = -2$

$x + 2y = 14$



So, $x = 2$.

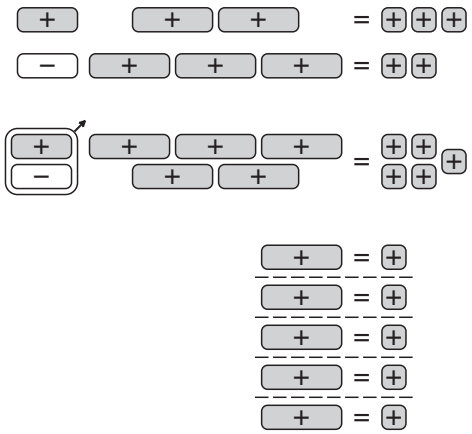


So, $y = 6$.

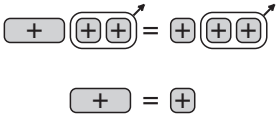
The solution to the system is (2, 6).

4. $x + 2y = 3$

$-x + 3y = 2$



So, $y = 1$.



So, $x = 1$.

The solution to the system is (1, 1).

5. You cannot solve this problem using only the steps above because neither of the variables will cancel out after you add the equations. So you would still have 2 variables and be unable to solve for one of them in step 3.

Guided Practice for the lesson "Solve Linear Systems by Adding or Subtracting"

1. $4x - 3y = 5$

$-2x + 3y = -7$

$2x = -2$

$x = -1$

$4(-1) - 3y = 5$

$-3y = 9$

$y = -3$

The solution is $(-1, -3)$.

Check:

$4(-1) - 3(-3) \stackrel{?}{=} 5$ $\qquad -2(-1) + 3(-3) \stackrel{?}{=} -7$

$5 = 5$ ✓ $\qquad -7 = -7$ ✓

2. $-5x - 6y = 8$

$5x + 2y = 4$

$-4y = 12$

$y = -3$

$5x + 2(-3) = 4$

$5x = 10$

$x = 2$

The solution is $(2, -3)$.

Check:

$-5(2) - 6(-3) \stackrel{?}{=} 8$ $\quad$ $5(2) + 2(-3) \stackrel{?}{=} 4$

$-10 + 18 \stackrel{?}{=} 8$ $\quad$ $10 - 6 \stackrel{?}{=} 4$

$8 = 8$ ✓ $\quad$ $4 = 4$ ✓

3. $6x - 4y = 14$

$-3x + 4y = 1$

$3x = 15$

$x = 5$

$6(5) - 4y = 14$

$-4y = -16$

$y = 4$

The solution is (5, 4).

Check:

$6(5) - 4(4) \stackrel{?}{=} 14$ $\quad$ $-3(5) + 4(4) \stackrel{?}{=} 1$

$14 = 14$ ✓ $\quad$ $1 = 1$ ✓

4. $7x - 2y = 5$

$7x - 3y = 4$

$y = 1$

$7x - 2(1) = 5$

$7x = 7$

$x = 1$

The solution is (1, 1).

Check:

$7(1) - 2(1) \stackrel{?}{=} 5$ $\quad$ $7(1) - 3(1) \stackrel{?}{=} 4$

$5 = 5$ ✓ $\quad$ $4 = 4$ ✓

5. $3x + 4y = -6$ $\quad$ $3x + 4y = -6$

$2y = 3x + 6$ $\quad$ $-3x + 2y = 6$

$6y = 0$

$y = 0$

$2(0) = 3x + 6$

$-6 = 3x$

$-2 = x$

The solution is $(-2, 0)$.

Check:

$3(-2) + 4(0) \stackrel{?}{=} -6$ $\quad$ $2(0) \stackrel{?}{=} 3(-2) + 6$

$-6 = -6$ ✓ $\quad$ $0 = 0$ ✓

6. $2x + 5y = 12$ $\quad$ $2x + 5y = 12$

$5y = 4x + 6$ $\quad$ $-4x + 5y = 6$

$6x = 6$

$x = 1$

$2(1) + 5y = 12$

$5y = 10$

$y = 2$

The solution is (1, 2).

Check:

$2(1) + 5(2) \stackrel{?}{=} 12$ $\quad$ $5(2) \stackrel{?}{=} 4(1) + 6$

$12 = 12$ ✓ $\quad$ $10 = 10$ ✓

7. Upstream: $\quad$ Downstream:

$d = rt$ $\quad$ $d = rt$

$10 = r(5)$ $\quad$ $10 = r(2)$

$2 = r$ $\quad$ $5 = r$

Let x = speed of kayak in still water

Let y = speed of current

$x - y = 2$ ← upstream

$x + y = 5$ ← downstream

$2x = 7$

$x = 3.5$

$3.5 + y = 5$

$y = 1.5$

The speed of the kayak in still water is 3.5 miles per hour, and the speed of the current is 1.5 miles per hour.

Exercises for the lesson "Solve Linear Systems by Adding or Subtracting"

Skill Practice

1. Answers will vary.

2. *Sample answer:* Subtract the equations to eliminate the x variable. Solve for y.

$2x - y = 2$

$2x + 3y = 22$

$-4y = -20$

$y = 5$

Substitute 5 for y in Equation 1. Solve for x.

$2x - 5 = 2$

$2x = 7$

$x = 3.5$

The solution is (3.5, 5).

3. $x + 2y = 13$

$-x + y = 5$

$3y = 18$

$y = 6$

$x + 2(6) = 13$

$x = 1$

The solution is (1, 6).

4. $9x + y = 2$

$-4x - y = -17$

$5x = -15$

$x = -3$

$9(-3) + y = 2$

$y = 29$

The solution is $(-3, 29)$.

5. $-3x - y = 8$

$7x + y = -12$

$4x = -4$

$x = -1$

$-3(-1) - y = 8$

$-y = 5$

$y = -5$

The solution is $(-1, -5)$.

6. $3x - y = 30$

$-3x + 7y = 6$

$6y = 36$

$y = 6$

$3x - 6 = 30$

$3x = 36$

$x = 12$

The solution is (12, 6).

7. $-9x + 4y = -17$
$9x - 6y = 3$
$-2y = -14$
$y = 7$
$9x - 6(7) = 3$
$9x = 45$
$x = 5$
The solution is (5, 7).

8. $-3x - 5y = -7$
$-4x + 5y = 14$
$-7x = 7$
$x = -1$
$-3(-1) - 5y = -7$
$-5y = -10$
$y = 2$
The solution is (−1, 2).

9. $x + y = 1$
$-2x + y = 4$
$3x = -3$
$x = -1$
$-1 + y = 1$
$y = 2$
The solution is (−1, 2).

10. $x - y = -4$
$x + 3y = 4$
$-4y = -8$
$y = 2$
$x - 2 = -4$
$x = -2$
The solution is (−2, 2).

11. $2x - y = 7$
$2x + 7y = 31$
$-8y = -24$
$y = 3$
$2x - 3 = 7$
$2x = 10$
$x = 5$
The solution is (5, 3).

12. $6x + y = -10$
$5x + y = -10$
$x = 0$
$6(0) + y = -10$
$y = -10$
The solution is (0, −10).

13. $5x + 6y = 50$
$-x + 6y = 26$
$6x = 24$
$x = 4$
$-4 + 6y = 26$
$6y = 30$
$y = 5$
The solution is (4, 5).

14. $4x - 9y = -21$
$4x + 3y = -9$
$-12y = -12$
$y = 1$
$4x - 9(1) = -21$
$4x = -12$
$x = -3$
The solution is (−3, 1).

15. C; (4, −2)
$4x + 9y = -2$
$11x + 9y = 26$
$-7x = -28$
$x = 4$
$4(4) + 9y = -2$
$9y = -18$
$y = -2$
The solution is (4, −2).

16. $2x - y = 32$
$y - 5x = 13$
$2x - y = 32$
$-5x + y = 13$
$-3x = 45$
$x = -15$
$2(-15) - y = 32$
$-30 - y = 32$
$-y = 62$
$y = -62$
The solution is (−15, −62).

17. $-8y + 6x = 36$
$6x - y = 15$
$-8y + 6x = 36$
$-y + 6x = 15$
$-7y = 21$
$y = -3$
$6x - (-3) = 15$
$6x = 12$
$x = 2$
The solution is (2, −3).

18. $2x - y = -11$
$y = -2x - 13$
$2x - y = -11$
$2x + y = -13$
$4x = -24$
$x = -6$
$y = -2(-6) - 13$
$y = -1$
The solution is (−6, −1).

19. $-x - y = 14$
$x = 5y - 38$
$-x - y = 14$
$x - 5y = -38$
$-6y = -24$
$y = 4$
$-x - 4 = 14$
$-x = 18$
$x = -18$
The solution is (−18, 4).

20. $11y - 3x = 18$
$-3x = -16y + 33$
$11y - 3x = 18$
$16y - 3x = 33$
$-5y = -15$
$y = 3$
$11(3) - 3x = 18$
$-3x = -15$
$x = 5$
The solution is (5, 3).

21. $-5x + y = -23$
$-y = 3x - 9$
$-5x + y = -23$
$-3x - y = -9$
$-8x = -32$
$x = 4$
$-5(4) + y = -23$
$y = -3$
The solution is (4, −3).

22. B; (3, 4)
$2x + y = 10$
$3y = 2x + 6$
$2x + y = 10$
$-2x + 3y = 6$
$4y = 16$
$y = 4$
$2x + 4 = 10$
$2x = 6$
$x = 3$

23. In this exercise the system must be solved by subtraction, not addition. $5x - (-x)$ is $6x$, not $4x$. $16 - 8$ is 8, not 24.
$5x - 7y = 16$
$-x - 7y = 8$
$6x = 8$
$x = 1\frac{1}{3}$

24. In rearranging the second equation, you must add $3x$ to both sides, making the second equation $3x + 5y = 60$.

$3x - 2y = -3$

$3x + 5y = 60$

$-7y = -63$

$y = 9$

25. $-x + \frac{1}{2}y = -19$

$x - y = 12$

$-\frac{1}{2}y = -7$

$y = 14$

$x - 14 = 12$

$x = 26$

The solution is (26, 14).

26. $\frac{1}{4}x - \frac{2}{3}y = 7$

$\frac{1}{2}x + \frac{2}{3}y = -4$

$\frac{3}{4}x = 3$

$x = 4$

$\frac{1}{4}(4) - \frac{2}{3}y = 7$

$-\frac{2}{3}y = 6$

$y = -9$

The solution is $(4, -9)$.

27. $8x - \frac{1}{2}y = -38$

$\frac{1}{4}x - \frac{1}{2}y = -7$

$\frac{31}{4}x = -31$

$x = -4$

$8(-4) - \frac{1}{2}y = -38$

$-\frac{1}{2}y = -6$

$y = 12$

The solution is $(-4, 12)$.

28. $5.2x + 3.5y = 54$

$-3.6x + 3.5y = 10$

$8.8x = 44$

$x = 5$

$5.2(5) + 3.5y = 54$

$26 + 3.5y = 54$

$3.5y = 28$

$y = 8$

The solution is (5, 8).

29. $1.3x - 3y = -17.6$

$-1.3x + 4.5y = 25.1$

$1.5y = 7.5$

$y = 5$

$1.3x - 3(5) = -17.6$

$1.3x - 15 = -17.6$

$1.3x = -2.6$

$x = -2$

The solution is $(-2, 5)$.

30. $-2.6x - 3.2y = 4.8$

$1.9x - 3.2y = -4.2$

$-4.5x = 9$

$x = -2$

$1.9(-2) - 3.2y = -4.2$

$-3.8 - 3.2y = -4.2$

$-3.2y = -0.4$

$y = 0.125$

The solution is $(-2, 0.125)$.

31. $\frac{4}{5}x + \frac{2}{5}y = 14$

$\frac{2}{5}y + \frac{1}{5}x = 11$

$\frac{4}{5}x + \frac{2}{5}y = 14$

$\frac{1}{5}x + \frac{2}{5}y = 11$

$\frac{3}{5}x = 3$

$x = 5$

$\frac{4}{5}(5) + \frac{2}{5}y = 14$

$\frac{2}{5}y = 10$

$y = 25$

The solution is (5, 25).

32. $2.7x + 1.5y = 36$

$3.5y = 2.7x - 6$

$2.7x + 1.5y = 36$

$-2.7x + 3.5y = -6$

$5y = 30$

$y = 6$

$2.7x + 1.5(6) = 36$

$2.7x = 27$

$x = 10$

The solution is (10, 6).

33. $4 - 4.8x = 1.7y$

$12.8 + 1.7y = -13.2x$

$-4.8x - 1.7y = -4$

$13.2x + 1.7y = -12.8$

$8.4x = -16.8$

$x = -2$

$4 - 4.8(-2) = 1.7y$

$13.6 = 1.7y$

$8 = y$

The solution is $(-2, 8)$.

34. (1, 2), (−4, 12)

$y = mx + b$

a. $2 = m(1) + b$

$12 = m(-4) + b$

b. $m + b = 2$

$-4m + b = 12$

$5m = -10$

$m = -2$

$-2 + b = 2$

$b = 4$

The slope is −2.

The y-intercept is 4.

c. $y = -2x + 4$

35. $2\ell + 2w = 14$

$2\ell = 4w - 1$

$2\ell + 2w = 14$

$2\ell - 4w = -1$

$6w = 15$

$w = 2.5$

$2\ell + 2(2.5) = 14$

$2\ell = 9$

$\ell = 4.5$

The length is 4.5 feet. The width is 2.5 feet.

36. Add equation 1 and equation 3.

$$\begin{aligned} x + 3y &= 8 \\ 5x - 3y &= -14 \\ \hline 6x &= -6 \\ x &= -1 \end{aligned}$$

Then solve for x to find $x = -1$. Substitute this value for x into equation 2 and solve for y.

$$\begin{aligned} x - 6y &= -19 \\ -1 - 6y &= -19 \\ -6y &= -18 \\ y &= 3 \end{aligned}$$

The solution to the system is $(-1, 3)$.

37.

$$\begin{aligned} ax + 2y &= 4 \\ ax - 3y &= -6 \\ \hline 5y &= 10 \\ y &= 2 \\ ax + 2(2) &= 4 \\ ax &= 0 \\ x &= 0 \end{aligned}$$

The solution is $(0, 2)$.

38. Rearrange Equation 2 so its terms are in the same order as Equation 1. Subtract the two equations.

$$\begin{aligned} x + 7y + 3z &= 29 \\ x - 2y + 3z &= -7 \end{aligned}$$

Solve for y. $9y = 36$

$$y = 4$$

Substitute 4 for y in equation 3. Solve for x.

$$\begin{aligned} 5(4) &= 10 - 2x \\ 10 &= -2x \\ -5 &= x \end{aligned}$$

Substitute these values for x and y into Equation 1 and solve for z.

$$\begin{aligned} -5 + 7(4) + 3z &= 29 \\ 23 + 3z &= 29 \\ 3z &= 6 \\ z &= 2 \end{aligned}$$

So, $x = -5$, $y = 4$, and $z = 2$.

Problem Solving

39. Let x = speed in still water.

Let y = speed of current.

$$\begin{aligned} x - y &= 4.3 \\ x + y &= 4.9 \\ \hline 2x &= 9.2 \\ x &= 4.6 \\ 4.6 + y &= 4.9 \\ y &= 0.3 \end{aligned}$$

The speed of the shell in still water is 4.6 meters per second. The speed of the current is 0.3 meters per second.

40.

$$\begin{aligned} x + 5y &= 22.45 \\ x + 7y &= 25.45 \\ \hline -2y &= -3 \\ y &= 1.5 \\ x + 5(1.5) &= 22.45 \\ x &= 14.95 \end{aligned}$$

The fee is \$14.95, and the cost per quart of oil is \$1.50.

41. Let x = cost of a monophonic ring tone.

Let y = cost of a polyphonic ring tone.

$$\begin{aligned} 3x + 2y &= 12.85 \\ x + 2y &= 8.95 \\ \hline 2x &= 3.90 \\ x &= 1.95 \\ 1.95 + 2y &= 8.95 \\ 2y &= 7 \\ y &= 3.5 \end{aligned}$$

A monophonic ring tone costs \$1.95, and a polyphonic ring tone costs \$3.50.

42. a. Let x = number of twigs.

Let y = number of flowers.

$$\begin{aligned} x + y &= 9 \\ x + 3y &= 15 \\ \hline -2y &= -6 \\ y &= 3 \\ x + 3 &= 9 \\ x &= 6 \end{aligned}$$

She used 6 twigs and 3 flowers.

b.

Number of flowers	Number of twigs	Total cost (\$)
0	9	9
1	8	11
2	7	13
3	**6**	**15**
4	5	17
5	4	19

43. a. $d = rt$

To Phoenix: $1800 = r(4.5)$

$400 = r$

The speed on the way to Phoenix is 400 miles per hour.

To Charlotte: $1800 = r(4)$

$450 = r$

The speed on the way back to Charlotte is 450 miles per hour.

b. Into the wind:

Speed of plane with no wind	$-$	Wind speed	$=$	Speed of plane

$s - w = 400$

Not into the wind:

Speed of plane with no wind	$+$	Wind speed	$=$	Speed of plane

$s + w = 450$

$s - w = 400$

$\underline{s + w = 450}$

$2s = 850$

$s = 425$

$425 + w = 450$

$w = 25$

The speed of the plane with no wind is 425 miles per hour. The wind speed is 25 miles per hour.

44. Let x = cost of cap-and-gown set.

Let y = cost of an extra tassel.

$215x + 72y = 3262$

$\underline{221x + 72y = 3346}$

$-6x = -84$

$x = 14$

$215(14) + 72y = 3262$

$72y = 252$

$y = 3.5$

A cap-and-gown set costs \$14, and an extra tassel costs \$3.50. So 218 cap-and-gown sets will cost:

$218(14) = \$3052$

And 56 extra tassels will cost:

$56(3.50) = \$196$

The total the third school will spend is

$\$3052 + \$196 = \$3248$.

45.

Ideal sleeve length	$+$	Allowable deviation	$=$	Maximum sleeve length

$x + y = 64.8$

Ideal sleeve length	$-$	Allowable deviation	$=$	Minimum sleeve length

$x - y = 62.2$

$x + y = 64.8$

$\underline{x - y = 62.2}$

$2x = 127$

$x = 63.5$

$63.5 + y = 64.8$

$y = 1.3$

The ideal sleeve length is 63.5 centimeters. The allowable deviation is 1.3 centimeters.

Lesson 6.4 Solve Linear Systems by Multiplying First

Guided Practice for the lesson "Solve Linear Systems by Multiplying First"

1. $6x - 2y = 1$

$-2x + 3y = -5$

$6x - 2y = 1$

$\times 3 \rightarrow \underline{-6x + 9y = -15}$

$7y = -14$

$y = -2$

$6x - 2(-2) = 1$

$6x = -3$

$x = -\frac{1}{2}$

The solution is $\left(-\frac{1}{2}, -2\right)$.

Check:

$6\left(-\frac{1}{2}\right) - 2(-2) \stackrel{?}{=} 1$

$1 = 1$ ✓

$-2\left(-\frac{1}{2}\right) + 3(-2) \stackrel{?}{=} -5$

$-5 = -5$ ✓

2. $2x + 5y = 3$

$3x + 10y = -3$

$\times 2 \rightarrow 4x + 10y = 6$

$\underline{3x + 10y = -3}$

$x = 9$

$2(9) + 5y = 3$

$5y = -15$

$y = -3$

The solution is $(9, -3)$.

Check:

$2(9) + 5(-3) \stackrel{?}{=} 3$

$3 = 3$ ✓

$3(9) + 10(-3) \stackrel{?}{=} -3$

$-3 = -3$ ✓

3. $3x - 7y = 5$

$9y = 5x + 5$

$3x - 7y = 5 \qquad \times 5 \rightarrow \quad 15x - 35y = 25$

$-5x + 9y = 5 \qquad \times 3 \rightarrow \underline{-15x + 27y = 15}$

$-8y = 40$

$y = -5$

$3x - 7(-5) = 5$

$3x = -30$

$x = -10$

The solution is $(-10, -5)$.

4. Let x = cost of soccer ball

Let y = cost of bag

$10x + 2y = 155 \qquad \times 3 \rightarrow 30x + 6y = 465$

$12x + 3y = 189 \qquad \times 2 \rightarrow \underline{24x + 6y = 378}$

$6x = 87$

$x = 14.5$

$10(14.5) + 2y = 155$

$2y = 10$

$y = 5$

A soccer ball costs \$14.50, and the cost of a soccer ball bag is \$5.

Exercises for the lesson "Solve Linear Systems by Multiplying First"

Skill Practice

1. The least common multiple of 12 and 18 is 36.

2. Multiply all of the terms in equation 1 by 3 so that one of the variables can be eliminated.

$2x - 3y = -4 \qquad \times 3 \rightarrow 6x - 9y = -12$

$7x + 9y = -5 \qquad \underline{7x + 9y = -5}$

$13x = -17$

Add the two equations together to get $13x = -17$. Solve for x. Substitute that x value into equation 1 and solve for y.

3. $x + y = 2 \qquad \times 2 \rightarrow 2x + 2y = 4$

$2x + 7y = 9 \qquad \underline{2x + 7y = 9}$

$-5y = -5$

$y = 1$

$x + 1 = 2$

$x = 1$

The solution is $(1, 1)$.

4. $3x - 2y = 3 \qquad 3x - 2y = 3$

$-x + y = 1 \qquad \times 2 \rightarrow \underline{-2x + 2y = 2}$

$x = 5$

$3(5) - 2y = 3$

$-2y = -12$

$y = 6$

The solution is $(5, 6)$.

5. $4x + 3y = 8 \qquad 4x + 3y = 8$

$x - 2y = 13 \qquad \times 4 \rightarrow \underline{4x - 8y = 52}$

$11y = -44$

$y = -4$

$x - 2(-4) = 13$

$x = 5$

The solution is $(5, -4)$.

6. $10x - 9y = 46 \qquad 10x - 9y = 46$

$-2x + 3y = 10 \qquad \times 3 \rightarrow \underline{-6x + 9y = 30}$

$4x = 76$

$x = 19$

$-2(19) + 3y = 10$

$3y = 48$

$y = 16$

The solution is $(19, 16)$.

7. $8x - 5y = 11 \qquad 8x - 5y = 11$

$4x - 3y = 5 \qquad \times 2 \rightarrow \underline{8x - 6y = 10}$

$y = 1$

$4x - 3(1) = 5$

$4x = 8$

$x = 2$

The solution is $(2, 1)$.

8. $11x - 20y = 28 \qquad 11x - 20y = 28$

$3x + 4y = 36 \qquad \times 5 \rightarrow \underline{15x + 20y = 180}$

$26x = 208$

$x = 8$

$3(8) + 4y = 36$

$4y = 12$

$y = 3$

The solution is $(8, 3)$.

9. $4x - 3y = 8 \qquad \times 2 \rightarrow \quad 8x - 6y = 16$

$5x - 2y = -11 \qquad \times 3 \rightarrow \quad \underline{15x - 6y = -33}$

$-7x = 49$

$x = -7$

$4(-7) - 3y = 8$

$-3y = 36$

$y = -12$

The solution is $(-7, -12)$.

10. $-2x - 5y = 9 \qquad \times 3 \rightarrow \quad -6x - 15y = 27$

$3x + 11y = 4 \qquad \times 2 \rightarrow \quad \underline{6x + 22y = 8}$

$7y = 35$

$y = 5$

$-2x - 5(5) = 9$

$-2x = 34$

$x = -17$

The solution is $(-17, 5)$.

11. $7x - 6y = -1 \quad \times 2 \rightarrow \quad 14x - 12y = -2$

$5x - 4y = 1 \quad \times 3 \rightarrow \quad 15x - 12y = 3$

$-x = -5$

$x = 5$

$7(5) - 6y = -1$

$-6y = -36$

$y = 6$

The solution is (5, 6).

12. $7x + 3y = -12 \quad \times 5 \rightarrow \quad 35x + 15y = -60$

$2x + 5y = 38 \quad \times 3 \rightarrow \quad 6x + 15y = 114$

$29x = -174$

$x = -6$

$7(-6) + 3y = -12$

$3y = 30$

$y = 10$

The solution is $(-6, 10)$.

13. $9x - 8y = 4 \quad \times 2 \rightarrow \quad 18x - 16y = 8$

$2x - 3y = -4 \quad \times 9 \rightarrow \quad 18x - 27y = -36$

$11y = 44$

$y = 4$

$9x - 8(4) = 4$

$9x = 36$

$x = 4$

The solution is (4, 4).

14. $12x - 7y = -2 \quad \times 2 \rightarrow \quad 24x - 14y = -4$

$-8x + 11y = 14 \quad \times 3 \rightarrow \quad -24x + 33y = 42$

$19y = 38$

$y = 2$

$12x - 7(2) = -2$

$12x = 12$

$x = 1$

The solution is (1, 2).

15. $9x + 2y = 39 \quad \times 2 \rightarrow \quad 18x + 4y = 78$

$6x + 13y = -9 \quad \times 3 \rightarrow \quad 18x + 39y = -27$

$-35y = 105$

$y = -3$

$6x + 13(-3) = -9$

$6x = 30$

$x = 5$

The solution is $(5, -3)$.

16. $-7x + 10y = 11 \quad \times 3 \rightarrow \quad -21x + 30y = 33$

$-8x + 15y = 34 \quad \times 2 \rightarrow \quad -16x + 30y = 68$

$-5x = -35$

$x = 7$

$-7(7) + 10y = 11$

$10y = 60$

$y = 6$

The solution is (7, 6).

17. $-14x + 15y = 15 \quad \times 3 \rightarrow \quad -42x + 45y = 45$

$21x - 20y = -10 \quad \times 2 \rightarrow \quad 42x - 40y = -20$

$5y = 25$

$y = 5$

$-14x + 15(5) = 15$

$-14x = -60$

$x = 4.29$

The solution is (4.29, 5).

18. D; $(2, -3)$

$15x + 8y = 6 \quad \times 3 \rightarrow \quad 45x + 24y = 18$

$25x + 12y = 14 \quad \times 2 \rightarrow \quad 50x + 24y = 28$

$-5x = -10$

$x = 2$

$15(2) + 8y = 6$

$8y = -24$

$y = -3$

The solution is $(2, -3)$.

19. In order to eliminate one variable, these equations must be subtracted, not added. $4x - 5x = -x$, and $-18 - (-9) = -9$, so,

$-x = -9$

$x = 9.$

20. The error is that the constants on the right sides of both equations must also be multiplied by the least common multiple.

$27x + 24y = 33$

$28x + 24y = 36$

$-x = -3$

$x = 3$

21. $3x + 2y = 4$

$2y = 8 - 5x$

$3x + 2y = 4$

$5x + 2y = 8$

$-2x = -4$

$x = 2$

$3(2) + 2y = 4$

$2y = -2$

$y = -1$

The solution is $(2, -1)$.

22. $4x - 5y = 18$

$3x = y + 11$

$3x - 11 = y$

$4x - 5(3x - 11) = 18$

$4x - 15x + 55 = 18$

$-11x = -37$

$x = 3.36$

$3(3.36) - 11 = y$

$-0.92 = y$

The solution is $(3.36, -0.92)$.

23. $8x - 9y = -15$

$-4x = 19 + y$

$-4x - 19 = y$

$8x - 9(-4x - 19) = -15$

$8x + 36x + 171 = -15$

$44x = -186$

$x = -4.23$

$-4(-4.23) - 19 = y$

$-2.08 = y$

The solution is $(-4.23, -2.08)$.

24. $0.3x + 0.1y = -0.1$

$-x + y = 3$

$y = x + 3$

$0.3x + 0.1(x + 3) = -0.1$

$0.3x + 0.1x + 0.3 = -0.1$

$0.4x = -0.4$

$x = -1$

$y = -1 + 3$

$y = 2$

The solution is $(-1, 2)$.

25. $4.4x - 3.6y = 7.6$

$x - y = 1$

$x = y + 1$

$4.4(y + 1) - 3.6y = 7.6$

$4.4y + 4.4 - 3.6y = 7.6$

$0.8y = 3.2$

$y = 4$

$x = 4 + 1 = 5$

The solution is $(5, 4)$.

26. $3x - 2y = -20$

$x + 1.2y = 6.4$

$x = 6.4 - 1.2y$

$3(6.4 - 1.2y) - 2y = -20$

$19.2 - 3.6y - 2y = -20$

$-5.6y = -39.2$

$y = 7$

$x = 6.4 - 1.2(7)$

$x = -2$

The solution is $(-2, 7)$.

27. $0.2x - 1.5y = -1$

$x - 4.5y = 1$

$x = 4.5y + 1$

$0.2(4.5y + 1) - 1.5y = -1$

$0.9y + 0.2 - 1.5y = -1$

$-0.6y = -1.2$

$y = 2$

$x = 4.5(2) + 1$

$x = 10$

The solution is $(10, 2)$.

28. $1.5x - 3.5y = -5 \quad \times 4 \rightarrow \quad 6x - 14y = -20$

$-1.2x + 2.5y = 1 \quad \times 5 \rightarrow \quad -6x + 12.5y = 5$

$-1.5y = -15$

$y = 10$

$1.5x - 3.5(10) = -5$

$1.5x = 30$

$x = 20$

The solution is $(20, 10)$.

29. $4.9x + 2.4y = 7.4 \qquad 4.9x + 2.4y = 7.4$

$0.7x + 3.6y = -2.2 \quad \times 7 \rightarrow \quad 4.9x + 25.2y = -15.4$

$-22.8y = 22.8$

$y = -1$

$4.9x + 2.4(-1) = 7.4$

$4.9x = 9.8$

$x = 2$

The solution is $(2, -1)$.

30. $x + y = 0 \qquad x + y = 0$

$\frac{1}{2}x - \frac{1}{2}y = 2 \quad \times 2 \rightarrow \quad x - y = 4$

$2x = 4$

$x = 2$

$2 + y = 0$

$y = -2$

The solution is $(2, -2)$.

31. $3x + y = \frac{1}{3}$ $\times 3 \rightarrow$ $9x + 3y = 1$

$2x - 3y = \frac{8}{3}$ $2x - 3y = \frac{8}{3}$

$11x = \frac{11}{3}$

$x = \frac{1}{3}$

$3\left(\frac{1}{3}\right) + y = \frac{1}{3}$

$y = -\frac{2}{3}$

The solution is $\left(\frac{1}{3}, -\frac{2}{3}\right)$.

32. $\frac{3}{5}x - \frac{3}{4}y = -3$ $\times 2 \rightarrow$ $\frac{6}{5}x - \frac{3}{2}y = -6$

$\frac{2}{5}x + \frac{1}{3}y = 8$ $\times 3 \rightarrow$ $\frac{6}{5}x + y = 24$

$-\frac{5}{2}y = -30$

$y = 12$

$\frac{3}{5}x - \frac{3}{4}(12) = -3$

$\frac{3}{5}x = 6$

$x = 10$

The solution is (10, 12).

33. a. $2\ell + 2w = 18$ $\times 2 \rightarrow$ $4\ell + 4w = 36$

$6\ell + 4w = 46$ $6\ell + 4w = 46$

$-2\ell = -10$

$\ell = 5$

$2(5) + 2w = 18$

$2w = 8$

$w = 4$

The original length is 5 inches, and the original width is 4 inches.

b. $3(5) = 15$

$2(4) = 8$

The new length is 15 inches, and the new width is 8 inches.

34. $ax + 3y = 2$

$4x + 5y = 6$

If $a = 1$ you can solve the system by substitution. You can solve the first equation for x and substitute $2 - 3y$ for x in the second equation. If $a = 0$ you can solve the system by solving the first equation for y and substitute that value into the second equation. If $a = 4$ or $a = -4$, you can solve the system by addition or subtraction since the second equation contains the term $4x$. That term could be eliminated without use of multiplication.

35. $ax - by = 4$ $x = 4, y = 2$

$bx - ay = 10$

$4a - 2b = 4$

$4b - 2a = 10$

$4a - 2b = 4$ $\times 2 \rightarrow$ $8a - 4b = 8$

$-2a + 4b = 10$ $-2a + 4b = 10$

$6a = 18$

$a = 3$

$4(3) - 2b = 4$

$-2b = -8$

$b = 4$

So, $a = 3$, $b = 4$.

36. $ax - by = 4$ $x = 2$

$bx - ay = 10$ $y = 1$

$2a - b = 4$

$2b - a = 10$

$2a - b = 4$ $\times 2 \rightarrow$ $4a - 2b = 8$

$-a + 2b = 10$ $-a + 2b = 10$

$3a = 18$

$a = 6$

$2(6) - b = 4$

$-b = -8$

$b = 8$

So, $a = 6$, $b = 8$.

Problem Solving

37. Let x = number of hardcovers.

Let y = number of paperbacks.

$x + y = 8$ $\times 2 \rightarrow$ $2x + 2y = 16$

$4x + 2y = 26$ $4x + 2y = 26$

$-2x = -10$

$x = 5$

$5 + y = 8$

$y = 3$

She purchased 5 hardcover books.

38. Let x = cost for 1 song.

Let y = cost for 1 album.

$5x + y = 14.94$ $\times 2 \rightarrow$ $10x + 2y = 29.88$

$3x + 2y = 22.95$ $3x + 2y = 22.95$

$7x = 6.93$

$x = 0.99$

$5(0.99) + y = 14.94$

$y = 9.99$

The website charges \$.99 to download a song and \$9.99 to download an album.

39. Let x = number of pies.

Let y = batches of applesauce.

$5x + 4y = 169$ $5x + 4y = 169$

$3x + 2y = 95$ $\times 2 \rightarrow$ $6x + 4y = 190$

$-x = -21$

$x = 21$

$3(21) + 2y = 95$

$2y = 32$

$y = 16$

You can make 21 pies and 16 batches of applesauce.

40. a. Let x = student tickets sold.

Let y = adult tickets sold.

$x + y = 729 \quad \times 3 \rightarrow \quad 3x + 3y = 2187$

$3x + 5y = 2995 \quad \underline{3x + 5y = 2995}$

$-2y = -808$

$y = 404$

$x + 404 = 729$

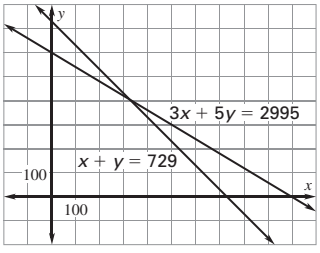
$x = 325$

There were 325 tickets sold to students and 404 tickets sold to adults.

b. $y = -x + 729$

$5y = -3x + 2995$

$y = -\frac{3}{5}x + 599$



The solution in part (a) is reasonable.

41. Let x = cost of small dish.

Let y = cost of large dish.

$3x + 5y = 28.20 \quad \times 4 \rightarrow \quad 12x + 20y = 112.8$

$4x + 3y = 23.30 \quad \times 3 \rightarrow \quad \underline{12x + 9y = 69.9}$

$11y = 42.9$

$y = 3.9$

$3x + 5(3.9) = 28.20$

$3x = 8.7$

$x = 2.9$

The cost of a small dish is \$2.90 and the cost of a large dish is \$3.90. The cost of 3 small dishes can be found by:

$3(2.90) = \$8.70$

The cost of 2 large dishes can be found by:

$2(3.90) = \$7.80$

The total cost is,

$\$8.70 + \$7.80 = \$16.50.$

42. Answers will vary.

43. Let x = amount invested in stocks.

Let y = amount invested in bonds.

$0.06x + 0.08y = 144 \qquad 0.06x + 0.08y = 144$

$x + y = 2000 \quad \times 0.06 \rightarrow \quad \underline{0.06x + 0.06y = 120}$

$0.02y = 24$

$y = 1200$

$x + 1200 = 2000$

$x = 800$

He invested \$800 in stocks and \$1200 in bonds.

44. Let t = time to reach destination.

$d = rt, t = \frac{d}{r}$

t going + t returning = t total

$\frac{45}{r} + \frac{45}{\frac{3}{4}r} = \frac{7}{4}$

$\frac{45}{r} + \frac{60}{r} = \frac{7}{4}$

$\frac{105}{r} = \frac{7}{4}$

$420 = 7r$

$60 = r$

$\frac{3}{4}(60) = 45$

The average speed going is 60 miles per hour. The average speed returning is 45 miles per hour.

Quiz for the lessons "Solve Linear Systems by Adding or Subtracting" and "Solve Linear Systems by Multiplying First"

1. $x + y = 4$

$\underline{-3x + y = -8}$

$4x = 12$

$x = 3$

$3 + y = 4$

$y = 1$

The solution is (3, 1).

2. $2x - y = 2$

$\underline{6x - y = -2}$

$-4x = 4$

$x = -1$

$2(-1) - y = 2$

$-y = 4$

$y = -4$

The solution is $(-1, -4)$.

3. $x + y = 5$

$\underline{-x + y = -3}$

$2y = 2$

$y = 1$

$x + 1 = 5$

$x = 4$

The solution is (4, 1).

4. $x + 3y = -10$

$\underline{-x + 5y = -30}$

$8y = -40$

$y = -5$

$x + 3(-5) = -10$

$x = 5$

The solution is $(5, -5)$.

5. $x + 3y = 10$ $\quad x + 3y = 10$

$3x - y = 13$ $\quad \times 3 \rightarrow 9x - 3y = 39$

$10x = 49$

$x = 4.9$

$4.9 + 3y = 10$

$y = 1.7$

The solution is (4.9, 1.7).

6. $x + 7y = 10$ $\quad x + 7(3.6) = 10$

$x + 2y = -8$ $\quad x = -15.2$

$5y = 18$

$y = 3.6$

The solution is (−15.2, 3.6).

7. $4x - y = -2$ $\quad \times 2 \rightarrow 8x - 2y = -4$

$3x + 2y = 7$ $\quad 3x + 2y = 7$

$11x = 3$

$x = 0.27$

$4(0.27) - y = -2$

$y = 3.08$

The solution is (0.27, 3.08).

8. $x + 3y = 1$ $\quad \times 2 \rightarrow 2x + 6y = 2$

$5x + 6y = 14$ $\quad 5x + 6y = 14$

$-3x = -12$

$x = 4$

$4 + 3y = 1$

$3y = -3$

$y = -1$

The solution is (4, −1).

9. $3x + y = 21$

$x + y = 1$

$2x = 20$

$x = 10$

$10 + y = 1$

$y = -9$

The solution is (10, −9).

10. $2x - 3y = -5$ $\quad \times 2 \rightarrow 4x - 6y = -10$

$5x + 2y = 16$ $\quad \times 3 \rightarrow 15x + 6y = 48$

$19x = 38$

$x = 2$

$2(2) - 3y = -5$

$-3y = -9$

$y = 3$

The solution is (2, 3).

11. $7x + 2y = 13$ $\quad \times 3 \rightarrow 21x + 6y = 39$

$4x + 3y = 13$ $\quad \times 2 \rightarrow 8x + 6y = 26$

$13x = 13$

$x = 1$

$7(1) + 2y = 13$

$2y = 6$

$y = 3$

The solution is (1, 3).

12. $\frac{1}{3}x + 5y = -3$ $\quad \times 3 \rightarrow x + 15y = -9$

$-\frac{2}{3}x + 6y = -10$ $\quad \times \frac{3}{2} \rightarrow -x + 9y = -15$

$24y = -24$

$y = -1$

$\frac{1}{3}x + 5(-1) = -3$

$\frac{1}{3}x = 2$

$x = 6$

The solution is (6, −1).

Graphing Calculator Activity for the lesson "Solve Linear Systems by Multiplying First"

Practice 1

1. $-x + y = 9$ Equation 1

$x + y = 1$ Equation 2

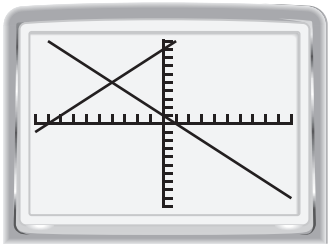
Step 1

Solve both equations for y.

$-x + y = 9$ $\quad x + y = 1$

$y = x + 9$ $\quad y = -x + 1$

Graph the two equations using a graphing calculator. Notice that the point of intersection of the graphs is the solution of the system.



The solution is (−4, 5).

Step 2

Add the two equations as you would if you were solving the system algebraically. Graph the resulting equation.

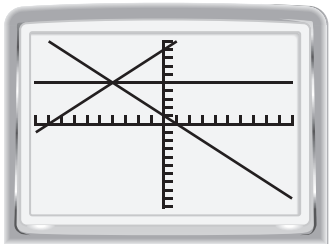
$-x + y = 9$ Equation 1

$x + y = 1$ Equation 2

$2y = 10$ Add.

$y = 5$ Solve for y.

Now graph the equation $y = 5$ on the same calculator screen with the two original equations.



Step 3

All three equations intersect at $(-4, 5)$.

So, $(-4, 5)$ is the solution of the system.

2. $6x - 7y = 4$ Equation 1

$x + 7y = 17$ Equation 2

Step 1

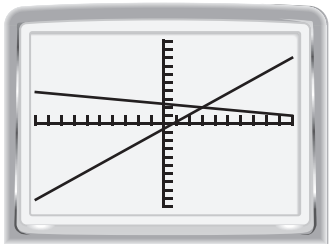
Solve both equations for y.

$6x - 7y = 4$ $\qquad$ $x + 7y = 17$

$-7y = -6x + 4$ $\qquad$ $7y = -x + 17$

$y = \frac{6x - 4}{7}$ $\qquad$ $y = \frac{-x + 17}{7}$

Graph the two equations using a graphing calculator. Notice that the point of intersection of the graphs is the solution of the system.



The solution is (3, 2).

Step 2

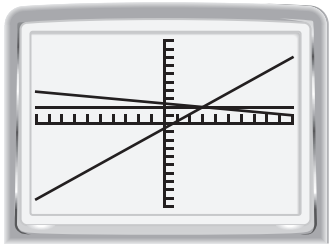
Add the two equations as you would if you were solving the system algebraically. Graph the resulting equation.

$6x - 7y = 4$	Equation 1
$x + 7y = 17$	Equation 2
$7x = 21$	Add.
$x = 3$	Solve for x.

Next solve for y when $x = 3$ in one of the equations.

$x + 7y = 17$	Equation 2
$3 + 7y = 17$	Substitute 3 for x.
$7y = 14$	Subtract 3 from each side.
$y = 2$	Divide each side by 7.

Now graph the equation $y = 2$ on the same calculator screen with the two original equations.



Step 3

All three equations intersect at (3, 2).

So, (3, 2) is the solution of the system.

3. $2x - 3y = 4$ Equation 1

$8x + 3y = 1$ Equation 2

Step 1

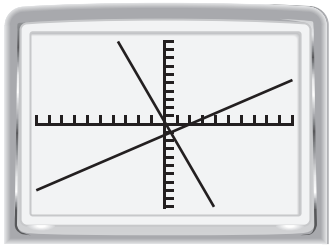
Solve both equations for y.

$2x - 3y = 4$ $\qquad$ $8x + 3y = 1$

$-3y = -2x + 4$ $\qquad$ $3y = -8x + 1$

$y = \frac{2}{3}x - \frac{4}{3}$ $\qquad$ $y = \frac{-8x + 1}{3}$

Graph the two equations using a graphing calculator. Notice that the point of intersection of the graphs is the solution of the system.



The solution is $\left(\frac{1}{2}, -1\right)$.

Step 2

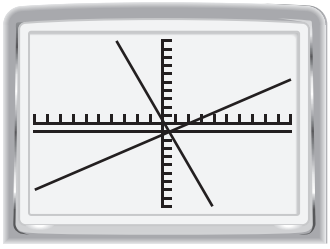
Add the two equations as you would if you were solving the system algebraically. Graph the resulting equation.

$2x - 3y = 4$	Equation 1
$8x + 3y = 1$	Equation 2
$10x = 5$	Add.
$x = \frac{1}{2}$	Solve for x.

Next solve for y when $x = \frac{1}{2}$ in one of the equations.

$2x - 3y = 4$	Equation 1
$2\left(\frac{1}{2}\right) - 3y = 4$	Substitute $\frac{1}{2}$ for x.
$1 - 3y = 4$	Simplify.
$-3y = 3$	Subtract 1 from each side.
$y = -1$	Divide each side by -3.

Now graph the equation $y = -1$ on the same calculator screen with the two original equations.



Step 3

All three equations intersect at $\left(\frac{1}{2}, -1\right)$.

So, $\left(\frac{1}{2}, -1\right)$ is the solution of the system.

Practice 2

4. $x - y = -5$ Equation 1

$4x + 3y = 1$ Equation 2

Step 1

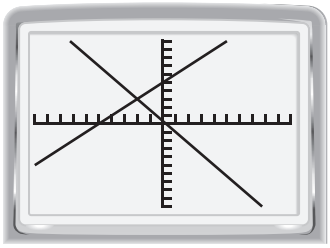
Solve each equation for y.

$x - y = -5$ $\qquad$ $4x + 3y = 1$

$-y = -x - 5$ $\qquad$ $3y = -4x + 1$

$y = x + 5$ $\qquad$ $y = \frac{-4x + 1}{3}$

Graph the two equations. The point of intersection of the graphs is the solution of the system.



The graphs intersect at $(-2, 3)$.

Step 2

Multiply Equation 1 by -4 so that you can eliminate the variable x by adding.

$x - y = -5 \quad \times -4 \quad -4x + 4y = 20$

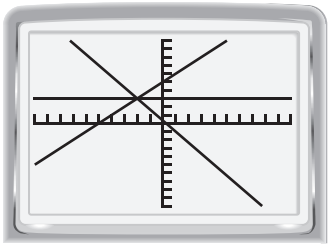
$4x + 3y = 1 \quad \rightarrow \quad 4x + 3y = 1$

$7y = 21$

$y = 3$

Step 3

Graph the equations $-4x + 4y = 20$, $4x + 3y = 1$, and $y = 3$ on the same graphing calculator screen with the original equations.



Step 4

All of the equations intersect at $(-2, 3)$.

So, $(-2, 3)$ is the solution of the system.

5. $2x - 5y = 3$ Equation 1

$-x + 2y = -2$ Equation 2

Step 1

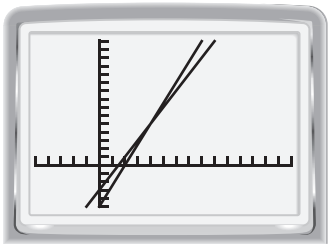
Solve each equation for y.

$2x - 5y = 3$ $\qquad$ $-x + 2y = -2$

$-5y = -2x + 3$ $\qquad$ $2y = x - 2$

$y = \frac{2x - 3}{5}$ $\qquad$ $y = \frac{1}{2}x - 1$

Graph the two equations. The point of intersection of the graphs is the solution of the system.



The graphs intersect at (4, 1).

Step 2

Multiply Equation 2 by 2 so that you can eliminate the variable x by adding.

$2x - 5y = 3 \quad \rightarrow \quad 2x - 5y = 3$

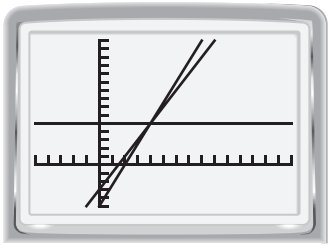
$-x + 2y = -2 \quad \times 2 \quad \underline{-2x + 4y = -4}$

$-y = -1$

$y = 1$

Step 3

Graph the equations $2x - 5y = 3$, $-2x + 4y = -4$, and $y = 1$ on the same graphing calculator screen with the original equations.



Step 4

All of the equations intersect at (4, 1).

So, (4, 1) is the solution of the system.

6. $3x + 5y = 3$ Equation 1

$x - y = 9$ Equation 2

Step 1

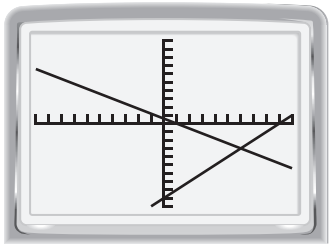
Solve each equation for y.

$3x + 5y = 3$ $\qquad$ $x - y = 9$

$5y = -3x + 3$ $\qquad$ $-y = -x + 9$

$y = \frac{-3x + 3}{5}$ $\qquad$ $y = x - 9$

Graph the two equations. The point of intersection of the graphs is the solution of the system.



The graphs intersect at $(6, -3)$.

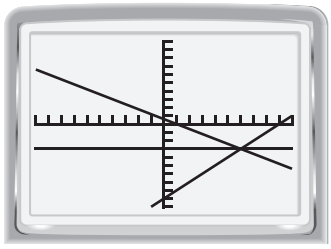
Step 2

Multiply Equation 2 by -3 so that you can eliminate the variable x by adding.

$$\begin{array}{lcr} 3x + 5y = 3 & \rightarrow & 3x + 5y = 3 \\ x - y = 9 & \times -3 & \underline{-3x + 3y = -27} \\ & & 8y = -24 \\ & & y = -3 \end{array}$$

Step 3

Graph the equations $3x + 5y = 3$, $-3x + 3y = -27$, and $y = -3$ on the same graphing calculator screen with the original equations.



Step 4

All of the equations intersect at $(6, -3)$.

So, $(6, -3)$ is the solution of the system.

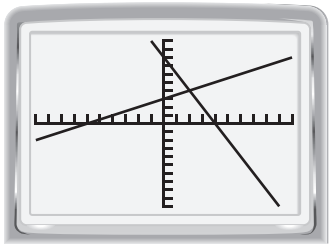
7. $x - 2y = -6$ Equation 1

$2x + y = 8$ Equation 2

Solve each equation for y.

$$x - 2y = -6 \qquad 2x + y = 8$$
$$-2y = -x - 6 \qquad y = -2x + 8$$
$$y = \frac{1}{2}x + 3$$

Graph the two equations. The point of intersection of the graphs is the solution of the system.



The solution using the graphing calculator is $(2, 4)$.

Multiply Equation 1 by -2 so that you can eliminate the variable x by adding.

$$\begin{array}{lcr} x - 2y = -6 & \times -2 & -2x + 4y = 12 \\ 2x + y = 8 & \rightarrow & \underline{2x + y = \ \ 8} \\ & & 5y = 20 \\ & & y = 4 \end{array}$$

Multiply Equation 2 by 2 so that you can eliminate the variable y by adding.

$$\begin{array}{lcr} x - 2y = -6 & \rightarrow & x - 2y = -6 \\ 2x + y = 8 & \times 2 & \underline{4x + 2y = 16} \\ & & 5x = 10 \\ & & x = 2 \end{array}$$

The solution $(2, 4)$ found using the graphing calculator has coordinates that are given by the equations resulting from using linear combinations to eliminate one of the variables from the original system.

Draw Conclusions

8. To check the solution of the system, you can add the equations and graph the sum. If the three lines intersect in one point, the solution is correct.

9. a. You get a false equation such as $5 = 3$.

b. The lines are parallel.

c. When you add the equations, you don't get an equation that can be graphed, so the method does not work.

Mixed Review of Problem Solving for the lessons "Solve Linear Systems by Graphing", "Solve Linear Systems by Substitution", "Solve Linear Systems by Adding or Subtracting", and "Solve Linear Systems by Multiplying First"

1. Let x = speed in still air.

Let y = speed in wind.

a. $\frac{15}{0.25} = 60, \frac{15}{0.2} = 75$

The average speed for the first flight is 60 km/h, and 75 km/h for the return.

b. $x - y = 60$

$x + y = 75$

c. $2x = 135$

$x = 67.5$

$67.5 + y = 75$

$y = 7.5$

The helicopter's average speed in still air is 67.5 kilometers per hour. The speed of the wind is 7.5 kilometers per hour.

2. Let x = cost of a pound of potato salad.

Let y = cost of a pound of coleslaw.

$1.8x + 1.4y = 9.70$

$x + 1.2y = 6.55$

$x = 6.55 - 1.2y$

$1.8(6.55 - 1.2y) + 1.4y = 9.70$

$11.79 - 2.16y + 1.4y = 9.70$

$-0.76y = -2.09$

$y = 2.75$

$x = 6.55 - 1.2(2.75)$

$x = 3.25$

$2(3.25) = 6.50$

$2(2.75) = 5.50$

Two pounds of potato salad cost \$6.50, and two pounds of coleslaw cost \$5.50, a total of \$12.

3. The point represented by the intersection of the graphs shows that the customers will have paid the same amount after 4 months.

4. Answers will vary.

5. $d = rt + h$

$d_1 = 7200(0.5) + 1705 = 5305$

$d_2 = 4000(0.5) + 3940 = 5940$

The balloon at Kirby Park will ascend to a height of 5305 feet. The balloon at Newman Park will ascend to a height of 5940 feet with regards to sea level. After about 42 minutes, the two balloons will be at the same distance from sea level.

6. a. $x + y = 500$

$0.1x + 0.3y = 0.2(500)$

b. $x = 500 - y$

$0.1(500 - y) + 0.3y = 100$

$50 - 0.1y + 0.3y = 100$

$0.2y = 50$

$y = 250$

$x + 250 = 500$

$x = 250$

To make the 20% acid and 80% water mix, 250 milliliters of the 10% acid and 90% water mix is combined with 250 milliliters of the 30% acid and 70% water mix.

c. $x + y = 500$

$0.1x + 0.3y = 0.15(500)$

$0.1(500 - y) + 0.3y = 75$

$50 - 0.1y + 0.3y = 75$

$0.2y = 25$

$y = 125$

$x + 125 = 500$

$x = 375$

The chemist does need more of the 10% acid and 90% water mix because this new mix has less acid than the first mix created, so more of the less acidic solution must be added.

Lesson 6.5 Solve Special Types of Linear Systems

Guided Practice for the lesson "Solve Special Types of Linear Systems"

1. $5x + 3y = 6$

$-5x - 3y = 3$

$0 = 9$

This is a false statement. Since the variables are eliminated and you are left with a false statement, regardless of the values of x and y. This tells you that the system has no solution.

2. $y = 2x - 4$

$-6x + 3y = -12$

$-6x + 3(2x - 4) = -12$

$-6x + 6x - 12 = -12$

$-12 = -12$

The variables are eliminated and you are left with a statement that is true regardless of the values of x and y. This tells you that the system has infinitely many solutions.

3. $x - 3y = -15$ $\qquad$ $2x - 3y = -18$

$-3y = -x - 15$ $\qquad$ $-3y = -2x - 18$

$y = \frac{1}{3}x + 5$ $\qquad$ $y = \frac{2}{3}x + 6$

The system has one solution.

4. Let $x =$ cost of regular print.

$45x + 30(x + 3) = 465$

$45x + 30x + 90 = 465$

$75x = 375$

$x = 5$

$5 + 3 = 8$

A glossy print costs \$8.00.

Exercises for the lesson "Solve Special Types of Linear Systems"

Skill Practice

1. A linear system with no solution is called an *inconsistent* system.

2. A linear system with infinitely many solutions is called a *consistent dependent* system.

3. The graph of a linear system with no solution is two parallel lines.

4. The graph of a linear system with infinitely many solutions is two lines that coincide producing a graph that appears to be a single line.

5. $x - 3y = -9$ $\qquad$ $x - y = -1$

$-3y = -x - 9$ $\qquad$ $-y = -x - 1$

$y = \frac{1}{3}x + 3$ $\qquad$ $y = x + 1$

Matches graph *B*. System has one solution.

6. $x - y = -4$ $\qquad -3x + 3y = 2$

$-y = -x - 4$ $\qquad 3y = 3x + 2$

$y = x + 4$ $\qquad y = x + \frac{2}{3}$

Matches graph *C*. System has no solution.

7. $x + 3y = -1$ $\qquad -2x - 6y = 2$

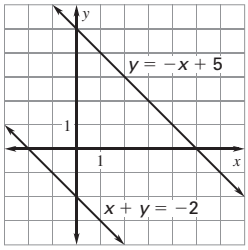
$3y = -x - 1$ $\qquad -6y = 2x + 2$

$y = -\frac{1}{3}x - \frac{1}{3}$ $\qquad y = -\frac{1}{3}x - \frac{1}{3}$

Matches graph *A*. System has infinitely many solutions.

8. $x + y = -2$ $\qquad y = -x + 5$

$y = -x - 2$

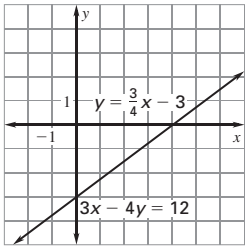


The system has no solution.

9. $3x - 4y = 12$ $\qquad y = \frac{3}{4}x - 3$

$-4y = -3x + 12$

$y = \frac{3}{4}x - 3$

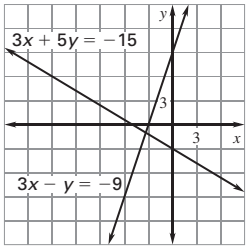


The system has infinitely many solutions.

10. $3x - y = -9$ $\qquad 3x + 5y = -15$

$-y = -3x - 9$ $\qquad 5y = -3x - 15$

$y = 3x + 9$ $\qquad y = -\frac{3}{5}x - 3$

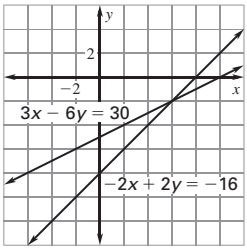


The system has one solution.

11. $-2x + 2y = -16$ $\qquad 3x - 6y = 30$

$2y = 2x - 16$ $\qquad -6y = -3x + 30$

$y = x - 8$ $\qquad y = \frac{1}{2}x - 5$

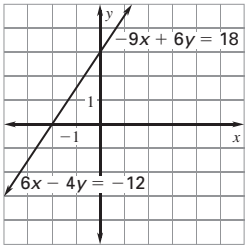


The system has one solution.

12. $-9x + 6y = 18$ $\qquad 6x - 4y = -12$

$6y = 9x + 18$ $\qquad -4y = -6x - 12$

$y = \frac{3}{2}x + 3$ $\qquad y = \frac{3}{2}x + 3$

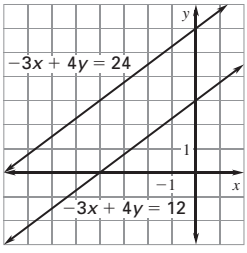


The system has infinitely many solutions.

13. $-3x + 4y = 12$ $\qquad -3x + 4y = 24$

$4y = 3x + 12$ $\qquad 4y = 3x + 24$

$y = \frac{3}{4}x + 3$ $\qquad y = \frac{3}{4}x + 6$



The system has no solution

14. The error is that, though the lines do not intersect in the graph shown, they are not parallel, so they will intersect at some point. Therefore the system has one solution.

15. $2x + 5y = 14$ $\quad \times 3 \rightarrow \quad 6x + 15y = 42$

$6x + 7y = 10$ $\qquad\qquad\quad 6x + 7y = 10$

$8y = 32$

$y = 4$

$2x + 5(4) = 14$

$2x = -6$

$x = -3$

The solution is $(-3, 4)$.

16. $-16x + 2y = -2$

$y = 8x - 1$

$-16x + 2(8x - 1) = -2$

$-16x + 16x - 2 = -2$

$-2 = -2$

The system has infinitely many solutions.

17. $3x - 2y = -5 \quad \times 4 \rightarrow \quad 12x - 8y = -20$

$4x + 5y = 47 \quad \times 3 \rightarrow \quad 12x + 15y = 141$

$-23y = -161$

$y = 7$

$3x - 2(7) = -5$

$3x = 9$

$x = 3$

The solution is (3, 7).

18. $5x - 5y = -3$

$y = x + 0.6$

$5x - 5(x + 0.6) = -3$

$5x - 5x - 3 = -3$

$-3 = -3$

The system has infinitely many solutions.

19. $x - y = 0 \rightarrow x = y$

$5x - 2y = 6$

$5x - 2x = 6$

$3x = 6$

$x = 2$

$2 - y = 0$

$y = 2$

The solution is (2, 2).

20. $x - 2y = 7$

$-x + 2y = 7$

$0 = 14$

The system has no solution.

21. $-18x + 6y = 24 \quad -18x + 6y = 24$

$3x - y = -2 \quad \times 6 \rightarrow \quad 18x - 6y = -12$

$0 = 12$

The system has no solution.

22. $4y + 5x = 15$

$x = 8y + 3$

$4y + 5(8y + 3) = 15$

$4y + 40y + 15 = 15$

$44y = 0$

$y = 0$

$x = 8(0) + 3$

$x = 3$

The solution is (3, 0).

23. $6x + 3y = 9 \quad 6x + 3y = 9$

$2x + 9y = 27 \quad \times 3 \rightarrow \quad 6x + 27y = 81$

$-24y = -72$

$y = 3$

$6x + 3(3) = 9$

$6x = 0$

$x = 0$

The solution is (0, 3).

24. C; $-x + y = 9$

$-x - y = 9$

The system has no solution.

25. D; $15x - 5y = 20$

$6x - 2y = 8$

26. $y = -6x - 2$

$12x + 2y = -6$

$2y = -12x - 6$

$y = -6x - 3$

The system has no solution.

27. $y = 7x + 13 \quad -21x + 3y = 39$

$3y = 21x + 39$

$y = 7x + 13$

The system has infinitely many solutions.

28. $4x + 3y = 27 \quad 4x - 3y = -27$

$3y = -4x + 27 \quad -3y = -4x - 27$

$y = -\frac{4}{3}x + 9 \quad y = \frac{4}{3}x + 9$

The system has one solution.

29. $9x - 15y = 24 \quad 6x - 10y = 16$

$-15y = -9x + 24 \quad -10y = -6x + 16$

$y = \frac{3}{5}x - \frac{8}{5} \quad y = \frac{3}{5}x - \frac{8}{5}$

The system has infinitely many solutions.

30. $0.3x + 0.4y = 2.4$

$0.4y = -0.3x + 2.4$

$y = -\frac{3}{4}x + 6$

$0.5x - 0.6y = 0.2$

$-0.6y = -0.5x + 0.2$

$y = \frac{5}{6}x - \frac{1}{3}$

The system has one solution.

31. $0.9x - 2.1y = 12.3$

$-2.1y = -0.9x + 12.3$

$y = \frac{3}{7}x - \frac{41}{7}$

$1.5x - 3.5y = 20.5$

$-3.5y = -1.5x + 20.5$

$y = \frac{3}{7}x - \frac{41}{7}$

The system has infinitely many solutions.

32. Answers will vary.

33. Answers will vary.

34. *Sample answer:*

$y = 2x + 5 \quad 2y = 4x + 10$

$y = 2x + 5$

35. *Sample answer:*

$px + qy = r$

$2x - 3y = 5$

a. $p = 2, q = -3, r \neq 5$

b. $p = 2, q = -3, r = 5$

c. $p = 3, q = 3, r = 15$

Problem Solving

36. $36x + 21y = 243$
$12x + 7y = 81 \quad \times 3 \rightarrow \quad 36x + 21y = 243$
$36x + 21y = 243$
$0 = 0$

There are infinitely many solutions, so you cannot determine the cost of one admission to the roller skating rink. You need more information.

37. Let x = cost of a coach ticket.

Let y = cost of a business class ticket.

$150x + 80y = 22{,}860$

$170x + 100y = 27{,}280$

$1.7x + y = 272.8$

$y = 272.8 - 1.7x$

$150x + 80(272.8 - 1.7x) = 22{,}860$

$150x + 21{,}824 - 136x = 22{,}860$

$14x = 1036$

$x = 74$

$y = 272.8 - 1.7(74)$

$y = 147$

There is enough information to determine that one coach ticket costs \$74.

38. a. Let x = memory needed for a picture.

Let y = memory needed for a 30 second movie.

$450x + 7y = 64 \quad \times 4 \rightarrow \quad 1800x + 28y = 256$
$1800x + 28y = 256$
$0 = 0$

There are infinitely many solutions, so you cannot determine the amount of memory used by a 30 second movie. You need more information.

b. Let x = memory used for a picture.

$50x$ = memory used for a movie.

$450x + 7(50x) = 64$

$450x + 350x = 64$

$800x = 64$

$x = 0.08$

$50(0.08) = 4$

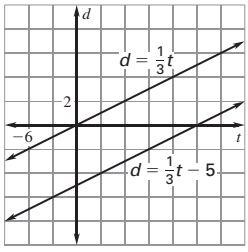
Now you can determine that a 30 second movie uses 4 megabytes of memory.

39. a. $d = rt$

$d_1 = \frac{10}{30}t \rightarrow d_1 = \frac{1}{3}t$

$d_2 = \frac{5}{15}(t - 15) \rightarrow d_2 = \frac{1}{3}t - 5$

b.



The lines are parallel, so they will never intersect. The second climber will never catch up to the first.

40. a. $y = 5x$

$y = 4(x - 10)$

$= 4x - 40$

b. $5x = 4x - 40$

$x = -40$

$y = 5(-40) = -200$

c. The solution does not make sense because you cannot fold a negative number of napkins.

41. Let d = distance from Salem to Lancaster.

Let x = wind speed.

$d = rt$

$d_1 = (160 + x)3$

$d_2 - 120 = (160 - x)3$

$(160 + x)3 - 120 = (160 - x)3$

$480 + 3x - 120 = 480 - 3x$

$3x + 360 = 480 - 3x$

$6x = 120$

$x = 20$

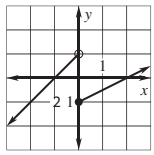
$d = (160 + 20)3$

$d = 540$

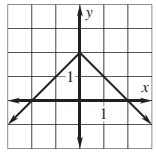
It is 540 miles from Salem to Lancaster.

Extension for the lesson "Solve Special Types of Linear Systems"

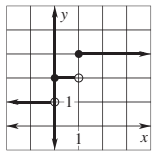
1.



2.



3.

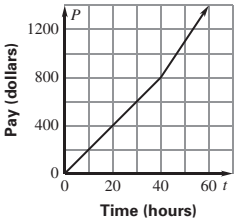


4. $y = \begin{cases} -x, \text{ if } x \le 0 \\ -x + 1, \text{ if } x > 0 \end{cases}$

5. $y = \begin{cases} 1, \text{ if } x < -1 \\ 2x, \text{ if } -1 \le x \le 0 \\ -\frac{1}{2}x + 2, \text{ if } x > 0 \end{cases}$

6. $y = \begin{cases} 4, \text{ if } 0 < x \le 1 \\ 3, \text{ if } 1 < x \le 2 \\ 2, \text{ if } 2 < x \le 3 \\ 1, \text{ if } 3 < x \le 4 \end{cases}$

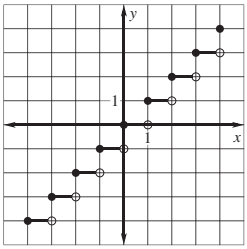
7. $P = \begin{cases} 20t, \text{ if } 0 \le t \le 40 \\ 30(t - 40) + 800, \text{ if } t > 40 \end{cases}$



When $t = 46$, $P =$ \$980.

8. The parent absolute value funtion is defined by two equations. For negative values of x, $|x| = -x$ and for nonnegative values of x, $|x| = x$; $y = \begin{cases} -x, \text{ if } x < 0 \\ x, \text{ if } x \geq 0 \end{cases}$.

9.



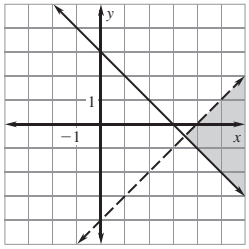
Yes; yes; the function is a piecewise function because, for every integer value of n, there is a unique equation that applies to the part of the domain defined by $n \leq x < n + 1$. The function is a step function because it is defined by a constant value over each part of its domain.

Lesson 6.6 Solve Systems of Linear Inequalities

Guided Practice for the lesson "Solve Systems of Linear Inequalities"

1. $y < x - 4$

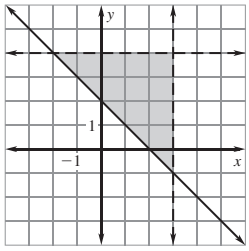
$y \geq x + 3$



2. $y \geq -x + 2$

$y < 4$

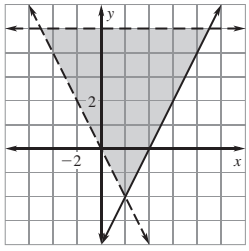
$x < 3$



3. $y \geq x - 4$

$y < 5$

$y > -x$



4. $x \leq 3$

$y > \frac{2}{3}x - 1$

5. $x < 2$

$y \leq 4$

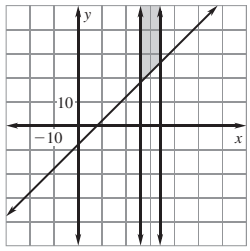
6. $x - y \leq 8 \rightarrow y \geq x - 8$

$x \geq 26$

$x \leq 34$

$y \geq 0$

Since the point does fall in the shaded area, this bat can be used by a senior league player.



Exercises for the lesson "Solve Systems of Linear Inequalities"

Skill Practice

1. A solution of a system of linear inequalities is an ordered pair that is a solution of each inequality in the system.

2. Graph both inequalities in the same coordinate plane. Use a dashed line for Inequality 1 and a solid line for Inequality 2. Find the intersection of the half-planes and shade that area.

3. (1, 1) is not a solution.

4. (0, 6) is a solution.

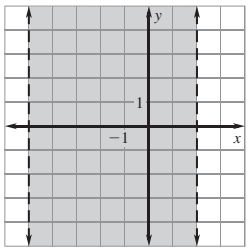
5. (3, −1) is not a solution.

6. Matches graph C.

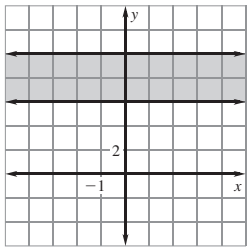
7. Matches graph A.

8. Matches graph B.

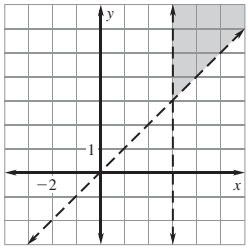
9.



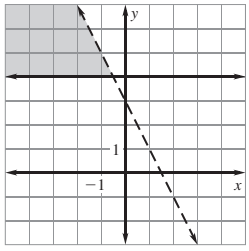
10.



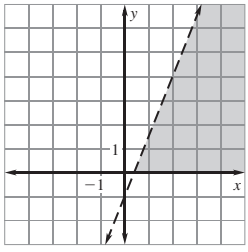
11.



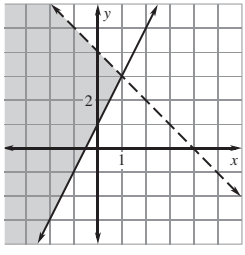
12.



13.



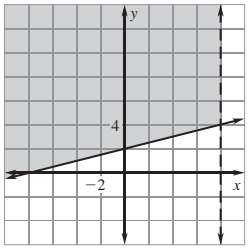
14.



15. $x - 4y \le 8, x < 8$

$-4y \le -x + 8$

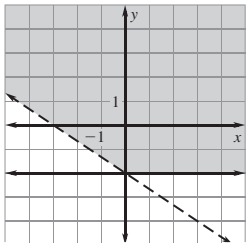
$y \ge \frac{1}{4}x - 2$



16. $2x + 3y > -6, y \ge -2$

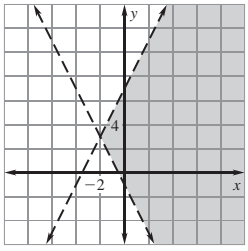
$3y > -2x - 6$

$y > -\frac{2}{3}x - 2$



17. $y - 2x < 7$

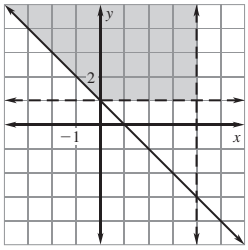
$y < 2x + 7$



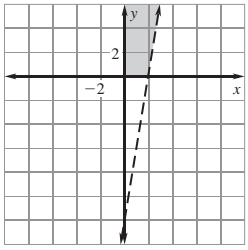
$y + 2x > -1$

$y > -2x - 1$

18. $x < 4, y > 1, y \ge -x + 1$



19. $x \ge 0, y \ge 0,$



$6x - y < 12$

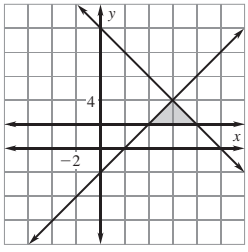
$-y < -6x + 12$

$y > 6x - 12$

20. $x + y \le 10$

$y \le -x + 10$

$y \ge 2$



$x - y \ge 2$

$-y \ge -x + 2$

$y \le x - 2$

21. D; (3, 2)

$2x - y \le 5, \ x + 2y > 2$

Try (1, −1).

$3 \le 5$ ✓ $\quad -2 \not> 2$

Try (4, 1).

$7 \not\le 5$ $\quad 6 > 2$ ✓

Try (2, 0).

$4 \le 5$ ✓ $\quad 2 \not> 2$

Try (3, 2).

$4 \le 5$ ✓ $\quad 7 > 2$

22. B; $y < 2x$

$2x + 3y > 6$

Check (3, 1).

$1 < 2(3)$ $\quad 2(3) + 3(1) > 6$

$1 < 6$ ✓ $\quad 9 > 6$ ✓

23. Inequality 1 says that $x + y$ is less than 3, not greater than, so the shaded region should be below that line, rather than above.

24. $x > 1, x < 4$

25. $y > -1, y < 4$

26. $y \ge -3, y < 2$

27. $y > x - 2$

$y \le 5x + 1$

28. $x \le 0$

$y \ge 2x + 2$

$y \le 2x + 5$

$y \ge 0$

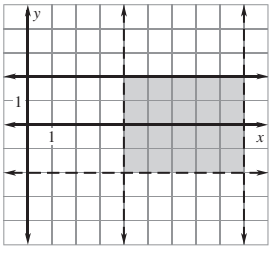
29. $y > -6$

$y \le x - 3$

$y > -2x - 1$

30. $x > 4, x < 9,$

$y \le 2, y > -2$



31. $x + y < 4$

$y < -x + 4$

$x - y \le 3$

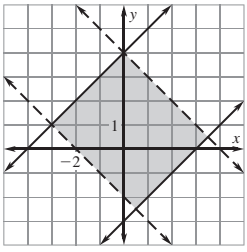
$y \ge x - 3$

$x + y > -2$

$y > -x - 2$

$x - y \ge -4$

$y \le x + 4$



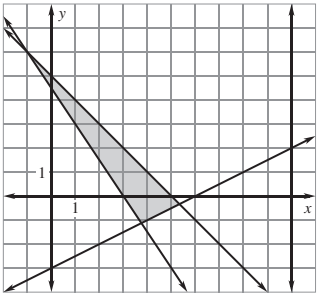
32. $x \le 10$ $\qquad 3x + 2y \ge 9$

$y \ge -\frac{3}{2}x + \frac{9}{2}$

$x - 2y \le 6$ $\qquad x + y \le 5$

$-2y \le -x + 6$ $\qquad y \le -x + 5$

$y \ge \frac{1}{2}x - 3$

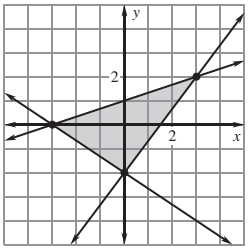


33. No, the system has no solutions because no point makes both inequalities true.

34. $x \ge 2$ $\qquad y \ge 1$

$x \le 6$ $\qquad y \le 4$

35.



$y \ge -\frac{2}{3}x - 2$

$y \ge \frac{4}{3}x - 2$

$y \le \frac{1}{3}x + 1$

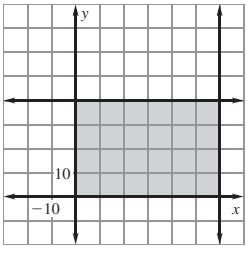
Problem Solving

36. $x \ge 0$ music score

$x \le 60$ music score

$y \ge 0$ visual score

$y \le 40$ visual score

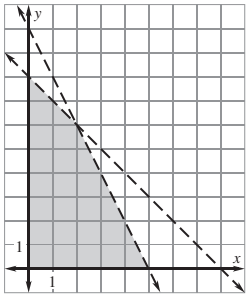


37. $x \ge 0, y \ge 0, x + y < 8,$

$14x + 7y < 70$

$7y < -14x + 70$

$y < -2x + 10$



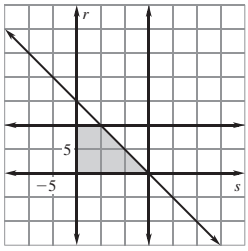
38. a. Let x = surfperch.

Let y = rockfish.

$x \ge 0, \quad y \ge 0$

$x \le 15, y \le 10$

$x + y \le 15$



b. (11, 9)

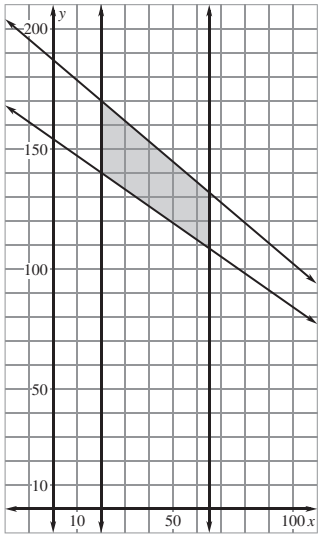
You cannot catch 11 surfperch and 9 rockfish in one day.

39. a. $x \ge 20$

$x \le 65$

$y \ge 0.7(220 - x)$ $\qquad y \le 0.85(220 - x)$

$y \ge 154 - 0.7x$ $\qquad y \le 187 - 0.85x$



b. By finding the points on the graph, you can see that his heart rate does not stay in the suggested range. It falls below it.

40. Let x = 3 in. by 5 in. pictures.

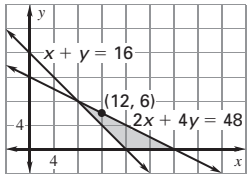
Let y = 4 in. by 6 in. pictures.

a. $x + y \ge 16 \rightarrow y \ge -x + 16$

$\frac{8}{4}x + \frac{8}{2}y \le 48$

$4y \le -2x + 48$

$y \le -\frac{1}{2}x + 12$



b. Because the point falls within the solution region, you are able to buy 12 pictures that are 3 inches by 5 inches and 6 pictures that are 4 inches by 6 inches.

41. a. $x \geq 0$

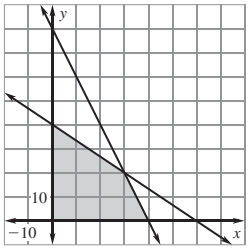
$y \geq 0$

$0.5x + 0.25y \leq 20$

$2x + 3y \leq 120$

$0.25y \leq -0.5x + 20$ $\qquad 3y \leq -2x + 120$

$y \leq -2x + 80$ $\qquad y \leq -\frac{2}{3}x + 40$



b. (0, 0), (0, 40), (30, 20), (40, 0)

c. $R = 10x + 8y$

$R = 10(0) + 8(0) = 0$

$R = 10(0) + 8(40) = 320$

$R = 10(30) + 8(20) = 460$

$R = 10(40) + 8(0) = 400$

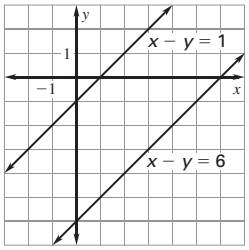
The vertex (30, 20) results in the maximum revenue.

Quiz for the lesson "Solve Special Types of Linear Systems" and "Solve Systems of Linear Inequalities"

1. $x - y = 1$ $\qquad x - y = 6$

$-y = -x + 1$ $\qquad -y = -x + 6$

$y = x - 1$ $\qquad y = x - 6$

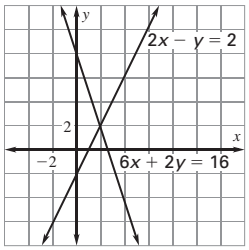


The system has no solution.

2. $6x + 2y = 16$ $\qquad 2x - y = 2$

$2y = -6x + 16$ $\qquad -y = -2x + 2$

$y = -3x + 8$ $\qquad y = 2x - 2$

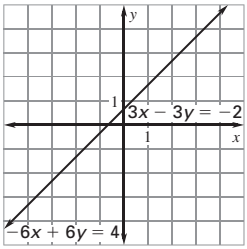


The system has one solution.

3. $3x - 3y = -2$ $\qquad -6x + 6y = 4$

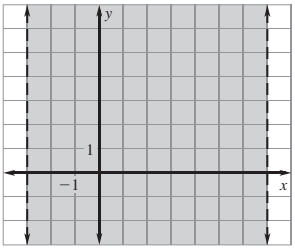
$-3y = -3x - 2$ $\qquad 6y = 6x + 4$

$y = x + \frac{2}{3}$ $\qquad y = x + \frac{2}{3}$

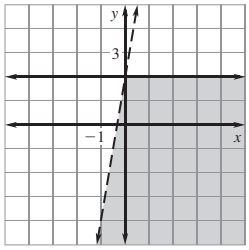


The system has infinitely many solutions.

4. $x > -3$ $\qquad x < 7$



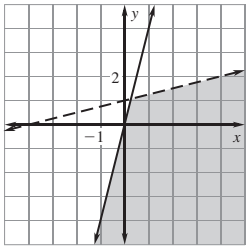
5. $y \leq 2$ $\qquad y < 6x + 2$



6. $4x \geq y$ $\qquad -x + 4y < 4$

$4y < x + 4$

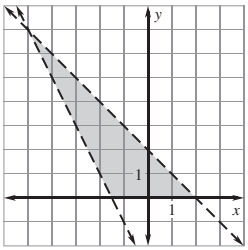
$y < \frac{1}{4}x + 1$



7. $x + y < 2$ $\qquad 2x + y > -3$

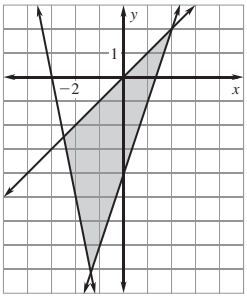
$y < -x + 2$ $\qquad y > -2x - 3$

$y \geq 0$

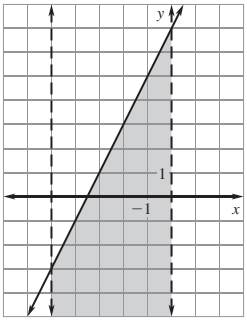


8. $y \geq 3x - 4$ $\qquad y \leq x$

$y \geq -5x - 15$



9. $x > -5, x < 0, y \leq 2x + 7$



Mixed Review of Problem Solving for the lessons "Solve Special Types of Linear Systems" and "Solve Systems of Linear Inequalities"

1. a. Let x = number of bricks.

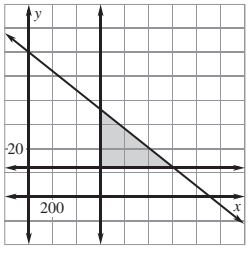
Let y = number of bags of sand.

$x \geq 600, \quad y \geq 12$

$2x + 50y \leq 3000$

b. $50y \leq -2x + 3000$

$y \leq -\frac{1}{25}x + 60$



c. 700 bricks and 20 bags of sand can be delivered in one trip.

2. Let x = cost of a gallon of ceiling paint.

Let y = cost of a gallon of wall paint.

a.
$$\begin{array}{rll} 2x + 4y = 120 & & 2x + 4y = 120 \\ x + 2y = 60 & \times(-2) \rightarrow & -2x - 4y = -120 \\ & & \hline & & 0 = 0 \end{array}$$

b. There is not enough information to determine the cost of each type of paint. There are infinitely many solutions, so you need more information.

c.
$$\begin{aligned} x &= y + 3 \\ (y + 3) + 2y &= 60 \\ 3y &= 57 \\ y &= 19 \\ x &= 19 + 3 \\ x &= 22 \end{aligned}$$

A gallon of wall paint is \$19.
A gallon of ceiling paint is \$22.

3. Let x = cost of a CD.

Let y = cost of a DVD.

$$\begin{array}{rll} 4x + 2y = 78 & & 4x + 2y = 78 \\ 2x + y = 39 & \times(-2) \rightarrow & -4x - 2y = -78 \\ & & \hline & & 0 = 0 \end{array}$$

There are infinitely many solutions, so you need more information to determine the cost of one CD.

4. The graph shows that the two lines are parallel, which means they will never intersect. So, bus B will never catch up to bus A.

5. a. Let x = hours lifeguarding.

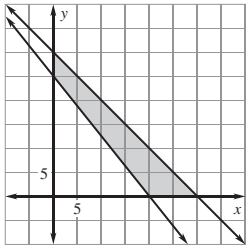
Let y = hours working at a retail store.

$10x + 8y \geq 200$

$x + y \leq 30$

$y \geq -\frac{5}{4}x + 25$

$y \leq -x + 30$



b. By locating the point (5, 15) on the graph, you can see it does not lie in the shaded region. So; you will not earn at least \$200 per week.

c. You can work 4 to 10 hours as a lifeguard to earn at least \$200 per week.

6. Answers will vary.

7. $y \geq 0, \quad x \geq 0$

$4x + 5y \leq 60$

$4(0) + 5y = 60$

$y = 12 \leftarrow y$-intercept

$4x + 5(0) = 60$

$x = 15 \leftarrow x$-intercept

Area $= \frac{1}{2}bh, b = 15, h = 12$

$A = \frac{1}{2}(15)(12) = 90$

The area of the triangle is 90 square feet.

Chapter Review for the chapter "Systems of Equations and Inequalities"

1. A *system of linear inequalities* consists of two or more linear inequalities in the same variables.

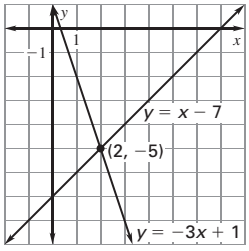
2. A *system of linear equations* consists of two or more linear equations in the same variables.

3. Graph each of the inequalities on the same coordinate plane. Shade the intersection of the two half-planes. To check your solution, choose a point in the shaded region and substitute its x and y-values into each inequality.

4. Answers will vary.

5. $y = -3x + 1$

$y = x - 7$



Check $(2, -5)$.

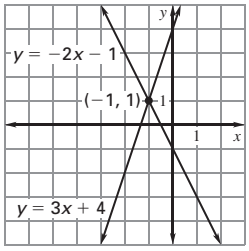
$-5 \stackrel{?}{=} -3(2) + 1$ $\qquad$ $-5 \stackrel{?}{=} 2 - 7$

$-5 = -5$ ✓ $\qquad$ $-5 = -5$ ✓

The solution is $(2, -5)$.

6. $y = 3x + 4$

$y = -2x - 1$



Check $(-1, 1)$.

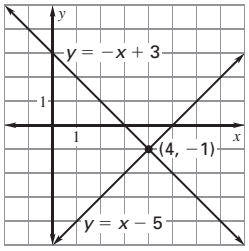
$1 \stackrel{?}{=} 3(-1) + 4$ $\qquad$ $1 \stackrel{?}{=} -2(-1) - 1$

$1 = 1$ ✓ $\qquad$ $1 = 1$ ✓

The solution is $(-1, 1)$.

7. $x + y = 3$ $\qquad$ $x - y = 5$

$y = -x + 3$ $\qquad$ $y = x - 5$



Check $(4, -1)$.

$4 + (-1) \stackrel{?}{=} 3$ $\qquad$ $4 - (-1) \stackrel{?}{=} 5$

$3 = 3$ ✓ $\qquad$ $5 = 5$ ✓

The solution is $(4, -1)$.

8. $y = 2x - 7$

$x + 2y = 1$

$x + 2(2x - 7) = 1$

$x + 4x - 14 = 1$

$5x = 15$

$x = 3$

$y = 2(3) - 7 = -1$

The solution is $(3, -1)$.

9. $x + 4y = 9$

$x - y = 4$

$x = y + 4$

$y + 4 + 4y = 9$

$5y = 5$

$y = 1$

$x = 1 + 4 = 5$

The solution is $(5, 1)$.

10. $2x + y = -15$

$y - 5x = 6$

$y = 5x + 6$

$2x + 5x + 6 = -15$

$7x = -21$

$x = -3$

$y = 5(-3) + 6 = -9$

The solution is $(-3, -9)$.

11. Let x = tubes of paint.

Let y = brushes.

$3x + 0.5y = 16$

$y = 2x$

$3x + 0.5(2x) = 16$

$3x + x = 16$

$4x = 16$

$x = 4$

$y = 2(4) = 8$

She purchases 4 tubes of paint and 8 brushes.

12. $x + 2y = 13$

$x - 2y = -7$

$2x = 6$

$x = 3$

$3 + 2y = 13$

$2y = 10$

$y = 5$

The solution is $(3, 5)$.

13. $4x - 5y = 14$

$-4x + y = -6$

$-4y = 8$

$y = -2$

$4x - 5(-2) = 14$

$4x = 4$

$x = 1$

The solution is $(1, -2)$.

14. $x + 7y = 12$

$-2x + 7y = 18$

$3x = -6$

$x = -2$

$-2 + 7y = 12$

$7y = 14$

$y = 2$

The solution is $(-2, 2)$.

15. $9x - 2y = 34$

$5x - 2y = 10$

$4x = 24$

$x = 6$

$9(6) - 2y = 34$

$-2y = -20$

$y = 10$

The solution is $(6, 10)$.

16. $3x = y + 1$
$2x - y = 9$
$3x - y = 1$
$2x - y = 9$
$x = -8$
$2(-8) - y = 9$
$y = -25$
The solution is $(-8, -25)$.

17. $4y = 11 - 3x$
$3x + 2y = -5$
$3x + 4y = 11$
$-2y = -16$
$y = 8$
$4(8) = 11 - 3x$
$21 = -3x$
$-7 = x$
The solution is $(-7, 8)$.

18. $-x + y = -4$ $\times 3 \rightarrow -3x + 3y = -12$
$2x - 3y = 5$ $2x - 3y = 5$
$-x = -7$
$x = 7$
$-7 + y = -4$
$y = 3$
The solution is $(7, 3)$.

19. $x + 6y = 28$ $\times 2 \rightarrow 2x + 12y = 56$
$2x - 3y = -19$ $2x - 3y = -19$
$15y = 75$
$y = 5$
$x + 6(5) = 28$
$x = -2$
The solution is $(-2, 5)$.

20. $3x - 5y = -7$ $\times 4 \rightarrow 12x - 20y = -28$
$-4x + 7y = 8$ $\times 3 \rightarrow -12x + 21y = 24$
$y = -4$
$3x - 5(-4) = -7$
$3x = -27$
$x = -9$
The solution is $(-9, -4)$.

21. $8x - 7y = -3$ $\times 3 \rightarrow 24x - 21y = -9$
$6x - 5y = -1$ $\times 4 \rightarrow 24x - 20y = -4$
$-y = -5$
$y = 5$
$6x - 5(5) = -1$
$6x = 24$
$x = 4$
The solution is $(4, 5)$.

22. $5x = 3y - 2$
$3x + 2y = 14$ $\times 3 \rightarrow 9x + 6y = 42$
$5x - 3y = -2$ $\times 2 \rightarrow 10x - 6y = -4$
$19x = 38$
$x = 2$
$5(2) = 3y - 2$
$12 = 3y$
$4 = y$
The solution is $(2, 4)$.

23. $11x = 2y - 1$
$3y = 10 + 8x$
$11x - 2y = -1$ $\times 3 \rightarrow 33x - 6y = -3$
$-8x + 3y = 10$ $\times 2 \rightarrow -16x + 6y = 20$
$17x = 17$
$x = 1$
$11(1) = 2y - 1$
$12 = 2y$
$6 = y$
The solution is $(1, 6)$.

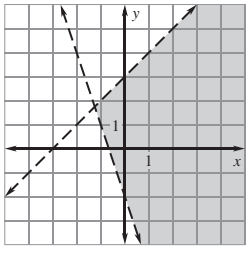
24. Let x = cost of 1 gallon of gasoline.
Let y = cost of 1 quart of oil.
$10x + y = 24.50$ $\times 2 \rightarrow 20x + 2y = 49$
$8x + 2y = 22$ $8x + 2y = 22$
$12x = 27$
$x = 2.25$
$10(2.25) + y = 24.50$
$y = 2$
One quart of oil costs \$2.00.

25. $x = 2y - 3$ $1.5x - 3y = 0$
$-2y = -x - 3$ $-3y = -1.5x$
$y = \frac{1}{2}x + \frac{3}{2}$ $y = \frac{1}{2}x$
The lines are parallel because they have the same slope but different y-intercepts. Parallel lines do not intersect, so the system has no solution.

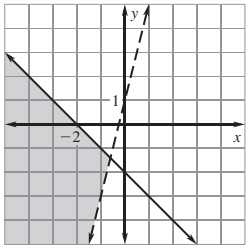
26. $-x + y = 8$ $x + 8 = y$
$y = x + 8$ $y = x + 8$
The lines are the same, so the system has infinitely many solutions.

27. $4x = 2y + 6$ $4x + 2y = 10$
$-2y = -4x + 6$ $2y = -4x + 10$
$y = 2x - 3$ $y = -2x + 5$
The lines have different slopes and different y-intercepts, so they intersect at one point. Therefore the system has one solution.

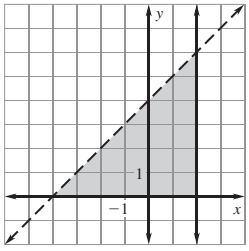
28. $y < x + 3$ $y > -3x - 2$



29. $y \le -x - 2 \qquad y > 4x + 1$



30. $y \ge 0, \quad x \le 2, \quad y < x + 4$



31. Let x = number of matinee tickets.

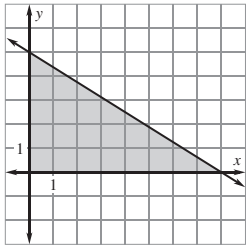
Let y = number of evening tickets.

$x \ge 0, \quad y \ge 0$

$5x + 8y \le 40$

$8y \le -5x + 40$

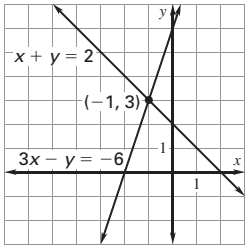
$y \le -\frac{5}{8}x + 5$



Chapter Test for the chapter "Systems of Equations and Inequalities"

1. $3x - y = -6 \qquad x + y = 2$

$y = 3x + 6 \qquad y = -x + 2$



Check $(-1, 3)$.

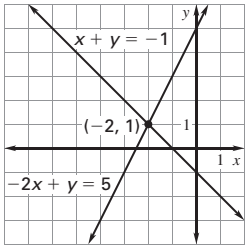
$3(-1) - 3 \stackrel{?}{=} -6 \qquad -1 + 3 \stackrel{?}{=} 2$

$-6 = -6$ ✓ $\qquad 2 = 2$ ✓

The solution is $(-1, 3)$.

2. $-2x + y = 5 \qquad x + y = -1$

$y = 2x + 5 \qquad y = -x - 1$



Check $(-2, 1)$.

$-2(-2) + 1 \stackrel{?}{=} 5 \qquad -2 + 1 \stackrel{?}{=} -1$

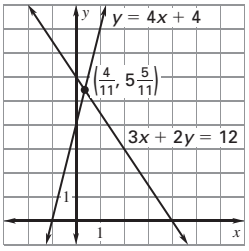
$5 = 5$ ✓ $\qquad -1 = -1$ ✓

The solution is $(-2, 1)$.

3. $y = 4x + 4 \qquad 3x + 2y = 12$

$2y = -3x + 12$

$y = -\frac{3}{2}x + 6$



Check (0.36, 5.45).

$5.45 \stackrel{?}{=} 4(0.36) + 4 \qquad 3(0.36) + 2(5.45) \stackrel{?}{=} 12$

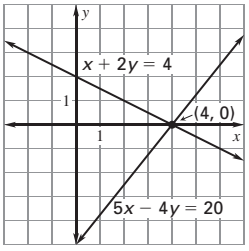
$5.45 \approx 5.44$ ✓ $\qquad 11.98 \approx 12$ ✓

The solution is (0.36, 5.45).

4. $5x - 4y = 20 \qquad x + 2y = 4$

$-4y = -5x + 20 \qquad 2y = -x + 4$

$y = \frac{5}{4}x - 5 \qquad y = -\frac{1}{2}x + 2$



Check (4, 0).

$5(4) - 4(0) \stackrel{?}{=} 20 \qquad 4 + 2(0) \stackrel{?}{=} 4$

$20 = 20$ ✓ $\qquad 4 = 4$ ✓

The solution is (4, 0).

5. $x + 3y = 9$

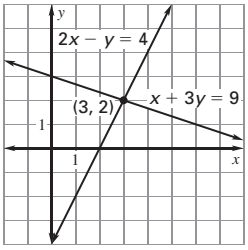
$3y = -x + 9$

$y = -\frac{1}{3}x + 3$

$2x - y = 4$

$-y = -2x + 4$

$y = 2x - 4$



Check (3, 2).

$3 + 3(2) \stackrel{?}{=} 9$

$9 = 9$ ✓

$2(3) - 2 \stackrel{?}{=} 4$

$4 = 4$ ✓

The solution is (3, 2).

6. $2x + 7y = 14$

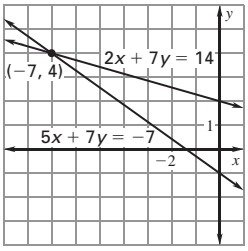
$7y = -2x + 14$

$y = -\frac{2}{7}x + 2$

$5x + 7y = -7$

$7y = -5x - 7$

$y = -\frac{5}{7}x - 1$



Check (−7, 4).

$2(-7) + 7(4) \stackrel{?}{=} 14$

$14 = 14$ ✓

$5(-7) + 7(4) \stackrel{?}{=} -7$

$-7 = -7$ ✓

The solution is (−7, 4).

7. $y = 5x - 7$

$-4x + y = -1$

$-4x + 5x - 7 = -1$

$x - 7 = -1$

$x = 6$

$y = 5(6) - 7$

$y = 23$

The solution is (6, 23).

8. $x = y - 11$

$x - 3y = 1$

$y - 11 - 3y = 1$

$-2y = 12$

$y = -6$

$x = -6 - 11$

$x = -17$

The solution is (−17, −6).

9. $3x + y = -19$

$x - y = 7$

$x = y + 7$

$3(y + 7) + y = -19$

$3y + 21 + y = -19$

$4y = -40$

$y = -10$

$x = -10 + 7$

$x = -3$

The solution is (−3, −10).

10. $15x + y = 70 \rightarrow y = -15x + 70$

$3x - 2y = -8$

$3x - 2(-15x + 70) = -8$

$3x + 30x - 140 = -8$

$33x = 132$

$x = 4$

$y = -15(4) + 70$

$y = 10$

The solution is (4, 10).

11. $3y + x = 17$

$x + y = 8 \rightarrow x = -y + 8$

$3y + (-y + 8) = 17$

$2y = 9$

$y = 4.5$

$x = -4.5 + 8$

$x = 3.5$

The solution is (3.5, 4.5).

12. $0.5x + y = 9 \rightarrow y = -0.5x + 9$

$1.6x + 0.2y = 13$

$1.6x + 0.2(-0.5x + 9) = 13$

$1.6x - 0.1x + 1.8 = 13$

$1.5x = 11.2$

$x = 7.47$

$y = -0.5(7.47) + 9$

$y = 5.27$

The solution is (7.47, 5.27).

13. $8x + 3y = -9$

$-8x + y = 29$

$4y = 20$

$y = 5$

$8x + 3(5) = -9$

$8x = -24$

$x = -3$

The solution is (−3, 5)

14. $x - 5y = -3$

$3x - 5y = 11$

$-2x = -14$

$x = 7$

$7 - 5y = -3$

$-5y = -10$

$y = 2$

The solution is (7, 2).

15. $4x + y = 17$

$7y = 4x - 9$

$4x + y = 17$

$-4x + 7y = -9$

$8y = 8$

$y = 1$

$4x + 1 = 17$

$4x = 16$

$x = 4$

The solution is (4, 1).

16. $3x + 2y = -5$
$x - y = 10 \quad \times 2 \rightarrow$

$$\begin{aligned} 3x + 2y &= -5 \\ 2x - 2y &= 20 \\ \hline 5x &= 15 \\ x &= 3 \end{aligned}$$

$3 - y = 10$
$-y = 7$
$y = -7$
The solution is $(3, -7)$.

17. $3y = x + 5 \rightarrow -x + 3y = 5$
$-x + 3y = 5 \quad \times 3 \rightarrow$

$$\begin{aligned} -3x + 9y &= 15 \\ -3x + 8y &= 8 \\ \hline y &= 7 \end{aligned}$$

$3(7) = x + 5$
$21 = x + 5$
$16 = x$
The solution is $(16, 7)$.

18. $6x - 5y = 9 \quad \times 3 \rightarrow \quad 18x - 15y = 27$
$9x - 7y = 15 \quad \times 2 \rightarrow \quad 18x - 14y = 30$

$$\begin{aligned} -y &= -3 \\ y &= 3 \end{aligned}$$

$6x - 5(3) = 9$
$6x = 24$
$x = 4$
The solution is $(4, 3)$.

19. $15x - 3y = 12$
$y = 5x - 4$
$15x - 3(5x - 4) = 12$
$15x - 15x + 12 = 12$
$0 = 0$
The system has infinitely many solutions.

20. $4x - y = -4 \quad \times 2 \rightarrow \quad 8x - 2y = -8$
$-8x + 2y = 2 \qquad\qquad -8x + 2y = 2$

$$0 = -6$$

The system has no solution.

21. $-12x + 3y = 18 \qquad\qquad -12x + 3y = 18$
$4x + y = -6 \quad \times 3 \rightarrow \quad 12x + 3y = -18$

$$\begin{aligned} 6y &= 0 \\ y &= 0 \end{aligned}$$

$4x + 0 = -6$
$x = -\frac{3}{2}$
The system has one solution.

22. $6x - 7y = 5 \quad \times 2 \rightarrow \quad 12x - 14y = 10$
$-12x + 14y = 10 \qquad -12x + 14y = 10$

$$0 = 20$$

The system has no solution.

23. $3x - 4y = 24$
$3x + 4y = 24$

$$\begin{aligned} 6x &= 48 \\ x &= 8 \end{aligned}$$

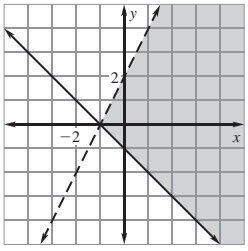
$3(8) - 4y = 24$
$y = 0$
The system has one solution.

24. $10x - 2y = 14 \quad \times 3 \rightarrow \quad 30x - 6y = 42$
$15x - 3y = 21 \quad \times 2 \rightarrow \quad 30x - 6y = 42$

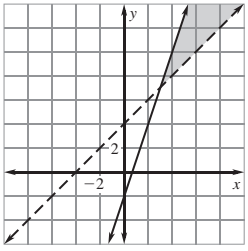
$$0 = 0$$

The system has infinitely many solutions.

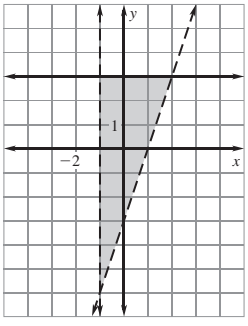
25. $y < 2x + 2 \qquad y \geq -x - 1$



26. $y \leq 3x - 2 \qquad y > x + 4$



27. $y \leq 3 \qquad x > -1$
$y > 3x - 3$



28. $x + 150y = 215$
$x + 120y = 176$

$$\begin{aligned} 30y &= 39 \\ y &= 1.3 \end{aligned}$$

$x + 150(1.3) = 215$
$x = 20$
The fee is \$20, and the cost per mile is \$1.30.

29. $2\ell + 2w = 58$

$\ell = 3w + 1$

$2(3w + 1) + 2w = 58$

$6w + 2 + 2w = 58$

$8w = 56$

$w = 7$

$\ell = 3(7) + 1 = 22$

The length is 22 inches and the width is 7 inches.

30. Let x = number of granola bars purchased.

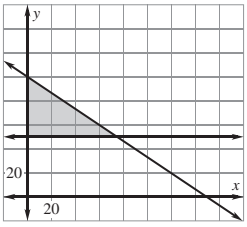
Let y = number of bottles of water purchased.

$x \geq 0, \quad y \geq 50$

$0.5x + 0.75y \leq 75$

$0.75y \leq -0.5x + 75$

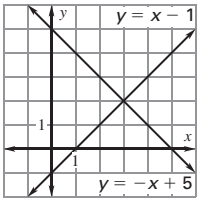
$y \leq -0.67x + 100$



Extra Practice for the chapter "Systems of Equations and Inequalities"

1. $y = x - 1$

$y = -x + 5$



The graphs appear to intersect at (3, 2).

$y = x - 1$ $\qquad$ $y = -x + 5$

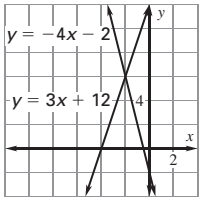
$2 \stackrel{?}{=} 3 - 1$ $\qquad$ $2 \stackrel{?}{=} -3 + 5$

$2 = 2$ ✓ $\qquad$ $2 = 2$ ✓

Because (3, 2) is a solution of each equation, it is a solution of the linear system.

2. $y = 3x + 12$

$y = -4x - 2$



The graphs appear to intersect at (−2, 6).

$y = 3x + 12$ $\qquad$ $y = -4x - 2$

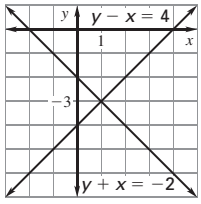
$6 \stackrel{?}{=} 3(-2) + 12$ $\qquad$ $6 \stackrel{?}{=} -4(-2) - 2$

$6 = 6$ ✓ $\qquad$ $6 = 6$ ✓

Because (−2, 6) is a solution to each equation, it is a solution of the linear system.

3. $x - y = 4 \rightarrow y = x - 4$

$x + y = -2 \rightarrow y = -x - 2$



The graphs appear to intersect at (1, −3).

$x - y = 4$ $\qquad$ $x + y = -2$

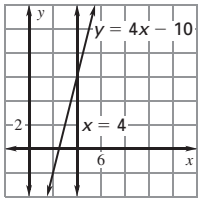
$1 - (-3) \stackrel{?}{=} 4$ $\qquad$ $1 + (-3) \stackrel{?}{=} -2$

$4 = 4$ ✓ $\qquad$ $-2 = -2$ ✓

Because (1, −3) is a solution of each equation, it is a solution of the linear system.

4. $4x - y = 10 \rightarrow y = 4x - 10$

$x = 4$



The graphs appear to intersect at (4, 6).

$4x - y = 10$ $\qquad$ $x = 4$

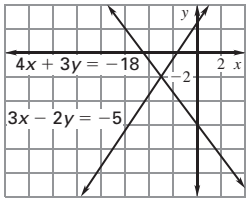
$4(4) - 6 \stackrel{?}{=} 10$ $\qquad$ $4 = 4$ ✓

$10 = 10$ ✓

Because (4, 6) is a solution of each equation, it is a solution of the linear system.

5. $3x - 2y = -5 \rightarrow y = \frac{3}{2}x + \frac{5}{2}$

$4x + 3y = -18 \rightarrow y = -\frac{4}{3}x - 6$



The graphs appear to intersect at (−3, −2).

$3x - 2y = -5$ $\qquad$ $4x + 3y = -18$

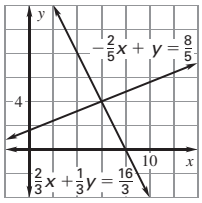
$3(-3) - 2(-2) \stackrel{?}{=} -5$ $\qquad$ $4(-3) + 3(-2) \stackrel{?}{=} -18$

$-5 = -5$ ✓ $\qquad$ $-18 = -18$ ✓

Because (−3, −2) is a solution of each equation, it is a solution of the linear system.

6. $\frac{2}{3}x + \frac{1}{3}y = \frac{16}{3} \rightarrow y = -2x + 16$

$-\frac{2}{5}x + y = \frac{8}{5} \rightarrow y = \frac{2}{5}x + \frac{8}{5}$



The graphs appear to intersect at (6, 4).

$\frac{2}{3}x + \frac{1}{3}y = \frac{16}{3}$ $\qquad$ $-\frac{2}{5}x + y = \frac{8}{5}$

$\frac{2}{3}(6) + \frac{1}{3}(4) \stackrel{?}{=} \frac{16}{3}$ $\qquad$ $-\frac{2}{5}(6) + 4 \stackrel{?}{=} \frac{8}{5}$

$\frac{16}{3} = \frac{16}{3}$ ✓ $\qquad$ $\frac{8}{5} = \frac{8}{5}$ ✓

Because (6, 4) is a solution of each equation, it is a solution of the linear system.

7. $y = 2x + 6$

$x = y - 3$

$x = (2x + 6) - 3$

$x = -3$

$y = 2(-3) + 6 = 0$

The solution is (−3, 0).

8. $y = 3x + 5$

$x + y = -1$

$x + (3x + 5) = -1$

$x = -\frac{3}{2}$

$y = 3\left(-\frac{3}{2}\right) + 5 = \frac{1}{2}$

The solution is $\left(-\frac{3}{2}, \frac{1}{2}\right)$.

9. $x = 2y - 5$

$2x - y = 11$

$2(2y - 5) - y = 11$

$y = 7$

$x = 2(7) - 5 = 9$

The solution is (9, 7).

10. $2x - y = 0 \rightarrow y = 2x$

$x + 3y = -56$

$x + 3(2x) = -56$

$x = -8$

$y = 2(-8) = -16$

The solution is (−8, −16).

11. $1.5x - 2.5y = 22$

$x - y = 10 \rightarrow x = y + 10$

$1.5(y + 10) - 2.5y = 22$

$y = -7$

$x = -7 + 10 = 3$

The solution is (3, −7).

12. $\frac{1}{2}x + \frac{3}{4}y = 5$

$x - \frac{1}{2}y = 6 \rightarrow x = \frac{1}{2}y + 6$

$\frac{1}{2}\left(\frac{1}{2}y + 6\right) + \frac{3}{4}y = 5$

$y = 2$

$x = \frac{1}{2}(2) + 6 = 7$

The solution is (7, 2).

13. $x + 2y = 2$

$-x + 3y = 13$

$5y = 15$

$y = 3$

$x + 2(3) = 2$

$x = -4$

The solution is (−4, 3).

14. $3x - 4y = -16 \rightarrow 3x - 4y = -16$

$x - 4y = -40 \rightarrow -x + 4y = 40$

$2x = 24$

$x = 12$

$12 - 4y = -40$

$y = 13$

The solution is (12, 13).

15. $3x + 2y = -31 \rightarrow 3x + 2y = -31$

$5x + 2y = -49 \rightarrow -5x - 2y = 49$

$-2x = 18$

$x = -9$

$3(-9) + 2y = -31$

$y = -2$

The solution is (−9, −2).

16. $5x + 4y = 6 \rightarrow 5x + 4y = 6$

$7x + 4y = 14 \rightarrow -7x - 4y = -14$

$-2x = -8$

$x = 4$

$5(4) + 4y = 6$

$y = -\frac{7}{2}$

The solution is $\left(4, -\frac{7}{2}\right)$.

17. $10y - 3x = -41 \rightarrow -3x + 10y = -41$

$3x - 5y = 16 \rightarrow 3x + -5y = 16$

$5y = -25$

$y = -5$

$3x - 5(-5) = 16$

$x = 3$

The solution is $(3, -5)$.

18. $4x - 3y = 39 \rightarrow 4x - 3y = 39$

$7y = 4x - 79 \rightarrow -4x + 7y = -79$

$4y = -40$

$y = -10$

$4x - 3(-10) = 39$

$x = \frac{9}{4}$

The solution is $\left(\frac{9}{4}, -10\right)$.

19. $x + y = -3 \rightarrow -5x - 5y = 15$

$5x + 7y = -9 \rightarrow 5x + 7y = -9$

$2y = 6$

$y = 3$

$x + 3 = -3$

$x = -6$

The solution is $(-6, 3)$.

20. $5x + 2y = -19 \rightarrow -10x - 4y = 38$

$10x - 7y = -16 \rightarrow 10x - 7y = -16$

$-11y = 22$

$y = -2$

$5x + 2(-2) = -19$

$x = -3$

The solution is $(-3, 2)$.

21. $8x - 3y = 61 \rightarrow 8x - 3y = 61$

$2x - 5y = -23 \rightarrow -8x + 20y = 92$

$17y = 153$

$y = 9$

$2x - 5(9) = -23$

$x = 11$

The solution is $(11, 9)$.

22. $4x - 3y = -2 \rightarrow -12x + 9y = 6$

$6x + 4y = 31 \rightarrow 12x + 8y = 62$

$17y = 68$

$y = 4$

$4x - 3(4) = -2$

$x = \frac{5}{2}$

The solution is $\left(\frac{5}{2}, 4\right)$.

23. $5x - 2y = 53 \rightarrow 15x - 6y = 159$

$2x + 6y = 11 \rightarrow 2x + 6y = 11$

$17x = 170$

$x = 10$

$5(10) - 2y = 53$

$y = -\frac{3}{2}$

The solutin is $\left(10, -\frac{3}{2}\right)$.

24. $15x - 8y = 6 \rightarrow -45x + 24y = -13$

$25x - 12y = 16 \rightarrow 50x - 24y = 32$

$5x = 14$

$x = \frac{14}{5}$

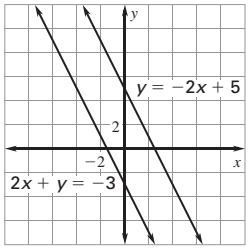
$15\left(\frac{14}{5}\right) - 8y = 6$

$y = \frac{9}{2}$

The solution is $\left(\frac{14}{5}, \frac{9}{2}\right)$.

25. $2x + y = -3 \rightarrow y = -2x - 3$

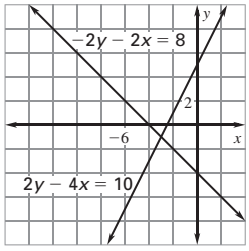
$y = -2x + 5$



The lines are parallel so this linear system has no solutions.

26. $2y - 4x = 10 \rightarrow y = 2x + 5$

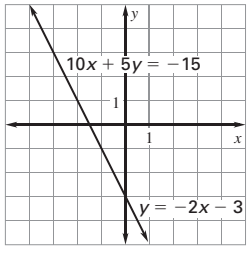
$-2y - 2x = 8 \rightarrow y = -x - 4$



The lines appear to intersect at the point $(-3, -1)$, so the linear system has one solution.

27. $10x + 5y = -15 \rightarrow y = -2x - 3$

$y = -2x - 3$



The equations represent the same line so the linear system has infinitely many solutions.

28. $y - 3x = 5$

$x = y - 5$

$y - 3(y - 5) = 5$

$y = 5$

$x = 5 - 5 = 0$

The solution is (0, 5).

29. $2y - 3x = 36$

$y = 3x - 12$

$2(3x - 12) - 3x = 36$

$x = 20$

$y = 3(20) - 12 = 48$

The solution is (20, 48).

30. $5x + 5y = -32 \rightarrow 15x + 15y = -96$

$3x + 3y = 14 \rightarrow -15x - 15y = -70$

$0 = -166$

Only a false statement is left. This tells you the system has no solution.

31. $4x + 6y = 11$

$y = -\frac{2}{3}x + 7$

$4x + 6\left(-\frac{2}{3}x + 7\right) = 11$

$42 = 11$

Only a false statement remains. This tells you the system has no solution.

32. $3y - 3x = 12$

$y = x - 4$

$3(x - 4) - 3x = 12$

$-12 = 12$

Only a false statement remains. This tells you the system has no solution.

33. $x + 2y = -30$

$y = \frac{1}{2}x + 15$

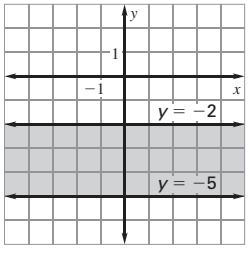
$x + 2\left(\frac{1}{2}x + 15\right) = -30$

$x = -30$

$y = \frac{1}{2}(-30) + 15 = 0$

The solution is $(-30, 0)$.

34.



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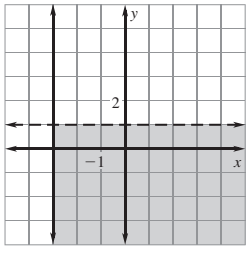

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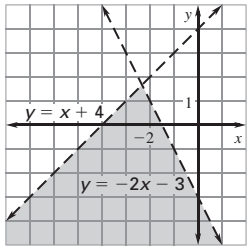

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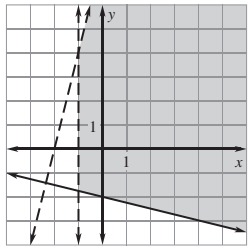

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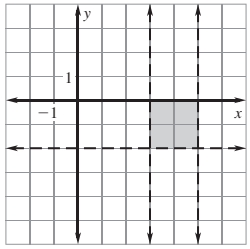

39.