Chapter 8 Polynomials and Factoring

Prerequisite Skills for the chapter "Polynomials and Factoring"

1. Terms that have the same variable part are called *like terms*.

2. For a function $f(x)$, a *zero* is an x-value for which $f(x) = 0$.

3. Factors of 121: 1, 11, 121

Factors of 77: 1, 7, 11, 77

The GCF is 11.

4. Factors of 96: 1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 96

Factors of 32: 1, 2, 4, 8, 16, 32

The GCF is 32.

5. Factors of 81: 1, 3, 9, 27, 81

Factors of 42: 1, 2, 3, 6, 7, 14, 21, 42

The GCF is 3.

6. Factors of 12: 1, 2, 3, 4, 6, 12

Factors of 56: 1, 2, 4, 7, 8, 14, 28, 56

The GCF is 4.

7. $3x + (-6x) = 3x - 6x = -3x$

8. $5 + 4x + 2 = 4x + 7$

9. $4(2x - 1) + x = 8x - 4 + x = 9x - 4$

10. $-(x + 4) - 6x = -x - 4 - 6x = -7x - 4$

11. $(3xy)^3 = 3^3x^3y^3 = 27x^3y^3$

12. $xy^2 \cdot xy^3 = x^{1+1}y^{2+3} = x^2y^5$

13. $(x^5)^3 = x^{15}$

14. $(-x)^3 = (-1)^3 \cdot x^3 = -x^3$

Lesson 8.1 Add and Subtract Polynomials

Guided Practice for the lesson "Add and Subtract Polynomials"

1. $5y - 2y^2 + 9 = -2y^2 + 5y + 9$

The greatest degree is 2, so the degree of the polynomial is 2, and the leading coefficient is -2.

2. $y^3 - 4y + 3$ is a polynomial because each of its terms has whole number exponents. The greatest degree is 3, so the degree of the polynomial is 3. There are 3 terms, so the polynomial is a trinomial.

3. $(5x^3 + 4x - 2x) + (4x^2 + 3x^3 - 6)$

$= (5x^3 + 3x^3) + 4x^2 + (4x - 2x) - 6$

$= 8x^3 + 4x^2 + 2x - 6$

4. $$\begin{array}{r} (4x^2 - 7x \quad\;\;) \\ \underline{-(5x^2 + 4x - 9)} \end{array} \rightarrow \begin{array}{r} 4x^2 - \;\;7x \quad\; \\ \underline{-5x^2 - \;\;4x + 9} \\ -x^2 - 11x + 9 \end{array}$$

5. $N - A = (-488t^2 + 5430t + 24{,}700)$

$\quad - (-318t^2 + 3040t + 25{,}600)$

$= -488t^2 + 5430t + 24{,}700 + 318t^2$

$\quad - 3040t - 25{,}600$

$= (-488t^2 + 318t^2) + (5430t - 3040t)$

$\quad + (24{,}700 - 25{,}600)$

$= -170t^2 + 2390t - 900$

For 2001, let $t = 6$:

$N - A = -170(6)^2 + 2390(6) - 900 = 7320$

The difference in attendance at National and American League baseball games in 2001 was 7,320,000 people.

Exercises for the lesson "Add and Subtract Polynomials"

Skill Practice

1. A number, a variable, or the product of one or more variables is called a *monomial*.

2. Yes; A polynomial is a monomial or a sum of monomials. Because 6 is a monomial, it is also a polynomial.

3. $9m^5$

Degree: 5

Leading coefficient: 9

4. $2 - 6y = -6y + 2$

Degree: 1

Leading coefficient: -6

5. $2x^2y^2 - 8xy$

Degree: 4

Leading coefficient: 2

6. $5n^3 + 2n - 7$

Degree: 3

Leading coefficient: 5

7. $5z + 2z^3 - z^2 + 3z^4 = 3z^4 + 2z^3 - z^2 + 5z$

Degree: 4 Leading coefficient: 3

8. $-2h^2 + 2h^4 - h^6 = -h^6 + 2h^4 - 2h^2$

Degree: 6 Leading coefficient: -1

9. C; $-4x^3 + 6x^4 - 1 = 6x^4 - 4x^3 - 1$

The greatest degree is 4, so the degree of the polynomial is 4.

10. D; $3s^{-2}$ has an exponent that is not a whole number.

11. -4^x is not a polynomial because it has a variable exponent.

12. $w^{-3} + 1$ is not a polynomial because not all of the exponents are whole numbers.

13. $3x - 5$ is a polynomial that is a first degree binomial.

14. $\frac{4}{5}f^2 - \frac{1}{2}f + \frac{2}{3}$ is a polynomial that is a second degree trinomial.

15. $6 - n^2 + 5n^3$ is a polynomial that is a third degree trinomial.

16. $10y^4 - 3y^2 + 11$ is a polynomial that is a fourth degree trinomial.

17. $$\begin{array}{r} 5a^2 - 3 \\ \underline{+\ 8a^2 - 1} \\ 13a^2 - 4 \end{array}$$

18. $(h^2 + 4h - 4) + (5h^2 - 8h + 2)$

$= (h^2 + 5h^2) + (4h - 8h) + (-4 + 2)$

$= 6h^2 - 4h - 2$

19. $$\begin{array}{r} 4m^2 - \;\;m + 2 \\ \underline{+\ -3m^2 + 10m + 7} \\ m^2 + \;9m + 9 \end{array}$$

20. $$\begin{array}{r} 7k^2 + \;\;2k - 6 \\ \underline{+\ 3k^2 - 11k - 8} \\ 10k^2 - 9k - 14 \end{array}$$

21. $(6c^2 + 3c + 9) - (3c - 5) = 6c^2 + 3c + 9 - 3c + 5$

$= 6c^2 + (3c - 3c) + (9 + 5) = 6c^2 + 14$

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22. $(3x^2 - 8) - (4x^3 + x^2 - 15x + 1)$
$= 3x^2 - 8 - 4x^3 - x^2 + 15x - 1$
$= -4x^3 + (3x^2 - x^2) + 15x + (-8 - 1)$
$= -4x^3 + 2x^2 + 15x - 9$

23. $$\begin{array}{r} (-n^2 + 2n \quad\quad\;\;) \\ -\,(2n^3 - n^2 + \;\; n + 12) \\ \hline \end{array} \rightarrow \begin{array}{r} -n^2 + 2n \quad\quad\;\; \\ +\; -2n^3 + n^2 - \;n - 12 \\ \hline -2n^3 \quad\quad\;\; + \;\; n - 12 \end{array}$$
$= -2n^3 + n - 12$

24. $$\begin{array}{r} (9b^3 - 13b^2 + b) \quad\quad\;\; \\ -(\quad -13b^2 - 5b + 14) \\ \hline \end{array} \rightarrow \begin{array}{r} 9b^3 - 13b^2 + \;\; b \quad\quad\; \\ + \quad\quad\;\; 13b^2 + 5b - 14 \\ \hline 9b^3 \quad\quad\quad\;\; + 6b - 14 \end{array}$$
$= 9b^3 + 6b - 14$

25. $(4d - 6d^3 + 3d^2) - (9d^3 + 7d - 2)$
$= 4d - 6d^3 + 3d^2 - 9d^3 - 7d + 2$
$= (-6d^3 - 9d^3) + 3d^2 + (4d - 7d) + 2$
$= -15d^3 + 3d^2 - 3d + 2$

26. $(9p^2 - 6p^3 + 3 - 11p) + (7p^3 - 3p^2 + 4)$
$= (-6p^3 + 7p^3) + (9p^2 - 3p^2) - 11p + (3 + 4)$
$= p^3 + 6p^2 - 11p + 7$

27. The like terms in the trinomials were not aligned, so the terms were added incorrectly.
$$\begin{array}{r} x^3 - 4x^2 \quad\quad\;\; + 3 \\ +\; -3x^3 \quad\quad\;\; + 8x - 2 \\ \hline -2x^3 - 4x^2 + 8x + 1 \end{array}$$

28. The negative sign wasn't distributed in the second polynomial before the terms were grouped.
$(6x^2 - 5x) - (2x^2 + 3x - 2)$
$= 6x^2 - 5x - 2x^2 - 3x + 2$
$= (6x^2 - 2x^2) + (-5x - 3x) + 2$
$= 4x^2 - 8x + 2$

29. The degree of $1 - 3x + 5x^2$ is 2 , the degree of $3x^5$ is 5, the degree of $x + 2x^3 + x^2$ is 3, and the degree of $12x + 1$ is 1. In decreasing order of degree, the polynomials are $3x^5$, $x + 2x^3 + x^2$, $1 - 3x + 5x^2$, and $12x + 1$.

30. $p(x) = (2x + 6) + (x + 8) + (9x - 6)$
$= (2x + x + 9x) + (6 + 8 - 6)$
$= 12x + 8$

31. $p(x) = (2x) + (3x - 2) + (2x + 1) + (5x - 2)$
$= (2x + 3x + 2x + 5x) + (-2 + 1 - 2)$
$= 12x - 3$

32. $(3r^2s + 5rs + 3) + (-8rs^2 - 9rs - 12)$
$= 3r^2s - 8rs^2 + (5rs - 9rs) + (3 - 12)$
$= 3r^2s - 8rs^2 - 4rs - 9$

33. $$\begin{array}{r} x^2 + 11xy - 3y^2 \\ +\; -2x^2 - \;\; xy + 4y^2 \\ \hline -x^2 + 10xy + \; y^2 \end{array}$$

34. $$\begin{array}{r} (\;5mn + 3m - 9n) \\ -\,(13mn + 2m \quad\quad\;\;) \\ \hline \end{array} \rightarrow \begin{array}{r} 5mn + 3m - 9n \\ +\; -13mn - 2m \quad\quad\; \\ \hline -8mn + \;\; m - 9n \end{array}$$

35. $(8a^2b - 6a) - (2a^2b - 4b + 19)$
$= 8a^2b - 6a - 2a^2b + 4b - 19$
$= (8a^2b - 2a^2b) - 6a + 4b - 19$
$= 6a^2b - 6a + 4b - 19$

36. **a.** $x + (x + 1) = (x + x) + 1 = 2x + 1$

b. The sum $2x + 1$ will always represent an odd number because 2 times any number is even, and when 1 is added the number becomes odd.

Problem Solving

37. $$\begin{array}{rl} C = & -0.0182t^3 + 0.522t^2 - 2.59t \;\; + 47 \\ +\; -B = & \;\;\, 0.0262t^3 - 0.376t^2 + 0.574t - \;\; 9.67 \\ \hline C - B = & \;\;\, 0.0080t^3 + 0.146t^2 - 2.016t + 37.33 \end{array}$$
For 2002, let $t = 10$:
$C - B = 0.008(10)^3 + 0.146(10)^2 - 2.016(10) + 37.33$
$= 39.77$
About 39,770,000 more people camped than backpacked in 2002.

38. $$\begin{array}{rl} I = & -137.63t^2 + 2705.2t \;\; + 15{,}111 \\ +\; -D = & \quad\quad\quad\quad\quad\; -442.14t - 14{,}433 \\ \hline I - D = & -137.63t^2 + 2263.06t + \quad 678 \end{array}$$
For 2002, let $t = 12$:
$\text{I} - \text{D} = -137.63(12)^2 + 2263.06(12) + 678$
$= 8016$
The difference in average costs for a new imported car and a new domestic car in 2002 was about \$8016.

39. **a.** $T = A + J$
$= (9.5t^3 - 58t^2 + 66t + 500) + (-15t^2 + 64t + 360)$
$= 9.5t^3 + (-58t^2 - 15t^2) + (66t + 64t) + (500 + 360)$
$= 9.5t^3 - 73t^2 + 130t + 860$

b. For 2002, let $t = 4$:
$T = 9.5(4)^3 - 73(4)^2 + 130(4) + 860 = 820$
For 1998, let $t = 0$:
$T = 9.5(0)^3 - 73(0)^2 + 130(0) + 860 = 860$
860 million books were sold in 1998, while 820 million books were sold in 2002. So, more books were sold in 1998.

40. $T = B + R$
$= (-18.53t^2 + 975.8t + 48{,}140) + (80.8t + 8049)$
$= -18.53t^2 + 1056.6t + 56{,}189$
For 2012, let $t = 27$:
$B = -18.53(27)^2 + 975.8(27) + 48{,}140 \approx 60{,}978$
$T = -18.53(27)^2 + 1056.6(27) + 56{,}189 \approx 71{,}209$
$\frac{B}{T} = \frac{60{,}978}{71{,}209} \approx 0.856$
About 85.6% of students are expected to enroll in public schools in 2012.

41. **a.** $D = W + L = (-0.44t^2 + 34t + 4.7) + (15t + 15)$
$= -0.44t^2 + 49t + 19.7$

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b. Let $t = 21$:

$D = -0.44(21)^2 + 49(21) + 19.7 = 855$

Cy Young had about 855 decisions during his career.

c. Let $t = 21$:

$W = -0.44(21)^2 + 34(21) + 4.7 = 525$

Cy Young had about 525 wins during his career.

$P = \frac{\text{wins}}{\text{decisions}} = \frac{525}{855} \approx 0.61$

By finding his total wins and dividing it by his total decisions, you can conclude that about 61% of his decisions were wins.

42. a. Let t represent the number of years since 1970, P represent the total U.S. energy production (in quadrillion BTU), and the C represent the total U.S. energy consumption (in quadrillion BTU).

Total amount	=	Initial amount	+	Rate of increase	•	Number of years
P	=	63.5	+	0.2813	•	t
C	=	67.86	+	0.912	•	t

Two equations for total energy production and consumption are $P = 0.2813t + 63.5$ and $C = 0.912t + 67.86$.

b. $C - P = 0.912t + 67.86 - (0.2813t + 63.5)$

$= (0.912t - 0.2813t) + (67.86 - 63.5)$

$= 0.6307t + 4.36$

For 1970, let $t = 0$:

$C - P = 0.6307(0) + 4.36 = 4.36$

In 1970, 4.36 quadrillion more BTU of energy was consumed than produced in the U.S.

For 2001, let $t = 31$:

$C - P = 0.6307(31) + 4.36 \approx 23.91$

In 2001, about 23.91 quadrillion more BTU of energy was consumed than produced in the U.S.

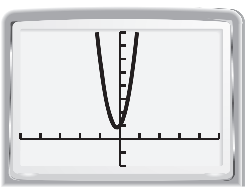
The change in the amount of energy consumed from 1970 to 2001 was about $(0.912(31) + 67.86) - (0.912(0) + 67.86) = 28.272$ quadrillion BTU.

Graphing Calculator Activity for the lesson "Add and Subtract Polynomials"

1. $(6x^2 + 4x - 1) + (x^2 - 2x + 2)$

$= (6x^2 + x^2) + (4x - 2x) + (-1 + 2)$

$= 7x^2 + 2x + 1$



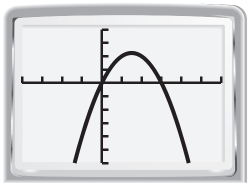
The thick curve coincides with the thin curve, so the sum is correct.

2. $(3x^2 - 2x + 1) - (4x^2 - 5x + 1)$

$= 3x^2 - 2x + 1 - 4x^2 + 5x - 1$

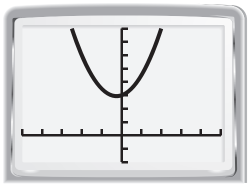
$= (3x^2 - 4x^2) + (-2x + 5x) + (1 - 1)$

$= -x^2 + 3x$



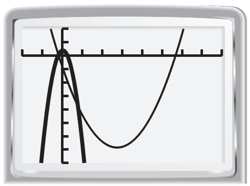
The thick curve coincides with the thin curve, so the difference is correct.

3.



The thick curve coincides with the thin curve, so the sum is correct.

4.



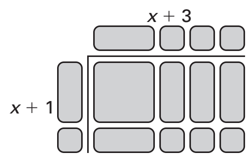
The thick curve does not coincide with the thin curve so the difference is incorrect.

The correct answer is $(-4x^2 - 5x - 1) - (-5x^2 + 6x + 3)$
$= -4x^2 - 5x - 1 + 5x^2 - 6x - 3 = x^2 - 11x - 4.$

Lesson 8.2 Multiply Polynomials

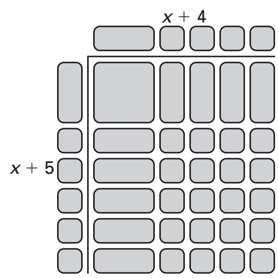
Investigating Algebra Activity for the lesson "Multiply Polynomials"

1.



$(x + 1)(x + 3) = x^2 + 4x + 3$

2.



$(x + 5)(x + 4) = x^2 + 9x + 20$

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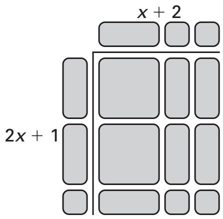
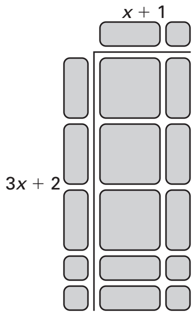
3.



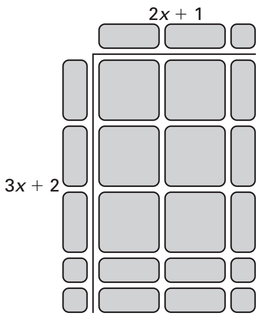
$(2x + 1)(x + 2)$
$= 2x^2 + 5x + 2$

4.



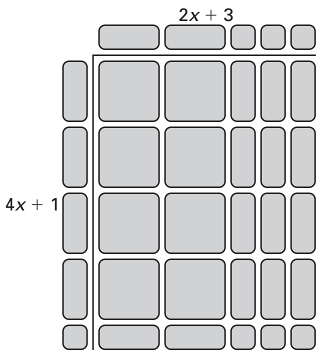
$(3x + 2)(x + 1)$
$= 3x^2 + 5x + 2$

5.



$(3x + 2)(2x + 1) = 6x^2 + 7x + 2$

6.



$(4x + 1)(2x + 3) = 8x^2 + 14x + 3$

7. $x(2x + 1) = 2x(x) + x = 2x^2 + x$

$3(2x + 1) = 3(2x) + 3(1) = 6x + 3$

$2x^2 + x + 6x + 3 = 2x^2 + 7x + 3$

The answer suggests you can find the product $(x + 3)(2x + 1)$ by multiplying one polynomial by each term of the other polynomial and adding the products.

Guided Practice for the lesson "Multiply Polynomials"

1. $x(7x^2 + 4) = x(7x^2) + x(4) = 7x^3 + 4x$
2. $(a + 3)(2a + 1) = 2a^2 + a + 6a + 3 = 2a^2 + 7a + 3$

	$2a$	1
a	$2a^2$	a
3	$6a$	3

3. $(4n - 1)(n + 5) = [4n + (-1)](n + 5)$

	n	5
$4n$	$4n^2$	$20n$
-1	$-n$	-5

$(4n - 1)(n + 5) = 4n^2 + 20n - n - 5 = 4n^2 + 19n - 5$

4.
$$\begin{array}{r} x^2 + 2x + 1 \\ x + 2 \\ \hline 2x^2 + 4x + 2 \\ x^3 + 2x^2 + x \phantom{{}+2} \\ \hline x^3 + 4x^2 + 5x + 2 \end{array}$$

5. $(3y^2 - y + 5)(2y - 3)$
$= 3y^2(2y - 3) - y(2y - 3) + 5(2y - 3)$
$= 6y^3 - 9y^2 - 2y^2 + 3y + 10y - 15$
$= 6y^3 - 11y^2 + 13y - 15$

6. $(4b - 5)(b - 2)$
$= 4b(b) + 4b(-2) + (-5)(b) + (-5)(-2)$
$= 4b^2 + (-8b) + (-5b) + 10$
$= 4b^2 - 13b + 10$

7. C; Area $=$ length • width
$= (x + 5)(x + 9)$
$= x^2 + 9x + 5x + 45$
$= x^2 + 14x + 45$

8. a. Area $=$ length • width
$= (2x + 10)(2x + 9)$
$= 4x^2 + 18x + 20x + 90$
$= 4x^2 + 38x + 90$

b. When $x = 4$: $A = 4(4)^2 + 38(4) + 90$
$= 64 + 152 + 90$
$= 306$

The combined area is 306 square feet when the width of the walkway is 4 feet.

Exercises for the lesson "Multiply Polynomials"

Skill Practice

1. The FOIL pattern can be used to multiply any two *binomials*.
2. The letters in FOIL stand for First, Outer, Inner, and Last, and these represent the order in which to multiply the terms in the binomials.
3. $x(2x^2 - 3x + 9) = x(2x^2) + x(-3x) + x(9)$
$= 2x^3 - 3x^2 + 9x$
4. $4y(-y^3 - 2y - 1) = 4y(-y^3) + 4y(-2y) + 4y(-1)$
$= -4y^4 - 8y^2 - 4y$
5. $z^2(4z^4 + z^3 - 11z^2 - 6)$
$= z^2(4z^4) + z^2(z^3) + z^2(-11z^2) + z^2(-6)$
$= 4z^6 + z^5 - 11z^4 - 6z^2$

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6. $3c^3(8c^4 - c^2 - 3c + 5)$
$= 3c^3(8c^4) + 3c^3(-c^2) + 3c^3(-3c) + 3c^3(5)$
$= 24c^7 - 3c^5 - 9c^4 + 15c^3$

7. $-a^5(-9a^2 + 5a + 13)$
$= -a^5(-9a^2) + (-a^5)(5a) + (-a^5)(13)$
$= 9a^7 - 5a^6 - 13a^5$

8. $-5b^3(4b^5 - 2b^3 + b - 11)$
$= -5b^3(4b^5) + (-5b^3)(-2b^3) + (-5b^3)(b) + (-5b^3)(-11)$
$= -20b^8 + 10b^6 - 5b^4 + 55b^3$

9. $(x + 2)(x - 3)$

	x	-3
x	x^2	$-3x$
2	$2x$	-6

$(x + 2)(x - 3) = x^2 - 3x + 2x - 6 = x^2 - x - 6$

10. $(y - 5)(2y + 3)$

	$2y$	3
y	$2y^2$	$3y$
-5	$-10y$	-15

$(y - 5)(2y + 3) = 2y^2 + 3y - 10y - 15$
$= 2y^2 - 7y - 15$

11. $(4b - 3)(b - 7)$

	b	-7
$4b$	$4b^2$	$-28b$
-3	$-3b$	$+21$

$(4b - 3)(b - 7) = 4b^2 - 28b - 3b + 21$
$= 4b^2 - 31b + 21$

12. $(5s + 2)(s + 8)$

	s	8
$5s$	$5s^2$	$40s$
2	$2s$	16

$(5s + 2)(s + 8) = 5s^2 + 40s + 2s + 16$
$= 5s^2 + 42s + 16$

13. $(3k - 1)(4k + 9)$

	$4k$	9
$3k$	$12k^2$	$27k$
-1	$-4k$	-9

$(3k - 1))(4k + 9) = 12k^2 + 27k - 4k - 9$
$= 12k^2 + 23k - 9$

14. $(8n - 5)(3n - 6)$

	$3n$	-6
$8n$	$24n^2$	$-48n$
-5	$-15n$	30

$(8n - 5)(3n - 6) = 24n^2 - 48n - 15n + 30$
$= 24n^2 - 63n + 30$

15. A 5 rather than a -5 was used in the table of products. $(x - 5)(3x + 1)$ should have been rewritten as $[x + (-5)](3x + 1)$.

	$3x$	1
x	$3x^2$	x
-5	$-15x$	-5

$(x - 5)(3x + 1) = 3x^2 + x - 15x - 5 = 3x^2 - 14x - 5$

16. The multiplication was done correctly, but when the sum of the products was found, the exponents were added together instead of just the coefficients.

$$\begin{array}{r} 2x^2 - 3x - 4 \\ x + 7 \\ \hline 14x^2 - 21x - 28 \\ 2x^3 - 3x^2 - 4x \quad\quad \\ \hline 2x^3 + 11x^2 - 25x - 28 \end{array}$$

17. $(y + 6)(y - 5) = y(y + 6) - 5(y + 6)$
$= y^2 + 6y - 5y - 30$
$= y^2 + y - 30$

18. $(5x - 8)(2x - 5) = 2x(5x - 8) - 5(5x - 8)$
$= 10x^2 - 16x - 25x + 40$
$= 10x^2 - 41x + 40$

19. $(7w + 5)(11w - 3) = 11w(7w + 5) - 3(7w + 5)$
$= 77w^2 + 55w - 21w - 15$
$= 77w^2 + 34w - 15$

20. $(b - 2)(b^2 - b + 1) = b^2(b - 2) - b(b - 2) + 1(b - 2)$
$= b^3 - 2b^2 - b^2 + 2b + b - 2$
$= b^3 - 3b^2 + 3b - 2$

21. $(s + 4)(s^2 + 6s - 5) = s^2(s + 4) + 6s(s + 4) - 5(s + 4)$
$= s^3 + 4s^2 + 6s^2 + 24s - 5s - 20$
$= s^3 + 10s^2 + 19s - 20$

22. $(-r + 7)(2r^2 - r - 9)$
$= 2r^2(-r + 7) - r(-r + 7) - 9(-r + 7)$
$= -2r^3 + 14r^2 + r^2 - 7r + 9r - 63$
$= -2r^3 + 15r^2 + 2r - 63$

23. $(5x + 2)(-3x^2 + 4x - 1)$
$= -3x^2(5x + 2) + 4x(5x + 2) - 1(5x + 2)$
$= -15x^3 - 6x^2 + 20x^2 + 8x - 5x - 2$
$= -15x^3 + 14x^2 + 3x - 2$

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24.
$$\begin{array}{r} y^2 + 8y - 6 \\ 4y - 3 \\ \hline -3y^2 - 24y + 18 \\ 4y^3 + 32y^2 - 24y \quad \\ \hline 4y^3 + 29y^2 - 48y + 18 \end{array}$$

25.
$$\begin{array}{r} 6z^2 + z - 1 \\ 9z - 5 \\ \hline -30z^2 - 5z + 5 \\ 54z^3 + 9z^2 - 9z \quad \\ \hline 54z^3 - 21z^2 - 14z + 5 \end{array}$$

26. B; $(2x - 9)(4x + 1) = 2x(4x) + 2x(1) - 9(4x) - 9(1)$
$= 8x^2 + 2x - 36x - 9$
$= 8x^2 - 34x - 9$

27. $(2r - 1)(5r + 3) = 2r(5r) + 2r(3) - 1(5r) - 1(3)$
$= 10r^2 + 6r - 5r - 3$
$= 10r^2 + r - 3$

28. $(7a - 2)(3a - 4) = 7a(3a) + 7a(-4) - 2(3a) - 2(-4)$
$= 21a^2 - 28a - 6a + 8$
$= 21a^2 - 34a + 8$

29. $(4m + 9)(2m + 7) = 4m(2m) + 4m(7) + 9(2m) + 9(7)$
$= 8m^2 + 28m + 18m + 63$
$= 8m^2 + 46m + 63$

30. $(8t + 11)(6t - 1) = 8t(6t) + 8t(-1) + 11(6t) + 11(-1)$
$= 48t^2 - 8t + 66t - 11$
$= 48t^2 + 58t - 11$

31. $(4x - 5)(12x - 7)$
$= 4x(12x) + 4x(-7) - 5(12x) - 5(-7)$
$= 48x^2 - 28x - 60x + 35$
$= 48x^2 - 88x + 35$

32. $(8z + 3)(5z + 4) = 8z(5z) + 8z(4) + 3(5z) + 3(4)$
$= 40z^2 + 32z + 15z + 12$
$= 40z^2 + 47z + 12$

33. $p(2p - 3) + (p - 3)(p + 3)$
$= p(2p) + p(-3) + p(p) + p(3) - 3(p) - 3(3)$
$= 2p^2 - 3p + p^2 + 3p - 3p - 9$
$= 3p^2 - 3p - 9$

34. $x^2(7x + 5) - (2x + 6)(x - 1)$
$= x^2(7x) + x^2(5) - [2x(x) + 2x(-1) + 6(x) + 6(-1)]$
$= 7x^3 + 5x^2 - (2x^2 - 2x + 6x - 6)$
$= 7x^3 + 5x^2 - 2x^2 - 4x + 6$
$= 7x^3 + 3x^2 - 4x + 6$

35. $-3c^2(c + 11) - (4c - 5)(3c - 2)$
$= -3c^2(c) - 3c^2(11) - [4c(3c) + 4c(-2) - 5(3c)$
$- 5(-2)]$
$= -3c^3 - 33c^2 - (12c^2 - 8c - 15c + 10)$
$= -3c^3 - 33c^2 - 12c^2 + 23c - 10$
$= -3c^3 - 45c^2 + 23c - 10$

36. $2w^3(2w^3 - 7w - 1) + w(5w^2 + 2w)$
$= 2w^3(2w^3) + 2w^3(-7w) + 2w^3(-1) + w(5w^2)$
$+ w(2w)$
$= 4w^6 - 14w^4 - 2w^3 + 5w^3 + 2w^2$
$= 4w^6 - 14w^4 + 3w^3 + 2w^2$

37. Area = length • width
$= (2x - 9)(x + 5)$
$= 2x^2 + 10x - 9x - 45$
$= 2x^2 + x - 45$

38. Area = length • width
$= (x + 6)(2x)$
$= 2x(x) + 2x(6)$
$= 2x^2 + 12x$

39. Area = length • width
$= (x + 5)(x + 3)$
$= x^2 + 3x + 5x + 15$
$= x^2 + 8x + 15$

40. Area = Area of rectangle − Area of triangle
= length • width $- \frac{1}{2}$ • base • height
$= (x + 6)(x + 5) - \frac{1}{2}(x + 6)(x + 5)$
$= \frac{1}{2}(x + 6)(x + 5)$
$= \frac{1}{2}(x^2 + 5x + 6x + 30)$
$= \frac{1}{2}(x^2 + 11x + 30)$
$= \frac{1}{2}x^2 + \frac{11}{2}x + 15$

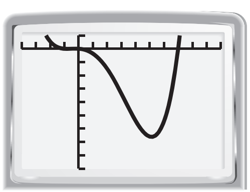
41. Area = Area of outer rectangle
− Area of inner rectangle
= length • width − length • width
$= 10 \cdot 8 - 3x \cdot 2x$
$= 80 - 6x^2$
$= -6x^2 + 80$

42. Area = Area of outer rectangle − Area of inner rectangle
= length • width − length • width
$= (x + 1)(x + 1) - (x - 7)(5)$
$= x^2 + x + x + 1 - 5x + 35$
$= x^2 - 3x + 36$

43. $f(x) \cdot g(x) = (x - 11)(2x + 12)$
$= 2x^2 + 12x - 22x - 132$
$= 2x^2 - 10x - 132$

44. C;
$f(x) \cdot g(x) = -2x^2(x^3 - 5x^2 + 2x - 1)$
$= -2x^2(x^3) - 2x^2(-5x^2) - 2x^2(2x)$
$- 2x^2(-1)$
$= -2x^5 + 10x^4 - 4x^3 + 2x^2$

45. $(x^2 - 7x)(2x^2 + 3x + 1)$
$= 2x^2(x^2 - 7x) + 3x(x^2 - 7x) + 1(x^2 - 7x)$
$= 2x^4 - 14x^3 + 3x^3 - 21x^2 + x^2 - 7x$
$= 2x^4 - 11x^3 - 20x^2 - 7x$



The thin curve represents $y = (x^2 - 7x)(2x^2 + 3x + 1)$ and the thick curve represents $y = 2x^4 - 11x^3 - 20x^2 - 7x$. Because the graphs coincide, the product is correct.

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46. $(x - y)(3x + 4y) = x(3x) + x(4y) - y(3x) - y(4y)$

$= 3x^2 + 4xy - 3xy - 4y^2$

$= 3x^2 + xy - 4y^2$

47. $(x^2y + 9y)(2x + 3y)$

$= x^2y(2x) + x^2y(3y) + 9y(2x) + 9y(3y)$

$= 2x^3y + 3x^2y^2 + 18xy + 27y^2$

48. $(x^2 - 5xy + y^2)(4xy) = x^2(4xy) - 5xy(4xy) + y^2(4xy)$

$= 4x^3y - 20x^2y^2 + 4xy^3$

Problem Solving

49. a. Area = length • width

$= (2x + 20)(2x + 22)$

$= 4x^2 + 44x + 40x + 440$

$= 4x^2 + 84x + 440$

b. When $x = 4$: Area $= 4(4)^2 + 84(4) + 440 = 840$

When the width of the frame is 4 inches, the combined area is 840 square inches.

50. a. Area = length • width

$= (x + 40)(2x + 20)$

$= 2x^2 + 20x + 80x + 800$

$= 2x^2 + 100x + 800$

b. When $x = 5$: Area $= 2(5)^2 + 100(5) + 800$

$= 50 + 500 + 800$

$= 1350$

When the walkway is 5 feet wide, the combined area is 1350 square feet.

51. a. When $t = 0$:

$R = -336(0)^2 + 1730(0) + 12{,}300 = 12{,}300$

$P = 0.00351(0)^2 - 0.0249(0) + 0.171 = 0.171$

When $t = 0$, $R \cdot P$ represents the amount of money (in millions of dollars) that people between the ages 15 and 19 years old spent on sound recordings in the U.S. in 1997.

b. $T = R \cdot P$

$$\begin{array}{rr} = & -336t^2 \quad + \quad 1730t + 12{,}300 \\ & \underline{0.00351t^2 - 0.0249t + 0.171} \\ & -57.46t^2 + 295.83t + 2103.3 \\ & 8.37t^3 - 43.08t^2 - 306.27t \\ & \underline{-1.18t^4 + 6.07t^3 + 43.17t^2} \end{array}$$

$T = -1.18t^4 + 14.4t^3 - 57.4t^2 - 10.4t + 2103.3$

c. For 2002, let $t = 5$:

$T = -1.18(5)^4 + 14.4(5)^3 - 57.4(5)^2 - 10.4(5)$

$+ 2103.3 = 1678.8$

In 2002, people between 15 and 19 years old spent about \$1678.8 million on sound recordings in the U.S.

52. a. $N = H \cdot P$

$= (1570t + 89{,}000)(0.0013t + 0.094)$

$= 1570t(0.0013t) + 1570t(0.094) + 89{,}000(0.0013t)$

$+ 89{,}000(0.094)$

$= 2.041t^2 + 147.58t + 115.7t + 8366$

$= 2.041t^2 + 263.28t + 8366$

The number of vacant houses is found by multiplying the percent of housing units that are vacant by the total number of housing units.

b. For 2002, let $t = 22$: $N = 2.041(22)^2 + 263.28(22)$

$+ 8366 \approx 15{,}146$

About 15,146 housing units were vacant in 2002.

53. a. $T = t + 90$; The data points (5, 95), (6, 96), (7, 97), (8, 98), and (9, 99) lie along a line that has a slope of 1. Using one of the points and the slope, the y-intercept is found to be 90. The other points, (0, 92), (10, 101), and (11, 102), lie close to the line $T = t + 90$.

b. $V = T \cdot P$

$= (t + 90)(-0.0015t^2 + 0.032t + 0.069)$

$= -0.0015t^2(t + 90) + 0.032t(t + 90) + 0.069$

$(t + 90)$

$= -0.0015t^3 - 0.135t^2 + 0.032t^2 + 2.88t + 0.069t$

$+ 6.21$

$= -0.0015t^3 - 0.103t^2 + 2.949t + 6.21$

c. For 2002, let $x = 12$:

$V = -0.0015(12)^3 - 0.103(12)^2 + 2.949(12)$

$+ 6.21 \approx 24.17$

About 24.17 million households had a VCR and a television in 2002.

For 2005, let $x = 15$:

$= -0.0015(15)^3 - 0.103(15)^2 + 2.949(15)$

$+ 6.21 \approx 22.21$

About 22.21 million households had a VCR and a television in 2005.

54. a. For 2000, let $t = 10$:

$P = -0.05(10)^2 + 0.25(10) + 38 = 35.5$

In 2000, 35.5% of the total energy consumed in the U.S. was consumed for industrial purposes.

b. P must be multipled by 0.01 so that it represents a percent that is written in decimal form.

$T = C \cdot 0.01P$

$= (1.5t + 84)[0.01(-0.05t^2 + 0.25t + 38)]$

$= (1.5t + 84)(-0.0005t^2 + 0.0025t + 0.38)$

$\approx -0.00075t^3 - 0.04t^2 + 0.78t + 32$

Lesson 8.3 Find Special Products of Polynomials

Guided Practice for the lesson "Find Special Products of Polynomials"

1. $(x + 3)^2 = x^2 + 2(x)(3) + 3^2 = x^2 + 6x + 9$

2. $(2x + 1)^2 = (2x)^2 + 2(2x)(1) + 1^2 = 4x^2 + 4x + 1$

3. $(4x - y)^2 = (4x)^2 - 2(4x)(y) + (y)^2 = 16x^2 - 8xy + y^2$

4. $(3m + n)^2 = (3m)^2 + 2(3m)(n) + n^2 = 9m^2 + 6mn + n^2$

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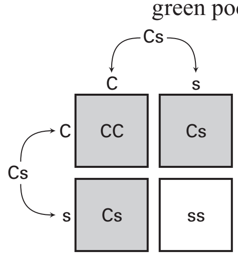
5. $(x + 10)(x - 10) = x^2 - 10^2 = x^2 - 100$
6. $(2x + 1)(2x - 1) = (2x)^2 - 1^2 = 4x^2 - 1$
7. $(x + 3y)(x - 3y) = x^2 - (3y)^2 = x^2 - 9y^2$
8. Use the square of a binomial pattern to find the product $(20 + 1)^2$.
9. 25% result in red patches.

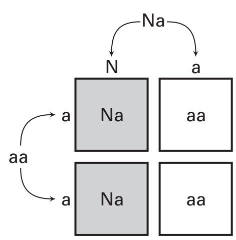
Exercises for the lesson "Find Special Products of Polynomials"

Skill Practice

1. *Sample answer:* $x + 9, x - 9$
2. Square the first term in the binomial, add 2 times the product of the two terms, and then add the square of the second term.
3. $(x + 8)^2 = x^2 + 2(x)(8) + 8^2 = x^2 + 16x + 64$
4. $(a + 6) = a^2 + 2(a)(6) + 6^2 = a^2 + 12a + 36$
5. $(2y + 5)^2 = (2y)^2 + 2(2y)(5) + 5^2 = 4y^2 + 20y + 25$
6. $(t - 7)^2 = t^2 - 2(t)(7) + (7)^2 = t^2 - 14t + 49$
7. $(n - 11)^2 = n^2 - 2(n)(11) + (11)^2 = n^2 - 22n + 121$
8. $(6b - 1)^2 = (6b)^2 - 2(6b)(1) + (1)^2 = 36b^2 - 12b + 1$
9. The middle term in the trinomial product is missing, $(s - 3)^2 = s^2 - 2(s)(3) + (3)^2 = s^2 - 6s + 9$
10. The second term was not multiplied by 2.
$(2d - 10)^2 = 4d^2 - 2(2d)(10) + (10)^2$
$= 4d^2 - 40d + 100$
11. $(t + 4)(t - 4) = t^2 - 4^2 = t^2 - 16$
12. $(m - 6)(m + 6) = m^2 - 6^2 = m^2 - 36$
13. $(2x + 1)(2x - 1) = (2x)^2 - 1^2 = 4x^2 - 1$
14. $(3x - 1)(3x + 1) = (3x)^2 - 1^2 = 9x^2 - 1$
15. $(7 + w)(7 - w) = 7^2 - w^2 = 49 - w^2$
16. $(3s - 8)(3s + 8) = (3s)^2 - 8^2 = 9s^2 - 64$
17. *B*; $(7x + 3)(7x - 3) = (7x)^2 - 3^2 = 49x^2 - 9$
18. *D*; $(5n - 3)^2 = (5n)^2 - 2(5n)(3) + (3)^2$
$= 25n^2 - 30n + 9$
19. *Sample answer:* 16 • 24 can be written as the sum and difference pattern $(20 - 4)(20 + 4)$.
20. *Sample answer:* 28 • 32 can be written as the sum and difference pattern $(30 - 2)(30 + 2)$.
21. *Sample answer:* 17^2 can be written as the square of a binomial pattern $(20 - 3)^2$.
22. *Sample answer:* 44^2 can be written as the square of a binomial pattern $(40 + 4)^2$.
23. $(r + 9s)^2 = r^2 + 2(r)(9s) + (9s)^2 = r^2 + 18rs + 81s^2$
24. $(6x + 5)^2 = (6x)^2 + 2(6x)(5) + 5^2 = 36x^2 + 60x + 25$
25. $(3m + 11n)(3m - 11n) = (3m)^2 - (11n)^2$
$= 9m^2 - 121n^2$
26. $(7a + 8b)(7a - 8b) = (7a)^2 - (8b)^2 = 49a^2 - 64b^2$
27. $(3m - 7n)^2 = (3m)^2 - 2(3m)(7n) + (7n)^2$
$= 9m^2 - 42mn + 49n^2$
28. $(13 - 2x)^2 = 13^2 - 2(13)(2x) + (2x)^2$
$= 169 - 52x + 4x^2$
29. $(3f - 9)(3f + 9) = (3f)^2 - 9^2 = 9f^2 - 81$
30. $(9 - 4t)(9 + 4t) = 9^2 - (4t)^2 = 81 - 16t^2$
31. $(3x + 8y)^2 = (3x)^2 + 2(3x)(8y) + (8y)^2$
$= 9x^2 + 48xy + 64y^2$
32. $(-x - 2y)^2 = (-x)^2 - 2(-x)(2y) + (2y)^2$
$= x^2 + 4xy + 4y^2$
33. $(2a - 5b)(2a + 5b) = (2a)^2 - (5b)^2 = 4a^2 - 25b^2$
34. $(6x + y)(6x - y) = (6x)^2 - y^2 = 36x^2 - y^2$
35. $f(x) \cdot g(x) = (3x + 0.5)(3x - 0.5) = (3x)^2 - (0.5)^2$
$= 9x^2 - 0.25$
36. $\left(f(x)\right)^2 = (3x + 0.5)^2 = (3x)^2 + 2(3x)(0.5) + (0.5)^2$
$= 9x^2 + 3x + 0.25$
37. $\left(g(x)\right)^2 = (3x - 0.5)^2 = (3x)^2 - 2(3x)(0.5) + (0.5)^2$
$= 9x^2 - 3x + 0.25$
38. $x + 11$ and $x - 11$; using the sum and difference pattern, $(x + 11)(x - 11) = x^2 - 121$.
39. $(a + b)^3 = (a + b)(a + b)^2 = (a + b)\left(a^2 + 2ab + b^2\right)$
$= a^2(a + b) + 2ab(a + b) + b^2(a + b)$
$= a^3 + a^2b + 2a^2b + 2ab^2 + ab^2 + b^3$
$= a^3 + 3a^2b + 3ab^2 + b^3$

Problem Solving

40. **a.** In the Punnett square, 1 of the 4 possible gene combinations results in a yellow pod. So, 25% of the possible gene combinations result in a yellow pod.
b. $(0.5G + 0.5y)^2 = (0.5G)^2 + 2(0.5G)(0.5y) + (0.5y)^2$
$= 0.25G^2 + 0.5Gy + 0.25y^2$
↑ 25% *GG* green pod; ↑ 50% *Gy* green pod; ↑ 25% *yy* yellow pod
41. **a.**


b. $(0.5C + 0.5s)^2 = (0.5C)^2 + 2(0.5C)(0.5s) + (0.5s)^2$
$= 0.25C^2 + 0.5Cs + 0.25s^2$
c. 25% + 50% = 75% of the possible gene combinations will result in a curved thumb.
42. 50%; In the Punnett square, half of the possible combinations are *aa*, which result in an albino python.


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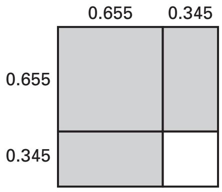
43. a. Find the sum of the areas of the regions that contain at least one complete pass.



Area $= (0.655)^2 + (0.655)(0.345) + (0.345)(0.655)$

$= 0.880975$

About 88.1% of the possible outcomes result in at least one complete pass.

b. $(0.655C + 0.345I)^2 =$

$0.429025C^2 + 0.45195CI + 0.119025I^2$

$\uparrow$ 42.9% CC, 2 complete

$\uparrow$ 45.2% CI, 1 complete, 1 incomplete

$\uparrow$ 11.9% II, 2 incomplete

42.9% + 45.2% = 88.1% of the outcomes will have at least one completed pass, and 11.9% of the outcomes will have two incomplete passes.

44. a. $r = 6 - w$

b. $A = \pi r^2 = \pi(6 - w)^2 = \pi(36 - 12w + w^2)$

$= \pi w^2 - 12\pi w + 36\pi$

c. The least possible area occurs when $w = 4$:

$A = \pi(4)^2 - 12\pi(4) + 36\pi = 4\pi$

The greatest possible area occurs when $w = 0.5$:

$A = \pi(0.5)^2 - 12\pi(0.5) + 36\pi = 30.25\pi$

The least possible area is 4π mm^2 and the greatest possible area is 30.25π mm^2.

45. $A =$ length • width

$= (25 - x)(25 + x)$

$= 25^2 - x^2 = 625 - x^2$

The area of the original square was $25^2 = 625$ square feet. Because x^2 is always positive, the area of the new region will always be less than 625 square feet.

Quiz for the lessons "Add and Subtract Polynomials", Multiply Polynomials", and "Find Special Products of Polynomials"

1. $(x^2 - 3x + 5) + (-2x^2 + 11x + 1)$

$= (x^2 - 2x^2) + (-3x + 11x) + (5 + 1)$

$= -x^2 + 8x + 6$

2. $(8y^3 - 7y^2 + y) - (9y^2 - 5y + 7)$

$= 8y^3 - 7y^2 + y - 9y^2 + 5y - 7$

$= 8y^3 + (-7y^2 - 9y^2) + (y + 5y) - 7$

$= 8y^3 - 16y^2 + 6y - 7$

3. $(2r + 11)(r - 6) = 2r(r) + 2r(-6) + 11(r) + 11(-6)$

$= 2r^2 - 12r + 11r - 66$

$= 2r^2 - r - 66$

4. $(m + 3)(-2m^2 + 5m - 1)$

$= -2m^2(m + 3) + 5m(m + 3) - (m + 3)$

$= -2m^3 - 6m^2 + 5m^2 + 15m - m - 3$

$= -2m^3 - m^2 + 14m - 3$

5. $(2 + 8p)(2 - 10p)$

$= 2(2) + 2(-10p) + 8p(2) + 8p(-10p)$

$= 4 - 20p + 16p - 80p^2 = 4 - 4p - 80p^2$

6. $(15 - 2s)^2 = 15^2 - 2(15)(2s) + (2s)^2$

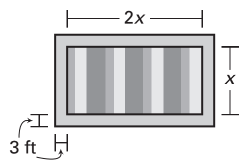
$= 225 - 60s + 4s^2$

7. $(5w + 9z)^2 = (5w)^2 + 2(5w)(9z) + (9z)^2$

$= 25w^2 + 90wz + 81z^2$

8. $(5x - 4y)(5x + 4y) = (5x)^2 - (4y)^2 = 25x^2 - 16y^2$

9.



$A =$ length • width

$= (2x + 6)(x + 6)$

$= 2x^2 + 12x + 6x + 36$

$= 2x^2 + 18x + 36$

Lesson 8.4 Solve Polynomial Equations in Factored Form

Guided Practice for the lesson "Solve Polynomial Equations in Factored Form"

1. $(x - 5)(x - 1) = 0$

$x - 5 = 0$ or $x - 1 = 0$

$x = 5$ or $x = 1$

The solutions are 5 and 1.

Check:

$(5 - 5)(5 - 1) \stackrel{?}{=} 0$ $\qquad$ $(1 - 5)(1 - 1) \stackrel{?}{=} 0$

$0 \cdot 4 \stackrel{?}{=} 0$ $\qquad$ $-4 \cdot 0 \stackrel{?}{=} 0$

$0 = 0$ ✓ $\qquad$ $0 = 0$ ✓

2. $14m + 35n = 7(2m + 5n)$

3. $a^2 + 5a = 0$

$a(a + 5) = 0$

$a = 0$ or $a + 5 = 0$

$a = -5$

The solutions are 0 and -5.

4. $3s^2 - 9s = 0$

$3s(s - 3) = 0$

$3s = 0$ or $s - 3 = 0$

$s = 0$ or $s = 3$

The solutions are 0 and 3.

5. $4x^2 - 2x = 0$

$2x(2x - 1) = 0$

$2x = 0$ or $2x - 1 = 0$

$x = 0$ or $x = \frac{1}{2}$

The solutions are 0 and $\frac{1}{2}$.

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6. $h = -16t^2 + vt + s$

$h = -16t^2 + 12t + 0$

$h = -16t^2 + 12t$

$0 = -16t^2 + 12t$

$0 = 4t(-4t + 3)$

$4t = 0$ or $-4t + 3 = 0$

$t = 0$ or $t = \frac{3}{4}$

The armadillo lands on the ground $\frac{3}{4}$ of a second after it jumps.

Exercises for the lesson "Solve Polynomial Equations in Factored Form"

Skill Practice

1. The vertical motion model is used to describe the height of a projectile. The height (in feet) is $h = -16t^2 + vt + s$ where t represents the time (in seconds), v represents the initial vertical velocity (in feet per second), and s represents the initial height (in feet).

2. Set $3x$ and $x - 7$ equal to zero and solve each equation for x.

3. $(x - 5)(x + 3) = 0$

$x - 5 = 0$ or $x + 3 = 0$

$x = 5$ or $x = -3$

The solutions are 5 and -3.

4. $(y + 9)(y - 1) = 0$

$y + 9 = 0$ or $y - 1 = 0$

$y = -9$ or $y = 1$

The solutions are -9 and 1.

5. $(z - 13)(z - 14) = 0$

$z - 13 = 0$ or $z - 14 = 0$

$z = 13$ or $z = 14$

The solutions are 13 and 14.

6. $(c + 6)(c + 8) = 0$

$c + 6 = 0$ or $c + 8 = 0$

$c = -6$ or $c = -8$

The solutions are -6 and -8.

7. $(d - 7)\left(d + \frac{4}{3}\right) = 0$

$d - 7 = 0$ or $d + \frac{4}{3} = 0$

$d = 7$ or $d = -\frac{4}{3}$

The solutions are 7 and $-\frac{4}{3}$.

8. $\left(g - \frac{1}{8}\right)(g + 18) = 0$

$g - \frac{1}{8} = 0$ or $g + 18 = 0$

$g = \frac{1}{8}$ or $g = -18$

The solutions are $\frac{1}{8}$ and -18.

9. $(m - 3)(4m + 12) = 0$

$m - 3 = 0$ or $4m + 12 = 0$

$m = 3$ or $m = -3$

The solutions are 3 and -3.

10. $(2n - 14)(3n + 9) = 0$

$2n - 14 = 0$ or $3n + 9 = 0$

$2n = 14$ or $3n = -9$

$n = 7$ or $n = -3$

The solutions are 7 and -3.

11. $(3n + 11)(n + 1) = 0$

$3n + 11 = 0$ or $n + 1 = 0$

$n = -\frac{11}{3}$ or $n = -1$

The solutions are $-\frac{11}{3}$ and -1.

12. $(3x + 1)(x + 6) = 0$

$3x + 1 = 0$ or $x + 6 = 0$

$x = -\frac{1}{3}$ or $x = -6$

The solutions are $-\frac{1}{3}$ and -6.

13. $(2y + 5)(7y - 5) = 0$

$2y + 5 = 0$ or $7y - 5 = 0$

$2y = -5$ or $7y = 5$

$y = -\frac{5}{2}$ or $y = \frac{5}{7}$

The solutions are $-\frac{5}{2}$ and $\frac{5}{7}$.

14. $(8z - 6)(12z + 14) = 0$

$8z - 6 = 0$ or $12z + 14 = 0$

$8z = 6$ or $12z = -14$

$z = \frac{3}{4}$ or $z = -\frac{7}{6}$

The solutions are $\frac{3}{4}$ and $-\frac{7}{6}$.

15. C; $(y - 12)(y + 6) = 0$

$y - 12 = 0$ or $y + 6 = 0$

$y = 12$ or $y = -6$

The roots are 12 and -6 .

16. The step of setting each factor to zero was left out.

$z - 15 = 0$ or $z + 21 = 0$

$z = 15$ or $z = -21$

17. $2x + 2y = 2(x + y)$

18. $6x^2 - 15y = 3(2x^2 - 5y)$

19. $3s^4 + 16s = s(3s^3 + 16)$

20. $5d^6 + 2d^5 = d^5(5d + 2)$

21. $7w^5 - 35w^2 = 7w^2(w^3 - 5)$

22. $9m^7 - 3m^2 = 3m^2(3m^5 - 1)$

23. $15n^3 + 25n = 5n(3n^2 + 5)$

24. $12a^5 + 8a = 4a(3a^4 + 2)$

25. $\frac{5}{2}x^6 - \frac{1}{2}x^4 = \frac{1}{2}x^4(5x^2 - 1)$

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26. The GCF is $3x^3$, not $3x$.

$18x^8 - 9x^4 - 6x^3 = 3x^3(6x^5 - 3x - 2)$

27. $b^2 + 6b = 0$

$b(b + 6) = 0$

$b = 0$ or $b + 6 = 0$

$b = -6$

The solutions are 0 and -6.

28. $5w^2 - 5w = 0$

$5w(w - 1) = 0$

$5w = 0$ or $w - 1 = 0$

$w = 0$ or $w = 1$

The solutions are 0 and 1.

29. $-10n^2 + 35n = 0$

$5n(-2n + 7) = 0$

$5n = 0$ or $-2n + 7 = 0$

$n = 0$ or $n = \frac{7}{2}$

The solutions are 0 and $\frac{7}{2}$.

30. $2x^2 + 15x = 0$

$x(2x + 15) = 0$

$x = 0$ or $2x + 15 = 0$

$x = -\frac{15}{2}$

The solutions are 0 and $-\frac{15}{2}$.

31. $18c^2 + 6c = 0$

$6c(3c + 1) = 0$

$6c = 0$ or $3c + 1 = 0$

$c = 0$ or $c = -\frac{1}{3}$

The solutions are 0 and $-\frac{1}{3}$.

32. $-32y^2 - 24y = 0$

$-8y(4y + 3) = 0$

$-8y = 0$ or $4y + 3 = 0$

$y = 0$ or $y = -\frac{3}{4}$

The solutions are 0 and $-\frac{3}{4}$.

33. $3k^2 = 6k$

$3k^2 - 6k = 0$

$3k(k - 2) = 0$

$3k = 0$ or $k - 2 = 0$

$k = 0$ or $k = 2$

The solutions are 0 and 2.

34. $6h^2 = 3h$

$6h^2 - 3h = 0$

$3h(2h - 1) = 0$

$3h = 0$ or $2h - 1 = 0$

$h = 0$ or $h = \frac{1}{2}$

The solutions are 0 and $\frac{1}{2}$.

35. $4s^2 = 10s$

$4s^2 - 10s = 0$

$2s(2s - 5) = 0$

$2s = 0$ or $2s - 5 = 0$

$s = 0$ or $s = \frac{5}{2}$

The solutions are 0 and $\frac{5}{2}$.

36. $-42z^2 = 14z$

$-42z^2 - 14z = 0$

$-14z(3z + 1) = 0$

$14z = 0$ or $3z + 1 = 0$

$z = 0$ or $z = -\frac{1}{3}$

The solutions are 0 and $-\frac{1}{3}$.

37. $28m^2 = -8m$

$28m^2 + 8m = 0$

$4m(7m + 2) = 0$

$4m = 0$ or $7m + 2 = 0$

$m = 0$ or $m = -\frac{2}{7}$

The solutions are 0 and $-\frac{2}{7}$.

38. $-12p^2 = -30p$

$-12p^2 + 30p = 0$

$6p(-2p + 5) = 0$

$6p = 0$ or $-2p + 5 = 0$

$p = 0$ or $p = \frac{5}{2}$

The solutions are 0 and $\frac{5}{2}$.

39. C; $4x^2 = x$

$4x^2 - x = 0$

$x(4x - 1) = 0$

$x = 0$ or $4x - 1 = 0$

$x = \frac{1}{4}$

The solutions are 0 and $\frac{1}{4}$.

40. $20x^2y^2 - 4xy = 4xy(5xy - 1)$

41. $8a^2b - 6ab^2 = 2ab(4a - 3b)$

42. $18s^2t^5 - 2s^3t = 2s^2t(9t^4 - s)$

43. $v^3 - 5v^2 + 9v = v(v^2 - 5v + 9)$

44. $-2g^4 + 14g^2 + 6g = -2g(g^3 - 7g - 3)$

45. $6q^5 - 21q^4 - 15q^2 = 3q^2(2q^3 - 7q^2 - 5)$

46. $f(x) = x^2 - 15x$

$0 = x^2 - 15x$

$0 = x(x - 15)$

$x = 0$ or $x - 15 = 0$

$x = 15$

The zeros are 0 and 15.

47. $f(x) = -2x^2 + x$

$0 = -2x^2 + x$

$0 = x(-2x + 1)$

$x = 0$ or $-2x + 1 = 0$

$x = \frac{1}{2}$

The zeros are 0 and $\frac{1}{2}$.

48. $f(x) = 3x^2 - 27x$

$0 = 3x^2 - 27x$

$0 = 3x(x - 9)$

$3x = 0$ or $x - 9 = 0$

$x = 0$ or $x = 9$

The zeros are 0 and 9.

49. $ab = 0$ or $ab = 0$

$b = \frac{0}{a}$ $a = \frac{0}{b}$

$b = 0$ $a = 0$

Either $a = 0$ or $b = 0$.

50. $z = x^2 - xy$

$0 = x^2 - xy$

$0 = x(x - y)$

$x = 0$ or $x - y = 0$

$x = y$

Either $x = 0$ and $y =$ any real number, or x and y are any real numbers where $x = y$.

51. Let $v = 11$ and $s = 0$.

$h = -16t^2 + vt + s$

$0 = -16t^2 + 11t$

$0 = t(-16t + 11)$

$t = 0$ or $-16t + 11 = 0$

$t = \frac{11}{16}$

The cat lands on the ground $\frac{11}{16}$ of a second after it jumps.

52. a. Let $v = 10$ and $s = 0$.

$h = -16t^2 + vt + s$

$h = -16t^2 + 10t$

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b. When $t = 0.3125$

$h = -16(0.3125)^2 + 10(0.3125)$

$= -1.5625 + 3.125$

$= 1.5625$

The spittle bug can jump 1.5625 feet.

53. $h = -16t^2 + 4.5t$

$0 = -16t^2 + 4.5t$

$0 = t(-16t + 4.5)$

$t = 0$ or $-16t + 4.5 = 0$

$-16t = -4.5$

$t = 0.28125$

The zeros are 0 and 0.28125. These are the times (in seconds) when the penguin leaves the water and reenters the water.

54. Let $v = 3.6$ and $s = 0$.

$h = -4.9t^2 + vt + s$

$h = -4.9t^2 + 3.6t$

$0 = -4.9t^2 + 3.6t$

$0 = t(-4.9t + 3.6)$

$t = 0$ or $-4.9t + 3.6 = 0$

$t \approx 0.73$

The ball lands after about 0.73 second.

55. a. Let $v = 4.9$ and $s = 0$.

$h = -4.9t^2 + vt + s$

$h = -4.9t^2 + 4.9t$

b. The rabbit lands when $h = 0$.

$0 = -4.9t^2 + 4.9t$

$0 = 4.9t(-t + 1)$

$4.9t = 0$ or $-t + 1 = 0$

$t = 0$ or $t = 1$

The rabbit lands 1 second after it leaves the ground, so a reasonable domain is $0 \le t \le 1$.

56. B; $w(w + 2) = 2w(w - 1)$

$w^2 + 2w = 2w^2 - 2w$

$-w^2 + 4w = 0$

$w(-w + 4) = 0$

$w = 0$ or $-w + 4 = 0$

$w = 4$

57. a. An equation that relates the areas of the tabletops is $w(w + 2) = w(10 - w)$.

b. $w(w + 2) = w(10 - w)$

$w^2 + 2w = 10w - w^2$

$2w^2 - 8w = 0$

$2w(w - 4) = 0$

$2w = 0$ or $w - 4 = 0$

$w = 0$ or $w = 4$

The value of w is 4 feet.

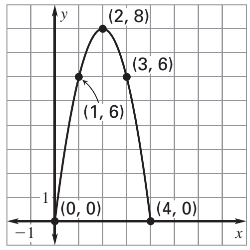
c. Area $= 2[w(2 + w)] = 2(4)(2 + 4) = 48$

The combined area of the tabletops is 48 square feet.

58. a.

x	0	1	2	3	4
y	0	6	8	6	0

b.



c. The distance between the zeros is 4, so the base is 4 feet wide.

59. a. $y = -0.5x(x - 8)$

$0 = -0.5x(x - 8)$

$-0.5x = 0$ or $x - 8 = 0$

$x = 0$ or $x = 8$

The distance between the zeros is 8, so the base is 8 feet wide.

b. The highest point occurs above the center of the base, at $x = 4$:

$y = -0.5(4)(4 - 8)$

$y = 8$

The highest point is 8 feet above the floor.

Mixed Review of Problem Solving for the lessons "Add and Subtract Polynomials", "Multiply Polynomials", "Find Special Products of Polynomials", and "Solve Polynomial Equations in Factored Form"

1. a. $A = \text{length} \cdot \text{width}$

$= (2x + 72)(2x + 48)$

$= 4x^2 + 96x + 144x + 3456$

$= 4x^2 + 240x + 3456$

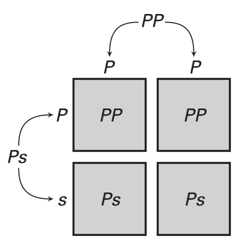
b. When $x = 4$:

$A = 4(4)^2 + 240(4) + 3456 = 4480$

When the fringe is 4 inches, the total area of the blanket with fringe is 4480 square inches.

2. *Sample answer:*

a.



b. Four of the four possible gene combinations, or 100%, result in pinto coloring.

c. $\left(\frac{1}{2}P + \frac{1}{2}s\right)\left(\frac{1}{2}P + \frac{1}{2}P\right) = \left(\frac{1}{2}P + \frac{1}{2}s\right)P$

$= \frac{1}{2}P^2 + \frac{1}{2}Ps$

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Because any gene combination with a P results in pinto coloring, both terms represent pinto offspring. The sum of the coefficients, $\frac{1}{2} + \frac{1}{2} = 1$, shows that 100% of the offspring will be pinto.

3. a. The football with the initial vertical velocity of $\frac{44 \text{ ft}}{\text{sec}}$ is in the air for more time.

b. When $v = 44$:

$h = -16t^2 + 44t$

$0 = -16t^2 + 44t$

$0 = 4t(-4t + 11)$

$4t = 0$ or $4t + 11 = 0$

$t = \frac{11}{4} = 2.75$ seconds

When $v = 40$:

$h = -16t^2 + 40t$

$0 = -16t^2 + 40t$

$0 = 8t(-2t + 5)$

$8t = 0$ or $-2t + 5 = 0$

$t = \frac{5}{2} = 2.5$ seconds

The first football stays in the air $2.75 - 2.5 = 0.25$ second longer than the second football.

4. The highest power in the polynomial is 3, so the polynomial is of degree 3.

5. a. $T = C + S$

$= 0.067t^3 - 0.107t^2 + 0.27t + 3.5 + 0.416t + 1.24$

$= 0.067t^3 - 0.107t^2 + 0.686t + 4.74$

b. For 1992, let $t = 0$:

$T = 0.067(0)^3 - 0.107(0)^2 + 0.686(0) + 4.74 = 4.74$

For 2000, let $t = 8$:

$T = 0.067(8)^3 - 0.107(8)^2 + 0.686(8) + 4.74$

$= 37.684$

The total participation in these activities was 4.74 million in 1992 and about 37.684 million in 2000.

c. Average rate of change $= \frac{\text{change in participants}}{\text{change in time}}$

$= \frac{37.684 \text{ million} - 4.74 \text{ million}}{8 - 0}$

$= 4.118$

The average rate of change was about 4.118 million people each year.

6. a. $A = \pi R^2$

$= \pi (r + 2)^2$

$= \pi \left(r^2 + 4r + 4\right)$

$= \pi r^2 + 4\pi r + 4\pi$

b. When $2r = 3$, $r = \frac{3}{2}$:

$$A = \pi\left(\frac{3}{2} + 2\right)^2 = \pi\left(\frac{7}{2}\right)^2 = \frac{49}{4}\pi = 12.25\,\pi$$

The area of the rug is 12.25π square feet. Because the diameter of the inner circle is 3 feet, its radius is $\frac{3}{2}$ feet, and you can substitute this for r in the equation for area.

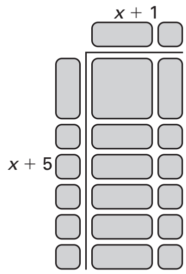
Lesson 8.5 Factor $x^2 + bx + c$

Investigating Algebra Activity for the lesson "Factor $x^2 + bx + c$"

1. $(x + 4)(x + 2) = x^2 + 2x + 4x + 8$

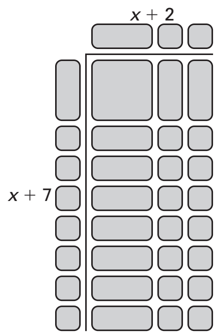
$= x^2 + 6x + 8$ ✓

2.



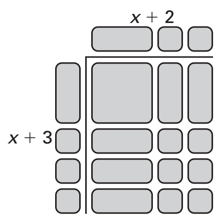
The side lengths of the rectangle represent the polynomials $x + 5$ and $x + 1$. So, $x^2 + 6x + 5 = (x + 1)(x + 5)$.

3.



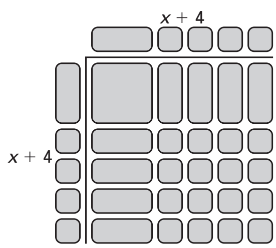
The side lengths of the rectangle represent the polynomials $x + 7$ and $x + 2$. So, $x^2 + 9x + 14 = (x + 7)(x + 2)$.

4.



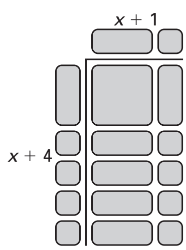
The side lengths of the rectangle represent the polynomials $x + 2$ and $x + 3$. So, $x^2 + 2x + 6 = (x + 2)(x + 3)$.

5.



The side lengths of the rectangle represent the polynomials $x + 4$ and $x + 4$. So, $x^2 + 8x + 16 = (x + 4)(x + 4)$.

6.



The side lengths of the rectangle represent the polynomials $x + 4$ and $x + 1$. So, $x^2 + 5x + 4 = (x + 1)(x + 4)$.

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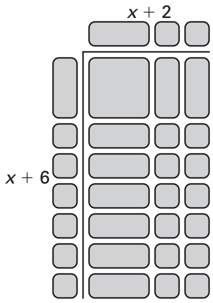
7. 

The side lengths of the rectangle represent the polynomials $x + 6$ and $x + 2$. So, $x^2 + 8x + 12 = (x + 6)(x + 2)$.

8. $p + q = 6$

$p \cdot q = 8$

Guided Practice for the lesson "Factor $x^2 + bx + c$"

1.

Factors of 2	Sum of factors	
2, 1	$2 + 1 = 3$	correct sum

The factors 2 and 1 have a sum of 3, so they are the correct factors.

$x^2 + 3x + 2 = (x + 2)(x + 1)$

Check: $(x + 2)(x + 1) = x^2 + x + 2x + 2$

$= x^2 + 3x + 2$ ✓

2.

Factors of 10	Sum of factors	
10, 1	$10 + 1 = 11$	✗
5, 2	$5 + 2 = 7$	correct sum

The factors 5 and 2 have a sum of 7, so they are the correct factors.

$a^2 + 7a + 10 = (a + 5)(a + 2)$

Check: $(a + 5)(a + 2) = a^2 + 2a + 5a + 10$

$= a^2 + 7a + 10$ ✓

3.

Factors of 14	Sum of factors	
14, 1	$14 + 1 = 15$	✗
7, 2	$7 + 2 = 9$	correct sum

The factors 7 and 2 have a sum of 9, so they are the correct factors.

$t^2 + 9t + 14 = (t + 7)(t + 2)$

Check: $(t + 7)(t + 2) = t^2 + 2t + 7t + 14$

$= t^2 + 9t + 14$ ✓

4.

Factors of 3	Sum of factors	
$-3, -1$	$-3 + (-1) = -4$	correct sum

$x^2 - 4x + 3 = (x - 3)(x - 1)$

5.

Factors of 12	Sum of factors	
$-12, -1$	$-12 + (-1) = -13$	✗
$-6, -2$	$-6 + (-2) = -8$	correct sum
$-4, -3$	$-4 + (-3) = -7$	✗

$t^2 - 8t + 12 = (t - 6)(t - 2)$

6.

Factors of -20	Sum of factors	
$-20, 1$	$-20 + 1 = -19$	✗
$20, -1$	$20 + (-1) = 19$	✗
$-10, 2$	$-10 + 2 = -8$	✗
$10, -2$	$10 + (-2) = 8$	✗
$-5, 4$	$-5 + 4 = -1$	✗
$5, -4$	$5 + (-4) = 1$	correct sum

$m^2 + m - 20 = (m + 5)(m - 4)$

7.

Factors of -16	Sum of factors	
$-16, 1$	$-16 + 1 = -15$	✗
$16, -1$	$16 + (-1) = 15$	✗
$-8, 2$	$-8 + 2 = -6$	✗
$8, -2$	$8 + (-2) = 6$	correct sum
$-4, 4$	$-4 + 4 = 0$	✗

$w^2 + 6w - 16 = (w + 8)(w - 2)$

8. $s^2 - 2s = 24$

$s^2 - 2s - 24 = 0$

$(s - 6)(s + 4) = 0$

$s - 6 = 0$ or $s + 4 = 0$

$s = 6$ or $s = -4$

The solutions of the equation are 6 and -4.

9. Area of 2 banners is $2(10) = 20$ square feet.

$A = \ell \cdot w$

$20 = (4 + w + 4) \cdot w$

$0 = w^2 + 8w - 20$

$0 = (w + 10)(w - 2)$

$w + 10 = 0$ or $w - 2 = 0$

$w = -10$ $w = 2$

The banner cannot have a negative width, so the width is 2 feet.

Exercises for the lesson "Factor $x^2 + bx + c$"

Skill Practice

1. The *factors* of $t^2 + 3t + 2$ are $t + 2$ and $t + 1$.

2. If $x^2 - 8x + 12 = (x + p)(x + q)$, p and q are negative. Because $p + q = -8$ is negative and $pq = 12$ is positive, p and q must both be negative.

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3. $x^2 + 4x + 3 = (x + 3)(x + 1)$

4. $a^2 + 6a + 8 = (a + 4)(a + 2)$

5. $b^2 - 17b + 72 = (b - 9)(b - 8)$

6. $s^2 - 10s + 16 = (s - 8)(s - 2)$

7. $z^2 + 8z - 48 = (z + 12)(z - 4)$

8. $w^2 + 18w + 56 = (w + 14)(w + 4)$

9. $y^2 - 7y - 18 = (y - 9)(y + 2)$

10. $n^2 - 9n + 14 = (n - 7)(n - 2)$

11. $x^2 + 3x - 70 = (x + 10)(x - 7)$

12. $f^2 + 4f - 32 = (f + 8)(f - 4)$

13. $m^2 - 7m - 120 = (m - 15)(m + 8)$

14. $d^2 - 20d + 99 = (d - 11)(d - 9)$

15. $p^2 + 20p + 64 = (p + 16)(p + 4)$

16. $x^2 + 6x - 72 = (x + 12)(x - 6)$

17. $c^2 + 15c + 44 = (c + 11)(c + 4)$

18. The product of p and q should be -60 not 60.

$s^2 - 17s - 60 = (s - 20)(s + 3)$

19. Because $b = -10$ is negative and $c = 24$ is positive, both p and q need to be negative. $m^2 - 10m + 24 = (m - 6)(m - 4)$

20. $x^2 - 10x + 21 = 0$

$(x - 7)(x - 3) = 0$

$x - 7 = 0$ or $x - 3 = 0$

$x = 7$ or $x = 3$

The solutions are 7 and 3.

21. $n^2 - 7n - 30 = 0$

$(n - 10)(n + 3) = 0$

$n - 10 = 0$ or $n + 3 = 0$

$n = 10$ or $n = -3$

The solutions are 10 and -3.

22. $w^2 - 15w + 44 = 0$

$(w - 11)(w - 4) = 0$

$w - 11 = 0$ or $w - 4 = 0$

$w = 11$ or $w = 4$

The solutions are 11 and 4.

23. $a^2 + 5a = 50$

$a^2 + 5a - 50 = 0$

$(a + 10)(a - 5) = 0$

$a + 10 = 0$ or $a - 5 = 0$

$a = -10$ or $a = 5$

The solutions are -10 and 5.

24. $r^2 + 2r = 24$

$r^2 + 2r - 24 = 0$

$(r + 6)(r - 4) = 0$

$r + 6 = 0$ or $r - 4 = 0$

$r = -6$ or $r = 4$

The solutions are -6 and 4.

25. $t^2 + 9t = -20$

$t^2 + 9t + 20 = 0$

$(t + 5)(t + 4) = 0$

$t + 5 = 0$ or $t + 4 = 0$

$t = -5$ or $t = -4$

The solutions are -5 and -4.

26. $y^2 - 2y - 8 = 7$

$y^2 - 2y - 15 = 0$

$(y - 5)(y + 3) = 0$

$y - 5 = 0$ or $y + 3 = 0$

$y = 5$ or $y = -3$

The solutions are 5 and -3.

27. $m^2 + 22 = -23m$

$m^2 + 23m + 22 = 0$

$(m + 22)(m + 1) = 0$

$m + 22 = 0$ or $m + 1 = 0$

$m = -22$ or $m = -1$

The solutions are -22 and -1.

28. $b^2 + 5 = 8b - 10$

$b^2 - 8b + 15 = 0$

$(b - 5)(b - 3) = 0$

$b - 5 = 0$ or $b - 3 = 0$

$b = 5$ or $b = 3$

The solutions are 5 and 3.

29. C; $x^2 - 8x = 240$

$x^2 - 8x - 240 = 0$

$(x - 20)(x + 12) = 0$

$x - 20 = 0$ or $x + 12 = 0$

$x = 20$ or $x = -12$

The solutions are 20 and -12.

30. $f(x) = x^2 + 11x + 18$

$0 = x^2 + 11x + 18$

$0 = (x + 9)(x + 2)$

$x + 9 = 0$ or $x + 2 = 0$

$x = -9$ or $x = -2$

The zeros are -9 and -2.

31. $g(x) = x^2 + 5x + 6$

$0 = x^2 + 5x + 6$

$0 = (x + 3)(x + 2)$

$x + 3 = 0$ or $x + 2 = 0$

$x = -3$ or $x = -2$

The zeros are -3 and -2.

32. $h(x) = x^2 - 18x + 32$

$0 = x^2 - 18x + 32$

$0 = (x - 16)(x - 2)$

$x - 16 = 0$ or $x - 2 = 0$

$x = 16$ or $x = 2$

The zeros are 16 and 2.

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33. $f(x) = x^2 - 14x + 45$

$0 = x^2 - 14x + 45$

$0 = (x - 9)(x - 5)$

$x - 9 = 0$ or $x - 5 = 0$

$x = 9$ or $x = 5$

The zeros are 9 and 5.

34. $h(x) = x^2 - 5x - 24$

$0 = x^2 - 5x - 24$

$0 = (x - 8)(x + 3)$

$x - 8 = 0$ or $x + 3 = 0$

$x = 8$ or $x = -3$

The zeros are 8 and -3.

35. $g(x) = x^2 - 14x - 51$

$0 = x^2 - 14x - 51$

$0 = (x - 17)(x + 3)$

$x - 17 = 0$ or $x + 3 = 0$

$x = 17$ or $x = -3$

The zeros are 17 and -3.

36. $g(x) = x^2 + 10x - 39$

$0 = x^2 + 10x - 39$

$0 = (x + 13)(x - 3)$

$x + 13 = 0$ or $x - 3 = 0$

$x = -13$ or $x = 3$

The zeros are -13 and 3.

37. $f(x) = -x^2 + 16x - 28$

$0 = -x^2 + 16x - 28$

$0 = x^2 - 16x + 28$

$0 = (x - 14)(x - 2)$

$x - 14 = 0$ or $x - 2 = 0$

$x = 14$ or $x = 2$

The zeros are 14 and 2.

38. $f(x) = -x^2 + 24x + 180$

$0 = -x^2 + 24x + 180$

$0 = x^2 - 24x - 180$

$0 = (x - 30)(x + 6)$

$x - 30 = 0$ or $x + 6 = 0$

$x = 30$ or $x = -6$

The zeros are 30 and -6.

39. $s(s + 1) = 72$

$s^2 + s = 72$

$s^2 + s - 72 = 0$

$(s + 9)(s - 8) = 0$

$s + 9 = 0$ or $s - 8 = 0$

$s = -9$ or $s = 8$

The solutions are -9 and 8.

40. $x^2 - 10(x - 1) = -11$

$x^2 - 10x + 10 = -11$

$x^2 - 10x + 21 = 0$

$(x - 7)(x - 3) = 0$

$x - 7 = 0$ or $x - 3 = 0$

$x = 7$ or $x = 3$

The solutions are 7 and 3.

41. $q(q + 19) = -34$

$q^2 + 19q = -34$

$q^2 + 19q + 34 = 0$

$(q + 17)(q + 2) = 0$

$q + 17 = 0$ or $q + 2 = 0$

$q = -17$ or $q = -2$

The solutions are -17 and -2.

42. $x = -4$ or $x = 6$

$x + 4 = 0$ or $x - 6 = 0$

$(x + 4)(x - 6) = 0$

$x^2 - 6x + 4x - 24 = 0$

$x^2 - 2x - 24 = 0$

You can obtain an equation by working backwards. Use the solutions to write factors of a polynomial. Then find and simplify the product to obtain a polynomial.

43. $A = \ell \cdot w$

$100 = x(x - 15)$

$0 = x^2 - 15x - 100$

$0 = (x - 20)(x + 5)$

$x - 20 = 0$ or $x + 5 = 0$

$x = 20$ or $x = -5$

The length cannot be negative, so $x = 20$. The dimensions are 20 inches by 5 inches.

44. $A = \ell \cdot w$

$34 = (x + 11)(x - 4)$

$34 = x^2 - 4x + 11x - 44$

$0 = x^2 + 7x - 78$

$0 = (x + 13)(x - 6)$

$x + 13 = 0$ or $x - 6 = 0$

$x = -13$ or $x = 6$

The dimensions cannot be negative, so $x = 6$. The dimensions are 17 meters by 2 meters.

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45. First, convert 702 square feet to square yards:

$$702 \text{ square feet} \cdot \frac{1 \text{ square yard}}{9 \text{ square feet}} = 78 \text{ square yards.}$$

$A = \frac{1}{2}b \cdot h$

$78 = \frac{1}{2}(x + 20)(x)$

$0 = \frac{1}{2}x^2 + 10x - 78$

$0 = x^2 + 20x - 156$

$0 = (x + 26)(x - 6)$

$x + 26 = 0$ or $x - 6 = 0$

$x = -26$ or $x = 6$

The height cannot be negative, so $x = 6$. The dimensions are 6 yards by 26 yards.

46. $A = \frac{1}{2}b \cdot h$

$119 = \frac{1}{2}(x + 6)(x + 3)$

$238 = x^2 + 3x + 6x + 18$

$0 = x^2 + 9x - 220$

$0 = (x + 20)(x - 11)$

$x + 20 = 0$ or $x - 11 = 0$

$x = -20$ or $x = 11$

A side length cannot be negative, so $x = 11$. The dimensions are 14 feet by 17 feet.

47. $x^2 - 4xy + 4y^2 = (x - 2y)(x - 2y)$

48. $y^2 - 6yz + 5z^2 = (y - 5z)(y - z)$

49. $c^2 + 13cd + 36d^2 = (c + 9d)(c + 4d)$

50. $r^2 + 15rs + 50s^2 = (r + 10s)(r + 5s)$

51. $a^2 + 2ab - 15b^2 = (a + 5b)(a - 3b)$

52. $x^2 + 8xy - 65y^2 = (x + 13y)(x - 5y)$

53. $m^2 - mn - 42n^2 = (m - 7n)(m + 6n)$

54. $u^2 - 3uv - 108v^2 = (u - 12v)(u + 9v)$

55. $g^2 + 4gh - 60h^2 = (g + 10h)(g - 6h)$

56.

Factors of 15	Sum of factors
15, 1	$15 + 1 = 16$
5, 3	$5 + 3 = 8$
−15, −1	$-15 + -1 = -16$
−5, −3	$-5 + -3 = -8$

For $x^2 + bx + 15$, $b = \pm 16$ or ± 8.

57.

Factors of 21	Sum of factors
−21, −1	$-21 + (-1) = -22$
−7, −3	$-7 + (-3) = -10$
21, 1	$21 + 1 = 22$
7, 3	$7 + 3 = 10$

For $x^2 - bx + 21$, $b = \pm 22$ or ± 10.

58.

Factors of −42	Sum of factors
−42, 1	$-42 + 1 = -41$
42, −1	$42 + (-1) = 41$
−21, 2	$-21 + 2 = -19$
21, −2	$21 + (-2) = 19$
−14, 3	$-14 + 3 = -11$
14, −3	$14 + (-3) = 11$
−7, 6	$-7 + 6 = -1$
7, −6	$7 + (-6) = 1$

For $x^2 + bx - 42$, $b = \pm 41, \pm 19, \pm 11$, or ± 1.

Problem Solving

59. $A = \ell \cdot w$

$30 = (s + 1)s$

$0 = s^2 + s - 30$

$0 = (s + 6)(s - 5)$

$s + 6 = 0$ or $s - 5 = 0$

$s = -6$ or $s = 5$

The width cannot be negative, so $s = 5$ cm. Area of the border $= 2s = 2(5) = 10$. The area of the border is 10 square centimeters.

60. a. $A = \ell \cdot w$

$= (x + 50)(x + 32)$

$= x^2 + 82x + 1600$

b. $2320 = x^2 + 82x + 1600$

$0 = x^2 + 82x - 720$

$0 = (x + 90)(x - 8)$

$x + 90 = 0$ or $x - 8 = 0$

$x = -90$ or $x = 8$

The width cannot be negative, so the width of the porch is 8 ft.

61. Area of resulting picture $= (x - 6)(x - 5)$

$20 = x^2 - 11x + 30$

$0 = x^2 - 11x + 10$

$0 = (x - 10)(x - 1)$

$x - 10 = 0$ or $x - 1 = 0$

$x = 10$ or $x = 1$

Reject $x = 1$ because it yields lengths $x - 5$ and $x - 6$ that are negative. The perimeter of the original square was $4x = 4(10) = 40$ inches.

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62. a. $A = 500w + 130w - w^2$

$= 630w - w^2$

b. $3125 = 630w - w^2$

$w^2 - 630w + 3125 = 0$

$(w - 625)(w - 5) = 0$

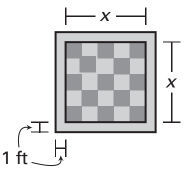
$w - 625 = 0$ or $w - 5 = 0$

$w = 625$ or $w = 5$

The width is 5 feet.

c. 625 feet does not make sense as a path width in this problem situation because it is wider than the length of either path. In the diagram w is also the side length of the overlapping square. If $w = 625$ feet, the square is larger than either part of the path. Only the solution $w = 5$ ft makes sense as a path width.

63. B;



$A = (x + 2)(x + 2)$

$25 = x^2 + 4x + 4$

$0 = x^2 + 4x - 21$

$0 = (x + 7)(x - 3)$

$x + 7 = 0$ or $x - 3 = 0$

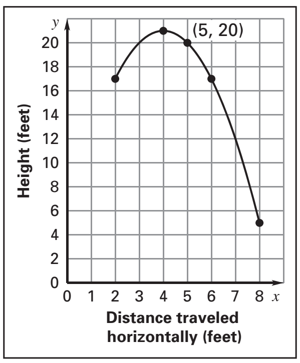
$x = -7$ or $x = 3$

The length cannot be negative, so $x = 3$ feet.

64. a.

x	2	4	6	8
y	17	21	17	5

b.



c. Yes; your friend is 5 feet away and 20 feet up. Because the projectile's path crosses that point, the keys will reach.

d. When $y = 20$:

$20 = -x^2 + 8x + 5$

$x^2 - 8x + 15 = 0$

$(x - 5)(x - 3) = 0$

$x - 5 = 0$ or $x - 3 = 0$

$x = 5$ or $x = 3$

This shows that the keys reach a height of 20 feet when $x = 3$ and $x = 5$. Your friend is 5 feet away from you, so the solution $x = 5$ makes sense.

65. a. $A = \ell \cdot w$

$120 = (x + 8)(x + 10)$

$120 = x^2 + 10x + 8x + 80$

$0 = x^2 + 18x - 40$

$0 = (x + 20)(x - 2)$

$x + 20 = 0$ or $x - 2 = 0$

$x = -20$ or $x = 2$

A side length cannot be negative, so $x = 2$. The dimensions of the stage are $x + 10 = 12$ ft by $x + 8 = 10$ ft.

b. $A = \ell \cdot w$

$360 = [(a + 2) + (a + 2) + 10][a + a + 12]$

$360 = (2a + 14)(2a + 12)$

$360 = 4a^2 + 24a + 28a + 168$

$0 = 4a^2 + 52a - 192$

$0 = a^2 + 13a - 48$

$0 = (a + 16)(a - 3)$

$a + 16 = 0$ or $a - 3 = 0$

$a = -16$ or $a = 3$

Geometric lengths cannot be negative, so $a = 3$. The overall dimensions of the room are $\ell = (a + 2) + (a + 2) + 10 = 20$ ft by $w = a + a + 12 = 18$ ft.

Problem Solving Workshop for the lesson "Factor $x^2 + bx + c$"

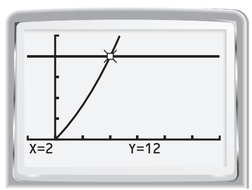
1. Method 1:

$A = \frac{1}{2}w^2$ $\quad A = 4w$ $\quad A = \frac{1}{2}w^2$

w	Area of triangle X $\left(\frac{1}{2}w^2\right)$	Area of rectangle $(4w)$	Area of triangle Y $\left(\frac{1}{2}w^2\right)$	Total area $(4w + w^2)$	
1	0.5	4	0.5	5	← Too small
2	2	8	2	12	← Correct area

The width of the counter should be 2 feet.

Method 2: Area = Area of triangle X + area of rectangle + area of triangle $Y = \frac{1}{2}w^2 + 4w + \frac{1}{2}w^2 = w^2 + 4w$



Graph $y_1 = x^2 + 4x$ and $y_2 = 12$. The intersection is at (2, 12), so the counter has a width of 2 feet.

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2. The trinomial was factored incorrectly as $(w + 2)(w - 6)$ instead of $(w - 2)(w + 6)$.

$12 = 4w + \frac{1}{2}w^2 + \frac{1}{2}w^2$

$0 = w^2 + 4w - 12$

$0 = (w - 2)(w + 6)$

$w - 2 = 0$ or $w + 6 = 0$

$w = 2$ or $w = -6$

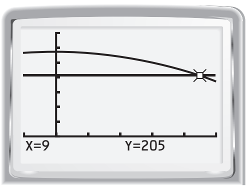
The width is 2 feet.

3. Method 1: Area $= 2(11 \cdot 13) - x^2 = 286 - x^2$

x	Area of fountain (x^2)	Total area $(286 - x^2)$
5	25	261
6	36	250
7	49	237
8	64	222
9	81	205 ← Correct area

The side length of the fountain is 9 ft.

Method 2: $A = 286 - x^2$; graph $y_1 = 286 - x^2$ and $y_2 = 205$.

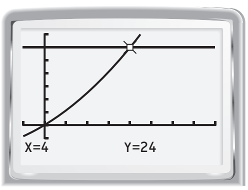


The graphs intersect at (9, 205), so the side length of the fountain is 9 feet.

4. Area of banner = Area of triangle + Area of rectangle

$= \frac{1}{2}w^2 + 4w$

Graph $y_1 = \frac{1}{2}x^2 + 4x$ and $y_2 = 24$.



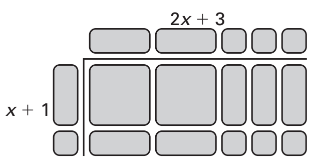
The graph intersects at (4, 24). The width of the banner is 4 feet. This answer was found using the graphing method.

Lesson 8.6 Factor $ax^2 + bx + c$

Investigating Algebra Activity for the lesson "Factor $ax^2 + bx + c$"

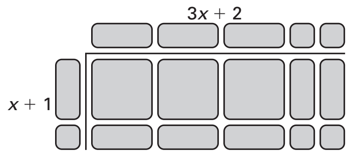
1. $(x + 3)(2x + 1) = 2x^2 + x + 6x + 3 = 2x^2 + 7x + 3$

2.



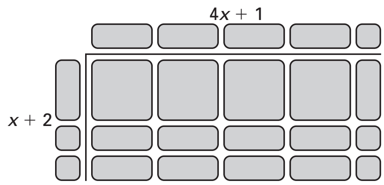
The side lengths of the rectangles represent the polynomials $2x + 3$ and $x + 1$. So, $2x^2 + 5x + 3 = (2x + 3)(x + 1)$.

3.



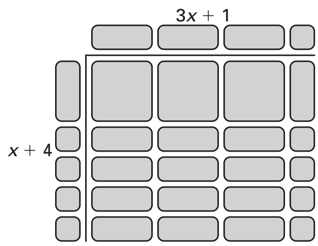
The side lengths of the rectangles represent the polynomials $3x + 2$ and $x + 1$. So, $3x^2 + 5x + 2 = (3x + 2)(x + 1)$.

4.



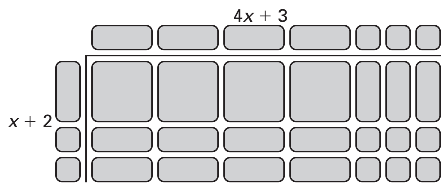
The side lengths of the rectangle represent the polynomials $4x + 1$ and $x + 2$. So, $4x^2 + 9x + 2 = (4x + 1)(x + 2)$.

5.



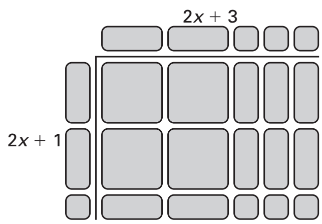
The side lengths of the rectangle represent the polynomials $3x + 1$ and $x + 4$. So, $3x^2 + 13x + 4 = (3x + 1)(x + 4)$.

6.



The side lengths of the rectangle represent the polynomials $4x + 3$ and $x + 2$. So, $4x^2 + 11x + 6 = (4x + 3)(x + 2)$.

7.



The side lengths of the rectangle represent the polynomials $2x + 3$ and $2x + 1$. So, $4x^2 + 8x + 3 = (2x + 3)(2x + 1)$.

8. $2x^2 + 11x + 5 = (2x + 1)(x + 5)$

The leading coefficient of the trinomial is the product of the leading coefficients of its binomial factors.

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Guided Practice for the lesson "Factor $ax^2 + bx + c$"

1. Because b is positive and c is positive, the factors of c are positive.

Factors of 3	Factors of 4	Possible factorization	Middle term when multiplied	
1, 3	1, 4	$(t + 1)(3t + 4)$	$7t$	✗
1, 3	2, 2	$(t + 2)(3t + 2)$	$8t$	Correct
1, 3	4, 1	$(t + 4)(3t + 1)$	$13t$	✗

Solution: $3t^2 + 8t + 4 = (t + 2)(3t + 2)$

2. Because b is negative and c is positive, both factors of c must be negative.

Factors of 4	Factors of 5	Possible factorization	Middle term when multiplied	
1, 4	−1, −5	$(s - 1)(4s - 5)$	$-9s$	Correct
1, 4	−5, −1	$(s - 5)(4s - 1)$	$-21s$	✗
2, 2	−1, −5	$(2s - 1)(2s - 5)$	$-12s$	✗

Solution: $4s^2 - 9s + 5 = (s - 1)(4s - 5)$

3. Because b is positive and c is negative, the factors of c will have opposite signs.

Factors of 2	Factors of −7	Possible factorization	Middle term when multiplied	
1, 2	1, −7	$(h + 1)(2h - 7)$	$-5h$	✗
1, 2	−1, 7	$(h - 1)(2h + 7)$	$5h$	✗
1, 2	−7, 1	$(h - 7)(2h + 1)$	$-13h$	✗
1, 2	7, −1	$(h + 7)(2h - 1)$	$13h$	Correct

Solution: $2h^2 + 13h - 7 = (h + 7)(2h - 1)$

4. $-2y^2 - 5y - 3 = -(2y^2 + 5y + 3)$

Because b and c are positive, both factors of c will be positive.

Factors of 2	Factors of 3	Possible factorization	Middle term when multiplied	
1, 2	1, 3	$(y + 1)(2y + 3)$	$5y$	Correct
1, 2	3, 1	$(y + 3)(2y + 1)$	$7y$	✗

Solution: $-2y^2 - 5y - 3 = -(y + 1)(2y + 3)$

5. $-5m^2 + 6m - 1 = -(5m^2 - 6m + 1)$

Because b is negative and c is positive, both factors of c must be negative.

Factors of 5	Factors of 1	Possible factorization	Middle term when multiplied	
1, 5	−1, −1	$(m - 1)(5m - 1)$	$-6m$	Correct

Solution: $-5m^2 + 6m - 1 = -(m - 1)(5m - 1)$

6. $-3x^2 - x + 2 = -(3x^2 + x - 2)$

Because b is positive and c is negative, the factors of c will have opposite signs.

Factors of 3	Factors of −2	Possible factorization	Middle term when multiplied	
1, 3	1, −2	$(x + 1)(3x - 2)$	x	Correct
1, 3	−1, 2	$(x - 1)(3x + 2)$	$-x$	✗
1, 3	−2, 1	$(x - 2)(3x + 1)$	$-5x$	✗
1, 3	2, −1	$(x + 2)(3x - 1)$	$5x$	✗

Solution: $-3x^2 - x + 2 = -(x + 1)(3x - 2)$

7. $h = -16t^2 + 38t + 5$

$0 = -16t^2 + 38t + 5$

$0 = -(16t^2 - 38t - 5)$

$0 = -(8t + 1)(2t - 5)$

$8t + 1 = 0$ or $2t - 5 = 0$

$t = -\frac{1}{8}$ or $t = \frac{5}{2}$

A negative solution does not make sense, so disregard $-\frac{1}{8}$. The discus hits the ground after $\frac{5}{2} = 2.5$ seconds.

8. $0 = -16t^2 + 29t + 6$

$0 = -(16t^2 - 29t - 6)$

$0 = -(16t + 3)(t - 2)$

$16t + 3 = 0$ or $t - 2 = 0$

$t = -\frac{3}{16}$ or $t = 2$

A negative solution does not make sense, so disregard $-\frac{3}{16}$. The shot put hits the ground after 2 seconds.

9. *B*; $(2w + 1)w = 6$

$2w^2 + w = 6$

$2w^2 + w - 6 = 0$

$(2w - 3)(w + 2) = 0$

$2w - 3 = 0$ or $w + 2 = 0$

$w = \frac{3}{2}$ or $w = -2$

A negative width does not make sense, so disregard -2. The width of the rectangle is $\frac{3}{2}$ inches.

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Skill Practice for the lesson "Factor $ax^2 + bx + c$"

1. Another word for the solutions of $x^2 + 2x + 1 = 0$ is *roots*.

2. To use a graph to check a factorization, plot the original equation and the factored equation on the same axes. If the graphs coincide then the factorization is correct.

3. $6x^2 - x - 2 \qquad x^2 - x - 2$

$(2x + 1)(3x - 2) \quad (x + 1)(x - 2)$

When factoring $6x^2 - x - 2$, you have to consider the factors of the leading coefficients and constant term; when factoring $x^2 - x - 2$, you only have to consider the factors of the constant term.

4. $-x^2 + x + 20 = -(x^2 - x - 20) = -(x + 4)(x - 5)$

5. $-y^2 + 2y + 8 = -(y^2 - 2y - 8) = -(y + 2)(y - 4)$

6. $-a^2 + 12a - 27 = -(a^2 - 12a + 27)$
$= -(a - 9)(a - 3)$

7. $5w^2 - 6w + 1 = (5w - 1)(w - 1)$

8. $-3p^2 - 10p - 3 = -(3p^2 + 10p + 3)$
$= -(3p + 1)(p + 3)$

9. $6s^2 - s - 5 = (6s + 5)(s - 1)$

10. $2t^2 + 5t - 63 = (2t - 9)(t + 7)$

11. $2c^2 - 7c + 3 = (2c - 1)(c - 3)$

12. $3n^2 - 17n + 10 = (3n - 2)(n - 5)$

13. $-2h^2 + 5h + 3 = -(2h^2 - 5h - 3)$
$= -(2h + 1)(h - 3)$

14. $-6k^2 - 13k - 6 = -(6k^2 + 13k + 6)$
$= -(2k + 3)(3k + 2)$

15. $10x^2 - 3x - 27 = (2x + 3)(5x - 9)$

16. $4m^2 + 9m + 5 = (4m + 5)(m + 1)$

17. $3z^2 + z - 14 = (3z + 7)(z - 2)$

18. $4a^2 + 9a - 9 = (4a - 3)(a + 3)$

19. $4n^2 + 16n + 15 = (2n + 3)(2n + 5)$

20. $-5b^2 + 7b - 2 = -(5b^2 - 7b + 2)$
$= -(5b - 2)(b - 1)$

21. $6y^2 - 5y - 4 = (3y - 4)(2y + 1)$

22. B; $8x^2 - 10x + 3 = (2x - 1)(4x - 3)$

23. $2x^2 - 3x - 35 = 0$

$(2x + 7)(x - 5) = 0$

$2x + 7 = 0$ or $x - 5 = 0$

$x = -\frac{7}{2}$ or $x = 5$

24. $3w^2 + 22w + 7 = 0$

$(3w + 1)(w + 7) = 0$

$3w + 1 = 0$ or $w + 7 = 0$

$w = -\frac{1}{3}$ or $w = -7$

25. $4s^2 + 11s - 3 = 0$

$(4s - 1)(s + 3) = 0$

$4s - 1 = 0$ or $s + 3 = 0$

$s = \frac{1}{4}$ or $s = -3$

26. $7a^2 + 2a = 5$

$7a^2 + 2a - 5 = 0$

$(a + 1)(7a - 5) = 0$

$a + 1 = 0$ or $7a - 5 = 0$

$a = -1$ or $a = \frac{5}{7}$

27. $8t^2 - 2t = 3$

$8t - 2t - 3 = 0$

$(2t + 1)(4t - 3) = 0$

$2t + 1 = 0$ or $4t - 3 = 0$

$t = -\frac{1}{2}$ or $t = \frac{3}{4}$

28. $6m^2 - 5m = 14$

$6m^2 - 5m - 14 = 0$

$(6m + 7)(m - 2) = 0$

$6m + 7 = 0$ or $m - 2 = 0$

$m = -\frac{7}{6}$ or $m = 2$

29. $b(20b - 3) - 2 = 0$

$20b^2 - 3b - 2 = 0$

$(4b + 1)(5b - 2) = 0$

$4b + 1 = 0$ or $5b - 2 = 0$

$b = -\frac{1}{4}$ or $b = \frac{2}{5}$

30. $4(3y^2 - 7y + 4) = 1$

$12y^2 - 28y + 16 - 1 = 0$

$12y^2 - 28y + 15 = 0$

$(6y - 5)(2y - 3) = 0$

$6y - 5 = 0$ or $2y - 3 = 0$

$y = \frac{5}{6}$ or $y = \frac{3}{2}$

31. $p(3p + 14) = 5$

$3p^2 + 14p - 5 = 0$

$(p + 5)(3p - 1) = 0$

$p + 5 = 0$ or $3p - 1 = 0$

$p = -5$ or $p = \frac{1}{3}$

32. $4n^2 - 2n - 90 = 0$

$(2n + 9)(2n - 10) = 0$

$2n + 9 = 0$ or $2n - 10 = 0$

$n = -\frac{9}{2}$ or $n = 5$

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33. $10c^2 - 14c + 4 = 0$

$(5c - 2)(2c - 2) = 0$

$5c - 2 = 0$ or $2c - 2 = 0$

$c = \frac{2}{5}$ or $c = 1$

34. $-16k^2 + 8k + 24 = 0$

$-4(4k^2 - 2k - 6) = 0$

$-4(2k - 3)(2k + 2) = 0$

$2k - 3 = 0$ or $2k + 2 = 0$

$k = \frac{3}{2}$ or $k = -1$

35. $6r^2 - 15r = 99$

$6r^2 - 15r - 99 = 0$

$(2r - 11)(3r + 9) = 0$

$2r - 11 = 0$ or $3r + 9 = 0$

$r = \frac{11}{2}$ or $r = -3$

36. $56z^2 + 2 = 22z$

$56z^2 - 22z + 2 = 0$

$2(28z^2 - 11z + 1) = 0$

$2(7z - 1)(4z - 1) = 0$

$7z - 1 = 0$ or $4z - 1 = 0$

$z = \frac{1}{7}$ or $z = \frac{1}{4}$

37. $30x^2 + 25x = 20$

$30x^2 + 25x - 20 = 0$

$5(6x^2 + 5x - 4) = 0$

$5(3x + 4)(2x - 1) = 0$

$3x + 4 = 0$ or $2x - 1 = 0$

$x = -\frac{4}{3}$ or $x = \frac{1}{2}$

38. The equation was not first rewritten in the form $ax^2 + bx + c = 0$.

$5x^2 + x = 4$

$5x + x - 4 = 0$

$(5x - 4)(x + 1) = 0$

$5x - 4 = 0$ or $x + 1 = 0$

$x = \frac{4}{5}$ or $x = -1$

39. In the second step, the factors were set up incorrectly.

$12x^2 + 5x - 2 = 0$

$(4x - 1)(3x + 2) = 0$

$4x - 1 = 0$ or $3x + 2 = 0$

$x = \frac{1}{4}$ or $x = -\frac{2}{3}$

40. $6 = (5w + 7)w$

$0 = 5w^2 + 7w - 6$

$0 = (5w - 3)(w + 2)$

$5w - 3 = 0$ or $w + 2 = 0$

$w = \frac{3}{5}$ or $w = -2$

The width can't be negative, so the width is $\frac{3}{5}$ inch.

41. $3 = (4w + 1)w$

$0 = 4w^2 + w - 3$

$0 = (4w - 3)(w + 1)$

$4w - 3 = 0$ or $w + 1 = 0$

$w = \frac{3}{4}$ or $w = -1$

Reject the negative width.

Perimeter $= 2\ell + 2w$

$= 2(4w + 1) + 2w$

$= 8w + 2 + 2w$

$= 10w + 2$

$= 10\left(\frac{3}{4}\right) + 2 = \frac{19}{2} = 9\frac{1}{2}$

The perimeter is $9\frac{1}{2}$ inches.

42. $g(x) = 2x^2 + x - 1$

$0 = (2x - 1)(x + 1)$

$2x - 1 = 0$ or $x + 1 = 0$

$x = \frac{1}{2}$ or $x = -1$

43. $f(x) = -x^2 + 12x - 35$

$0 = -(x - 12x + 35)$

$0 = -(x - 5)(x - 7)$

$x - 5 = 0$ or $x - 7 = 0$

$x = 5$ or $x = 7$

44. $h(x) = -3x^2 + 2x + 5$

$0 = -(3x^2 - 2x - 5)$

$0 = -(3x - 5)(x + 1)$

$3x - 5 = 0$ or $x + 1 = 0$

$x = \frac{5}{3}$ or $x = -1$

45. $f(x) = 3x^2 + x - 14$

$0 = (3x + 7)(x - 2)$

$3x + 7 = 0$ or $x - 2 = 0$

$x = -\frac{7}{3}$ or $x = 2$

46. $g(x) = 8x^2 - 6x - 14$

$0 = 2(4x^2 - 3x - 7)$

$0 = 2(x + 1)(4x - 7)$

$x + 1 = 0$ or $4x - 7 = 0$

$x = -1$ or $x = \frac{7}{4}$

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47. $f(x) = 12x^2 - 24x - 63$

$$0 = 3(4x^2 - 8x - 21)$$
$$0 = 3(2x - 7)(2x + 3)$$
$$2x - 7 = 0 \quad \text{or} \quad 2x + 3 = 0$$
$$x = \frac{7}{2} \quad \text{or} \quad x = -\frac{3}{2}$$

48.
$$0.3x^2 - 0.7x - 4.0 = 0$$
$$10(0.3x^2 - 0.7x - 4.0) = 0$$
$$3x^2 - 7x - 40 = 0$$
$$(3x + 8)(x - 5) = 0$$
$$3x + 8 = 0 \quad \text{or} \quad x - 5 = 0$$
$$x = -\frac{8}{3} \quad \text{or} \quad x = 5$$

49.
$$0.8x^2 - 1.8x - 0.5 = 0$$
$$10(0.8x^2 - 1.8x - 0.5) = 0$$
$$8x^2 - 18x - 5 = 0$$
$$(4x + 1)(2x - 5) = 0$$
$$4x + 1 = 0 \quad \text{or} \quad 2x - 5 = 0$$
$$x = -\frac{1}{4} \quad \text{or} \quad x = \frac{5}{2}$$

50.
$$0.4x^2 - 0.4x = 9.9$$
$$0.4x^2 - 0.4x - 9.9 = 0$$
$$10(0.4x^2 - 0.4x - 9.9) = 0$$
$$4x^2 - 4x - 99 = 0$$
$$(2x + 9)(2x - 11) = 0$$
$$2x + 9 = 0 \quad \text{or} \quad 2x - 11 = 0$$
$$x = -\frac{9}{2} \quad \text{or} \quad x = \frac{11}{2}$$

51. C;
$$0.4x^2 - 1.1x = 2$$
$$0.4x^2 - 1.1x - 2 = 0$$
$$10(0.4x^2 - 1.1x - 2) = 0$$
$$4x^2 - 11x - 20 = 0$$
$$(4x + 5)(x - 4) = 0$$
$$4x + 5 = 0 \quad \text{or} \quad x - 4 = 0$$
$$x = -\frac{5}{4} \quad \text{or} \quad x = 4$$

52.
$$(x + 3)(x - 2) = 0$$
$$x^2 - 2x + 3x - 6 = 0$$
$$x^2 + x - 6 = 0$$

Knowing the solutions, two binomials can be formed, multiplied, and set equal to zero.

53.
$$\left(x + \frac{1}{2}\right)(x - 5) = 0$$
$$(2x + 1)(x - 5) = 0$$
$$2x^2 - 10x + x - 5 = 0$$
$$2x^2 - 9x - 5 = 0$$

Knowing the solutions, two binomials can be formed, multiplied, and set equal to zero.

54.
$$\left(x + \frac{3}{4}\right)\left(x + \frac{1}{3}\right) = 0$$
$$(4x + 3)(3x + 1) = 0$$
$$12x^2 + 4x + 9x + 3 = 0$$
$$12x^2 + 13x + 3 = 0$$

Knowing the solutions, two binomials can be formed, multiplied, and set equal to zero.

55. $2x^2 - 11xy + 5y^2 = (2x - y)(x - 5y)$

56. $3x^2 + 2xy - 8y^2 = (3x - 4y)(x + 2y)$

57. $6x^3 - 10x^2y - 56xy^2 = 2x(3x^2 - 5xy - 28y^2)$

$$= 2x(3x + 7y)(x - 4y)$$

Problem Solving

58. Let $h = 0$, $v = 9$, and $s = 46$.

$$h = -16t^2 + vt + s$$
$$0 = -16t^2 + 9t + 46$$
$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$t = \frac{-9 \pm \sqrt{81 - 4(-16)(46)}}{2(-16)}$$
$$t = 2 \quad \text{or} \quad t \approx -1.44$$

Reject the negative solution. The diver's center of gravity enters the water after 2 seconds.

59. **a.** Area = length • width

$$= (2 + 3(2x) + 2(1) + 2)(2 + 4x + 2)$$
$$= (6x + 6)(4x + 4)$$
$$= 24x^2 + 48x + 24$$

b.
$$24x^2 + 48x + 24 = 96$$
$$24x^2 + 48x - 72 = 0$$
$$24(x^2 + 2x - 3) = 0$$
$$24(x - 1)(x + 3) = 0$$
$$x = 1 \quad \text{or} \quad x = -3$$

Reject the negative solution. The pictures will be $2x = 2$ centimeters wide and $4x = 4$ centimeters long.

60. You can use the vertical motion model to write an equation for the height (in feet) of the ball. Let $v = 31$ because the initial velocity is 31 feet per second, let $s = 6$ because the initial height is 6 feet, and let $h = 4$ because you catch the ball at a height of 4 feet. Solving this equation for t will give the time (in seconds) that the ball is in the air before it is caught.

$$h = -16t^2 + vt + s$$
$$4 = -16t^2 + 31t + 6$$
$$0 = -16t^2 + 31t + 2$$
$$0 = -(16t^2 - 31t - 2)$$
$$0 = -(16t + 1)(t - 2)$$
$$16t + 1 = 0 \quad \text{or} \quad t - 2 = 0$$
$$t = -\frac{1}{16} \quad \text{or} \quad t = 2$$

A negative solution does not make sense in this situation, so disregard $-\frac{1}{16}$. You catch the ball after 2 seconds.

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61. $A = (2w + 8)w$

$2170 = 2w^2 + 8w$

$0 = 2w^2 + 8w - 2170$

$0 = 2(w^2 + 4w - 1085)$

$0 = 2(w - 31)(w + 35)$

$w = 31$ or $w = -35$

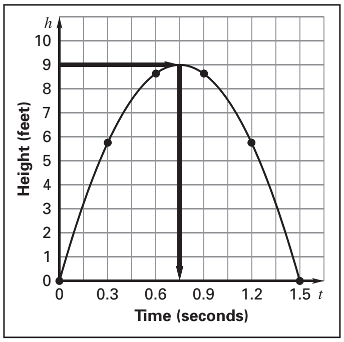
Reject the negative width. The width is 31 meters and the length is $\ell = 2w + 8 = 70$ meters.

62. a. $h = -16t^2 + 24t$

b.

t	0	0.3	0.6	0.9	1.2	1.5
h	0	5.76	8.64	8.64	5.76	0

c.



From the graph, you can estimate that the serval reaches a height of 9 feet after about 0.75 seconds. Using the equation from part (a), let $h = 9$.

$h = -16t^2 + 24t$

$9 = -16t^2 + 24t$

$0 = -16t^2 + 24t - 9$

$0 = -(16t^2 - 24t + 9)$

$0 = -(4t - 3)(4t - 3)$

$4t - 3 = 0$

$t = \frac{3}{4}$, or 0.75

63. a. $h = -16t^2 + 4t$

b. $3 \text{ in.} \cdot \frac{1 \text{ ft}}{12 \text{ in.}} = \frac{1}{4} \text{ ft}$

$\frac{1}{4} = -16t^2 + 4t$

$0 = \left(-16t^2 + 4t - \frac{1}{4}\right)4$

$0 = -64t^2 + 16t - 1$

$0 = -(64t^2 - 16t + 1)$

$0 = -(8t - 1)(8t - 1)$

$8t - 1 = 0$

$t = \frac{1}{8}$

The cricket is 3 inches off the ground after $\frac{1}{8}$ second.

c. No; because the solution in part (b) gives only one value of *t*, the cricket reaches a height of 3 inches only once. This happens only at the highest point of the cricket's jump; all other heights are reached twice, once on the way up and once on the way down.

Quiz for the lessons "Solve Polynomial Equations in Factored Form", "Factor $x^2 + bx + c$", and "Factor $ax^2 + bx + c$"

1. $16a^2 - 40b = 8(2a^2 - 5b)$

2. $9xy^2 + 6x^2y = 3xy(3y + 2x)$

3. $4n^4 - 22n^3 - 8n^2 = 2n^2(2n^2 - 11n - 4)$

4. $3x^2 + 6xy - 3y^2 = 3(x^2 + 2xy - y^2)$

5. $12abc^2 - 6a^2c = 6ac(2bc - a)$

6. $-36s^3 + 18s^2 - 54s = -18s(2s^2 - s + 3)$

7. $r^2 + 15r + 56 = (r + 7)(r + 8)$

8. $s^2 - 6s + 5 = (s - 1)(s - 5)$

9. $w^2 + 6w - 40 = (w + 10)(w - 4)$

10. $-a^2 + 9a + 22 = -(a^2 - 9a - 22)$

$= -(a + 2)(a - 11)$

11. $2x^2 - 9x + 4 = (2x - 1)(x - 4)$

12. $5m^2 + m - 6 = (5m + 6)(m - 1)$

13. $6h^2 - 19h + 3 = (6h - 1)(h - 3)$

14. $-7y^2 - 23y - 6 = -(7y^2 + 23y + 6)$

$= -(7y + 2)(y + 3)$

15. $18c^2 + 12c - 6 = 6(3c^2 + 2c - 1) = 6(3c - 1)(c + 1)$

16. $(4p - 7)(p + 5) = 0$

$4p - 7 = 0$ or $p + 5 = 0$

$p = \frac{7}{4}$ or $p = -5$

17. $-8u^2 + 28u = 0$

$-4u(2u - 7) = 0$

$-4u = 0$ or $2u - 7 = 0$

$u = 0$ or $u = \frac{7}{2}$

18. $51x^2 = -17x$

$51x^2 + 17x = 0$

$17x(3x + 1) = 0$

$17x = 0$ or $3x + 1 = 0$

$x = 0$ or $x = -\frac{1}{3}$

19. $b^2 - 11b = -24$

$b^2 - 11b + 24 = 0$

$(b - 8)(b - 3) = 0$

$b - 8 = 0$ or $b - 3 = 0$

$b = 8$ or $b = 3$

20. $m^2 + 12m = -35$

$m^2 + 12m + 35 = 0$

$(m + 7)(m + 5) = 0$

$m + 7 = 0$ or $m + 5 = 0$

$m = -7$ or $m = -5$

21. $q^2 + 19 = -20q$

$q^2 + 20q + 19 = 0$

$(q + 1)(q + 19) = 0$

$q + 1 = 0$ or $q + 19 = 0$

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$q = -1$ or $q = -19$

22. $3t^2 - 11t + 10 = 0$

$(3t - 5)(t - 2) = 0$

$3t - 5 = 0$ or $t - 2 = 0$

$t = \frac{5}{3}$ or $t = 2$

23. $4y^2 + 31y = 8$

$4y^2 + 31y - 8 = 0$

$(4y - 1)(y + 8) = 0$

$4y - 1 = 0$ or $y + 8 = 0$

$y = \frac{1}{4}$ or $y = -8$

24. $14s^2 + 12s = 2$

$14s^2 + 12s - 2 = 0$

$2(7s^2 + 6s - 1) = 0$

$2(7s - 1)(s + 1) = 0$

$7s - 1 = 0$ or $s + 1 = 0$

$s = \frac{1}{7}$ or $s = -1$

25. a. $h = -16t^2 + 72t + 3$

b. $84 = -16t^2 + 72t + 3$

$0 = -16t^2 + 72t - 81$

$0 = -(16t^2 - 72t + 81)$

$0 = -(4t - 9)(4t - 9)$

$4t - 9 = 0$

$t = \frac{9}{4}$

The ball is 84 feet above the ground after $\frac{9}{4} = 2.25$ seconds.

Lesson 8.7 Factor Special Products

Guided Practice for the lesson "Factor Special Products"

1. $4y^2 - 64 = (2y)^2 - 8^2 = (2y + 8)(2y - 8)$

2. $h^2 + 4h + 4 = h^2 + 2(h \cdot 2) + 2^2 = (h + 2)^2$

3. $2y^2 - 20y + 50 = 2(y^2 - 10y + 25)$

$= 2(y^2 - 2(y \cdot 5) + 5^2)$

$= 2(y - 5)^2$

4. $3x^2 + 6xy + 3y^2 = 3(x^2 + 2xy + y^2) = 3(x + y)^2$

5. $a^2 + 6a + 9 = 0$

$a^2 + 2(a \cdot 3) + 3^2 = 0$

$(a + 3)^2 = 0$

$a + 3 = 0$

$a = -3$

6. $w^2 - 14w + 49 = 0$

$w^2 - 2(w \cdot 7) + 7^2 = 0$

$(w - 7)^2 = 0$

$w - 7 = 0$

$w = 7$

7. $n^2 - 81 = 0$

$n^2 - 9^2 = 0$

$(n + 9)(n - 9) = 0$

$n + 9 = 0$ or $n - 9 = 0$

$n = -9$ or $n = 9$

8. $0 = -16t^2 + 16$

$0 = -16(t^2 - 1)$

$0 = -16(t + 1)(t - 1)$

$t + 1 = 0$ or $t - 1 = 0$

$t = -1$ or $t = 1$

Disregard the negative solution. The sponge lands on the ground 1 second after it is dropped.

Exercises for the lesson "Factor Special Products"

Skill Practice

1. The polynomial $9n^2 + 6n + 1$ is called a *perfect square* trinomial.

2. Write the binomial in the form $a^2 - b^2$ and then factor it as $(a + b)(a - b)$.

3. $x^2 - 25 = (x + 5)(x - 5)$

4. $n^2 - 64 = n^2 - 8^2 = (n + 8)(n - 8)$

5. $81c^2 - 4 = (9c)^2 - 2^2 = (9c + 2)(9c - 2)$

6. $49 - 121p^2 = 7^2 - (11p)^2 = (7 + 11p)(7 - 11p)$

7. $-3m^2 + 48n^2 = -3(m^2 - 16n^2) = -3(m^2 - (4n)^2)$

$= -3(m + 4n)(m - 4n)$

8. $225x^2 - 144y^2 = (15x)^2 - (12y)^2$

$= (15x + 12y)(15x - 12y)$

9. $x^2 - 4x + 4 = x^2 - 2(x \cdot 2) + 2^2 = (x - 2)^2$

10. $y^2 - 10y + 25 = y^2 - 2(y \cdot 5) + 5^2 = (y - 5)^2$

11. $49a^2 + 14a + 1 = (7a)^2 + 2(7a \cdot 1) + 1^2 = (7a + 1)^2$

12. $9t^2 - 12t + 4 = (3t)^2 - 2(3t \cdot 2) + 2^2 = (3t - 2)^2$

13. $m^2 + m + \frac{1}{4} = m^2 + 2\left(m \cdot \frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 = \left(m + \frac{1}{2}\right)^2$

14. $2x^2 + 12xy + 18y^2 = 2(x^2 + 6xy + 9y^2)$

$= 2(x^2 + 2(x \cdot 3y) + (3y)^2)$

$= 2(x + 3y)^2$

15. $4c^2 - 400 = 4(c^2 - 100) = 4(c^2 - 10^2)$

$= 4(c - 10)(c + 10)$

16. $4f^2 - 36f + 81 = (2f)^2 - 2(2f \cdot 9) + 9^2 = (2f - 9)^2$

17. $-9r^2 + 4s^2 = -(9r^2 - 4s^2) = -((3r)^2 - (2s)^2)$

$= -(3r + 2s)(3r - 2s)$

18. $z^2 + 12z + 36 = z^2 + 2(z \cdot 6) + 6^2 = (z + 6)^2$

19. $72 - 32y^2 = 8(9 - 4y^2) = 8(3^2 - (2y)^2)$

$= 8(3 + 2y)(3 - 2y)$

20. $45r^2 - 120rs + 80s^2 = 5(9r^2 - 24rs + 16s^2)$

$= 5((3r)^2 - (2(3r \cdot 4s)) + (4s)^2)$

$= 5(3r - 4s)^2$

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21. The result is the difference of two squares, not a perfect square trinomial.

$$36x^2 - 81 = 9(4x^2 - 9)$$
$$= 9((2x)^2 - 3^2)$$
$$= 9(2x + 3)(2x - 3)$$

22. The result is a perfect square trinomial, not the difference of two squares.

$$y^2 - 6y + 9 = y^2 - 2(y \cdot 3) + 3^2 = (y - 3)^2$$

23. C; $-45x^2 + 20y^2 = -5(9x^2 - 4y^2) = -5((3x)^2 - (2y)^2)$
$$= -5(3x + 2y)(3x - 2y)$$

24. A; $16m^2 - 8mn + n^2 = (4m)^2 - 2(4m \cdot n) + n^2$
$$= (4m - n)^2$$

25.
$$x^2 + 8x + 16 = 0$$
$$x^2 + 2(x \cdot 4) + 4^2 = 0$$
$$(x + 4)^2 = 0$$
$$x + 4 = 0$$
$$x = -4$$

26.
$$16a^2 - 8a + 1 = 0$$
$$(4a)^2 - 2(4a \cdot 1) + 1^2 = 0$$
$$(4a - 1)^2 = 0$$
$$4a - 1 = 0$$
$$a = \frac{1}{4}$$

27.
$$4w^2 - 36 = 0$$
$$(2w)^2 - (6)^2 = 0$$
$$(2w + 6)(2w - 6) = 0$$
$$2w + 6 = 0 \quad \text{or} \quad 2w - 6 = 0$$
$$w = -3 \quad \text{or} \quad w = 3$$

28.
$$32 - 18m^2 = 0$$
$$2(16 - 9m^2) = 0$$
$$2((4)^2 - (3m)^2) = 0$$
$$2(4 + 3m)(4 - 3m) = 0$$
$$4 + 3m = 0 \quad \text{or} \quad 4 - 3m = 0$$
$$m = -\frac{4}{3} \quad \text{or} \quad m = \frac{4}{3}$$

29.
$$27c^2 + 108c + 108 = 0$$
$$27(c^2 + 4c + 4) = 0$$
$$27(c^2 + 2(c \cdot 2) + 2^2) = 0$$
$$27(c + 2)^2 = 0$$
$$c + 2 = 0$$
$$c = -2$$

30.
$$-2h^2 - 28h - 98 = 0$$
$$-2(h^2 + 14h + 49) = 0$$
$$-2(h^2 + 2(h \cdot 7) + 7^2) = 0$$
$$-2(h + 7)^2 = 0$$
$$h + 7 = 0$$
$$h = -7$$

31.
$$6p^2 = 864$$
$$6p^2 - 864 = 0$$
$$6(p^2 - 144) = 0$$
$$6(p^2 - 12^2) = 0$$
$$6(p + 12)(p - 12) = 0$$
$$p + 12 = 0 \quad \text{or} \quad p - 12 = 0$$
$$p = -12 \quad \text{or} \quad p = 12$$

32.
$$-3t^2 = -108$$
$$-3t^2 + 108 = 0$$
$$-3(t^2 - 36) = 0$$
$$-3(t^2 - 6^2) = 0$$
$$-3(t + 6)(t - 6) = 0$$
$$t + 6 = 0 \quad \text{or} \quad t - 6 = 0$$
$$t = -6 \quad \text{or} \quad t = 6$$

33.
$$8k^2 = 98$$
$$8k^2 - 98 = 0$$
$$2(4k^2 - 49) = 0$$
$$2((2k)^2 - 7^2) = 0$$
$$2(2k + 7)(2k - 7) = 0$$
$$2k + 7 = 0 \quad \text{or} \quad 2k - 7 = 0$$
$$k = -\frac{7}{2} \quad \text{or} \quad k = \frac{7}{2}$$

34.
$$-\frac{4}{3}x + \frac{4}{9} = -x^2$$
$$x^2 - \frac{4}{3}x + \frac{4}{9} = 0$$
$$x^2 - 2\left(x \cdot \frac{2}{3}\right) + \left(\frac{2}{3}\right)^2 = 0$$
$$\left(x - \frac{2}{3}\right)^2 = 0$$
$$x - \frac{2}{3} = 0$$
$$x = \frac{2}{3}$$

35.
$$y^2 - \frac{5}{3}y = \frac{-25}{36}$$
$$y^2 - \frac{5}{3}y + \frac{25}{36} = 0$$
$$y^2 - 2\left(y \cdot \frac{5}{6}\right) + \left(\frac{5}{6}\right)^2 = 0$$
$$\left(y - \frac{5}{6}\right)^2 = 0$$
$$y - \frac{5}{6} = 0$$
$$y = \frac{5}{6}$$

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36. $\frac{2}{9} = 8n^2$

$\frac{2}{9} - 8n^2 = 0$

$2\left(\frac{1}{9} - 4n^2\right) = 0$

$2\left(\left(\frac{1}{3}\right)^2 - (2n)^2\right) = 0$

$2\left(\frac{1}{3} + 2n\right)\left(\frac{1}{3} - 2n\right) = 0$

$\frac{1}{3} + 2n = 0$ or $\frac{1}{3} - 2n = 0$

$n = -\frac{1}{6}$ or $n = \frac{1}{6}$

37. $-9c^2 = -16$

$9c^2 - 16 = 0$

$(3c)^2 - (4)^2 = 0$

$(3c + 4)(3c - 4) = 0$

$3c + 4 = 0$ or $3c - 4 = 0$

$c = -\frac{4}{3}$ or $c = \frac{4}{3}$

38. $-20s - 3 = 25s^2 + 1$

$-25s^2 - 20s - 4 = 0$

$-(25s^2 + 20s + 4) = 0$

$-((5s)^2 + 2(5s \cdot 2) + 2^2) = 0$

$-(5s + 2)^2 = 0$

$5s + 2 = 0$

$s = -\frac{2}{5}$

39. $y^4 - 2y^3 + y^2 = 0$

$y^2(y^2 - 2y + 1) = 0$

$y^2(y^2 - 2(y \cdot 1) + 1^2) = 0$

$y^2(y - 1)^2 = 0$

$y^2 = 0$ or $y - 1 = 0$

$y = 0$ or $y = 1$

40. $x^2 + kx + 36 = x^2 \pm 2(x \cdot 6) + 6^2$

$kx = \pm 2(x \cdot 6)$

$kx = \pm 12x$

$k = \pm 12$

41. $4x^2 + kx + 9 = (2x)^2 \pm 2(2x \cdot 3) + 3^2$

$kx = \pm 2(2x \cdot 3)$

$kx = \pm 12x$

$k = \pm 12$

42. $16x^2 + kx + 4 = (4x)^2 \pm 2(4x \cdot 2) + 2^2$

$kx = \pm 2(4x \cdot 2)$

$kx = \pm 16x$

$k = \pm 16$

43. $25x^2 + 10x + k = (5x)^2 + 2(5x \cdot \sqrt{k}) + (\sqrt{k})^2$

$10x = 2(5x \cdot \sqrt{k})$

$10x = 10x\sqrt{k}$

$1 = \sqrt{k}$

$1 = k$

44. $49x^2 - 84x + k = (7x)^2 - 2(7x \cdot \sqrt{k}) + (\sqrt{k})^2$

$-84x = -2(7x \cdot \sqrt{k})$

$-84x = -14x\sqrt{k}$

$6 = \sqrt{k}$

$36 = k$

45. $4x^2 - 48x + k = (2x)^2 - 2(2x \cdot \sqrt{k}) + (\sqrt{k})^2$

$-48x = -2(2x \cdot \sqrt{k})$

$-48x = -4x\sqrt{k}$

$12 = \sqrt{k}$

$144 = k$

Problem Solving

46. $h = -16t^2 + 25$

$0 = -((4t)^2 - 5^2)$

$0 = -(4t + 5)(4t - 5)$

$4t + 5 = 0$ or $4t - 5 = 0$

$t = -\frac{5}{4}$ or $t = \frac{5}{4}$

Disregard the negative solution. The paintbrush lands after $\frac{5}{4}$ seconds.

47. $h = -16t^2 + 100$

$0 = -((4t)^2 - 10^2)$

$0 = -(4t + 10)(4t - 10)$

$4t + 10 = 0$ or $4t - 10 = 0$

$t = -\frac{5}{2}$ or $t = \frac{5}{2}$

Disregard the negative solution. The hickory nut lands after $\frac{5}{2}$ seconds.

48. a. $h = -16t^2 + 8t$

b. $1 = -16t^2 + 8t$

$0 = -16t^2 + 8t - 1$

$0 = -((4t)^2 - 8t + 1^2)$

$0 = -((4t)^2 - 2(4t \cdot 1) + 1^2)$

$0 = -(4t - 1)^2$

$4t - 1 = 0$

$t = \frac{1}{4}$

The grasshopper is 1 foot off the ground after $\frac{1}{4}$ second.

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49. $h = -16t^2 + 56t + 5$

$54 = -16t^2 + 56t + 5$

$0 = -16t^2 + 56t - 49$

$0 = -\left((4t)^2 - 2(4t \cdot 7) + 7^2\right)$

$0 = -(4t - 7)^2$

$4t - 7 = 0$

$t = \frac{7}{4}$

The ball reaches a maximum height of 54 feet only once, 1.75 seconds after it is thrown.

50. a. $y = -\frac{1}{4}x^2 + 3x$

x	0	2	5	8	11
y	0	5	8.75	8	2.75

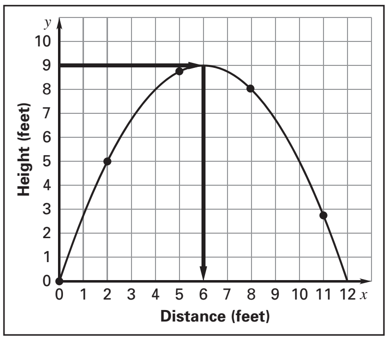
b. The equation makes sense for values of $x \le 12$. For $x > 12$, y is negative.

c. Plotting the points in the table in part (a) and sketching a curve through the points, you can estimate that the arch reaches a height of 9 feet at about 6 feet from the left end.

Check: $y = -\frac{1}{4}x^2 + 3x$

$y = -\frac{1}{4}(6)^2 + 3(6)$

$y = 9$



51. a. Total area $= (2d)^2 = 4d^2$

Mirror area $= 3^2 = 9$

Stained glass area $= 4d^2 - 9$

b. $91 = 4d^2 - 9$

$0 = 4d^2 - 100$

$0 = (2d)^2 - 10^2$

$0 = (2d + 10)(2d - 10)$

$2d + 10 = 0$ or $2d - 10 = 0$

$d = -5$ or $d = 5$

Disregard the negative solution. The side length of the frame is $2d = 10$ inches.

52. a.

n	**nth odd integer**	**Sum of First n odd integers**	**Sum as a power**
1	1	1	1^2
2	3	$1 + 3 = 4$	2^2
3	5	$1 + 3 + 5 = 9$	3^2
4	7	$1 + 3 + 5 + 7 = 16$	4^2
5	9	$1 + 3 + 5 + 7 + 9 = 25$	5^2

b. The sum of the first n odd integers is n^2. The sum of the first 10 integers is $10^2 = 100$.

c. 11 is the 6^{th} odd integer and 21 is the 11^{th} odd integer. So, find the sum of the first 11 odd integers and subtract the sum of the first 5 odd integers.

$11^2 - 5^2 = 121 - 25 = 96$

d. The number of chairs in the first row, 15, is the 8th odd integer, and the number of chairs in the last row is the nth odd integer.

Total number of chairs = Sum of first n odd integers − Sum of first 7 odd integers

$120 = n^2 - 7^2$

$0 = n^2 - 169$

$0 = n^2 - 13^2$

$0 = (n + 13)(n - 13)$

$n + 13 = 0$ or $n - 13 = 0$

$n = -13$ or $n = 13$

You will need $13 - 7 = 6$ rows of chairs.

Lesson 8.8 Factor Polynomials Completely

Guided Practice for the lesson "Factor Polynomials Completely"

1. $x(x - 2) + (x - 2) = (x - 2)(x + 1)$

2. $a^3 + 3a^2 + a + 3 = \left(a^3 + 3a^2\right) + (a + 3)$

$= a^2(a + 3) + (a + 3)$

$= (a + 3)\left(a^2 + 1\right)$

3. $y^2 + 2x + yx + 2y = \left(y^2 + 2y\right) + (yx + 2x)$

$= y(y + 2) + x(y + 2)$

$= (y + 2)(y + x)$

4. $3x^3 - 12x = 3x\left(x^2 - 4\right) = 3x(x + 2)(x - 2)$

5. $2y^3 - 12y^2 + 18y = 2y\left(y^2 - 6y + 9\right)$

$= 2y\left(y^2 - 2(y \cdot 3) + 3^2\right)$

$= 2y(y - 3)^2$

6. $m^3 - 2m^2 - 8m = m\left(m^2 - 2m - 8\right) = m(m - 4)(m + 2)$

7. $w^3 - 8w^2 + 16w = 0$

$w\left(w^2 - 8w + 16\right) = 0$

$w\left(w^2 - 2(w \cdot 4) + 4^2\right) = 0$

$w(w - 4)^2 = 0$

$w = 0$ or $w - 4 = 0$

$w = 0$ or $w = 4$

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8. $x^3 - 25x = 0$

$x(x^2 - 25) = 0$

$x(x + 5)(x - 5) = 0$

$x = 0$ or $x + 5 = 0$ or $x - 5 = 0$

$x = 0$ or $x = -5$ or $x = 5$

9. $c^3 - 7c^2 + 12c = 0$

$c(c^2 - 7c + 12) = 0$

$c(c - 4)(c - 3) = 0$

$c = 0$ or $c - 4 = 0$ or $c - 3 = 0$

$c = 0$ or $c = 4$ or $c = 3$

10. Volume (cubic feet) = Length (feet) • Width (feet) • Height (feet)

$72 = x \cdot (x - 1) \cdot (x + 9)$

$72 = x(x - 1)(x + 9)$

$72 = x(x^2 + 8x - 9)$

$72 = x^3 + 8x^2 - 9x$

$0 = x^3 + 8x^2 + 9x - 72$

$0 = x^2(x + 8) - 9(x + 8)$

$0 = (x + 8)(x^2 - 9)$

$0 = (x + 8)(x + 3)(x - 3)$

$x + 8 = 0$ or $x + 3 = 0$ or $x - 3 = 0$

$x = -8$ or $x = -3$ or $x = 3$

The length cannot be negative, so 3 is the correct value of x. The dimensions of the box are 3 feet long, $3 - 1 = 2$ feet wide, and $3 + 9 = 12$ feet high.

Exercises for the lesson "Factor Polynomials Completely"

Skill Practice

1. A polynomial is factored completely if it is written as a product of unfactorable polynomials with integer coefficients.

2. A polynomial is unfactorable if it cannot be written as the product of polynomials with integer coefficients.

3. $x(x - 8) + (x - 8) = (x - 8)(x + 1)$

4. $5y(y + 3) - 2(y + 3) = (y + 3)(5y - 2)$

5. $6z(z - 4) - 7(z - 4) = (z - 4)(6z - 7)$

6. $10(a - 6) - 3a(a - 6) = (a - 6)(10 - 3a)$

7. $b^2(b + 5) - 3(b + 5) = (b + 5)(b^2 - 3)$

8. $7c^2(c + 9) + 2(c + 9) = (c + 9)(7c^2 + 2)$

9. $x(13 + x) - (x + 13) = (x + 13)(x - 1)$

10. $y^2(y - 4) + 5(4 - y) = y^2(y - 4) - 5(-4 + y)$

$= (y - 4)(y^2 - 5)$

11. $12(z - 1) - 5z^2(1 - z) = 12(z - 1) + 5z^2(-1 + z)$

$= (z - 1)(12 + 5z^2)$

12. C; $x^2(x - 8) + 5(8 - x) = x^2(x - 8) - 5(-8 + x)$

$= (x - 8)(x^2 - 5)$

13. $x^3 + x^2 + 2x + 2 = x^2(x + 1) + 2(x + 1)$

$= (x + 1)(x^2 + 2)$

14. $y^3 - 9y^2 + y - 9 = y^2(y - 9) + (y - 9)$

$= (y - 9)(y^2 + 1)$

15. $z^3 - 4z^2 + 3z - 12 = z^2(z - 4) + 3(z - 4)$

$= (z - 4)(z^2 + 3)$

16. $c^3 + 7c^2 + 5c + 35 = c^2(c + 7) + 5(c + 7)$

$= (c + 7)(c^2 + 5)$

17. $a^3 + 13a^2 - 5a - 65 = a^2(a + 13) - 5(a + 13)$

$= (a + 13)(a^2 - 5)$

18. $2s^3 - 3s^2 + 18s - 27 = s^2(2s - 3) + 9(2s - 3)$

$= (2s - 3)(s^2 + 9)$

19. $5n^3 - 4n^2 + 25n - 20 = n^2(5n - 4) + 5(5n - 4)$

$= (5n - 4)(n^2 + 5)$

20. $x^2 + 8x - xy - 8y = x(x + 8) - y(x + 8)$

$= (x + 8)(x - y)$

21. $y^2 + y + 5xy + 5x = y(y + 1) + 5x(y + 1)$

$= (y + 1)(y + 5x)$

22. The error occured when the negative signs were forgotten.

$a^3 + 8a^2 - 6a - 48 = a^2(a + 8) - 6(a + 8)$

$= (a + 8)(a^2 - 6)$

23. $x^4 - x^2 = x^2(x^2 - 1) = x^2(x + 1)(x - 1)$

24. $36a^4 - 4a^2 = 4a^2(9a^2 - 1) = 4a((3a)^2 - 1)$

$= 4a(3a + 1)(3a - 1)$

25. $3n^5 - 48n^3 = 3n^3(n^2 - 16) = 3n^3(n + 4)(n - 4)$

26. $4y^6 - 16y^4 = 4y^4(y^2 - 4) = 4y^4(y + 2)(y - 2)$

27. $75c^9 - 3c^7 = 3c^7(25c^2 - 1) = 3c^7(5c + 1)(5c - 1)$

28. $72p - 2p^3 = 2p(36 - p^2) = 2p(6 + p)(6 - p)$

29. $32s^4 - 8s^2 = 8s^2(4s^2 - 1) = 8s^2(2s + 1)(2s - 1)$

30. $80z^8 - 45z^6 = 5z^6(16z^2 - 9) = 5z^6(4z + 3)(4z - 3)$

31. $m^2 - 5m - 35$ Cannot be factored.

32. $6g^3 - 24g^2 + 24g = 6g(g^2 - 4g + 4)$

$= 6g(g^2 - 2(g \cdot 2) + 2^2)$

$= 6g(g - 2)^2$

33. $3w^4 + 24w^3 + 48w^2 = 3w^2(w^2 + 8w + 16)$

$= 3w^2(w^2 + 2(w \cdot 4) + 4^2)$

$= 3w^2(w + 4)^2$

34. $3r^5 + 3r^4 - 90r^3 = 3r^3(r^2 + r - 30)$

$= 3r^3(r + 6)(r - 5)$

35. $b^3 - 5b^2 - 4b + 20 = b^2(b - 5) - 4(b - 5)$

$= (b - 5)(b^2 - 4)$

$= (b - 5)(b + 2)(b - 2)$

36. $h^3 + 4h^2 - 25h - 100 = h^2(h + 4) - 25(h + 4)$

$= (h + 4)(h^2 - 25)$

$= (h + 4)(h + 5)(h - 5)$

37. $9t^3 + 18t - t^2 - 2 = t^2(9t - 1) + 2(9t - 1)$

$= (9t - 1)(t^2 + 2)$

38. $2x^5y - 162x^3y = 2x^3y(x^2 - 81) = 2x^3y(x + 9)(x - 9)$

39. $7a^3b^3 - 63ab^3 = 7ab^3(a^2 - 9) = 7ab^3(a + 3)(a - 3)$

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40. $-4s^3t^3 + 24s^2t^2 - 36st = -4st(s^2t^2 - 6st + 9)$
$= -4st((st)^2 - 2(st \cdot 3) + 3^2)$
$= -4st(st - 3)^2$

41. D; $3x^6 - 75x^4 = 3x^4(x^2 - 25) = 3x^4(x - 5)(x + 5)$

42. The polynomial was not factored completely. The factor $x^2 - 9$ is the difference of two squares.
$x^3 - 6x^2 - 9x + 54 = x^2(x - 6) - 9(x - 6)$
$= (x - 6)(x^2 - 9)$
$= (x + 6)(x + 3)(x - 3)$

43. $x^3 + x^2 - 4x - 4 = 0$
$x^2(x + 1) - 4(x + 1) = 0$
$(x + 1)(x^2 - 4) = 0$
$(x + 1)(x + 2)(x - 2) = 0$
$x + 1 = 0$ or $x + 2 = 0$ or $x - 2 = 0$
$x = -1$ or $x = -2$ or $x = 2$

44. $a^3 - 11a^2 - 9a + 99 = 0$
$a^2(a - 11) - 9(a - 11) = 0$
$(a - 11)(a^2 - 9) = 0$
$(a - 11)(a + 3)(a - 3) = 0$
$a - 11 = 0$ or $a + 3 = 0$ or $a - 3 = 0$
$a = 11$ or $a = -3$ or $a = 3$

45. $4y^3 - 7y^2 - 16y + 28 = 0$
$y^2(4y - 7) - 4(4y - 7) = 0$
$(4y - 7)(y^2 - 4) = 0$
$(4y - 7)(y + 2)(y - 2) = 0$
$4y - 7 = 0$ or $y + 2 = 0$ or $y - 2 = 0$
$y = \frac{7}{4}$ or $y = -2$ or $y = 2$

46. $5n^3 - 30n^2 + 40n = 0$
$5n(n^2 - 6n + 8) = 0$
$5n(n - 2)(n - 4) = 0$
$5n = 0$ or $n - 2 = 0$ or $n - 4 = 0$
$n = 0$ or $n = 2$ or $n = 4$

47. $3b^3 + 24b^2 + 45b = 0$
$3b(b^2 + 8b + 15) = 0$
$3b(b + 5)(b + 3) = 0$
$3b = 0$ or $b + 5 = 0$ or $b + 3 = 0$
$b = 0$ or $b = -5$ or $b = -3$

48. $2t^5 + 2t^4 - 144t^3 = 0$
$2t^3(t^2 + t - 72) = 0$
$2t^3(t + 9)(t - 8) = 0$
$2t^3 = 0$ or $t + 9 = 0$ or $t - 8 = 0$
$t = 0$ or $t = -9$ or $t = 8$

49. $z^3 - 81z = 0$
$z(z^2 - 81) = 0$
$z(z + 9)(z - 9) = 0$
$z = 0$ or $z + 9 = 0$ or $z - 9 = 0$
$z = 0$ or $z = -9$ or $z = 9$

50. $c^4 - 100c^2 = 0$
$c^2(c^2 - 100) = 0$
$c^2(c + 10)(c - 10) = 0$
$c^2 = 0$ or $c + 10 = 0$ or $c - 10 = 0$
$c = 0$ or $c = -10$ or $c = 10$

51. $12s - 3s^3 = 0$
$3s(4 - s^2) = 0$
$3s(2 + s)(2 - s) = 0$
$3s = 0$ or $2 + s = 0$ or $2 - s = 0$
$s = 0$ or $s = -2$ or $s = 2$

52. $2x^3 - 10x^2 + 40 = 8x$
$2x^3 - 10x^2 - 8x + 40 = 0$
$2x^2(x - 5) - 8(x - 5) = 0$
$(x - 5)(2x^2 - 8) = 0$
$2(x - 5)(x^2 - 4) = 0$
$2(x - 5)(x + 2)(x - 2) = 0$
$x - 5 = 0$ or $x + 2 = 0$ or $x - 2 = 0$
$x = 5$ or $x = -2$ or $x = 2$

53. $3p + 1 = p^2 + 3p^3$
$3p^3 + p^2 - 3p - 1 = 0$
$p^2(3p + 1) - (3p + 1) = 0$
$(3p + 1)(p^2 - 1) = 0$
$(3p + 1)(p - 1)(p + 1) = 0$
$3p + 1 = 0$ or $p + 1 = 0$ or $p - 1 = 0$
$p = -\frac{1}{3}$ or $p = -1$ or $p = 1$

54. $m^3 - 3m^2 = 4m - 12$
$m^3 - 3m^2 - 4m + 12 = 0$
$m^2(m - 3) - 4(m - 3) = 0$
$(m - 3)(m^2 - 4) = 0$
$(m - 3)(m + 2)(m - 2) = 0$
$m - 3 = 0$ or $m - 2 = 0$ or $m + 2 = 0$
$m = 3$ or $m = 2$ or $m = -2$

55. No. When the polynomial is factored completely, the equation becomes $(x + 2)(x^2 + 3) = 0$. The factor $x^2 + 3$ yields no real solutions when it is set equal to zero. So, the only real solution of the equation is $x = -2$.
$x^3 + 2x^2 + 3x + 6 = 0$
$x^2(x + 2) + 3(x + 2) = 0$
$(x + 2)(x^2 + 3) = 0$
$x + 2 = 0$ or $x^2 + 3 = 0 \leftarrow$ No real solution
$x = -2$

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56. $(x + 4)(x - 1)(x) = 12$

$(x^2 + 3x - 4)x = 12$

$x^3 + 3x^2 - 4x - 12 = 0$

$x^2(x + 3) - 4(x + 3) = 0$

$(x + 3)(x^2 - 4) = 0$

$(x + 3)(x + 2)(x - 2) = 0$

$x + 3 = 0$ or $x + 2 = 0$ or $x - 2 = 0$

$x = -3$ or $x = -2$ or $x = 2$

$x = 2$

$x + 4 = 6$

$x - 1 = 1$

The length is 6 inches, the width is 1 inch, and the height is 2 inches.

57. First, convert 2592 cubic feet to cubic yards:

2592 cubic feet • $\frac{\text{1 cubic yard}}{\text{27 cubic feet}}$ = 96 cubic yards.

$(x + 8)(x)(x - 2) = 96$

$(x^2 + 6x - 16)x = 96$

$x^3 + 6x^2 - 16x - 96 = 0$

$x^2(x + 6) - 16(x + 6) = 0$

$(x + 6)(x^2 - 16) = 0$

$(x + 6)(x + 4)(x - 4) = 0$

$x + 6 = 0$ or $x + 4 = 0$ or $x - 4 = 0$

$x = -6$ or $x = -4$ or $x = 4$

$x = 4$

$x + 8 = 12$

$x - 2 = 2$

The length is 12 feet, the width is 4 feet, and the height is 2 feet.

58. $x^3 + 2x^2y - x - 2y = x^2(x + 2y) - (x + 2y)$

$= (x + 2y)(x^2 - 1)$

$= (x + 2y)(x + 1)(x - 1)$

59. $8b^3 - 4b^2a - 18b + 9a = 4b^2(2b - a) - 9(2b - a)$

$= (2b - a)(4b^2 - 9)$

$= (2b - a)(2b + 3)(2b - 3)$

60. $4s^2 - s + 12st - 3t = s(4s - 1) + 3t(4s - 1)$

$= (4s - 1)(s + 3t)$

61. $6x^2 + 5x - 4$

$6(-4) = -24$

$-24 = 8(-3)$ and $8 - 3 = 5$

$6x^2 + 5x - 4 = 6x^2 + 8x - 3x - 4$

$= 2x(3x + 4) - (3x + 4)$

$= (3x + 4)(2x - 1)$

62. $10s^2 + 19s + 6$

$10(6) = 60$

$60 = 15(4)$ and $15 + 4 = 19$

$10s^2 + 19s + 6 = 10s^2 + 15s + 4s + 6$

$= 5s(2s + 3) + 2(2s + 3)$

$= (2s + 3)(5s + 2)$

63. $12n^2 - 13n + 3$

$12(3) = 36$

$36 = -9(-4)$ and $-9 - 4 = -13$

$12n^2 - 13n + 3 = 12n^2 - 9n - 4n + 3$

$= 3n(4n - 3) - (4n - 3)$

$= (4n - 3)(3n - 1)$

64. $16a^2 + 14a + 3$

$16(3) = 48$

$48 = 8(6)$ and $8 + 6 = 14$

$16a^2 + 14a + 3 = 16a^2 + 8a + 6a + 3$

$= 8a(2a + 1) + 3(2a + 1)$

$= (2a + 1)(8a + 3)$

65. $21w^2 + 8w - 4$

$21(-4) = -84$

$-84 = 14(-6)$ and $14 - 6 = 8$

$21w^2 + 8w - 4 = 21w^2 + 14w - 6w - 4$

$= 7w(3w + 2) - 2(3w + 2)$

$= (3w + 2)(7w - 2)$

66. $15y^2 - 31y + 10$

$15(10) = 150$

$150 = -25(-6)$ and $-25 - 6 = -31$

$15y^2 - 31y + 10 = 15y^2 - 25y - 6y + 10$

$= 5y(3y - 5) - 2(3y - 5)$

$= (3y - 5)(5y - 2)$

67. Rewrite the middle term as $ab + ab$ and group the terms.

$a^2 + 2ab + b^2 = a^2 + ab + ab + b^2$ Rewrite $2ab$ as $ab + ab$.

$= (a^2 + ab) + (ab + b^2)$ Group terms.

$= a(a + b) + b(a + b)$ Factor each group.

$= (a + b)(a + b)$ Distributive property.

$= (a + b)^2$ Simplify.

Problem Solving

68. Volume (cubic inches) = π • Radius^2 (square inches) • Height (inches)

$24\pi = \pi \cdot r^2 \cdot 6$

$24\pi = 6\pi r^2$

$0 = 6\pi r^2 - 24\pi$

$0 = 6\pi(r^2 - 4)$

$0 = 6\pi(r + 2)(r - 2)$

$r + 2 = 0$ or $r - 2 = 0$

$r = -2$ or $r = 2$

The radius cannot be negative, so the radius is 2 inches.

69. a. Volume (cubic inches) = Length (inches) • Width (inches) • Height (inches)

$= 4 \cdot w \cdot (w + 4)$

A polynomial that represents the volume of the birdhouse is $4w^2 + 16w$.

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b. $128 = 4w(w + 4)$

$0 = 4w^2 + 16w - 128$

$0 = 4(w^2 + 4w - 32)$

$0 = 4(w + 8)(w - 4)$

$w + 8 = 0$ or $w - 4 = 0$

$w = -8$ or $w = 4$

The width must be positive, so $w = -8$ can be discarded.

$w = 4$

$w + 4 = 8$

The dimensions of the birdhouse are 4 inches long by 4 inches wide by 8 inches high.

70. Volume (cubic inches) = Length (inches) • Width (inches) • Height (inches)

$1152 = (2w + 4)(w)(18 - w)$

$0 = -2w^3 + 32w^2 + 72w - 1152$

$0 = -2w^2(w - 16) + 72(w - 16)$

$0 = (w - 16)(72 - 2w^2)$

$w - 16 = 0$ or $72 - 2w^2 = 0$

$w = 16$ or $w = 6$

When $w = 16$:

Height $\stackrel{?}{>}$ Width

$18 - w \stackrel{?}{>} w$

$18 - 16 \not> 16$

When $w = 6$:

Height $\stackrel{?}{>}$ Width

$18 - w \stackrel{?}{>} w$

$18 - 6 > 6$ ✓

The solution $w = 6$ gives a height that is greater than the width, so 6 is the correct value of w.

Length $= 2w + 4 = 16$

Width $= w = 6$

Height $= 18 - w = 12$

The dimensions of the gift bag are 16 inches long by 6 inches wide by 12 inches high.

71. a. $h = -4.9t^2 + 3.9t + 1$

$0 = -4.9t^2 + 3.9t + 1$

$0 = -(4.9t^2 - 3.9t - 1)$

$0 = -(4.9t + 1)(t - 1)$

$4.9t + 1 = 0$ or $t - 1 = 0$

$t \approx -0.2$ or $t = 1$

The zeros are 1 and approximately -0.2.

b. The zeros are the times when $h = 0$, or when the ball touches the ground. A negative time does not make sense in this situation, so $t \approx -0.2$ can be discarded. The pallino will land 1 second after being thrown.

72. a. Let $y = 22.5$.

$y = -10x^2 + 30x$

$22.5 = -10x^2 + 30x$

$0 = (-10x^2 + 30x - 22.5)(10)$

$0 = -100x^2 + 300x - 225$

$0 = -25(4x^2 - 12x + 9)$

$0 = -25(2x - 3)^2$

$2x - 3 = 0$

$x = \frac{3}{2} = 1.5$

When the robot's height is 22.5 feet, its horizontal distance from its starting point is 1.5 feet.

b. The robot will be on the ground when the vertical distance (y) is zero.

$y = -10x^2 + 30x$

$0 = -10x^2 + 30x$

$0 = -10x(x - 3)$

$-10x = 0$ or $x - 3 = 0$

$x = 0$ or $x = 3$

The robot's starting position is the origin, so $x = 0$ can be discarded. When the robot lands, it will have traveled 3 feet horizontally.

73. a. Volume (cubic inches) = Length (inches) • Width (inches) • Height (inches)

$V = (9 - h) \cdot (4 + h) \cdot h$

$V = -h^3 + 5h^2 + 36h$

b. $180 = -h^3 + 5h^2 + 36h$

$0 = -h^3 + 5h^2 + 36h - 180$

$0 = -h^2(h - 5) + 36(h - 5)$

$0 = -(h - 5)(h^2 - 36)$

$0 = -(h - 5)(h + 6)(h - 6)$

$h - 5 = 0$ or $h + 6 = 0$ or $h - 6 = 0$

$h = 5$ or $h = -6$ or $h = 6$

The height cannot be negative, so $h = -6$ can be discarded. Possible dimensions:

$h = 5$; height = 5 inches

$9 - h = 4$; length = 4 inches

$4 + h = 9$; width = 9 inches

$h = 6$, height = 6 inches

$9 - h = 3$, length = 3 inches

$4 + h = 10$, width = 10 inches

c.

Surface area (Square inches) = 2 • Length (in.) • Width (in.) + 2 • Length (in.) • Height (in.) + 2 • Height (in.) • Width (in)

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Possible box 1:

$$SA = 2(4 \cdot 9) + 2(4 \cdot 5) + 2(5 \cdot 9)$$
$$= 72 + 40 + 90$$
$$= 202$$

Possible box 2:

$$SA = 2(3 \cdot 10) + 2(3 \cdot 6) + 2(6 \cdot 10)$$
$$= 60 + 36 + 120$$
$$= 216$$

Box 1 has a surface area of 202 square inches; Box 2 has a surface area of 216 square inches. So, the box with the smallest possible surface area should have the dimensions of box 1: height = 5 inches; length = 4 inches; width = 9 inches.

74. a. The surface area of a cube is six times the area of one face of the cube. The area of one face of the cube is s^2 where s = the side length.

$$A = 6s^2$$
$$54 = 6s^2$$
$$0 = 6s^2 - 54$$
$$0 = 6(s^2 - 9)$$
$$0 = 6(s + 3)(s - 3)$$
$$s + 3 = 0 \quad \text{or} \quad s - 3 = 0$$
$$s = -3 \quad \text{or} \quad s = 3$$

The side length of a cube cannot be negative, so $s = 3$ inches.

b. Volume of a cube $= s^3$

When $s = 3$:

$$V = 3^3$$
$$V = 27$$

The greatest possible volume is 27 cubic inches. The actual volume inside the cube may be less, depending on the thickness of the plastic. The thickness would cause the inside side length to be less than 3, so when this length is cubed to find the volume, it would be less than 27.

Quiz for the lessons "Factor Special Products" and "Factor Polynomials Completely"

1. $x^2 - 400 = x^2 - 20^2 = (x + 20)(x - 20)$

2. $18 - 32z^2 = 2(9 - 16z^2)$
$$= 2[3^2 - (4z)^2]$$
$$= 2(3 + 4z)(3 - 4z)$$

3. $169x^2 - 25y^2 = (13x)^2 - (5y)^2 = (13x + 5y)(13x - 5y)$

4. $n^2 - 6n + 9 = n^2 - 2(n \cdot 3) + 3^2 = (n - 3)^2$

5. $100a^2 + 20a + 1 = (10a)^2 + 2(10a \cdot 1) + 1^2$
$$= (10a + 1)^2$$

6. $8r^2 - 40rs + 50s^2 = 2(4r^2 - 20rs + 25s^2)$
$$= 2((2r)^2 - 2(2r \cdot 5s) + (5s)^2)$$
$$= 2(2r - 5s)^2$$

7. $3x^5 - 75x^3 = 3x^3(x^2 - 25) = 3x^3(x + 5)(x - 5)$

8. $72s^4 - 8s^2 = 8s^2(9s^2 - 1) = 8s^2(3s + 1)(3s - 1)$

9. $3x^4y - 300x^2y = 3x^2y(x^2 - 100)$
$$= 3x^2y(x + 10)(x - 10)$$

10. $a^3 - 4a^2 - 21a = a(a^2 - 4a - 21) = a(a - 7)(a + 3)$

11. $2h^4 + 28h^3 + 98h^2 = 2h^2(h^2 + 14h + 49)$
$$= 2h^2(h^2 + 2(h \cdot 7) + 7^2)$$
$$= 2h^2(h + 7)^2$$

12. $z^3 - 4z^2 - 16z + 64 = z^2(z - 4) - 16(z - 4)$
$$= (z - 4)(z^2 - 16)$$
$$= (z - 4)(z - 4)(z + 4)$$
$$= (z - 4)^2(z + 4)$$

13. $x^2 + 10x + 25 = 0$
$$(x + 5)^2 = 0$$
$$x + 5 = 0$$
$$x = -5$$

14.
$$48 - 27m^3 = 0$$
$$3(16 - 9m^2) = 0$$
$$3(4 + 3m)(4 - 3m) = 0$$
$$4 + 3m = 0 \quad \text{or} \quad 4 - 3m = 0$$
$$m = -\frac{4}{3} \quad \text{or} \quad m = \frac{4}{3}$$

15.
$$w^3 - w^2 - 4w + 4 = 0$$
$$w^3 - 4w - w^2 + 4 = 0$$
$$w(w^2 - 4) - (w^2 - 4) = 0$$
$$(w^2 - 4)(w - 1) = 0$$
$$(w + 2)(w - 2)(w - 1) = 0$$
$$w + 2 = 0 \quad \text{or} \quad w - 2 = 0 \quad \text{or} \quad w - 1 = 0$$
$$w = -2 \quad \text{or} \quad w = 2 \quad \text{or} \quad w = 1$$

16. $4x^3 - 28x^2 + 40x = 0$
$$4x(x^2 - 7x + 10) = 0$$
$$4x(x - 5)(x - 2) = 0$$
$$4x = 0 \quad \text{or} \quad x - 5 = 0 \quad \text{or} \quad x - 2 = 0$$
$$x = 0 \quad \text{or} \quad x = 5 \quad \text{or} \quad x = 2$$

17. $3x^5 - 6x^4 - 45x^3 = 0$
$$3x^3(x^2 - 2x - 15) = 0$$
$$3x^3(x - 5)(x + 3) = 0$$
$$3x^3 = 0 \quad \text{or} \quad x - 5 = 0 \quad \text{or} \quad x + 3 = 0$$
$$x = 0 \quad \text{or} \quad x = 5 \quad \text{or} \quad x = -3$$

18.
$$x^3 - 121x = 0$$
$$x(x^2 - 121) = 0$$
$$x(x + 11)(x - 11) = 0$$
$$x = 0 \quad \text{or} \quad x + 11 = 0 \quad \text{or} \quad x - 11 = 0$$
$$x = -11 \quad \text{or} \quad x = 11$$

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19. **a.** Volume of a cylinder $= \pi r^2 h = 8\pi r^2$

b. $72\pi = 8\pi r^2$

$0 = 8\pi r^2 - 72\pi$

$0 = 8\pi(r^2 - 9)$

$0 = 8\pi(r + 3)(r - 3)$

$r + 3 = 0$ or $r - 3 = 0$

$r = -3$ or $r = 3$

A geometric length must be positive, so $r = -3$ can be discarded. The radius of the cylinder is 3 inches.

Mixed Review of Problem Solving for the lessons "Factor $x^2 + bx + c$", "Factor $ax^2 + bx + c$", "Factor Special Products", and "Factor Polynomials Completely"

1. **a.** Area = length • width $= (w + 5) \cdot w = w^2 + 5w$

b. $150 = w^2 + 5w$

$0 = w^2 + 5w - 150$

$0 = (w + 15)(w - 10)$

$w + 15 = 0$ or $w - 10 = 0$

$w = -15$ or $w = 10$

The width cannot be negative, so $w = -15$ can be discarded. So, the width is 10 feet and the length is $10 + 5 = 15$ feet.

2. **a.** Volume = length • width • height

$= (x + 9)(x)(x - 4)$

$= x^3 + 5x^2 - 36x$

b. $180 = x^3 + 5x^2 - 36x$

$0 = x^3 + 5x^2 - 36x - 180$

$0 = x^2(x + 5) - 36(x + 5)$

$0 = (x + 5)(x^2 - 36)$

$0 = (x + 5)(x + 6)(x - 6)$

$x + 5 = 0$ or $x + 6 = 0$ or $x - 6 = 0$

$x = -5$ or $x = -6$ or $x = 6$

Because $x = 6$ is the only positive solution, the width is 6 inches, the length is $6 + 9 = 15$ inches , and the height is $6 - 4 = 2$ inches.

3. **a.** Area = length • width $= (x - 4)^2 = x^2 - 8x + 16$

b. $100 = x^2 - 8x + 16$

$0 = x^2 - 8x - 84$

$0 = (x - 14)(x + 6)$

$x - 14 = 0$ or $x + 6 = 0$

$x = 14$ or $x = -6$

Length cannot be negative, so $x = -6$ can be discarded. The side length of the original piece of wood was 14 inches. So, the original area was $x^2 = 14^2 = 196$ square inches.

4. *Sample answer:*

A cannonball that is shot straight up from the ground at 48 feet per second can be modeled using the vertical motion model $h = -16t^2 + 48t$.

When $h = 0$:

$0 = -16t(t - 3)$

$-16t = 0$ or $t - 3 = 0$

$t = 0$ or $t = 3$

These t-values represent the times (in seconds) that the cannonball is at height 0 ft, or on the ground. Because $t = 3$, it takes 3 seconds for the ball to return to the ground.

5. **a.** $h = -16t^2 + vt + s$

$h = -16t^2 + 80t + 3$

b. $99 = -16t^2 + 80t + 3$

$0 = -16t^2 + 80t - 96$

$0 = -16(t^2 - 5t + 6)$

$0 = -16(t - 3)(t - 2)$

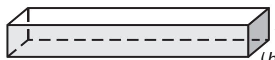
$t - 3 = 0$ or $t - 2 = 0$

$t = 3$ or $t = 2$

The ball reaches 99 feet after 2 seconds,

c. Yes, the ball reaches 99 feet twice, once at $t = 2$ seconds and once at $t = 3$ seconds.

6. **a.**



b. Volume = length • width • height

$= (h + 25)(h - 1)(h)$

$= h^3 + 24h^2 - 25h$

c. $600 = h^3 + 24h^2 - 25h$

$0 = h^3 + 24h^2 - 25h - 600$

$0 = h^2(h + 24) - 25(h + 24)$

$0 = (h + 24)(h^2 - 25)$

$0 = (h + 24)(h + 5)(h - 5)$

$h + 24 = 0$ or $h + 5 = 0$ or $h - 5 = 0$

$h = -24$ or $h = -5$ or $h = 5$

Because height cannot be negative, the height is 5 inches. The area of the top is the length $(h + 25)$ times the width $(h - 1)$. So, $A = 30 \cdot 4 = 120$ square inches.

7. No, the initial height of the ball is not given so there is not enough information given to solve for t when using the vertical motion model.

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8. Let $v = 0$, $s = 144$, and $h = 0$:

$$h = -16t^2 + vt + s$$
$$h = -16t^2 + 144$$
$$0 = -16(t^2 - 9)$$
$$0 = -16(t + 3)(t - 3)$$
$$t + 3 = 0 \quad \text{or} \quad t - 3 = 0$$
$$t = -3 \quad \text{or} \quad t = 3$$

A negative solution does not make sense in this situation, so $t = -3$ can be discarded. So, it takes 3 seconds for the ball to reach the ground.

9.
$$y = -0.005x^2 + 0.6x$$
$$10 = -0.005x^2 + 0.6x$$
$$0 = (-0.005x^2 + 0.6x - 10)(-1000)$$
$$0 = 5x^2 - 600x + 10{,}000$$
$$0 = 5(x^2 - 120x + 2000)$$
$$0 = 5(x - 100)(x - 20)$$
$$x - 100 = 0 \quad \text{or} \quad x - 20 = 0$$
$$x = 100 \quad \text{or} \quad x = 20$$

The ball reaches a height of 10 feet twice, once 20 feet away from the kicker and again 100 feet away from the kicker. Because the ball hits the goal post on the way down, the kicker must be 100 feet away from the goal post.

Chapter Review for the chapter "Polynomials and Factoring"

1. The greatest degree of the terms in a polynomial is called the *degree of the polynomial*.

2. No, for a term to be a monomial it must have whole number exponents. Because -1 is not a whole number, $2x^{-1}$ is not a monomial.

3. A polynomial is factored completely if it is written as a product of unfactorable polynomials with integer coefficients. *Sample answer:* $(x - 2)^2(x + 3)$.

4. B; The polynomial $5x - 22$ has two terms, so it is a binomial.

5. A; The polynomial $-11x^3$ has only one term, so it is a monomial.

6. C; The polynomial $x^2 + x + 1$ has three terms, so it is a trinomial.

7. $(9x + 6x^3 - 8x^2) + (-5x^3 + 6x)$

$= (6x^3 - 8x^2 + 9x) + (-5x^3 + 6x)$

$$\begin{array}{r} 6x^3 - 8x^2 + 9x \\ + \; -5x^3 \qquad\quad + 6x \\ \hline x^3 - 8x^2 + 15x \end{array}$$

8. $(7a^3 - 4a^2 - 2a + 1) + (a^3 - 1) =$
$$\begin{array}{r} 7a^3 - 4a^2 - 2a + 1 \\ + \; a^3 \qquad\qquad\quad - 1 \\ \hline 8a^3 - 4a^2 - 2a \end{array}$$

9. $(11y^5 + 3y^2 - 4) + (y^2 - y + 1) =$
$$\begin{array}{r} 11y^5 + 3y^2 \qquad - 4 \\ + \qquad\quad y^2 - y + 1 \\ \hline 11y^5 + 4y^2 - y - 3 \end{array}$$

10. $(3n^2 - 4n + 1) - (8n^2 - 4n + 17)$

$$\begin{array}{r} 3n^2 - 4n + 1 \\ -(8n^2 - 4n + 17) \end{array} \rightarrow \begin{array}{r} 3n^2 - 4n + 1 \\ + \; -8n^2 + 4n - 17 \\ \hline -5n^2 \qquad - 16 \end{array}$$

11. $(2s^3 + 8) - (-3s^3 + 7s - 5)$

$$\begin{array}{r} 2s^3 \qquad + 8 \\ -(-3s^3 + 7s - 5) \end{array} \rightarrow \begin{array}{r} 2s^3 \qquad + 8 \\ + \; 3s^3 - 7s + 5 \\ \hline 5s^3 - 7s + 13 \end{array}$$

12. $(-k^2 + 7k + 5) - (2k^4 - 3k^3 - 6)$

$$\begin{array}{r} -k^2 + 7k + 5 \\ -(2k^4 - 3k^3 \qquad\qquad - 6) \end{array} \rightarrow \begin{array}{r} -k^2 + 7k + 5 \\ + \; -2k^4 + 3k^3 \qquad\qquad + 6 \\ \hline -2k^4 + 3k^3 - k^2 + 7k + 11 \end{array}$$

13. $(x^2 - 2x + 1)(x - 3) = x^2(x - 3) - 2x(x - 3) + 1(x - 3)$

$= x^3 - 3x^2 - 2x^2 + 6x + x - 3$

$= x^3 - 5x^2 + 7x - 3$

14. $(y^2 + 5y + 4)(3y + 2)$

$= y^2(3y + 2) + 5y(3y + 2) + 4(3y + 2)$

$= 3y^3 + 2y^2 + 15y^2 + 10y + 12y + 8$

$= 3y^3 + 17y^2 + 22y + 8$

15.
$$\begin{array}{r} x - 4 \\ \times \quad x + 2 \\ \hline 2x - 8 \\ x^2 - 4x \quad \\ \hline x^2 - 2x - 8 \end{array}$$

16. $(5b^2 - b - 7)(b + 6) = 5b^2(b + 6) - b(b + 6) - 7(b + 6)$

$= 5b^3 + 30b^2 - b^2 - 6b - 7b - 42$

$= 5b^3 + 29b^2 - 13b - 42$

17.
$$\begin{array}{r} z + 8 \\ \times \quad z - 11 \\ \hline -11z - 88 \\ z^2 + 8z \quad \\ \hline z^2 - 3z - 88 \end{array}$$

18.
$$\begin{array}{r} 2a - 1 \\ \times \quad a - 3 \\ \hline -6a + 3 \\ 2a^2 - a \quad \\ \hline 2a^2 - 7a + 3 \end{array}$$

19.
$$\begin{array}{r} 6n + 7 \\ \times \quad 3n + 1 \\ \hline 6n + 7 \\ 18n^2 + 21n \quad \\ \hline 18n^2 + 27n + 7 \end{array}$$

20.
$$\begin{array}{r} 4n - 5 \\ \times \quad 7n - 3 \\ \hline -12n + 15 \\ 28n^2 - 35n \quad \\ \hline 28n^2 - 47n + 15 \end{array}$$

21.
$$\begin{array}{r} 3x - 2 \\ \times \quad x + 4 \\ \hline 12x - 8 \\ 3x^2 - 2x \quad \\ \hline 3x^2 + 10x - 8 \end{array}$$

22. $(x + 11)^2 = x^2 + 2(11x) + 11^2 = x^2 + 22x + 121$

23. $(6y + 1)^2 = (6y)^2 + 2(6y) + 1^2 = 36y^2 + 12y + 1$

24. $(2x - y)^2 = (2x)^2 - 2(2xy) + y^2 = 4x^2 - 4xy + y^2$

25. $(4a - 3)^2 = (4a)^2 - 2(12a) + 3^2 = 16a^2 - 24a + 9$

26. $(k + 7)(k - 7) = k^2 - 7^2 = k^2 - 49$

27. $(3s + 5)(3s - 5) = (3s)^2 - 5^2 = 9s^2 - 25$

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28. $2a^2 + 26a = 0$

$2a(a + 13) = 0$

$2a = 0 \quad \text{or} \quad a + 13 = 0$

$a = 0 \quad \text{or} \quad a = -13$

29. $3t^2 - 33t = 0$

$3t(t - 11) = 0$

$3t = 0 \quad \text{or} \quad t - 11 = 0$

$t = 0 \quad \text{or} \quad t = 11$

30. $8x^2 - 4x = 0$

$4x(2x - 1) = 0$

$4x = 0 \quad \text{or} \quad 2x - 1 = 0$

$x = 0 \quad \text{or} \quad x = \frac{1}{2}$

31. $m^2 = 9m$

$m^2 - 9m = 0$

$m(m - 9) = 0$

$m = 0 \quad \text{or} \quad m - 9 = 0$

$m = 9$

32. $5y^2 = -50y$

$5y^2 + 50y = 0$

$5y(y + 10) = 0$

$5y = 0 \quad \text{or} \quad y + 10 = 0$

$y = 0 \quad \text{or} \quad y = -10$

33. $21h^2 = 7h$

$21h^2 - 7h = 0$

$7h(3h - 1) = 0$

$7h = 0 \quad \text{or} \quad 3h - 1 = 0$

$h = 0 \quad \text{or} \quad h = \frac{1}{3}$

34. $n^2 + 15n + 26 = (n + 2)(n + 13)$

35. $s^2 + 10s - 11 = (s - 1)(s + 11)$

36. $b^2 - 5b - 14 = (b + 2)(b - 7)$

37. $a^2 + 5a - 84 = (a - 7)(a + 12)$

38. $t^2 - 24t + 135 = (t - 9)(t - 15)$

39. $x^2 + 4x - 32 = (x - 4)(x + 8)$

40. $p^2 + 9p + 14 = (p + 2)(p + 7)$

41. $c^2 + 8c + 15 = (c + 3)(c + 5)$

42. $y^2 - 10y + 21 = (y - 3)(y - 7)$

43. $7x^2 - 8x = -1$

$7x^2 - 8x + 1 = 0$

$(7x - 1)(x - 1) = 0$

$7x - 1 = 0 \quad \text{or} \quad x - 1 = 0$

$x = \frac{1}{7} \quad \text{or} \quad x = 1$

44. $4n^2 + 3 = 7n$

$4n^2 - 7n + 3 = 0$

$(4n - 3)(n - 1) = 0$

$4n - 3 = 0 \quad \text{or} \quad n - 1 = 0$

$n = \frac{3}{4} \quad \text{or} \quad n = 1$

45. $3s^2 + 4s + 4 = 8$

$3s^2 + 4s - 4 = 0$

$(3s - 2)(s + 2) = 0$

$3s - 2 = 0 \quad \text{or} \quad s + 2 = 0$

$s = \frac{2}{3} \quad \text{or} \quad s = -2$

46. $6z^2 + 13z = 5$

$6z^2 + 13z - 5 = 0$

$(3z - 1)(2z + 5) = 0$

$3z - 1 = 0 \quad \text{or} \quad 2z + 5 = 0$

$z = \frac{1}{3} \quad \text{or} \quad z = -\frac{5}{2}$

47. $-4r^2 = 18r + 18$

$0 = 4r^2 + 18r + 18$

$0 = 2(2r^2 + 9r + 9)$

$0 = 2(2r + 3)(r + 3)$

$2r + 3 = 0 \quad \text{or} \quad r + 3 = 0$

$r = -\frac{3}{2} \quad \text{or} \quad r = -3$

48. $9a^2 = 6a + 24$

$9a^2 - 6a - 24 = 0$

$3(3a^2 - 2a - 8) = 0$

$3(3a + 4)(a - 2) = 0$

$3a + 4 = 0 \quad \text{or} \quad a - 2 = 0$

$a = -\frac{4}{3} \quad \text{or} \quad a = 2$

49. $h = -16t^2 + vt + s$

$h = -16t^2 + 46t + 6$

$0 = -16t^2 + 46t + 6$

$0 = -2(8t^2 - 23t - 3)$

$0 = -2(8t + 1)(t - 3)$

$8t + 1 = 0 \quad \text{or} \quad t - 3 = 0$

$t = -\frac{1}{8} \quad \text{or} \quad t = 3$

A negative solution does not makes sense in this situation, so disregard $-\frac{1}{8}$. The ball lands on the ground after 3 seconds.

50. Area = length • width

$21 = (2w - 1)(w)$

$0 = 2w^2 - w - 21$

$0 = (2w - 7)(w + 3)$

$2w - 7 = 0 \quad \text{or} \quad w + 3 = 0$

$w = \frac{7}{2} \quad \text{or} \quad w = -3$

The width cannot be negative, so disregard -3. The length of the rectangle is $2w - 1 = 2\left(\frac{7}{2}\right) - 1 = 6$ inches.

51. $z^2 - 225 = z^2 - 15^2 = (z + 15)(z - 15)$

52. $a^2 - 16y^2 = a^2 - (4y)^2 = (a + 4y)(a - 4y)$

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53. $12 - 48n^2 = 12(1 - 4n^2)$
$= 12(1 - (2n)^2)$
$= 12(1 + 2n)(1 - 2n)$

54. $x^2 + 20x + 100 = x^2 + 2(x \cdot 10) + 10^2 = (x + 10)^2$

55. $16p^2 - 8p + 1 = (4p)^2 - 2(4p \cdot 1) + 1^2 = (4p - 1)^2$

56. $-2y^2 + 32y - 128 = -2(y^2 - 16y + 64)$
$= -2(y^2 - 2(y \cdot 8) + 8^2)$
$= -2(y - 8)^2$

57. $h = -16t^2 + vt + s$
$h = -16t^2 + 16$
$0 = -16t^2 + 16$
$0 = -16(t^2 - 1)$
$0 = -16(t + 1)(t - 1)$
$t + 1 = 0$ or $t - 1 = 0$
$t = -1$ or $t = 1$

A negative answer does not make sense in this situation. The penny lands after 1 second.

58. $a^3 + 6a - 5a^2 - 30 = a(a^2 + 6) - 5(a^2 + 6)$
$= (a^2 + 6)(a - 5)$

59. $y^2 + 3y + yx + 3x = y(y + 3) + x(y + 3)$
$= (y + 3)(y + x)$

60. $x^3 - 11x^2 - x + 11 = x^2(x - 11) - (x - 11)$
$= (x - 11)(x^2 - 1)$
$= (x - 11)(x + 1)(x - 1)$

61. $5s^4 - 125s^2 = 5s^2(s^2 - 25) = 5s^2(s + 5)(s - 5)$

62. $147n^5 - 3n^3 = 3n^3(49n^2 - 1) = 3n^3(7n + 1)(7n - 1)$

63. $2z^3 + 2z^2 - 60z = 2z(z^2 + z - 30) = 2z(z + 6)(z - 5)$

64. $x^3 + 5x^2 - x - 5 = x^2(x + 5) - (x + 5)$
$= (x + 5)(x^2 - 1)$
$= (x + 5)(x + 1)(x - 1)$

65. $2b^3 + 3b^2 - 8b - 12 = b^2(2b + 3) - 4(2b + 3)$
$= (2b + 3)(b^2 - 4)$
$= (2b + 3)(b + 2)(b - 2)$

66. $x^3 + x^2 - 6x - 6 = x^2(x + 1) - 6(x + 1)$
$= (x + 1)(x^2 - 6)$

Chapter Test for the chapter "Polynomials and Factoring"

1.
$$\begin{array}{r} a^2 - 4a + 6 \\ -\ 3a^2 + 13a + 1 \\ \hline -\ 2a^2 + 9a + 7 \end{array}$$

2. $(5x^2 - 2) + (8x^3 + 2x^2 - x + 9)$
$= 8x^3 + (5x^2 + 2x^2) - x + (-2 + 9)$
$= 8x^3 + 7x^2 - x + 7$

3.
$$\begin{array}{r} 15n^2 + 7n - 1 \\ -\ (\ 4n^2 - 3n - 8) \\ \hline \end{array} \rightarrow \begin{array}{r} 15n^2 + 7n - 1 \\ +\ -4n^2 + 3n + 8 \\ \hline 11n^2 + 10n + 7 \end{array}$$

4. $(9c^3 - 11c^2 + 2c) - (-6c^2 - 3c + 11)$
$= 9c^3 - 11c^2 + 2c + 6c^2 + 3c - 11$
$= 9c^3 + (-11c^2 + 6c^2) + (2c + 3c) - 11$
$= 9c^3 - 5c^2 + 5c - 11$

5.
$$\begin{array}{r} 2z + 9 \\ \times \quad z - 7 \\ \hline -\ 14z - 63 \\ 2z^2 + 9z \quad\quad \\ \hline 2z^2 - 5z - 63 \end{array}$$

6.
$$\begin{array}{r} 5m - 8 \\ \times \quad 5m - 7 \\ \hline -\ 35m + 56 \\ 25m^2 - 40m \quad\quad \\ \hline 25m^2 - 75m + 56 \end{array}$$

7. $(b + 2)(-b^2 + 4b - 3)$
$= -b^2(b + 2) + 4b(b + 2) - 3(b + 2)$
$= -b^3 - 2b^2 + 4b^2 + 8b - 3b - 6$
$= -b^3 + 2b^2 + 5b - 6$

8.
$$\begin{array}{r} 5 + 7y \\ \times \quad 1 - 9y \\ \hline -\ 45y - 63y^2 \\ 5 + 7y \quad\quad \\ \hline 5 - 38y - 63y^2 \end{array}$$

9. $(2x^2 - 3x + 5)(x - 4)$
$= 2x^2(x - 4) - 3x(x - 4) + 5(x - 4)$
$= 2x^3 - 8x^2 - 3x^2 + 12x + 5x - 20$
$= 2x^3 - 11x^2 + 17x - 20$

10. $(5p - 6)(5p + 6) = (5p)^2 - 6^2 = 25p^2 - 36$

11. $(12 - 3g)^2 = 12^2 - 2(12 \cdot 3g) + (3g)^2$
$= 144 - 72g + 9g^2$

12. $(2s + 9t)^2 = (2s)^2 + 2(2s \cdot 9t) + (9t)^2$
$= 4s^2 + 36st + 81t^2$

13. $(11a - 4b)(11a + 4b) = (11a)^2 - (4b)^2$
$= 121a^2 - 16b^2$

14. $x^2 + 8x + 7 = (x + 1)(x + 7)$

15. $2n^2 - 11n + 15 = (2n - 5)(n - 3)$

16. $-12r^2 + 5r + 3 = -(12r^2 - 5r - 3)$
$= -(3r + 1)(4r - 3)$

17. $t^2 - 10t + 25 = t^2 - 2(5 \cdot t) + 5^2 = (t - 5)^2$

18. $-3n^2 + 75 = -3(n^2 - 25)$
$= -3(n^2 - 5^2)$
$= -3(n + 5)(n - 5)$

19. $3x^2 + 29x - 44 = (3x - 4)(x + 11)$

20. $x^2 - 49 = x^2 - 7^2 = (x + 7)(x - 7)$

21. $2a^4 + 21a^3 + 49a^2 = a^2(2a^2 + 21a + 49)$
$= a^2(2a + 7)(a + 7)$

22. $y^3 + 2y^2 - 81y - 162 = y^2(y + 2) - 81(y + 2)$
$= (y + 2)(y^2 - 81)$
$= (y + 2)(y + 9)(y - 9)$

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23. $25a = 10a^2$

$0 = 10a^2 - 25a$

$0 = 5a(2a - 5)$

$5a = 0$ or $2a - 5 = 0$

$a = 0$ or $a = \frac{5}{2}$

24. $21z^2 + 85z - 26 = 0$

$(3z + 13)(7z - 2) = 0$

$3z + 13 = 0$ or $7z - 2 = 0$

$z = -\frac{13}{3}$ or $z = \frac{2}{7}$

25. $x^2 - 22x = -121$

$x^2 - 22x + 121 = 0$

$x^2 - 2(11 \cdot x) + 11^2 = 0$

$(x - 11)^2 = 0$

$x - 11 = 0$

$x = 11$

26. $a^2 - 11a + 24 = 0$

$(a - 3)(a - 8) = 0$

$a - 3 = 0$ or $a - 8 = 0$

$a = 3$ or $a = 8$

27. $t^2 + 7t = 60$

$t^2 + 7t - 60 = 0$

$(t + 12)(t - 5) = 0$

$t + 12 = 0$ or $t - 5 = 0$

$t = -12$ or $t = 5$

28. $4x^2 = 22x + 42$

$4x^2 - 22x - 42 = 0$

$2(2x^2 - 11x - 21) = 0$

$2(2x + 3)(x - 7) = 0$

$2x + 3 = 0$ or $x - 7 = 0$

$x = -\frac{3}{2}$ or $x = 7$

29. $56b^2 + b = 1$

$56b^2 + b - 1 = 0$

$(8b - 1)(7b + 1) = 0$

$8b - 1 = 0$ or $7b + 1 = 0$

$b = \frac{1}{8}$ or $b = -\frac{1}{7}$

30. $n^3 - 121n = 0$

$n(n^2 - 121) = 0$

$n(n + 11)(n - 11) = 0$

$n = 0$ or $n + 11 = 0$ or $n - 11 = 0$

$n = -11$ or $n = 11$

31. $a^3 + a^2 = 64a + 64$

$a^3 + a^2 - 64a - 64 = 0$

$a^2(a + 1) - 64(a + 1) = 0$

$(a + 1)(a^2 - 64) = 0$

$(a + 1)(a + 8)(a - 8) = 0$

$a + 1 = 0$ or $a + 8 = 0$ or $a - 8 = 0$

$a = -1$ or $a = -8$ or $a = 8$

32. a. $h = -16t^2 + vt + s$

$h = -16t^2 + 4t$

b. $0 = -16t^2 + 4t$

$0 = -4t(4t - 1)$

$-4t = 0$ or $4t - 1 = 0$

$t = 0$ or $t = \frac{1}{4}$

The solution $t = 0$ represents the time at the beginning of the jump. The cricket will land back on the ground in $\frac{1}{4}$ of a second.

33. a. Area of poster 1 = Area of poster 2

$(3w)(w) = 2w(w + 2)$

$3w^2 = 2w^2 + 4w$

b. $3w^2 = 2w^2 + 4w$

$w^2 - 4w = 0$

$w(w - 4) = 0$

$w = 0$ or $w - 4 = 0$

$w = 4$

Because the width cannot be zero, the width of poster 1 is 4 feet. So, the dimensions of the posters are:

Poster 1: 4 feet by 12 feet

Poster 2: 6 feet by 8 feet

34. $h = -16t^2 + vt + s$

$h = -16t^2 + 225$

$0 = -16t^2 + 225$

$0 = 15^2 - (4t)^2$

$0 = (15 + 4t)(15 - 4t)$

$15 + 4t = 0$ or $15 - 4t = 0$

$t = -\frac{15}{4}$ or $t = \frac{15}{4} = 3.75$

A negative solution does not make sense in this situation, so $t = -\frac{15}{4}$ can be disregarded. The drop of paint reaches the ground in 3.75 seconds.

35. a. Volume = length • width • height

$= (x + 6)(x - 2)(x - 1)$

$= (x^2 + 4x - 12)(x - 1)$

$= x^3 + 3x^2 - 16x + 12$

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b. $60 = x^3 + 3x^2 - 16x + 12$

$0 = x^3 + 3x^2 - 16x - 48$

$0 = x^2(x + 3) - 16(x + 3)$

$0 = (x + 3)(x^2 - 16)$

$0 = (x + 3)(x + 4)(x - 4)$

$x + 3 = 0$ or $x + 4 = 0$ or $x - 4 = 0$

$x = -3$ or $x = -4$ or $x = 4$

A dimension cannot be negative, so $x = -3$ and $x = -4$ can be disregarded. Because $x = 4$ inches, the dimensions of the box are 10 inches long, 2 inches wide and 3 inches high.

Extra Practice for the chapter "Polynomials and Factoring"

1. $(6x^2 + 7) + (x^2 - 9) = (6x^2 + x^2) + (7 - 9) = 7x^2 - 2$

2. $(8y^2 - 3y - 10) + (-11y^2 + 2y - 7)$

$= (8y^2 - 11y^2) + (-3y + 2y) + (-10 - 7)$

$= -3y^2 - y - 17$

3. $(10m^2 - 7m + 2) - (3m^2 - 2m + 5)$

$= 10m^2 - 7m + 2 - 3m^2 + 2m - 5$

$= (10m^2 - 3m^2) + (-7m + 2m) + (2 - 5)$

$= 7m^2 - 5m - 3$

4. $(2t^3 - 3t^2 + 5t) - (6t^3 + 3t^2 - 5t)$

$= 2t^3 - 3t^2 + 5t - 6t^3 - 3t^2 + 5t$

$= (2t^3 - 6t^3) + (-3t^2 - 3t^2) + (5t + 5t)$

$= -4t^3 - 6t^2 + 10t$

5. $(6b^3 + 12b^2 - b) - (15b^2 + 7b - 8)$

$= 6b^3 + 12b^2 - b - 15b^2 - 7b + 8$

$= 6b^3 + (12b^2 - 15b^2) + (-b - 7b) + 8$

$= 6b^3 - 3b^2 - 8b + 8$

6. $(r^2 - 8 + 4r^3 + 5r) - (7r^3 - 3r^2 + 5)$

$= 4r^3 + r^2 + 5r - 8 - 7r^3 + 3r^2 - 5$

$= (4r^3 - 7r^3) + (r^2 + 3r^2) + 5r + (-8 - 5)$

$= -3r^3 + 4r^2 + 5r - 13$

7. $5x^4(2x^3 - 3x^2 + 5x - 1)$

$= 5x^4(2x^3) - 5x^4(3x^2) + 5x^4(5x) - 5x^4(1)$

$= 10x^7 - 15x^6 + 25x^5 - 5x^4$

8. $(x^2 + 4x + 2)(x + 7)$

$= x^2(x + 7) + 4x(x + 7) + 2(x + 7)$

$= x^3 + 7x^2 + 4x^2 + 28x + 2x + 14$

$= x^3 + 11x^2 + 30x + 14$

9. $(2x + 3)(4x + 2)$

$= 2x(4x) + 2x(2) + 3(4x) + 3(2)$

$= 8x^2 + 4x + 12x + 6 = 8x^2 + 16x + 6$

10. $(2x^2 - 5x + 6)(3x - 2)$

$= 2x^2(3x - 2) - 5x(3x - 2) + 6(3x - 2)$

$= 6x^3 - 4x^2 - 15x^2 + 10x + 18x - 12$

$= 6x^3 - 19x^2 + 28x - 12$

11. $(3x - 7)(x + 5) = 3x(x) + 3x(5) - 7(x) - 7(5)$

$= 3x^2 + 15x - 7x - 35 = 3x^2 + 8x - 35$

12. $(9t - 2)(2t - 3) = 9t(2t) - 9t(3) - 2(2t) - 2(-3)$

$= 18t^2 - 27t - 4t + 6 = 18t^2 - 31t + 6$

13. $(x + 10)^2 = x^2 + 2(x)(10) + 10^2 = x^2 + 20x + 100$

14. $(m + 8)(m - 8) = m^2 - 8^2 = m^2 - 64$

15. $(4x - 2)(4x + 2) = (4x)^2 - 2^2 = 16x^2 - 4$

16. $(3x - 4y)(3x + 4y) = (3x)^2 - (4y)^2 = 9x^2 - 16y^2$

17. $(6 - 3t)(6 + 3t) = 6^2 - (3t)^2 = 36 - 9t^2$

18. $(-11x - 4y)^2 = (-11x)^2 + 2(-11x)(-4y) + (4y)^2$

$= 121x^2 + 88xy + 16y^2$

19. $(m + 8)(m - 2) = 0$

$m + 8 = 0$ or $m - 2 = 0$

$m = -8$ or $m = 2$

The solutions are -8 and 2.

20. $(2y - 6)(y + 3) = 0$

$2y - 6 = 0$ or $y + 3 = 0$

$y = 3$ or $y = -3$

The solutions are -3 and 3.

21. $(5y - 3)(2y - 4) = 0$

$5y - 3 = 0$ or $2y - 4 = 0$

$y = \frac{3}{5}$ or $y = 2$

The solutions are $\frac{3}{5}$ and 2.

22. $3b^2 + 9b = 0$

$3b(b + 3) = 0$

$3b = 0$ or $b + 3 = 0$

$b = 0$ or $b = -3$

The solutions are -3 and 0.

23. $-12m^2 - 3m = 0$

$-3m(4m + 1) = 0$

$-3m = 0$ or $4m + 1 = 0$

$m = 0$ or $m = -\frac{1}{4}$

The solutions are $-\frac{1}{4}$ and 0.

24. $14k^2 = 28k$

$14k^2 - 28k = 0$

$14k(k - 2) = 0$

$14k = 0$ or $k - 2 = 0$

$k = 0$ or $k = 2$

The solutions are 0 and 2.

25. $y^2 + 7y + 12$

Because b and c are both positive, the factors must both be positive.

Using an organized list, the factors 3 and 4 have a sum of 7, so they are the correct values.

$y^2 + 7y + 12 = (y + 3)(y + 4)$

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26. $x^2 - 12x + 35$

Because b is negative and c is positive, both factors must be negative.

Using an aranged list, the factors -5 and -7 have a sum of -12, so they are the correct values.

$x^2 - 12x + 35 = (x - 5)(x - 7)$

27. $x^2 + 5x - 36$

Because c is negative, the factors have opposite signs.

Using an organized list, the factors 9 and -4 have a sum of 5, so they are the correct values.

$x^2 + 5x - 36 = (x + 9)(x - 4)$

28. $q^2 + 3q - 40$

Because c is negative, the factors have opposite signs.

Using an organized list, the factors 8 and -5 have a sum of 3, so they are the correct values.

$q^2 + 3q - 40 = (q + 8)(q - 5)$

29. $m^2 - 29m + 100$

Because b is positive and c is negative, the factors are both negative.

Using an organized list, the factors -25 and -4 have a sum of -29, so they are the correct values.

$m^2 - 29m + 100 = (m - 25)(m - 4)$

30. $y^2 + 14y - 72$

Because c is negative, the factors have opposite signs.

Using an organized list the factors 18 and -4 have a sum of 14, so they are the correct values.

$y^2 + 14y - 72 = (y + 18)(y - 4)$

31. $m^2 - 7m + 10 = 0$

$(m - 5)(m - 2) = 0$

$m - 5 = 0$ or $m - 2 = 0$

$m = 5$ or $m = 2$

The solutions are 2 and 5.

32. $p^2 - 7p = 18$

$p^2 - 7b - 18 = 0$

$(p - 9)(p + 2) = 0$

$p - 9 = 0$ or $p + 2 = 0$

$p = 9$ or $p = -2$

The solutions are -2 and 9.

33. $z^2 - 13z + 24 = -12$

$z^2 - 13z + 36 = 0$

$(z - 9)(z - 4) = 0$

$z - 9 = 0$ or $z - 4 = 0$

$z = 9$ or $z = 4$

The solutions are 4 and 9.

34. $n^2 + 8 = 6n$

$n^2 - 6n + 8 = 0$

$(n - 2)(n - 4) = 0$

$n - 2 = 0$ or $n - 4 = 0$

$n = 2$ or $n = 4$

The solutions are 2 and 4.

35. $r^2 - 15r = -8r - 10$

$r^2 - 7r + 10 = 0$

$(r - 5)(r - 2) = 0$

$r - 5 = 0$ or $r - 2 = 0$

$r = 5$ or $r = 2$

The solutions are 2 and 5.

36. $c^2 - 8 = -13c + 6$

$c^2 + 13c - 14 = 0$

$(c + 14)(c - 1) = 0$

$c + 14 = 0$ or $c - 1 = 0$

$c = -14$ or $c = 1$

The solutions are -14 and 1.

37. $-x^2 + 5x - 6 = -(x^2 - 5x + 6)$

Because b is negative and c is positive, both factors of c must be negative.

Using a table to find the correct factorization, $-x^2 + 5x - 6 = -(x - 2)(x - 3)$.

38. $3k^2 - 10k + 8$

Because b is negative and c is positive, both factors of c must be negative.

Using a table to find the correct factorization,

$3k^2 - 10k + 8 = (k - 2)(3k - 4)$.

39. $4k^2 - 12k + 5$

Because b is negative and c is positive, both factors of c must be negative.

Using a table to find the correct factorization,

$4k^2 - 12k + 5 = (2k - 1)(2k - 5)$.

40. $6t^2 - 5t - 6$

Because c is negative, the factors of c have different signs.

Using a table to find the correct factorization, $6t^2 - 5t - 6 = (2t - 3)(3t + 2)$.

41. $-3s^2 - 7s - 2 = -(3s^2 + 7s + 2)$

Because b is positive and c is positive, both factors of c must be positive.

Using a table to find the correct factorization, $-3s^2 - 7s - 2 = -(s + 2)(3s + 1)$.

42. $2x^2 - 5x + 3$

Because b is negative and c is positive, both factors of c must be negative.

Using a table to find the correct factorization, $2v^2 - 5v + 3 = (v - 1)(2v - 3)$

43. $-3x^2 + 14x - 8 = 0$

$(-3x + 2)(x - 4) = 0$

$-3x + 2 = 0$ or $x - 4 = 0$

$x = \frac{2}{3}$ or $x = 4$

The solutions are $\frac{2}{3}$ and 4.

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44. $8t^2 + 6t = 9$

$8t^2 + 6t - 9 = 0$

$(4t - 3)(2t + 3) = 0$

$4t - 3 = 0$ or $2t + 3 = 0$

$t = \frac{3}{4}$ or $t = -\frac{3}{2}$

The solutions are $-\frac{3}{2}$ and $\frac{3}{4}$.

45. $2x^2 + 3x - 2 = 0$

$(2x - 1)(x + 2) = 0$

$2x - 1 = 0$ or $x + 2 = 0$

$x = \frac{1}{2}$ or $x = -2$

The solutions are -2 and $\frac{1}{2}$.

46. $3p^2 - 28 = 17p$

$3p^2 - 17p - 28 = 0$

$(3p + 4)(p - 7) = 0$

$3p + 4 = 0$ or $p - 7 = 0$

$p = -\frac{4}{3}$ or $p = 7$

The solutions are $-\frac{4}{3}$ and 7.

47. $16m^2 - 1 = -15m$

$16m^2 + 15m - 1 = 0$

$(16m - 1)(m + 1) = 0$

$16m - 1 = 0$ or $m + 1 = 0$

$m = \frac{1}{16}$ or $m = -1$

The solutions are -1 and $\frac{1}{16}$.

48. $t(6t - 7) = 3$

$6t^2 - 7t = 3$

$6t^2 - 7t - 3 = 0$

$(3t + 1)(2t - 3) = 0$

$3t + 1 = 0$ or $2t - 3 = 0$

$t = -\frac{1}{3}$ or $t = \frac{3}{2}$

The solutions are $-\frac{1}{3}$ and $\frac{3}{2}$.

49. $y^2 - 36 = y^2 - 6^2 = (y + 6)(y - 6)$

50. $9y^2 - 49 = (3y)^2 - 7^2 = (3y + 7)(3y - 7)$

51. $12y^2 - 27 = 3(4y^2 - 9) = 3[(2y)^2 - 3^2]$

$= 3(2y + 3)(2y - 3)$

52. $x^2 - 8x + 16 = x^2 - 2(x \cdot 4) + 4^2 = (x - 4)^2$

53. $4x^2 - 12x + 9 = (2x)^2 - 2(2x \cdot 3) + 3^2 = (2x - 3)^2$

54. $27x^2 - 36x + 12 = 3(9x^2 - 12x + 4)$

$= 3[(3x)^2 - 2(3x \cdot 2) + 2^2] = 3(3x - 2)^2$

55. $g^2 + 10g + 25 = g^2 + 2(g \cdot 5) + 5^2 = (g + 5)^2$

56. $9b^2 + 24b + 16 = (3b)^2 + 2(3b \cdot 4) + 4^2 = (3b + 4)^2$

57. $4w^2 + 28w + 49 = (2w)^2 + 2(2w \cdot 7) + 7^2$

$= (2w + 7)^2$

58. $2x^2 + 8x + 6 = 2(x^2 + 4x + 3) = 2(x + 1)(x + 3)$

59. $3z^2 - 16z + 5 = (3z - 1)(z - 5)$

60. $5m^2 - 23m + 12 = (5m - 3)(m - 4)$

61. $3y^2 + 15y^2 + 2y + 10 = (3y^3 + 15y^2) + (2y + 10)$

$= 3y^2(y + 5) + 2(y + 5)$

$= (y + 5)(3y^2 + 2)$

62. $30z^3 - 14z^2 - 8z = 2z(15z^2 - 7z - 4)$

$= 2z(3z + 1)(5z - 4)$

63. $98m^3 - 18\text{m} = 2m(49m^2 - 9)$

$= 2m[(7m)^2 - 3^2]$

$= 2m(7m + 3)(7m - 3)$

64. $8h^2k - 32k = 8k(h^2 - 4)$

$= 8k(h^2 - 2^2)$

$= 8k(h + 2)(h - 2)$

65. $2h^3 - 3h^2 - 18h + 27 = (2h^3 - 3h^2) + (-18h + 27)$

$= h^2(2h - 3) - 9(2h - 3)$

$= (2h - 3)(h^2 - 9)$

$= (2h - 3)(h + 3)(h - 3)$

66. $-12z^3 + 12z^2 - 3z = -3z(4z^2 - 4z + 1)$

$= -3z[(2z)^2 - 2(2z \cdot 1) + 1^2]$

$= -3z(2z - 1)^2$

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