# Chapter 7 Exponents and Exponential Functions

***Prerequisite Skills for the chapter "Exponents and Exponential Functions"***

**1.** $13^8$: exponent = 8, base = 13

**2.** An expression that represents repeated multiplication of the same factor is called a *power*.

**3.** $10^2 = 10 \cdot 10 = 100$

**4.** $3^3 = 3 \cdot 3 \cdot 3 = 27$

**5.** $\left(\frac{5}{6}\right)^2 = \frac{5^2}{6^2} = \frac{25}{36}$

**6.** $\left(\frac{1}{2}\right)^3 = \frac{1^3}{2^3} = \frac{1}{8}$

**7.** 6.01, 6.12, 6.2

**8.** 0.0098, 0.073, 0.101

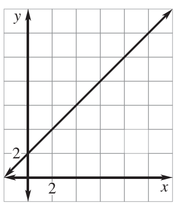
**9.** $4\% = 04\% = 0.04$

**10.** $0.5\% = 005\% = 0.005$

**11.** $13.8\% = 13.8\% = 0.138$

**12.** $145\% = 145\% = 1.45$

**13.** Let $x$ be the input, or independent variable, and let $y$ be the output, or dependent variable. Notice that each output is 2 more than the corresponding input. So, a rule for the function is $y = x + 2$. To graph the function, plot a point for each ordered pair. Draw a line through the points.




## Lesson 7.1 Apply Exponent Properties Involving Products

***Investigating Algebra Activity for the lesson "Apply Exponent Properties Involving Products"***

*Explore 1*

| Expression | Expression as repeated multiplication |
|---|---|
| $7^4 \cdot 7^5$ | $(7 \cdot 7 \cdot 7 \cdot 7) \cdot (7 \cdot 7 \cdot 7 \cdot 7 \cdot 7)$ |
| $(-4)^2 \cdot (-4)^3$ | $[(-4) \cdot (-4)] \cdot [(-4) \cdot (-4) \cdot (-4)]$ |
| $x^1 \cdot x^5$ | $(x) \cdot (x \cdot x \cdot x \cdot x \cdot x)$ |

| Expression | Number of factors | Simplified expression |
|---|---|---|
| $7^4 \cdot 7^5$ | 9 | $7^9$ |
| $(-4)^2 \cdot (-4)^3$ | 5 | $(-4)^5$ |
| $x^1 \cdot x^5$ | 6 | $x^6$ |

*Sample answer:* The exponent of the simplified expression is equal to the sum of the exponents in the first column.

*Explore 2*

| Expression | Expanded Expression |
|---|---|
| $(5^3)^2$ | $(5^3) \cdot (5^3)$ |
| $[(-6)^2]^4$ | $(-6)^2 \cdot (-6)^2 \cdot (-6)^2 \cdot (-6)^2$ |
| $(a^3)^3$ | $(a^3) \cdot (a^3) \cdot (a^3)$ |

| Expression | Expression as repeated multiplication |
|---|---|
| $(5^3)^2$ | $(5 \cdot 5 \cdot 5) \cdot (5 \cdot 5 \cdot 5)$ |
| $[(-6)^2]^4$ | $(-6)(-6)(-6)(-6)(-6)(-6)(-6)(-6)$ |
| $(a^3)^3$ | $(a \cdot a \cdot a) \cdot (a \cdot a \cdot a) \cdot (a \cdot a \cdot a)$ |

| Expression | Number of factors | Simplified Expression |
|---|---|---|
| $(5^3)^2$ | 6 | $5^6$ |
| $[(-6)^2]^4$ | 8 | $(-6)^8$ |
| $(a^3)^3$ | 9 | $a^9$ |

*Sample answer:* The exponent of the simplied expression is equal to the product of the exponents in the first column.

*Draw Conclusions*

**1.** $5^2 \cdot 5^3 = 5^{2+3} = 5^5$

**2.** $(-6)^1 \cdot (-6)^4 = (-6)^{1+4} = (-6)^5$

**3.** $m^6 \cdot m^4 = m^{6+4} = m^{10}$

**4.** $10^{3(3)} = 10^{3 \cdot 3} = 10^9$

**5.** $[(-2)^3]^4 = (-2)^{3 \cdot 4} = (-2)^{12}$

**6.** $c^{2(6)} = c^{2 \cdot 6} = c^{12}$

**7.** If $a$ is a real number and $m$ and $n$ are positive integers, then $a^m \cdot a^n = a^{m+n}$.

**8.** If $a$ is a real number and $m$ and $n$ are positive integers, then $(a^m)^n = a^{mn}$.

***Guided Practice for the lesson "Apply Exponent Properties Involving Products"***

**1.** $3^2 \cdot 3^7 = 3^{2+7} = 3^9$

**2.** $5 \cdot 5^9 = 5^{1+9} = 5^{10}$

**3.** $(-7)^2(-7) = (-7)^{2+1} = (-7)^3$

**4.** $x^2 \cdot x^6 \cdot x = x^{2+6+1} = x^9$

**5.** $(4^2)^7 = 4^{2 \cdot 7} = 4^{14}$

**6.** $[(-2)^4]^5 = (-2)^{4 \cdot 5} = (-2)^{20}$

**7.** $(n^3)^6 = n^{3 \cdot 6} = n^{18}$

**8.** $[(m+1)^5]^4 = (m+1)^{5 \cdot 4} = (m+1)^{20}$

**9.** $(42 \cdot 12)^2 = 42^2 \cdot 12^2$

**10.** $(-3n)^2 = (-3 \cdot n)^2 = (-3)^2 \cdot n^2 = 9n^2$

**11.** $(9m^3n)^4 = (9 \cdot m^3 \cdot n)^4 = 9^4 \cdot m^{3 \cdot 4} \cdot n^4 = 6561m^{12}n^4$

**12.** $5 \cdot (5x^2)^4 = 5 \cdot (5 \cdot x^2)^4 = 5 \cdot 5^4 \cdot x^{2 \cdot 4} = 5^{1+4} \cdot x^8$
$= 3125x^8$

**13.** $10^2 \cdot 10^4 = 10^{2+4} = 10^6$

About $10^6$, or 1,000,000, bees were studied in Idaho.

***Exercises for the lesson "Apply Exponent Properties Involving Products"***

***Skill Practice***

**1.** The *order of magnitude* of the quantity 93,534,004 people is the power of 10 nearest the quantity, or $10^8$ people.

**2.** When powers have the same base, their product is the base raised to the sum of the exponents.

**3.** $4^2 \cdot 4^6 = 4^{2+6} = 4^8$

**4.** $8^5 \cdot 8^2 = 8^{5+2} = 8^7$

5. $3^3 \cdot 3 = 3^{3+1} = 3^4$

6. $9 \cdot 9^5 = 9^{1+5} = 9^6$

7. $(-7)^4(-7)^5 = (-7)^{4+5} = (-7)^9$

8. $(-6)^6(-6) = (-6)^{6+1} = (-6)^7$

9. $2^4 \cdot 2^9 \cdot 2 = 2^{4+9+1} = 2^{14}$

10. $(-3)^2(-3)^{11}(-3) = (-3)^{2+11+1} = (-3)^{14}$

11. $(3^5)^2 = 3^{5 \cdot 2} = 3^{10}$

12. $(7^4)^3 = 7^{4 \cdot 3} = 7^{12}$

13. $[(-5)^3]^4 = (-5)^{3 \cdot 4} = (-5)^{12}$

14. $[(-8)^9]^2 = (-8)^{9 \cdot 2} = (-8)^{18}$

15. $(15 \cdot 29)^3 = 15^3 \cdot 29^3$

16. $(17 \cdot 16)^4 = 17^4 \cdot 16^4$

17. $(132 \cdot 9)^6 = 132^6 \cdot 9^6$

18. $((-14) \cdot 22)^5 = (-14)^5 \cdot 22^5$

19. $x^4 \cdot x^2 = x^{4+2} = x^6$

20. $y^9 \cdot y = y^{9+1} = y^{10}$

21. $z^2 \cdot z \cdot z^3 = z^{2+1+3} = z^6$

22. $a^4 \cdot a^3 \cdot a^{10} = a^{4+3+10} = a^{17}$

23. $(x^5)^2 = x^{5 \cdot 2} = x^{10}$

24. $(y^4)^6 = y^{4 \cdot 6} = y^{24}$

25. $[(b-2)^2]^6 = (b-2)^{2 \cdot 6} = (b-2)^{12}$

26. $[(d+9)^7]^3 = (d+9)^{7 \cdot 3} = (d+9)^{21}$

27. $(-5x)^2 = (-5)^2 \cdot x^2 = 25x^2$

28. $-(5x)^2 = -(5^2 \cdot x^2) = -25x^2$

29. $(7xy)^2 = 7^2 \cdot x^2 \cdot y^2 = 49x^2y^2$

30. $(5pq)^3 = 5^3 \cdot p^3 \cdot q^3 = 125p^3q^3$

31. $(-10x^6)^2 \cdot x^2 = (-10)^2 \cdot x^{6 \cdot 2} \cdot x^2$
$= 100 \cdot x^{12} \cdot x^2$
$= 100 \cdot x^{12+2}$
$= 100x^{14}$

32. $(-8m^4)^2 \cdot m^3 = (-8)^2 \cdot m^{4 \cdot 2} \cdot m^3$
$= 64 \cdot m^8 \cdot m^3$
$= 64m^{8+3}$
$= 64m^{11}$

33. $6d^2 \cdot (2d^5)^4 = 6d^2 \cdot 2^4 \cdot d^{5 \cdot 4}$
$= 6d^2 \cdot 16 \cdot d^{20}$
$= 6 \cdot 16 \cdot d^{2+20}$
$= 96d^{22}$

34. $(-20x^3)^2(-x^7) = (-20)^2 \cdot x^{3 \cdot 2} \cdot (-x^7)$
$= 400 \cdot x^6 \cdot (-1) \cdot x^7$
$= 400 \cdot (-1) \cdot x^{6+7}$
$= -400x^{13}$

35. $-(2p^4)^3(-1.5p^7) = -(2^3 \cdot p^{4 \cdot 3}) \cdot (-1.5 \cdot p^7)$
$= -(8 \cdot p^{12}) \cdot (-1.5 \cdot p^7)$
$= (-8) \cdot (-1.5) \cdot p^{12} \cdot p^7$
$= 12p^{12+7}$
$= 12p^{19}$

36. $\left(\frac{1}{2}y^5\right)^3(2y^2)^4 = \left(\frac{1}{2}\right)^3 \cdot y^{5 \cdot 3} \cdot 2^4 \cdot y^{2 \cdot 4}$
$= \frac{1}{8} \cdot y^{15} \cdot 16 \cdot y^8$
$= 2y^{15+8}$
$= 2y^{23}$

37. $(3x^5)^3(2x^7)^2 = 3^3 \cdot x^{5 \cdot 3} \cdot 2^2 \cdot x^{7 \cdot 2}$
$= 27 \cdot x^{15} \cdot 4 \cdot x^{14}$
$= 108x^{15+14}$
$= 108x^{29}$

38. $(-10n)^2(-4n^3)^3 = (-10)^2 \cdot n^2 \cdot (-4)^3 \cdot n^{3 \cdot 3}$
$= 100 \cdot n^2 \cdot (-64) \cdot n^9$
$= -6400n^{2+9}$
$= -6400n^{11}$

39. The exponents were multiplied instead of added;
$c \cdot c^4 \cdot c^5 = c^1 \cdot c^4 \cdot c^5 = c^{1+4+5} = c^{10}$

40. B; $(-9)(-9)^5 = (-9)^{1+5} = (-9)^6$

41. D;
$(6x^5)^2 \cdot x^2 = 6^2 \cdot x^{5 \cdot 2} \cdot x^2$
$= 36 \cdot x^{10} \cdot x^2$
$= 36 \cdot x^{10+2}$
$= 36x^{12}$

42. $x^4 \cdot x^? = x^5$
$4 + ? = 5$
$? = 1$

43. $(y^8)^? = y^{16}$
$8 \cdot ? = 16$
$? = 2$

44. $(2z^?)^3 = 8z^{15}$
$2^3 \cdot z^{? \cdot 3} = 8z^{15}$
$8 \cdot z^{? \cdot 3} = 8z^{15}$
$? \cdot 3 = 15$
$? = 5$

45. $(3a^3)^? \cdot 2a^3 = 18a^9$
$3^? \cdot a^{3 \cdot ?} \cdot 2 \cdot a^3 = 18a^9$
$3^? \cdot 2 \cdot a^{3 \cdot ?} \cdot a^3 = 18a^9$
$a^{3 \cdot ? + 3} = a^9$
$3 \cdot ? + 3 = 9$
$3 \cdot ? = 6$
$? = 2$

46. $10^7$ people

47. $(-3x^2y)^3(11x^3y^5)^2$
$= (-3)^3 \cdot x^{2 \cdot 3} \cdot y^3 \cdot 11^2 \cdot x^{3 \cdot 2} \cdot y^{5 \cdot 2}$
$= -27 \cdot x^6 \cdot y^3 \cdot 121 \cdot x^6 \cdot y^{10}$
$= -3267 \cdot x^{6+6} \cdot y^{3+10}$
$= -3267x^{12}y^{13}$

48. $-(-xy^2z^3)^5(x^4yz)^2$
$= -[(-1)^5 \cdot x^5 \cdot y^{2 \cdot 5} \cdot z^{3 \cdot 5}](x^{4 \cdot 2} \cdot y^2 \cdot z^2)$
$= x^5 \cdot y^{10} \cdot z^{15} \cdot x^8 \cdot y^2 \cdot z^2$
$= x^{5+8} \cdot y^{10+2} \cdot z^{15+2}$
$= x^{13}y^{12}z^{17}$

49. $(-2s)(-5r^3st)^3(-2r^4st^7)^2$
$= -2s \cdot (-5)^3 \cdot r^{3 \cdot 3} \cdot s^3 \cdot t^3 \cdot (-2)^2 \cdot r^{4 \cdot 2} \cdot s^2 \cdot t^{7 \cdot 2}$
$= -2s \cdot (-125) \cdot r^9 \cdot s^3 \cdot t^3 \cdot 4 \cdot r^8 \cdot s^2 \cdot t^{14}$
$= (-2)(-125)(4) \cdot s^{1+3+2} \cdot r^{9+8} \cdot t^{3+14}$
$= 1000r^{17}s^6t^{17}$

**50.** *Sample answer:* $3(2x^4)^2$, $12x \cdot x^7$, $12(x^4)^2$

**51.** $(a \cdot b)^n = (a \cdot b) \cdot (a \cdot b) \cdot \ldots \cdot (a \cdot b)$, with $a \cdot b$ as a factor $n$ times. The associative and commutative properties of multiplication allow you to rewrite this expression as $a \cdot a \cdot \ldots \cdot a \cdot b \cdot b \cdot \ldots \cdot b$ with each factor appearing $n$ times. This expression is equal to $a^n \cdot b^n$.

### *Problem Solving*

**52.** $\frac{10^6 \text{ bubbles}}{1 \text{ cm}^3}\left(\frac{10^3 \text{ cm}^3}{1 \text{ quart}}\right) = 10^{6+3} = 10^9$ bubbles in 1 quart

**53.** $(10^{13})(10^{13}) = 10^{13+13} = 10^{26}$ m

**54.** $\frac{10^9 \text{ grains of sand}}{1 \text{ ft}^3}(10^7 \text{ ft}^3) = 10^{9+7} = 10^{16}$ grains of sand

**55. a.**

| Gold (ounces) | 10 | 100 | 1000 | 10,000 | 100,000 |
|---|---|---|---|---|---|
| Number of atoms | $10^{24}$ | $10^{25}$ | $10^{26}$ | $10^{27}$ | $10^{28}$ |

**b.** $\frac{10^{23} \text{ atoms}}{1 \text{ ounce}}(10^5 \text{ ounces}) = 10^{23+5} = 10^{28}$ atoms

**56. a.** $(10^2)(10) = 10^{2+1} = 10^3$

**b.** $(10^2 \text{ nanometers})(10^3) = 10^{2+3} = 10^5$ nanometers

**57.** $(10^9 \text{ meters})^3 = 10^{9 \cdot 3} = 10^{27} \text{ m}^3$

**58. a.** $(10^3)(10) = 10^{3+1} = 10^4$ ft

**b.** $V = (1)(10^3)^2(10^4) = 10^6(10^4) = 10^{6+4} = 10^{10} \text{ ft}^3$

**c.** The volume will be $10^2$ times as great because in the formula for volume, the radius is squared. So,

$$V = (1)(10^3 \cdot 10)^2(10^4)$$
$$= (10^4)^2(10^4)$$
$$= (10^8)(10^4)$$
$$= 10^{12} \text{ ft}^3$$

**59.** $2^{13}$; $2^{10}$; $2^{13+10} = 2^{23}$

## Lesson 7.2 Apply Exponent Properties Involving Quotients

### *Guided Practice for the lesson "Apply Exponent Properties Involving Quotients"*

**1.** $\frac{6^{11}}{6^5} = 6^{11-5} = 6^6$

**2.** $\frac{(-4)^9}{(-4)^2} = (-4)^{9-2} = (-4)^7$

**3.** $\frac{9^4 \cdot 9^3}{9^2} = \frac{9^7}{9^2} = 9^{7-2} = 9^5$

**4.** $\frac{1}{y^5} \cdot y^8 = \frac{y^8}{y^5} = y^{8-5} = y^3$

**5.** $\left(\frac{a}{b}\right)^2 = \frac{a^2}{b^2}$

**6.** $\left(-\frac{5}{y}\right)^3 = \left(\frac{-5}{y}\right)^3 = \frac{(-5)^3}{y^3} = \frac{-125}{y^3}$

**7.** $\left(\frac{x^2}{4y}\right)^2 = \frac{(x^2)^2}{(4y)^2} = \frac{x^4}{4^2y^2} = \frac{x^4}{16y^2}$

**8.** $\left(\frac{2s}{3t}\right)^3 \cdot \left(\frac{t^5}{16}\right) = \frac{(2s)^3}{(3t)^3} \cdot \frac{t^5}{16}$

$$= \frac{2^3s^3}{3^3t^3} \cdot \frac{t^5}{16}$$
$$= \frac{8s^3}{27t^3} \cdot \frac{t^5}{16}$$
$$= \frac{8s^3t^5}{432t^3} = \frac{s^3t^2}{54}$$

**9.**

| Step | Number of new branches | Length of new branch |
|---|---|---|
| 1 | $3 = 3^1$ | $\frac{1}{2} = \left(\frac{1}{2}\right)^1$ |
| 2 | $9 = 3^2$ | $\frac{1}{2}\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2$ |
| 3 | $27 = 3^3$ | $\frac{1}{2}\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3$ |
| 4 | $81 = 3^4$ | $\frac{1}{2}\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^4$ |

The length of the new branch added at Step 9 is

$\left(\frac{1}{2}\right)^9 = \frac{1^9}{2^9} = \frac{1}{512}$.

**10.** $\frac{\text{Luminosity of Canopus (watts)}}{\text{Luminosity of Sirius(watts)}} = \frac{10^{30}}{10^{28}} = 10^{30-28} = 10^2$

Canopus is about $10^2$ times as luminous as Sirius.

### *Exercises for the lesson "Apply Exponent Properties Involving Quotients"*

### *Skill Practice*

**1.** In the power $4^3$, 4 is the *base* and 3 is the *exponent*.

**2.** When powers have the same base, their quotient is the base raised to the difference of the exponents.

**3.** $\frac{5^6}{5^2} = 5^{6-2} = 5^4$

**4.** $\frac{2^{11}}{2^6} = 2^{11-6} = 2^5$

**5.** $\frac{3^9}{3^5} = 3^{9-5} = 3^4$

**6.** $\frac{(-6)^8}{(-6)^5} = (-6)^{8-5} = (-6)^3$

**7.** $\frac{(-4)^7}{(-4)^4} = (-4)^{7-4} = (-4)^3$

**8.** $\frac{(-12)^9}{(-12)^3} = (-12)^{9-3} = (-12)^6$

**9.** $\frac{10^5 \cdot 10^5}{10^4} = \frac{10^{5+5}}{10^4} = \frac{10^{10}}{10^4} = 10^{10-4} = 10^6$

**10.** $\frac{6^7 \cdot 6^4}{6^6} = \frac{6^{7+4}}{6^6} = \frac{6^{11}}{6^6} = 6^{11-6} = 6^5$

**11.** $\left(\frac{1}{3}\right)^5 = \frac{1^5}{3^5} = \frac{1}{3^5}$

**12.** $\left(\frac{3}{2}\right)^4 = \frac{3^4}{2^4}$

**13.** $\left(-\frac{5}{4}\right)^4 = \frac{(-5)^4}{4^4} = \frac{5^4}{4^4}$

**14.** $\left(-\frac{2}{5}\right)^5 = \frac{(-2)^5}{5^5} = \frac{-2^5}{5^5}$

**15.** $7^9 \cdot \frac{1}{7^2} = \frac{7^9}{7^2} = 7^{9-2} = 7^7$

**16.** $\frac{1}{9^5} \cdot 9^{11} = \frac{9^{11}}{9^5} = 9^{11-5} = 9^6$

**17.** $\left(\frac{1}{3}\right)^4 \cdot 3^{12} = \frac{1^4}{3^4} \cdot 3^{12} = \frac{1}{3^4} \cdot 3^{12} = \frac{3^{12}}{3^4} = 3^{12-4} = 3^8$

**18.** $4^9 \cdot \left(-\frac{1}{4}\right)^5 = 4^9 \cdot \frac{(-1)^5}{4^5}$

$= 4^9 \cdot (-1) \cdot \frac{1}{4^5}$

$= -\frac{4^9}{4^5}$

$= -4^{9-5}$

$= -4^4$

**19.** C; $\left(\frac{16^6}{16^3}\right)^2 = (16^{6-3})^2 = (16^3)^2 = 16^{3 \cdot 2} = 16^6$

**20.** The quotient of powers property was used incorrectly. The exponents should be subtracted, not added;

$\frac{9^5 \cdot 9^3}{9^4} = \frac{9^8}{9^4} = 9^4$

**21.** $\frac{1}{y^8} \cdot y^{15} = \frac{y^{15}}{y^8} = y^{15-8} = y^7$

**22.** $z^8 \cdot \frac{1}{z^7} = \frac{z^8}{z^7} = z^{8-7} = z$

**23.** $\left(\frac{a}{y}\right)^9 = \frac{a^9}{y^9}$

**24.** $\left(\frac{j}{k}\right)^{11} = \frac{j^{11}}{k^{11}}$

**25.** $\left(\frac{p}{q}\right)^4 = \frac{p^4}{q^4}$

**26.** $\left(-\frac{1}{x}\right)^5 = \frac{(-1)^5}{x^5} = -\frac{1}{x^5}$

**27.** $\left(-\frac{4}{x}\right)^3 = \frac{(-4)^3}{x^3} = -\frac{64}{x^3}$

**28.** $\left(-\frac{a}{b}\right)^4 = (-1)^4 \cdot \left(\frac{a^4}{b^4}\right) = \frac{a^4}{b^4}$

**29.** $\left(\frac{4c}{d^2}\right)^3 = \frac{(4c)^3}{(d^2)^3} = \frac{4^3c^3}{d^6} = \frac{64c^3}{d^6}$

**30.** $\left(\frac{a^7}{2b}\right)^5 = \frac{(a^7)^5}{(2b)^5} = \frac{a^{35}}{2^5b^5} = \frac{a^{35}}{32b^5}$

**31.** $\left(\frac{x^2}{3y^3}\right)^2 = \frac{(x^2)^2}{(3y^3)^2} = \frac{x^4}{9y^6}$

**32.** $\left(\frac{3x^5}{7y^2}\right)^3 = \frac{(3x^5)^3}{(7y^2)^3} = \frac{3^3x^{15}}{7^3y^6} = \frac{27x^{15}}{343y^6}$

**33.** $\left(\frac{3x^3}{2y}\right)^2 \cdot \frac{1}{x^2} = \frac{3^2x^6}{2^2y^2} \cdot \frac{1}{x^2} = \frac{9x^6}{4y^2} \cdot \frac{1}{x^2} = \frac{9x^6}{4y^2x^2} = \frac{9x^{6-2}}{4y^2} = \frac{9x^4}{4y^2}$

**34.** $\left(\frac{2x^3}{y}\right)^3 \cdot \frac{1}{6x^3} = \frac{(2x^3)^3}{y^3} \cdot \frac{1}{6x^3}$

$= \frac{2^3x^9}{y^3} \cdot \frac{1}{6x^3}$

$= \frac{8x^9}{6y^3x^3}$

$= \frac{8x^{9-3}}{6y^3}$

$= \frac{4x^6}{3y^3}$

**35.** $\frac{3}{8m^5} \cdot \left(\frac{m^4}{n^2}\right)^3 = \frac{3}{8m^5} \cdot \frac{(m^4)^3}{(n^2)^3}$

$= \frac{3}{8m^5} \cdot \frac{m^{12}}{n^6}$

$= \frac{3m^{12}}{8m^5n^6}$

$= \frac{3m^{12-5}}{8n^6}$

$= \frac{3m^7}{8n^6}$

**36.** $\left(\frac{-5}{x}\right)^2 \cdot \left(\frac{2x^4}{y^3}\right)^2 = (-1)^2 \cdot \left(\frac{5^2}{x^2}\right) \cdot \frac{(2x^4)^2}{(y^3)^2}$

$= 1 \cdot \left(\frac{25}{x^2}\right) \cdot \frac{4x^8}{y^6}$

$= \frac{100x^8}{x^2y^6}$

$= \frac{100x^{8-2}}{y^6}$

$= \frac{100x^6}{y^6}$

**37.** D; $\left(\frac{7x^3}{2y^4}\right)^2 = \frac{(7x^3)^2}{(2y^4)^2} = \frac{49x^6}{4y^8}$

**38.** $\frac{(-8)^7}{(-8)^?} = (-8)^3$

$(-8)^{7-?} = (-8)^3$

$7 - ? = 3$

$? = 4$

**39.** $\frac{7^? \cdot 7^2}{7^4} = 7^6$

$\frac{7^{?+2}}{7^4} = 7^6$

$7^{?+2-4} = 7^6$

$? - 2 = 6$

$? = 8$

**40.** $\frac{1}{p^5} \cdot p^? = p^9$

$\frac{p^?}{p^5} = p^9$

$p^{?-5} = p^9$

$? - 5 = 9$

$? = 14$

**41.** $\left(\frac{2c^3}{d^2}\right)^? = \frac{16c^{12}}{d^8}$

$\frac{(2c^3)^?}{(d^2)^?} = \frac{16c^{12}}{d^8}$

$\frac{2^?c^{3?}}{d^{2?}} = \frac{16c^{12}}{d^8}$

$3? = 12$

$? = 4$

**42.** $\left(\frac{2f^2g^3}{3fg}\right)^4 = \frac{2^4(f^2)^4(g^3)^4}{3^4f^4g^4}$

$= \frac{16f^8g^{12}}{81f^4g^4}$

$= \frac{16f^{8-4}g^{12-4}}{81}$

$= \frac{16f^4g^8}{81}$

**43.** $\frac{2s^3t^3}{st^2} \cdot \frac{(3st)^3}{s^2t} = \frac{2s^3t^3}{st^2} \cdot \frac{3^3s^3t^3}{s^2t}$

$= \frac{2s^3t^3}{st^2} \cdot \frac{27s^3t^3}{s^2t}$

$= \frac{2s^{3-1}t^{3-2}}{1} \cdot \frac{27s^{3-2}t^{3-1}}{1}$

$= 2s^2t^1 \cdot 27st^2$

$= 54s^{2+1}t^{1+2} = 54s^3t^3$

**44.** $\left(\frac{2m^5n}{4m^2}\right)^2 \cdot \left(\frac{mn^4}{5n}\right)^2 = \frac{(2m^5n)^2}{(4m^2)^2} \cdot \frac{(mn^4)^2}{(5n)^2}$

$= \frac{4m^{10}n^2}{16m^4} \cdot \frac{m^2n^8}{25n^2}$

$= \frac{m^{10-4}n^2}{4} \cdot \frac{m^2n^{8-2}}{25}$

$= \frac{m^6n^6}{4} \cdot \frac{m^2n^6}{25} = \frac{m^{6+2}n^{2+6}}{100} = \frac{m^8n^8}{100}$

**45.** $\left(\frac{3x^3y}{x^2}\right)^3 \cdot \left(\frac{y^2x^4}{5y}\right)^2 = \frac{(3x^3y)^3}{(x^2)^3} \cdot \frac{(y^2x^4)^2}{(5y)^2}$

$= \frac{27x^9y^3}{x^6} \cdot \frac{y^4x^8}{25y^2}$

$= 27x^{9-6}y^3 \cdot \frac{y^{4-2}x^8}{25}$

$= 27x^3y^3 \cdot \frac{y^2x^8}{25}$

$= \frac{27x^{3+8}y^{3+2}}{25}$

$= \frac{27x^{11}y^5}{25}$

**46.** *Sample answer*: $\frac{14^9}{14^2}; \frac{14^{11}}{14^4}; \frac{14^5 \cdot 14^4}{14^2}$

**47.** Let $m < n$. Given

$\frac{a^m}{a^n} = \frac{a^m}{a^n}\left(\frac{\frac{1}{a^m}}{\frac{1}{a^m}}\right)$ Identity property of multiplication.

$= \frac{1}{\frac{a^n}{a^m}}$ Multiply fractions.

$= \frac{1}{a^{n-m}}$ Quotient of powers property.

**48.** $\frac{b^x}{b^y} = b^9$

$b^{x-y} = b^9$

$x - y = 9$ Equation 1

$\frac{b^x \cdot b^2}{b^{3y}} = b^{13}$

$\frac{b^{x+2}}{b^{3y}} = b^{13}$

$b^{x+2-3y} = b^{13}$

$x + 2 - 3y = 13$

$x - 3y = 11$ Equation 2

Solve the system $x - y = 9$

$x - 3y = 11$

$x = 9 + y$ Revised Equation 1

$(9 + y) - 3y = 11$ Substitute revised Eqn 1 into Eqn 2.

$9 - 2y = 11$

$-2y = 2$

$y = -1$

$x - (-1) = 9$ Substitute $y = -1$ into Eqn 1.

$x + 1 = 9$

$x = 8$

### Problem Solving

**49. a.**

| steps | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| new squares | $4^1$ | $4^2$ | $4^3$ | $4^4$ |
| side length | $\left(\frac{1}{2}\right)^1$ | $\left(\frac{1}{2}\right)^2$ | $\left(\frac{1}{2}\right)^3$ | $\left(\frac{1}{2}\right)^4$ |

**b.** $\frac{4^4}{4^2} = 4^{4-2} = 4^2 = 16$ times more squares

**50.** $\frac{10^{13}}{10^8} = 10^{13-8} = \$10^5$ per capita GDP

**51.** $10^{13}\text{ km} \cdot \frac{10^3\text{ m}}{1\text{ km}} = 10^{16}\text{ m}$

$\frac{10^{16}\text{ m}}{10^4\text{ m/sec}} = 10^{16-4} = 10^{12}\text{ sec}$

$10^{12}\text{ sec} \cdot \frac{1\text{ min}}{60\text{ sec}} \cdot \frac{1\text{ h}}{60\text{ min}} \cdot \frac{1\text{ day}}{24\text{ h}} \cdot \frac{1\text{ yr}}{365\text{ days}} \approx 31{,}710\text{ yr}$

**52.** $2.512^{5-2} = 2.512^3 \approx 15.85$ times less bright

**53.** $31^{7-4} = 31^3 = 29{,}791$ times greater

**54. a.** $\frac{2^{40}}{2^{10}} = 2^{40-10} = 2^{30}$ gigabytes

**b.** $\frac{2^{50}}{2^{20}} = 2^{50-20} = 2^{30}$ megabytes

**c.** Multiply the number of bytes by 8.

### Quiz for the lessons "Apply Exponent Properties Involving Products" and "Apply Exponent Properties Involving Quotients"

**1.** $3^2 \cdot 3^6 = 3^{2+6} = 3^8$

**2.** $(5^4)^3 = 5^{4 \cdot 3} = 5^{12}$

**3.** $(32 \cdot 14)^7 = 32^7 \cdot 14^7$

**4.** $7^2 \cdot 7^6 \cdot 7 = 7^{2+6+1} = 7^9$

**5.** $(-4)(-4)^9 = (-4)^{1+9} = (-4)^{10}$

**6.** $\frac{7^{12}}{7^4} = 7^{12-4} = 7^8$

**7.** $\frac{(-9)^9}{(-9)^7} = (-9)^{9-7} = (-9)^2$

**8.** $\frac{3^7 \cdot 3^4}{3^6} = \frac{3^{7+4}}{3^6} = \frac{3^{11}}{3^6} = 3^{11-6} = 3^5$

**9.** $\left(\frac{5}{4}\right)^4 = \frac{5^4}{4^4}$

**10.** $x^2 \cdot x^5 = x^{2+5} = x^7$

**11.** $(3x^3)^2 = 3^2x^{3 \cdot 2} = 9x^6$

**12.** $-(7x)^2 = -7^2x^2 = -49x^2$

**13.** $(6x^5)^3 \cdot x = 6^3x^{5 \cdot 3} \cdot x = 216x^{15} \cdot x = 216x^{16}$

**14.** $(2x^5)^3(7x^7)^2 = 2^3x^{5 \cdot 3} \cdot 7^2x^{7 \cdot 2}$

$= 8x^{15} \cdot 49x^{14}$

$= 392x^{15 + 14}$

$= 392x^{29}$

**15.** $\frac{1}{x^9} \cdot x^{21} = \frac{x^{21}}{x^9} = x^{21 - 9} = x^{12}$

**16.** $\left(\frac{-4}{x}\right)^3 = \frac{(-4)^3}{x^3} = \frac{-64}{x^3}$

**17.** $\left(\frac{w}{v}\right)^6 = \frac{w^6}{v^6}$

**18.** $\left(\frac{x^3}{4}\right)^2 = \frac{(x^3)^2}{4^2} = \frac{x^6}{16}$

**19.** $\frac{10^{10}\text{ lb}}{10^6\text{ acres}} = 10^{10 - 6} = 10^4$, about $10^4$ pounds

## Lesson 7.3 Define and Use Zero and Negative Exponents

### *Investigating Algebra Activity for the lesson "Define and Use Zero and Negative Exponents"*

*Explore*

| Exponent, $n$ | Value of $2^n$ |
|---|---|
| 4 | 16 |
| 3 | 8 |
| 2 | 4 |
| 1 | 2 |

| Exponent, $n$ | Value of $3^n$ |
|---|---|
| 4 | 81 |
| 3 | 27 |
| 2 | 9 |
| 1 | 3 |

Each time the exponent is decreased by 1, the value is divided by the base.

| Exponent, $n$ | Power, $2^n$ |
|---|---|
| 3 | 8 |
| 2 | 4 |
| 1 | 2 |
| 0 | 1 |
| −1 | $\frac{1}{2}$ |
| −2 | $\frac{1}{4}$ |

| Exponent, $n$ | Power, $2^n$ |
|---|---|
| 3 | 27 |
| 2 | 9 |
| 1 | 3 |
| 0 | 1 |
| −1 | $\frac{1}{3}$ |
| −2 | $\frac{1}{9}$ |

*Draw Conclusions*

**1.**

| $n$ | $2^n$ | $3^n$ |
|---|---|---|
| −3 | $\frac{1}{8}$ | $\frac{1}{27}$ |
| −4 | $\frac{1}{16}$ | $\frac{1}{81}$ |
| −5 | $\frac{1}{32}$ | $\frac{1}{243}$ |

**2.** $a^0 = 1$

**3.**

| Power, $2^n$ | Exponent |
|---|---|
| 8 | $2^3$ |
| 4 | $2^2$ |
| 2 | $2^1$ |
| 1 | $2^0$ |
| $\frac{1}{2}$ | $\frac{1}{2^1}$ |
| $\frac{1}{4}$ | $\frac{1}{2^2}$ |

| Power, $3^n$ | Exponent |
|---|---|
| 27 | $3^3$ |
| 9 | $3^2$ |
| 3 | $3^1$ |
| 1 | $3^0$ |
| $\frac{1}{3}$ | $\frac{1}{3^1}$ |
| $\frac{1}{9}$ | $\frac{1}{3^2}$ |

### *Guided Practice for the lesson "Define and Use Zero and Negative Exponents"*

**1.** $\left(\frac{2}{3}\right)^0 = 1$

**2.** $(-8)^{-2} = \frac{1}{(-8)^2} = \frac{1}{64}$

**3.** $\frac{1}{2^{-3}} = \frac{1}{\frac{1}{2^3}} = \frac{1}{\frac{1}{8}} = 8$

**4.** $(-1)^0 = 1$

**5.** $\frac{1}{4^{-3}} = 4^3 = 64$

**6.** $(5^{-3})^{-1} = 5^{(-3)(-1)} = 5^3 = 125$

**7.** $(-3)^5 \cdot (-3)^{-5} = (-3)^{5 + (-5)} = (-3)^0 = 1$

**8.** $\frac{6^{-2}}{6^2} = 6^{-2 - 2} = 6^{-4} = \frac{1}{6^4} = \frac{1}{1296}$

**9.** $\frac{3xy^{-3}}{9x^3y} = \frac{3x}{9x^3yy^3} = \frac{3x}{9x^3y^4} = \frac{1}{3x^2y^4}$

**10.** $10^{-27} \cdot 10^4 = 10^{-27 + 4} = 10^{-23}$

The order of magnitude of the mass of a proton is $10^{-23}$ gram.

### *Exercises for the lesson "Define and Use Zero and Negative Exponents"*

### *Skill Practice*

**1.** *Sample answer:* I would use the product of powers property because the expression simplifies to $3^0$. By the definition of zero exponent, $3^0 = 1$.

**2.** *Sample answer:* The definition of negative exponents is defined only for nonzero bases.

**3.** $4^{-3} = \frac{1}{4^3} = \frac{1}{64}$

**4.** $7^{-3} = \frac{1}{7^3} = \frac{1}{343}$

**5.** $(-3)^{-1} = \frac{1}{(-3)^1} = -\frac{1}{3}$

**6.** $(-2)^{-6} = \frac{1}{(-2)^6} = \frac{1}{64}$

**7.** $2^0 = 1$

**8.** $(-4)^0 = 1$

**9.** $\left(\frac{3}{4}\right)^0 = 1$

**10.** $\left(-\frac{9}{16}\right)^0 = 1$

**11.** $\left(\frac{2}{7}\right)^{-2} = \frac{1}{\left(\frac{2}{7}\right)^2} = \frac{1}{\frac{4}{49}} = \frac{49}{4}$

**12.** $\left(\frac{4}{3}\right)^{-3} = \frac{1}{\left(\frac{4}{3}\right)^3} = \frac{1}{\frac{64}{27}} = \frac{27}{64}$

13. $0^{-3}$ = undefined

14. $0^{-2}$ = undefined

15. $2^{-2} \cdot 2^{-3} = 2^{-2+(-3)} = 2^{-5} = \frac{1}{2^5} = \frac{1}{32}$

16. $7^{-6} \cdot 7^4 = 7^{-6+4} = 7^{-2} = \frac{1}{7^2} = \frac{1}{49}$

17. $(2^{-1})^5 = 2^{-5} = \frac{1}{2^5} = \frac{1}{32}$

18. $(3^{-2})^2 = 3^{-2 \cdot 2} = 3^{-4} = \frac{1}{3^4} = \frac{1}{81}$

19. $\frac{1}{3^{-3}} = 3^3 = 27$

20. $\frac{1}{6^{-2}} = 6^2 = 36$

21. $\frac{3^{-3}}{3^2} = 3^{-3-2} = 3^{-5} = \frac{1}{3^5} = \frac{1}{243}$

22. $\frac{6^{-3}}{6^{-5}} = 6^{-3-(-5)} = 6^2 = 36$

23. $4\left(\frac{3}{2}\right)^{-1} = 4\left(\frac{3^{-1}}{2^{-1}}\right) = 4\left(\frac{2}{3}\right) = \frac{8}{3}$

24. $16\left(\frac{2^{-3}}{2^2}\right) = 16(2^{-3-2}) = 16(2^{-5}) = 16\left(\frac{1}{2^5}\right) = 16\left(\frac{1}{32}\right) = \frac{1}{2}$

25. $6^0 \cdot \left(\frac{1}{4^{-2}}\right) = 1 \cdot 4^2 = 16$

26. $3^{-2} \cdot \left(\frac{5}{7^0}\right) = \frac{1}{3^2} \cdot \left(\frac{5}{1}\right) = \frac{5}{9}$

27. $3^0$ is 1, not 0.
$-6 \cdot 3^0 = -6 \cdot 1 = -6$

28. $x^{-4} = \frac{1}{x^4}$

29. $2y^{-3} = \frac{2}{y^3}$

30. $(4g)^{-3} = \frac{1}{(4g)^3} = \frac{1}{4^3g^3} = \frac{1}{64g^3}$

31. $(-11h)^{-2} = \frac{1}{(-11h)^2} = \frac{1}{(-11)^2h^2} = \frac{1}{121h^2}$

32. $x^2y^{-3} = \frac{x^2}{y^3}$

33. $5m^{-3}n^{-4} = \frac{5}{m^3n^4}$

34. $(6x^{-2}y^3)^{-3} = 6^{-3}x^{-2 \cdot (-3)}y^{3 \cdot (-3)} = \frac{1}{6^3}x^6y^{-9} = \frac{x^6}{216y^9}$

35. $(-15fg^2)^0 = 1$

36. $\frac{r^{-2}}{s^{-4}} = \frac{s^4}{r^2}$

37. $\frac{x^{-5}}{y^2} = \frac{1}{x^5y^2}$

38. $\frac{1}{8x^{-2}y^{-6}} = \frac{x^2y^6}{8}$

39. $\frac{1}{15x^{10}y^{-8}} = \frac{y^8}{15x^{10}}$

40. $\frac{1}{(-2z)^{-2}} = (-2z)^2 = 4z^2$

41. $\frac{9}{(3d)^{-3}} = 9(3d)^3 = 9 \cdot 27d^3 = 243d^3$

42. $\frac{(3x)^{-3}y^4}{-x^2y^{-6}} = \frac{y^4y^6}{-x^2(3x)^3} = \frac{y^4y^6}{-27x^2x^3} = \frac{y^{4+6}}{-27x^{2+3}} = -\frac{y^{10}}{27x^5}$

43. $\frac{12x^8y^{-7}}{(4x^{-2}y^{-6})^2} = \frac{12x^8y^{-7}}{4^2x^{-4}y^{-12}} = \frac{12x^8x^4y^{12}}{16y^7} = \frac{3x^{8+4}y^{12-7}}{4} = \frac{3x^{12}y^5}{4}$

44. D; $\frac{8}{4x^{-4}} = \frac{8x^4}{4} = 2x^4$

45. *D;* $(-4 \cdot 2^0 \cdot 3)^{-2} = (-12)^{-2} = \frac{1}{(-12)^2} = \frac{1}{144}$

46. False, $\frac{a^{-3}}{a^{-4}} = a^{-3-(-4)} = a^1$;

counterexample: $\frac{2^{-3}}{2^{-4}} = \frac{2^4}{2^3} = 2 \left(\text{not } \frac{1}{2}\right)$

47. True, $\frac{a^{-1}}{b^{-1}} = \frac{\frac{1}{a}}{\frac{1}{b}} = \frac{b}{a}$

48. False, $a^{-1} + b^{-1} = \frac{1}{a} + \frac{1}{b} = \frac{b}{ab} + \frac{a}{ab} = \frac{b+a}{ab}$;

counterexample: $2^{-1} + 4^{-1} = \frac{1}{2} + \frac{1}{4} = \frac{3}{4} \left(\text{not } \frac{1}{6}\right)$

49. **a.** For $a > 1$, when $n < 0$, $-n > 0$, so $a^n$ is between 0 and 1, $a^{-n}$ is greater than 1, and $a^n < a^{-n}$. When $n = 0$, $a^n = a^0 = 1$ and $a^{-n} = a^0 = 1$, so $a^n = a^{-n}$. When $n > 0$, $-n < 0$, so $a^n$ is greater than 1, $a^{-n}$ is between 0 and 1, and $a^n > a^{-n}$.

**b.** For $0 < a < 1$, when $n < 0$, $-n > 0$, so $a^n$ is a whole number, $a^{-n}$ is a fraction, and $a^n > a^{-n}$. When $n = 0$, $a^n = a^0 = 1$ and $a^{-n} = a^0 = 1$, so $a^n = a^{-n}$. When $n > 0$, $-n < 0$, so $a^n$ is a fraction, $a^{-n}$ is a whole number, and $a^n < a^{-n}$.

### *Problem Solving*

50. $\frac{10^2}{10^{-4}} = 10^{2-(-4)} = 10^6$ grains of salt

51. $\frac{10^3}{10^{-2}} = 10^{3-(-2)} = 10^5$ grains of rice

52. $10^4 \text{ kg}\left(\frac{1000 \text{ g}}{1 \text{ kg}}\right) = 10^7 \text{ g}$

$\frac{10^7 \text{ g}}{10^{-4} \text{ g}} = 10^{7-(-4)} = 10^{11}$ times larger

53. $\frac{10^{-6} \text{ liter}}{10^7 \text{ red blood cells}} = \frac{10^{-2} \text{ liter}}{x}$

$10^{-6}x = 10^7 \cdot 10^{-2}$

$x = \frac{10^5}{10^{-6}}$

$x = 10^{5-(-6)} = 10^{11}$

The entire sample contained about $10^{11}$ red blood cells.

54. No; mass of giant fan palm $= 10^{-9} \cdot 10^{13} = 10^4$ g

$10^4 \text{ g}\left(\frac{1 \text{ kg}}{10^3 \text{ g}}\right) = 10$ kg, not 1 kg.

55. **a.**

| **Number of folds** | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| **Fraction of original area** | 1 | $\frac{1}{2}$ | $\frac{1}{4}$ | $\frac{1}{8}$ |

**b.** $\left(\frac{1}{2}\right)^n$ where $n$ is the number of folds

**56.** **a.** $t = \frac{(10^{-4})^2}{2(10^{-5})} = \frac{10^{-8}}{2(10^{-5})} = \frac{10^{-8+5}}{2} = \frac{10^{-3}}{2}$

$= \frac{1}{2} \times 10^{-3}$

$= 0.0005$ sec

**b.** $\frac{\text{cm}^2}{\frac{\text{cm}^2}{\text{sec}}} = \frac{\text{cm}^2 \cdot \text{sec}}{\text{cm}^2} = \text{sec}$

**57.** **a.** $I = 0.08Pd^{-2}$

$10^{-2} = 0.08P(30)^{-2}$

$\frac{10^{-2}}{(0.08)(30^{-2})} = P$

$\frac{30^2}{10^2(0.08)} = P$

$112.5 \text{ watts} = P$

**b.** $I = 0.08(112.5)d^{-2} = \frac{9}{d^2}$

**c.** The intensity is divided by 4.

**58.** **a.** $\frac{1 \text{ lb}}{10^4 \text{ BTU}} = \frac{10 \text{ lb}}{x}$

$x = 10 \cdot 10^4$

$x = 10^5$

Your stereo uses $10^5$ BTUs in 1 year.

**b.** $\frac{10^{-1} \text{ lb}}{10^6 \text{ BTU}} = \frac{x}{10^5 \text{ BTU}}$

$10^6 x = 10^{-1} \cdot 10^5$

$x = \frac{10^{-1} \cdot 10^5}{10^6}$

$x = 10^{-1+5-6}$

$x = 10^{-2} = 0.01$

0.01 pound of sulfur dioxide is added to the air.

### *Extension for the lesson "Define and Use Zero and Negative Exponents"*

**1.** $100^{3/2} = 100^{(1/2) \cdot 3} = (100^{1/2})^3 = (\sqrt{100})^3 = 10^3$

$= 1000$

**2.** $121^{-1/2} = \frac{1}{121^{1/2}} = \frac{1}{\sqrt{121}} = \frac{1}{11}$

**3.** $81^{-3/2} = 81^{(1/2) \cdot (-3)} = (81^{1/2})^{-3} = (\sqrt{81})^{-3} = 9^{-3} = \frac{1}{9^3}$

$= \frac{1}{729}$

**4.** $216^{2/3} = 216^{(1/3) \cdot 2} = (216^{1/3})^2 = (\sqrt[3]{216})^2 = 6^2 = 36$

**5.** $27^{-1/3} = \frac{1}{27^{1/3}} = \frac{1}{\sqrt[3]{27}} = \frac{1}{3}$

**6.** $343^{-2/3} = 343^{(1/3) \cdot (-2)} = (343^{1/3})^{-2} = (\sqrt[3]{343})^{-2} = 7^{-2}$

$= \frac{1}{7^2} = \frac{1}{49}$

**7.** $9^{7/2} \cdot 9^{-3/2} = 9^{(7/2) + (-3/2)} = 9^{4/2} = 9^2 = 81$

**8.** $\left(\frac{1}{16}\right)^{1/2}\left(\frac{1}{16}\right)^{-1/2} = \left(\frac{1}{16}\right)^{(1/2)+(-1/2)} = \left(\frac{1}{16}\right)^0 = 1$

**9.** $36^{5/2} \cdot \frac{36^{-1/2}}{(36^{-1})^{-7/2}} = 36^{5/2} \cdot \frac{36^{-1/2}}{36^{7/2}}$

$= 36^{5/2} \cdot 36^{-1/2 - 7/2}$

$= 36^{5/2} \cdot 36^{-8/2}$

$= 36^{(5/2) + (-8/2)}$

$= 36^{-3/2}$

$= (36^{1/2})^{-3}$

$= 6^{-3}$

$= \frac{1}{6^3}$

$= \frac{1}{216}$

**10.** $(27^{-1/3})^3 = 27^{-3/3} = 27^{-1} = \frac{1}{27}$

**11.** $(-64)^{-5/3}(-64)^{4/3} = (-64)^{(-5/3)+(4/3)}$

$= (-64)^{-1/3}$

$= \frac{1}{(-64)^{1/3}}$

$= \frac{1}{\sqrt[3]{-64}}$

$= -\frac{1}{4}$

**12.** $(-8)^{1/3}(-8)^{-2/3}(-8)^{1/3} = (-8)^{(1/3)+(-2/3)+(1/3)}$

$= (-8)^0 = 1$

**13.** For $0 < x < 1$, $x^{1/2} < x^{-1/2}$; for $x = 1$, $x^{1/2} = x^{-1/2}$; for $x > 1$, $x^{1/2} > x^{-1/2}$. *Samples:* $\left(\frac{1}{4}\right)^{1/2} = \frac{1}{2}$ and $\left(\frac{1}{4}\right)^{-1/2} = 2$; $4^{1/2} = 2$ and $4^{-1/2} = \frac{1}{2}$.

### *Mixed Review of Problem Solving for the lessons "Apply Exponent Properties Involving Products", "Apply Exponent Properties Involving Quotients", and "Define and Use Zero and Negative Exponents"*

**1.** $\frac{10^{12}}{10^9} = 10^{12-9} = 10^3 = 1000$ times faster

**2.** **a.** $V = s^3 = \left(\frac{9}{2}\right)^3 = \frac{9^3}{2^3} = \frac{729}{8}$ in.$^3$

**b.** Power of a quotient property.

**3.** **a.** Because $V = \frac{4}{3}\pi r^3$, the order of magnitude of the volume of the droplet is $(1)(1)(10^{-4})^3 = 10^{-12}$ cm$^3$.

**b.** $r = 10^{-4} \cdot 10^2 = 10^{-4+2} = 10^{-2}$

Because $V = \frac{4}{3}\pi r^3$, the order of magnitude of the volume of the raindrop is $(1)(1)(10^{-2})^3 = 10^{-6}$ cm$^3$.

**c.** Divide the volume of the raindrop by the volume of the droplet.

$\frac{10^{-6}}{10^{-12}} = 10^{-6-(-12)} = 10^6$ droplets

Quotient of powers property

**4.** $10^{-12} \cdot 10^{15} = 10^{-12+15} = 10^3 = 1000 \frac{\text{watts}}{\text{m}^2}$

**5. a.** $\frac{10^{-1}}{10^5} = 10^{-1-5} = 10^{-6}$ in.

$\frac{\text{in.}^3}{\text{in.}^2} = \text{in.}$

**b.** $10^7 \cdot 10^{-6} = 10^{7+(-6)} = 10 \text{ in.}^3$

**c.** $10^x \cdot 10^{-6} = 10^{x+(-6)} = 10^{x-6} \text{ in.}^3$

**6. a.** *Sample answer:* How many milliseconds are in a gigasecond?

**b.** *Sample answer:* How many megaseconds are in a gigasecond?

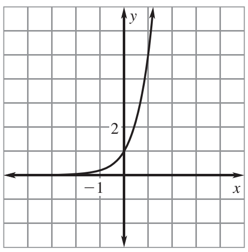
## Lesson 7.4 Write and Graph Exponential Growth Functions

### *Guided Practice for the lesson "Write and Graph Exponential Growth Functions"*

**1.** The $y$-values are multiplied by 3 for each increase of 1 in $x$, so $y = a \cdot 3^x$. When $x = 0$, $y = 27 = a$. So, a rule for the function is $y = 27 \cdot 3^x$.

**2.** $y = 5^x$

| **x** | $-2$ | $-1$ | 0 | 1 | 2 |
|---|---|---|---|---|---|
| **y** | $\frac{1}{25}$ | $\frac{1}{5}$ | 1 | 5 | 25 |

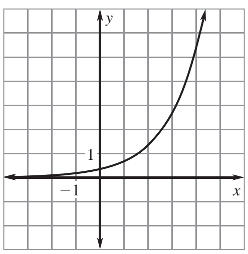



Domain: all real numbers

Range: all positive real numbers

**3.** $y = \frac{1}{3} \cdot 2^x$

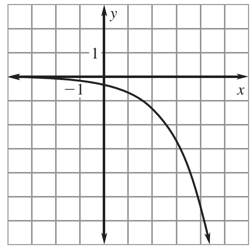
| **x** | $-2$ | $-1$ | 0 | 1 | 2 |
|---|---|---|---|---|---|
| **y** | $\frac{1}{12}$ | $\frac{1}{6}$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{4}{3}$ |




The graph of $y = \frac{1}{3} \cdot 2^x$ is a vertical shrink of the graph of $y = 2^x$.

**4.** $y = -\frac{1}{3} \cdot 2^x$

| **x** | $-2$ | $-1$ | 0 | 1 | 2 |
|---|---|---|---|---|---|
| **y** | $-\frac{1}{12}$ | $-\frac{1}{6}$ | $-\frac{1}{3}$ | $-\frac{2}{3}$ | $-\frac{4}{3}$ |




The graph of $y = -\frac{1}{3} \cdot 2^x$ is a vertical shrink with a reflection in the $x$-axis of the graph of $y = 2^x$.

**5.** $C = 11{,}000(1.069)^t = 11{,}000(1.069)^{10} \approx 21{,}437$

In 1994, the value of the car was about \$21,437.

**6.** $y = a(1 + r)^t = 250(1 + 0.035)^5 = 250(1.035)^5$

$\approx 296.92$

You will have \$296.92 in 5 years.

### *Exercises for the lesson "Write and Graph Exponential Growth Functions"*

### *Skill Practice*

**1.** In the exponential growth model $y = a(1 + r)^t$, the quantity $1 + r$ is called the *growth factor.*

**2.** The exponential function $y = ab^x$ (where $a > 0$) represents exponential growth when $b > 1$.

**3.** $y = 2 \cdot 5^x$ is a vertical stretch of $y = 5^x$. The $y$-values for $y = 2 \cdot 5^x$ are 2 times the corresponding $y$-values for $y = 5^x$.

**4.** The $y$-values are multiplied by 2 for each increase of 1 in $x$, so $y = a \cdot 2^x$. When $x = 0$, $y = 4 = a$. So, a rule for the function is $y = 4 \cdot 2^x$.

**5.** The $y$-values are multiplied by 5 for each increase of 1 in $x$, so $y = a \cdot 5^x$. When $x = 0$, $y = 125 = a$. So, a rule for the function is $y = 125 \cdot 5^x$.

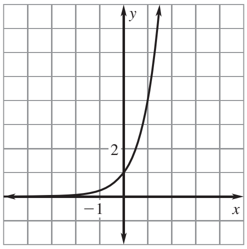
**6.** The $y$-values are multiplied by 2 for each increase of 1 in $x$, so $y = a \cdot 2^x$. When $x = 0$, $y = \frac{1}{2} = a$. So, a rule for the function is $y = \frac{1}{2} \cdot 2^x$.

**7.** The $y$-values are multiplied by 3 for each increase of 1 in $x$, so $y = a \cdot 3^x$. When $x = 0$, $y = \frac{1}{9} = a$. So, a rule for the function is $y = \frac{1}{9} \cdot 3^x$.

**8.** *Sample answer:* If the function is linear, for each increase of 1 in $x$, the corresponding $y$-values increase by a set amount. If the function is exponential, for each increase of 1 in $x$, the corresponding $y$-values are multiplied by a set amount.

**9.** $y = 4^x$

| **x** | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| **y** | $\frac{1}{16}$ | $\frac{1}{4}$ | 1 | 4 | 16 |

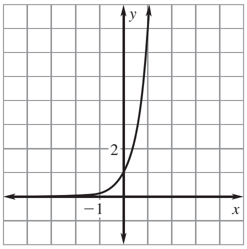



Domain: All real numbers

Range: $y > 0$

**10.** $y = 7^x$

| **x** | −3 | −2 | −1 | 0 | 1 |
|---|---|---|---|---|---|
| **y** | $\frac{1}{343}$ | $\frac{1}{49}$ | $\frac{1}{7}$ | 1 | 7 |

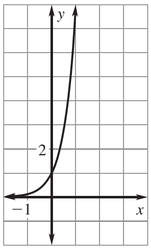



Domain: All real numbers

Range: $y > 0$

**11.** $y = 8^x$

| **x** | −3 | −2 | −1 | 0 | 1 |
|---|---|---|---|---|---|
| **y** | $\frac{1}{512}$ | $\frac{1}{64}$ | $\frac{1}{8}$ | 1 | 8 |

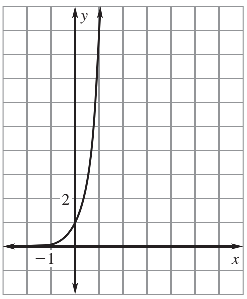



Domain: All real numbers

Range: $y > 0$

**12.** $y = 9^x$

| **x** | −3 | −2 | −1 | 0 | 1 |
|---|---|---|---|---|---|
| **y** | $\frac{1}{729}$ | $\frac{1}{81}$ | $\frac{1}{9}$ | 1 | 9 |

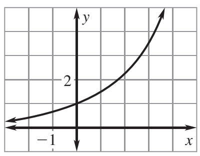



Domain: All real numbers

Range: $y > 0$

**13.** $y = (1.5)^x$

| **x** | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| **y** | 0.44 | 0.67 | 1 | 1.5 | 2.25 |

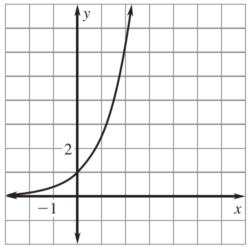



Domain: All real numbers

Range: $y > 0$

**14.** $y = (2.5)^x$

| **x** | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| **y** | 0.16 | 0.4 | 1 | 2.5 | 6.25 |

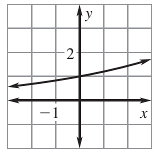



Domain: All real numbers

Range: $y > 0$

**15.** $y = (1.2)^x$

| **x** | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|
| **y** | 0.83 | 1 | 1.2 | 1.44 | 1.728 |

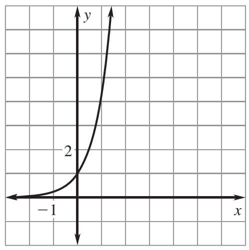



Domain: All real numbers

Range: $y > 0$

**16.** $y = (4.3)^x$

| **x** | −3 | −2 | −1 | 0 | 1 |
|---|---|---|---|---|---|
| **y** | 0.01 | 0.05 | 0.23 | 1 | 4.3 |

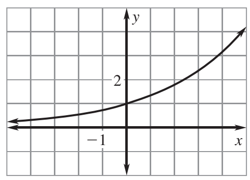



Domain: All real numbers

Range: $y > 0$

**17.** $y = \left(\frac{4}{3}\right)^x$

| **x** | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|
| **y** | $\frac{3}{4}$ | 1 | $\frac{4}{3}$ | $\frac{16}{9}$ | $\frac{64}{27}$ |

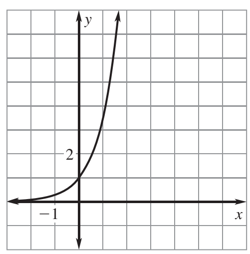



Domain: All real numbers

Range: $y > 0$

**18.** $y = \left(\frac{7}{2}\right)^x$

| **x** | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| **y** | $\frac{4}{49}$ | $\frac{2}{7}$ | 1 | $\frac{7}{2}$ | $\frac{49}{4}$ |

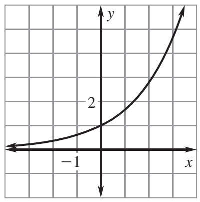



Domain: All real numbers

Range: $y > 0$

**19.** $y = \left(\frac{5}{3}\right)^x$

| **x** | $-1$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|
| **y** | $\frac{3}{5}$ | 1 | $\frac{5}{3}$ | $\frac{25}{9}$ | $\frac{125}{27}$ |

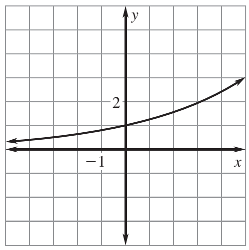



Domain: All real numbers

Range: $y > 0$

**20.** $y = \left(\frac{5}{4}\right)^x$

| **x** | $-1$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|
| **y** | $\frac{4}{5}$ | 1 | $\frac{5}{4}$ | $\frac{25}{16}$ | $\frac{125}{64}$ |




Domain: All real numbers

Range: $y > 0$

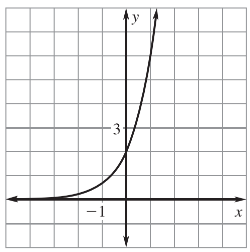
**21.** The error is that the percent increase was not written as a decimal.

$P = a(1 + r)^t = 0.27(1 + 0.02)^3 = 0.27(1.02)^3 \approx 0.29$

In 2002 the price of a pound of flour was about \$.29.

**22.** $y = 2 \cdot 3^x$

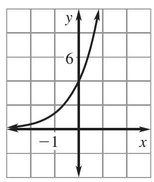
| **x** | $-2$ | $-1$ | 0 | 1 | 2 |
|---|---|---|---|---|---|
| **y** | $\frac{2}{9}$ | $\frac{2}{3}$ | 2 | 6 | 18 |




The graph of $y = 2 \cdot 3^x$ is a vertical stretch of the graph of $y = 3^x$.

**23.** $y = 4 \cdot 3^x$

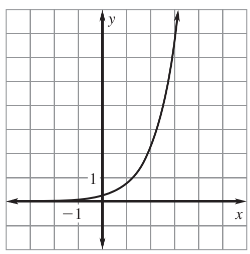
| **x** | $-3$ | $-2$ | $-1$ | 0 | 1 |
|---|---|---|---|---|---|
| **y** | $\frac{4}{27}$ | $\frac{4}{9}$ | $\frac{4}{3}$ | 4 | 12 |




The graph of $y = 4 \cdot 3^x$ is a vertical stretch of the graph of $y = 3^x$.

**24.** $y = \frac{1}{4} \cdot 3^x$

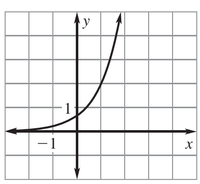
| **x** | $-1$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|
| **y** | $\frac{1}{12}$ | $\frac{1}{4}$ | $\frac{3}{4}$ | $\frac{9}{4}$ | $\frac{27}{4}$ |




The graph of $y = \frac{1}{4} \cdot 3^x$ is a vertical shrink of the graph of $y = 3^x$.

**25.** $y = \frac{2}{3} \cdot 3^x$

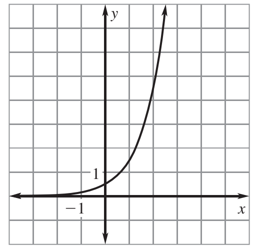
| **x** | $-2$ | $-1$ | 0 | 1 | 2 |
|---|---|---|---|---|---|
| **y** | $\frac{2}{27}$ | $\frac{2}{9}$ | $\frac{2}{3}$ | 2 | 6 |




The graph of $y = \frac{2}{3} \cdot 3^x$ is a vertical shrink of the graph of $y = 3^x$.

**26.** $y = 0.5 \cdot 3^x$

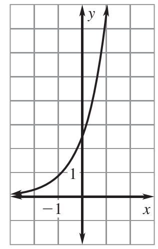
| **x** | $-2$ | $-1$ | 0 | 1 | 2 |
|---|---|---|---|---|---|
| **y** | $\frac{1}{18}$ | $\frac{1}{6}$ | $\frac{1}{2}$ | $\frac{3}{2}$ | $\frac{9}{2}$ |




The graph of $y = (0.5) \cdot 3^x$ is a vertical shrink of the graph of $y = 3^x$.

**27.** $y = 2.5 \cdot 3^x$

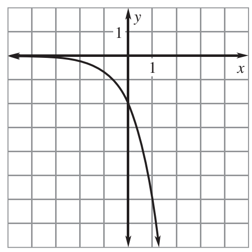
| **x** | $-3$ | $-2$ | $-1$ | 0 | 1 |
|---|---|---|---|---|---|
| **y** | 0.09 | 0.28 | 0.83 | 2.5 | 7.5 |




The graph of $y = 2.5 \cdot 3^x$ is a vertical stretch of the graph of $y = 3^x$.

**28.** $y = -2 \cdot 3^x$

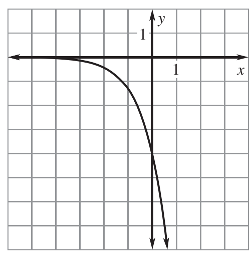
| **x** | $-2$ | $-1$ | 0 | 1 | 2 |
|---|---|---|---|---|---|
| **y** | $-\frac{2}{9}$ | $-\frac{2}{3}$ | $-2$ | $-6$ | $-18$ |




The graph of $y = -2 \cdot 3^x$ is a vertical stretch with a reflection in the $x$-axis of the graph of $y = 3^x$.

**29.** $y = -4 \cdot 3^x$

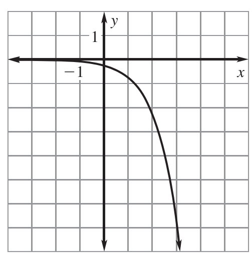
| **x** | $-3$ | $-2$ | $-1$ | 0 | 1 |
|---|---|---|---|---|---|
| **y** | $-\frac{4}{27}$ | $-\frac{4}{9}$ | $-\frac{4}{3}$ | $-4$ | $-12$ |




The graph of $y = -4 \cdot 3^x$ is a vertical stretch with a reflection in the $x$-axis of the graph of $y = 3^x$.

**30.** $y = -\frac{1}{4} \cdot 3^x$

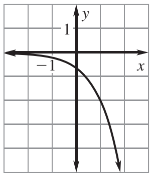
| **x** | $-2$ | $-1$ | 0 | 1 | 2 |
|---|---|---|---|---|---|
| **y** | $-\frac{1}{36}$ | $-\frac{1}{12}$ | $-\frac{1}{4}$ | $-\frac{3}{4}$ | $-\frac{9}{4}$ |




The graph of $y = -\frac{1}{4} \cdot 3^x$ is a vertical shrink with a reflection in the $x$-axis of the graph of $y = 3^x$.

**31.** $y = -\frac{2}{3} \cdot 3^x$

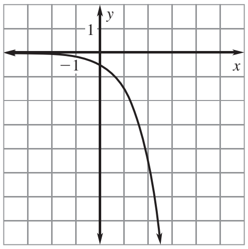
| x | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| y | $-\frac{2}{27}$ | $-\frac{2}{9}$ | $-\frac{2}{3}$ | −2 | −6 |




The graph of $y = -\frac{2}{3} \cdot 3^x$ is a vertical shrink with a reflection in the $x$-axis of the graph of $y = 3^x$.

**32.** $y = -0.5 \cdot 3^x$

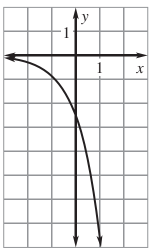
| x | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| y | $-\frac{1}{18}$ | $-\frac{1}{6}$ | $-\frac{1}{2}$ | $-\frac{3}{2}$ | $-\frac{9}{2}$ |




The graph of $y = -\frac{1}{2} \cdot 3^x$ is a vertical shrink with a reflection in the $x$-axis of the graph of $y = 3^x$.

**33.** $y = -2.5 \cdot 3^x$

| x | −3 | −2 | −1 | 0 | 1 |
|---|---|---|---|---|---|
| y | −0.09 | −0.28 | −0.83 | −2.5 | −7.5 |

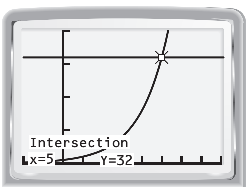



The graph of $y = -2.5 \cdot 3^x$ is a vertical stretch with a reflection in the $x$-axis of the graph of $y = 3^x$.

**34.** **a.**

| x | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| y | 2 | 4 | 8 | 16 | 32 | 64 |

**b.**




**c.** $32 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5$; because $2^5 = 2^x$, $x$ must equal 5.

**35.** 200%; *Sample answer*: A growth rate of 200% would create a growth factor of $1 + 2 = 3$, which would represent the tripling of the population every year.

**36.** Linear:

Slope $= \frac{6-2}{1-0} = 4$

Point-slope form: $y - 2 = 4(x - 0)$

$y = 4x + 2$

Exponential: An increase by 1 in $x$ means multiplication by 3 in $y$, so $y = a \cdot 3^x$.

When $x = 0$, $y = 2 = a$.

So, $y = 2 \cdot 3^x$.

**37.** The graphs are the same.

$f(x) = 2^{x+2} = 2^x \cdot 2^2 = 2^x \cdot 4 = 4 \cdot 2^x = g(x)$

## *Problem Solving*

**38.** $y = a(1 + r)^t$

**a.** $y = (125)(1 + 0.05)^1 = \$131.25$

**b.** $y = (125)(1 + 0.05)^2 = \$137.81$

**c.** $y = (125)(1 + 0.05)^5 = \$159.54$

**d.** $y = (125)(1 + 0.05)^{20} = \$331.66$

**39.** **a.** $r = 10\% = 0.1$

$a = 600$ (million)

$c = 600(1 + 0.1)^t$

$c = 600(1.1)^t$, where $c$ is the number of computers (in millions) and $t$ is the number of years since 2001.

**b.** $c = 600(1.1)^t = 600(1.1)^8 = 1286.153286$

There will be about 1,286,153,286 computers in use worldwide in 2009.

**40.** **a.** $r = 7\% = 0.07$

$a = 3{,}173{,}000$

$g = 3{,}173{,}000(1 + 0.07)^t$

$g = 3{,}173{,}000(1.07)^t$, where $g$ is the number of gas grills and $t$ is the number of years since 1985.

**b.** $g = 3{,}173{,}000(1.07)^t = 3{,}173{,}000(1.07)^{17}$

$= 10{,}022{,}920.66$

About 10,022,921 gas grills were shipped in 2002.

**41.** **a.** Tree 1:

$A = 154(1 + 0.06)^t$

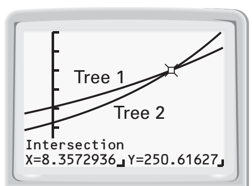
$A = 154(1.06)^t$

Tree 2:

$A = 113(1 + 0.1)^t$

$A = 113(1.1)^t$

**b.**




In about 8.4 years the trees will have the same basal area.

**42.** Yes; *Sample answer:* For each increase of 5 feet in length, the cost is multiplied by $\frac{7}{4}$.

**43.** C;

| Time | Blogs |
|---|---|
| 0 months | 600,000 |
| 6 months | 1,200,000 |
| 12 months | 2,400,000 |
| 18 months | 4,800,000 |

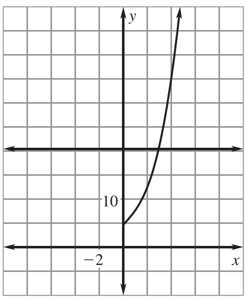
**44. a.** initial amount = 4.67 million

growth factor = 1.65

growth rate = 0.65

**b.**

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $y$ | 4.67 | 7.71 | 12.71 | 20.98 | 34.61 |




Domain: $0 \le x \le 10$

Range: 4.67 million $\le y \le$ 698.48 million

**c.** When $x = 3$, or in 1994, the number of Internet users worldwide was about 21 million.

**45.** $y = 25.96(1.059)^x$

When $x = 30$, $y \approx 145$ hertz.

**46. a.** growth per year $= \dfrac{62{,}947{,}714 - 12{,}866{,}020}{1890 - 1830}$

$= \dfrac{50{,}081{,}694}{60}$

$= 834{,}694.9$

A linear model is $y = 12{,}866{,}020 + 834{,}694.9x$.

The growth was 834,694.9 people per year.

**b.** Using the exponential regression feature on a graphing calculator, $y = 12{,}866{,}020(1.027)^x$.

The population growth was about 2.7% each year.

**c.** Linear model at $x = 20$:

$12{,}866{,}020 + 834{,}694.9(20) = 29{,}559{,}918$

Exponential model at $x = 20$:

$12{,}866{,}020(1.027)^{20} = 21{,}920{,}633.15$

Linear model at $x = 40$:

$12{,}866{,}020 + 834{,}694.9(40) = 46{,}253{,}816$

Exponential model at $x = 40$:

$12{,}866{,}020(1.027)^{40} = 37{,}347{,}536.99$

The exponential model is a better approximation of actual U.S. population for the time period 1850-1890.

**47.** $A = P\left(1 + \frac{r}{n}\right)^{nt} = 1000\left(1 + \frac{0.03}{1}\right)^{1(8)} = 1000(1.03)^8$

$= \$1266.77$

**48.** $A = P\left(1 + \frac{r}{n}\right)^{nt} = 1000\left(1 + \frac{0.03}{4}\right)^{4(8)} = 1000(1.0075)^{32}$

$= \$1270.11$

**49.** $A = P\left(1 + \frac{r}{n}\right)^{nt} = 1000\left(1 + \frac{0.03}{365}\right)^{365(8)}$

$= 1000(1.000082192)^{2920}$

$= \$1271.24$

**50.** Daily; in an account that is compounded daily, each day you earn interest on both the principal and the interest that was accrued on the previous days.

**51.** Use the *intersect* feature on your graphing calculator to determine that the graphs intersect when $x \approx 14.2066699$. So, the doubling time is about 14 years. To check, note that the calculator gives a value for $1.05^{14}$ of 1.979931599, or about 2.

### *Problem Solving Workshop for the lesson "Write and Graph Exponential Growth Functions"*

**1. a.** $b = 20(1 + 0.12)^t$

$b = 20(1.12)^t$

**b.** $b = 20(1.12)^{(1)} = \$22.40$

**c.** 2000; Using the fill down feature of the spreadsheet, you can see that when $t = 3$, $b = 28.1$. So the intercity bus fare was \$28.10 in 2000.

**2.** The error is that the growth rate is written in place of the growth factor. The function should be $b = 20(1.12)^t$.

**3. a.** $T = 7.5(1 + 0.039)^t$

$T = 7.5(1.039)^t$

b.

| Months since May, 1997, *t* | Number of transistors, *T* ( millions) |
|---|---|
| 0 | 7.5 |
| 1 | 7.7925 |
| 2 | 8.0964 |
| . . . | |
| 40 | 34.648 |
| 41 | 36 |
| 42 | 37.404 |

About 37.4 million transistors in a CPU were released by the company in November 2000.

**4. a.** $V = 150{,}000(1 + 0.065)^t$

$V = 150{,}000(1.065)^t$

**b.**

| Years since 2002, *t* | Value, *V* (dollars) |
|---|---|
| 0 | 150,000 |
| 1 | 159,750 |
| . . . | |
| 4 | 192,970 |
| 5 | 205,513 |
| 6 | 218,871 |

When $t = 5$, or in 2007, the value of the home was about \$200,000.

## Lesson 7.5 Write and Graph Exponential Decay Functions

### *Investigating Algebra Activity for the lesson "Write and Graph Exponential Decay Functions"*

| Stage | Number of pieces | Length of each new piece |
|---|---|---|
| 1 | 2 | $\frac{1}{2}$ |
| 2 | 4 | $\frac{1}{4}$ |
| 3 | 8 | $\frac{1}{8}$ |
| 4 | 16 | $\frac{1}{16}$ |
| 5 | 32 | $\frac{1}{32}$ |

**1. a.** Yes; For each increase of 1 in the stage, the number of pieces is multiplied by 2.

**b.** $y = 2^x$

**c.** $y = 2^{10}$ there are 1024 pieces of yarn at stage 10.

**2. a.** Yes; For each increase of 1 in the stage; the length of each piece is multiplied by $\frac{1}{2}$.

**b.** $y = \left(\frac{1}{2}\right)^x$

**c.** $y = \left(\frac{1}{2}\right)^{10} = \frac{1}{1024}$ units

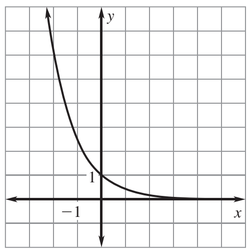
The length of each new piece of yarn at stage 10 is $\frac{1}{1024}$ units.

### *Guided Practice for the lesson "Write and Graph Exponential Decay Functions"*

**1.** Yes; The $y$-values are multipied by $\frac{1}{5}$ for each increase of 1 in $x$, so the table represents an exponential function of the form $y = a \cdot \left(\frac{1}{5}\right)^x$. When $x = 0$, $y = 1 = a$. So, $y = \left(\frac{1}{5}\right)^x$.

**2.** $y = (0.4)^x$

| *x* | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| *y* | 6.25 | 2.5 | 1 | 0.4 | 0.16 |

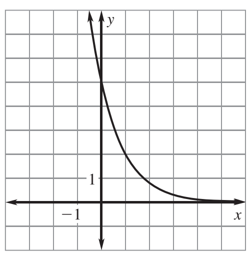



Domain: all real numbers

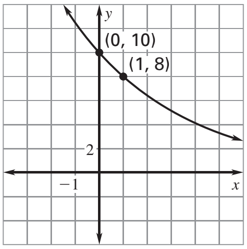
Range: all positive real numbers

**3.** $y = 5 \cdot (0.4)^x$

| *x* | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| *y* | 31.25 | 12.5 | 5 | 2 | 0.8 |




The graph of $y = 5 \cdot (0.4)^x$ is a vertical stretch of the graph of $y = (0.4)^x$.

4. 


The graph represents exponential decay. The $y$-intercept is 10, so $a = 10$,

$y = ab^x$

$8 = 10 \cdot b^1$

$0.8 = b$

A function rule is $y = 10(0.8)^x$.

5. $P = 41(0.995)^t = 41(0.995)^{47} \approx 32.4$

There will be about 32.4 million acres left in 2010.

## *Exercises for the lesson "Write and Graph Exponential Decay Functions"*

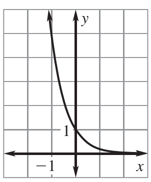
### *Skill Practice*

1. The decay factor in the exponential decay model $y = a(1 - r)^t$ is $1 - r$.
2. *Sample answer:* The graph of an exponential decay function falls from left to right while the graph of an exponential growth function rises from left to right.
3. The $y$-values are multiplied by 4 for each increase of 1 in $x$, so the table represents an exponential function of the form $y = a \cdot 4^x$, when $x = 0$, $y = 8 = a$. So, $y = 8 \cdot 4^x$.
4. The $y$-values are multiplied by $\frac{1}{5}$ for each increase of 1 in $x$, so the table represents an exponential function of the form $y = a \cdot \left(\frac{1}{5}\right)^x$. When $x = 0$, $y = 10 = a$.

   So, $y = 10 \cdot \left(\frac{1}{5}\right)^x$.
5. The $y$-values are multiplied by $\frac{1}{3}$ for each increase of 1 in $x$, so the table represents an exponential function of the form $y = a \cdot \left(\frac{1}{3}\right)^x$. When $x = 0$, $y = 2 = a$.

   So, $y = 2 \cdot \frac{1}{3^x}$.
6. The $y$-values are increased by 4 for each increase of 1 in $x$, so the function is not exponential.
7. $y = \left(\frac{1}{5}\right)^x$

| **x** | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|
| **y** | 5 | 1 | $\frac{1}{5}$ | $\frac{1}{25}$ | $\frac{1}{125}$ |

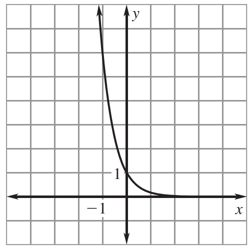



Domain: All real numbers

Range: $y > 0$

8. $y = \left(\frac{1}{6}\right)^x$

| **x** | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|
| **y** | 6 | 1 | $\frac{1}{6}$ | $\frac{1}{36}$ | $\frac{1}{216}$ |

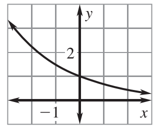



Domain: All real numbers

Range: $y > 0$

9. $y = \left(\frac{2}{3}\right)^x$

| **x** | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| **y** | $\frac{9}{4}$ | $\frac{3}{2}$ | 1 | $\frac{2}{3}$ | $\frac{4}{9}$ |

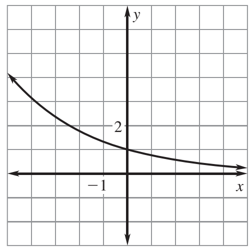



Domain: All real numbers

Range: $y > 0$

10. $y = \left(\frac{3}{4}\right)^x$

| **x** | −3 | −2 | −1 | 0 | 1 |
|---|---|---|---|---|---|
| **y** | $\frac{64}{27}$ | $\frac{16}{9}$ | $\frac{4}{3}$ | 1 | $\frac{3}{4}$ |

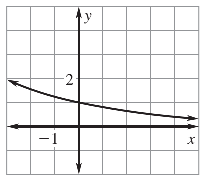



Domain: All real numbers

Range: $y > 0$

11. $y = \left(\frac{4}{5}\right)^x$

| **x** | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| **y** | $\frac{25}{16}$ | $\frac{5}{4}$ | 1 | $\frac{4}{5}$ | $\frac{16}{25}$ |

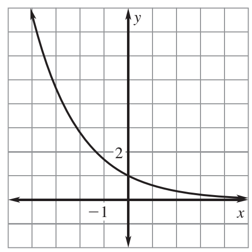



Domain: All real numbers

Range: $y > 0$

12. $y = \left(\frac{3}{5}\right)^x$

| **x** | −4 | −2 | 0 | 2 | 4 |
|---|---|---|---|---|---|
| **y** | $\frac{625}{81}$ | $\frac{25}{9}$ | 1 | $\frac{9}{25}$ | $\frac{81}{625}$ |

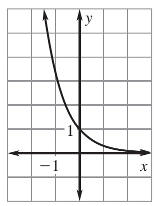



Domain: All real numbers

Range: $y > 0$

13. $y = (0.3)^x$

| **x** | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| **y** | 11.11 | 3.33 | 1 | 0.3 | 0.09 |

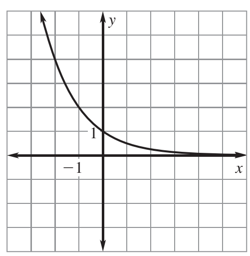



Domain: All real numbers

Range: $y > 0$

14. $y = (0.5)^x$

| **x** | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| **y** | 4 | 2 | 1 | 0.5 | 0.25 |

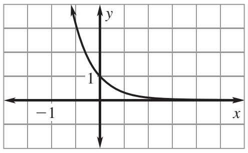



Domain: All real numbers

Range: $y > 0$

**15.** $y = (0.1)^x$

| **x** | $-1$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|
| **y** | 10 | 1 | 0.1 | 0.01 | 0.001 |

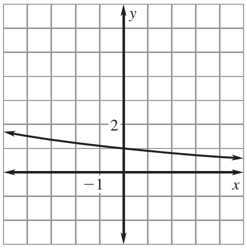


Domain: All real numbers

Range: $y > 0$

**16.** $y = (0.9)^x$

| **x** | $-6$ | $-4$ | $-2$ | 0 | 2 |
|---|---|---|---|---|---|
| **y** | 1.9 | 1.5 | 1.2 | 1 | 0.8 |

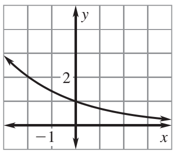


Domain: All real numbers

Range: $y > 0$

**17.** $y = (0.7)^x$

| **x** | $-4$ | $-2$ | 0 | 2 | 4 |
|---|---|---|---|---|---|
| **y** | 4.16 | 2.04 | 1 | 0.49 | 0.24 |

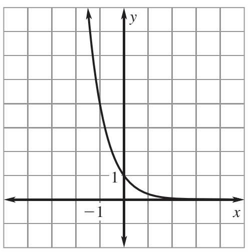


Domain: All real numbers

Range: $y > 0$

**18.** $y = (0.25)^x$

| **x** | $-2$ | $-1$ | 0 | 1 | 2 |
|---|---|---|---|---|---|
| **y** | 16 | 4 | 1 | 0.25 | 0.0625 |



Domain: All real numbers

Range: $y > 0$

**19.** D; When $x = 0$, $y = 4 = a$.

$y = 4 \cdot b^x$

$2 = 4 \cdot b^1$

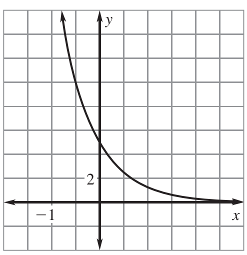
$2 = 4b$

$0.5 = b$

The function is $y = 4 \cdot (0.5)^x$.

**20.** $y = 5 \cdot \left(\frac{1}{4}\right)^x$

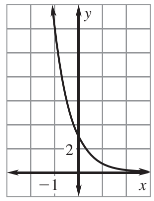
| **x** | $-2$ | $-1$ | 0 | 1 | 2 |
|---|---|---|---|---|---|
| **y** | 80 | 20 | 5 | $\frac{5}{4}$ | $\frac{5}{16}$ |



The graph of $y = 5 \cdot \left(\frac{1}{4}\right)^x$ is a vertical stretch of the graph of $y = \left(\frac{1}{4}\right)^x$.

**21.** $y = 3 \cdot \left(\frac{1}{4}\right)^x$

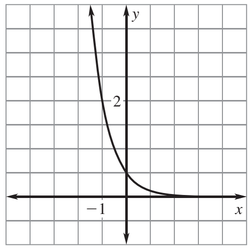
| **x** | $-2$ | $-1$ | 0 | 1 | 2 |
|---|---|---|---|---|---|
| **y** | 48 | 12 | 3 | $\frac{3}{4}$ | $\frac{3}{16}$ |



The graph of $y = 3 \cdot \left(\frac{1}{4}\right)^x$ is a vertical stretch of the graph of $y = \left(\frac{1}{4}\right)^x$.

**22.** $y = \frac{1}{2} \cdot \left(\frac{1}{4}\right)^x$

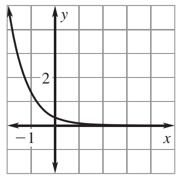
| **x** | $-2$ | $-1$ | 0 | 1 | 2 |
|---|---|---|---|---|---|
| **y** | 8 | 2 | $\frac{1}{2}$ | $\frac{1}{8}$ | $\frac{1}{32}$ |



The graph of $y = \frac{1}{2} \cdot \left(\frac{1}{4}\right)^x$ is a vertical shrink of the graph of $y = \left(\frac{1}{4}\right)^x$.

**23.** $y = \frac{1}{3} \cdot \left(\frac{1}{4}\right)^x$

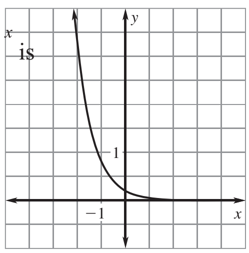
| **x** | $-2$ | $-1$ | 0 | 1 | 2 |
|---|---|---|---|---|---|
| **y** | $\frac{16}{3}$ | $\frac{4}{3}$ | $\frac{1}{3}$ | $\frac{1}{12}$ | $\frac{1}{48}$ |



The graph of $y = \frac{1}{3} \cdot \left(\frac{1}{4}\right)^x$ is a vertical shrink of the graph of $y = \left(\frac{1}{4}\right)^x$.

**24.** $y = 0.2 \cdot \left(\frac{1}{4}\right)^x$

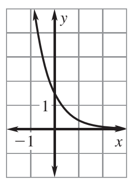
| **x** | $-2$ | $-1$ | 0 | 1 | 2 |
|---|---|---|---|---|---|
| **y** | 3.2 | 0.8 | 0.2 | 0.05 | 0.01 |



The graph of $y = 0.2 \cdot \left(\frac{1}{4}\right)^x$ is a vertical shrink of the graph of $y = \left(\frac{1}{4}\right)^x$.

**25.** $y = 1.5 \cdot \left(\frac{1}{4}\right)^x$

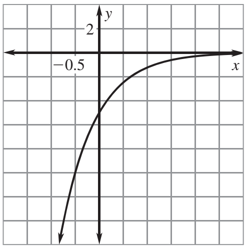
| **x** | $-2$ | $-1$ | 0 | 1 | 2 |
|---|---|---|---|---|---|
| **y** | 24 | 6 | $\frac{3}{2}$ | $\frac{3}{8}$ | $\frac{3}{32}$ |



The graph of $y = 1.5 \cdot \left(\frac{1}{4}\right)^x$ is a vertical stretch of the graph of $y = \left(\frac{1}{4}\right)^x$.

**26.** $y = -5 \cdot \left(\frac{1}{4}\right)^x$

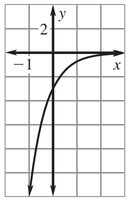
| **x** | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| **y** | −80 | −20 | −5 | $-\frac{5}{4}$ | $-\frac{5}{16}$ |




The graph of $y = -5 \cdot \left(\frac{1}{4}\right)^x$ is a vertical stretch with a reflection in the $x$-axis of the graph of $y = \left(\frac{1}{4}\right)^x$.

**27.** $y = -3 \cdot \left(\frac{1}{4}\right)^x$

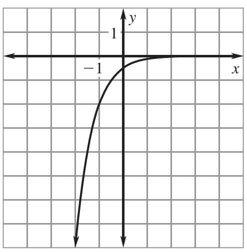
| **x** | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| **y** | −48 | −12 | −3 | $-\frac{3}{4}$ | $-\frac{3}{16}$ |




The graph of $y = -3 \cdot \left(\frac{1}{4}\right)^x$ is a vertical stretch with a reflection in the $x$-axis of the graph of $y = \left(\frac{1}{4}\right)^x$.

**28.** $y = -\frac{1}{2} \cdot \left(\frac{1}{4}\right)^x$

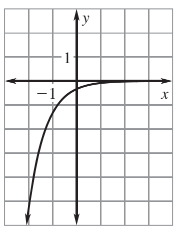
| **x** | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| **y** | −8 | −2 | $-\frac{1}{2}$ | $-\frac{1}{8}$ | $-\frac{1}{32}$ |




The graph of $y = -\frac{1}{2} \cdot \left(\frac{1}{4}\right)^x$ is a vertical shrink with a reflection in the $x$-axis of the graph of $y = \left(\frac{1}{4}\right)^x$.

**29.** $y = -\frac{1}{3} \cdot \left(\frac{1}{4}\right)^x$

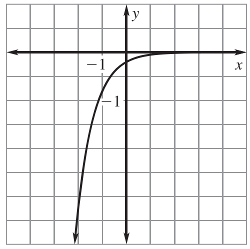
| **x** | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| **y** | $-\frac{16}{3}$ | $-\frac{4}{3}$ | $-\frac{1}{3}$ | $-\frac{1}{12}$ | $-\frac{1}{48}$ |




The graph of $y = -\frac{1}{3} \cdot \left(\frac{1}{4}\right)^x$ is a vertical shrink with a reflection in the $x$-axis of the graph of $y = \left(\frac{1}{4}\right)^x$.

**30.** $y = -0.2 \cdot \left(\frac{1}{4}\right)^x$

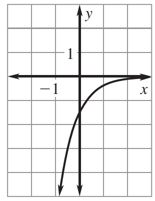
| **x** | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| **y** | −3.2 | −0.8 | −0.2 | −0.05 | −0.01 |




The graph of $y = -0.2 \cdot \left(\frac{1}{4}\right)^x$ is a vertical shrink with a reflection in the $x$-axis of the graph of $y = \left(\frac{1}{4}\right)^x$.

**31.** $y = -1.5 \cdot \left(\frac{1}{4}\right)^x$

| **x** | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| **y** | −24 | −6 | $-\frac{3}{2}$ | $-\frac{3}{8}$ | $-\frac{3}{32}$ |




The graph of $y = -1.5 \cdot \left(\frac{1}{4}\right)^x$ is a vertical stretch with a reflection in the $x$-axis of the graph of $y = \left(\frac{1}{4}\right)^x$.

**32.** A; $y = (0.2)^x$

Let $x = 0$: $y = (0.2)^0 = 1$

$y$-intercept: (0, 1)

**33.** C; $y = 5(0.2)^x$

Let $x = 0$: $y = 5(0.2)^0 = 5$

$y$-intercept: (0, 5)

**34.** B; $y = \frac{1}{2}(0.2)^x$

Let $x = 0$: $y = \frac{1}{2}(0.2)^0 = \frac{1}{2}$

$y$-intercept: $\left(0, \frac{1}{2}\right)$

**35.** $y = a(1 - r)^t$

initial amount $= a = 90{,}000$

decay rate $= r = 0.025$

decay factor $= 1 - r = 1 - 0.025 = 0.975$

$y = 90{,}000(0.975)^t$

**36.** D;

decay factor = 1 − decay rate

$0.97 = 1 - r$

$r = 1 - 0.97$

$r = 0.03$

**37.** The error is that the decay rate was placed where the decay factor should be. The equation should be:

$y = a(1 - r)^t = 25{,}000(1 - 0.14)^t = 25{,}000(0.86)^t$

**38.** The graph represents exponential decay because it falls from left to right. The $y$-intercept is 6, so $a = 6$.

$y = ab^x$

$4.8 = 6b^1$

$0.8 = b$

The function rule is $y = 6(0.8)^x$.

**39.** The graph represents exponential decay because it falls from left to right. The $y$-intercept is 8, so $a = 8$.

$y = ab^x$

$4.8 = 8b^1$

$0.6 = b$

The function rule is $y = 8(0.6)^x$.

**40.** The graph represents exponential growth because it rises from left to right. The $y$-intercept is 8, so $a = 8$.

$y = ab^x$

$12.8 = 8b^1$

$1.6 = b$

The function rule is $y = 8(1.6)^x$.

**41.** **a.** $m(x)$ is a vertical shrink of $f(x)$.

**b.** $n(x)$ is a vertical stretch with a reflection in the $x$-axis of $f(x)$.

**c.** $p(x)$ is a vertical translation of 1 unit up of $f(x)$.

**42.** $(0, 1), \left(2, \frac{1}{4}\right)$

$y$ is multiplied by $\frac{1}{2}$ for each increase of 1 in $x$, so $b = \frac{1}{2}$.

When $x = 0$, $y = 1 = a$. A function rule is $y = \left(\frac{1}{2}\right)^x$.

**43.** $(1, 20), (2, 4)$

$y$ is multiplied by $\frac{1}{5}$ for each increase of 1 in $x$, so $b = \frac{1}{5}$.

$y = a\left(\frac{1}{5}\right)^x$

$20 = a\left(\frac{1}{5}\right)^1$

$100 = a$

A function rule is $y = 100\left(\frac{1}{5}\right)^x$.

**44.** $\left(1, \frac{3}{2}\right), \left(2, \frac{3}{4}\right)$

$y$ is multiplied by $\frac{1}{2}$ for each increase of 1 in $x$, so $b = \frac{1}{2}$.

$y = a\left(\frac{1}{2}\right)^x$

$\frac{3}{2} = a\left(\frac{1}{2}\right)^1$

$3 = a$

A function rule is $y = 3\left(\frac{1}{2}\right)^x$.

**45.** To find $t$, divide the number of days, 40, by the half-life, 10; $t = \frac{40}{10} = 4$. Then $A = 100(0.5)^t = 100(0.5)^4 = 6.25$. The amount left after 40 days is 6.25 grams.

**46.** The graphs are the same graph.

$$f(x) = 4^{x-2} = 4^x \cdot 4^{(-2)} = 4^x \cdot \frac{1}{16} = \frac{1}{16} \cdot 4^x = g(x)$$

## *Problem Solving*

**47.** Let $y$ represent the value of the cell phone and $t$ represent the number of years since purchase.

$y = 125(1 - 0.2)^t = 125(0.8)^t = 125(0.8)^3 = 64$

The value of the cell phone after 3 years is \$64.

**48.** **a.** Initial amount $= a = 141{,}200$

Decay rate $= r = 0.11$

Decay Factor $= 1 - r = 1 - 0.11 = 0.89$

**b.** Let $B$ represent the number of bats and $t$ represent the number of years since 1983.

$y = 141{,}200(0.89)^t = 141{,}200(0.89)^{20} \approx 13{,}729$

There were about 13,729 bats in 2003.

**49.** No; in 2006, the boat will be worth

$B = 4000(0.93)^3 = \$3217.43$.

Selling the boat for \$3000 will be selling the boat for less than what it's worth.

**50.** **a.** Initial amount $= 128$

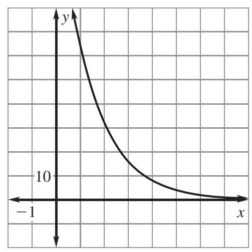
Decay factor $= \frac{1}{2}$

$y = 128\left(\frac{1}{2}\right)^x$

**b.**

| Rounds completed | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| Teams remaining | 128 | 64 | 32 | 16 | 8 | 4 | 2 | 1 |

**c.**




After round 5 there will be 4 teams left in the tournament.

**51.** **a.** Decay factor $= 0.9439$

Decay Rate $= 1 - 0.9439 = 0.0561$

**b.** $d = 1.516(0.9439)^1 \approx 1.431$ inches

**c.** The distance between the nut and the first fret is $d = \frac{1}{2}\left[1.516(0.9439)^1\right] = \frac{1}{2}(1.431) \approx 0.716$ inches.

The distance between the 12th and 13th frets is $d = 1.516(0.9439)^{13} \approx 0.716$ inches.

**52.** Let $y$ represent the remaining balance and $t$ represent the number of months since purchase.

$y = 1850(1 - 0.0225)^t = 1850(0.9775)^t$

After the 23rd month, the remaining balance is

$y = 1850(0.9775)^{23} = 1096.12$.

If the student buys the computer without paying interest, he should pay the remaining balance \$1096.12 after the 23rd month.

**53. a.** 1st athlete

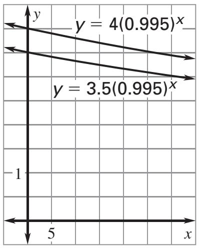
$y_1 = 4(1 - 0.005)^x = 4(0.995)^x$

2nd athlete

$y_2 = 3.5(1 - 0.005)^x = 3.5(0.995)^x$

**b.**

| **x** | 0 | 10 | 20 | 30 | 40 | 50 | 60 |
|---|---|---|---|---|---|---|---|
| $y_1$ | 4 | 3.80 | 3.62 | 3.44 | 3.27 | 3.11 | 2.96 |
| $y_2$ | 3.5 | 3.33 | 3.17 | 3.01 | 2.86 | 2.72 | 2.59 |




**c.** At about $x = 26$, $y \approx 3.5$. So the first athlete will be about $25 + 26 = 51$ years old when her maximal oxygen consumption is equal to 3.5 liters per minute.

### *Quiz for the lessons "Define and Use Zero and Negative Exponents", "Write and Graph Exponential Growth Functions" and "Write and Graph Exponential Decay Functions"*

**1.** $(-4x)^4 \cdot (-4)^{-6} = (-4)^4 \cdot x^4 \cdot (-4)^{-6}$

$= (-4)^{4+(-6)} \cdot x^4$

$= (-4)^{-2}x^4$

$= \dfrac{x^4}{(-4)^2}$

$= \dfrac{x^4}{16}$

**2.** $(-3x^7y^{-2})^{-3} = (-3)^{-3} \cdot (x^7)^{-3} \cdot (y^{-2})^{-3}$

$= (-3)^{-3} \cdot x^{-21} \cdot y^6$

$= \dfrac{y^6}{(-3)^3 \cdot x^{21}}$

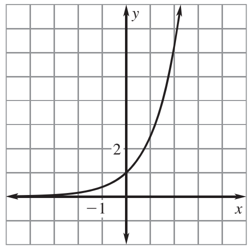
$= -\dfrac{y^6}{27x^{21}}$

**3.** $\dfrac{1}{(5z)^{-3}} = (5z)^3 = 5^3 \cdot z^3 = 125z^3$

**4.** $\dfrac{(6x)^{-2}y^5}{-x^3y^{-7}} = \dfrac{6^{-2}x^{-2}y^5}{-x^3y^{-7}} = \dfrac{y^5y^7}{-6^2x^2x^3} = -\dfrac{y^{12}}{36x^5}$

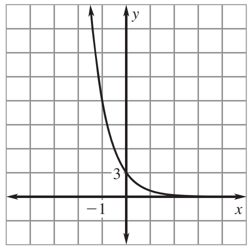
**5.** $y = \left(\frac{5}{2}\right)^x$

| **x** | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| **y** | $\frac{4}{25}$ | $\frac{2}{5}$ | 1 | $\frac{5}{2}$ | $\frac{25}{4}$ |



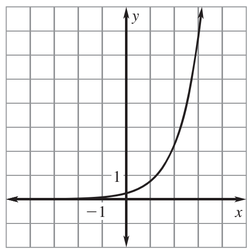

**6.** $y = 3 \cdot \left(\frac{1}{4}\right)^x$

| **x** | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| **y** | 48 | 12 | 3 | $\frac{3}{4}$ | $\frac{3}{16}$ |



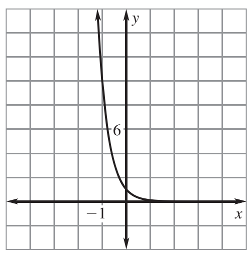

**7.** $y = \frac{1}{4} \cdot 3^x$

| **x** | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|
| **y** | $\frac{1}{12}$ | $\frac{1}{4}$ | $\frac{3}{4}$ | $\frac{9}{4}$ | $\frac{27}{4}$ |



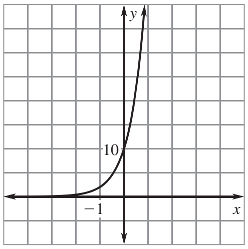

**8.** $y = (0.1)^x$

| **x** | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|
| **y** | 10 | 1 | 0.1 | 0.01 | 0.001 |



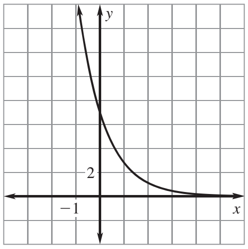

**9.** $y = 10 \cdot 5^x$

| **x** | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| **y** | 0.4 | 2 | 10 | 50 | 250 |




**10.** $y = 7(0.4)^x$

| **x** | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| **y** | 43.75 | 17.5 | 7 | 2.8 | 1.12 |




**11.** Let $y$ represent the value of the coin and $t$ represent the number of years since purchase.

$y = 25(1 + 0.08)^t = 25(1.08)^t$

When $t = 10$: $y = 25(1.08)^{10} = 53.97$

The value of the coin after 10 years is \$53.97.

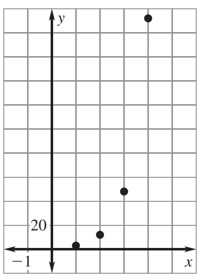
### *First Extension for the lesson "Write and Graph Exponential Decay Functions"*

**1.** The ratios of consecutive terms are:

$\frac{a_2}{a_1} = \frac{12}{3} = 4, \frac{a_3}{a_2} = \frac{48}{12} = 4, \frac{a_4}{a_3} = \frac{192}{48} = 4$

The common ratio is 4, so the sequence is geometric.

| **Position, *x*** | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| **Term, *y*** | 3 | 12 | 48 | 192 |




**2.** The difference of consecutive terms are:

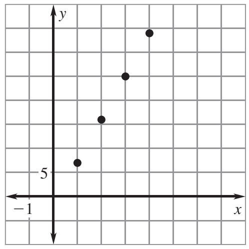
$a_2 - a_1 = 16 - 7 = 9$

$a_3 - a_2 = 25 - 16 = 9$

$a_4 - a_3 = 34 - 25 = 9$

The common difference is 9, so the sequence is arithmetic.

| **Position, *x*** | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| **Term, *y*** | 7 | 16 | 25 | 34 |




**3.** The difference of consecutive terms are:

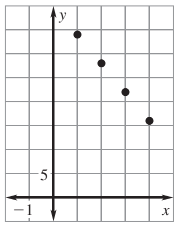
$a_2 - a_1 = 28 - 34 = -6$

$a_3 - a_2 = 22 - 28 = -6$

$a_4 - a_3 = 16 - 22 = -6$

The common difference is −6, so the sequence is arithmetic.

| **Position, *x*** | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| **Term, *y*** | 34 | 28 | 22 | 16 |

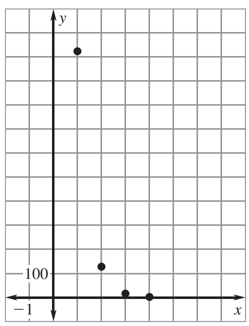



**4.** The ratios of consecutive terms are:

$\frac{a_2}{a_1} = \frac{128}{1024} = \frac{1}{8}, \frac{a_3}{a_2} = \frac{16}{128} = \frac{1}{8}, \frac{a_4}{a_3} = \frac{2}{16} = \frac{1}{8}$

The common ratio is $\frac{1}{8}$, so the sequence is geometric.

| **Position, *x*** | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| **Term, *y*** | 1024 | 128 | 16 | 2 |

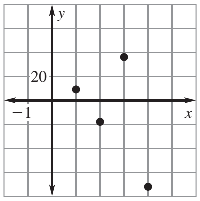



**5.** The ratios between consecutive terms are:

$\frac{a_2}{a_1} = \frac{-18}{9} = -2, \frac{a_3}{a_2} = \frac{36}{-18} = -2, \frac{a_4}{a_3} = \frac{-72}{36} = -2$

The common ratio is −2, so the sequence is geometric.

| Position, $x$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Term, $y$ | 9 | $-18$ | 36 | $-72$ |




**6.** The differences between consecutive terms are:

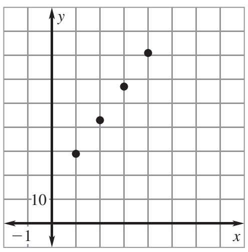
$a_2 - a_1 = 43 - 29 = 14$

$a_3 - a_2 = 57 - 43 = 14$

$a_4 - a_3 = 71 - 57 = 14$

The common difference is 14, so the sequence is arithmetic.

| Position, $x$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Term, $y$ | 29 | 43 | 57 | 71 |




**7.** $r = \frac{a_2}{a_1} = \frac{-5}{1} = -5$

$a_n = (-5)^{n-1}$

$a_7 = (-5)^{7-1} = 15{,}625$

**8.** $r = \frac{a_2}{a_1} = \frac{26}{13} = 2$

$a_n = 13 \cdot 2^{n-1}$

$a_7 = 13 \cdot 2^{7-1} = 832$

**9.** $r = \frac{a_2}{a_1} = \frac{72}{432} = \frac{1}{6}$

$a_n = 432 \cdot \left(\frac{1}{6}\right)^{n-1}$

$a_7 = 432 \cdot \left(\frac{1}{6}\right)^{7-1} = \frac{1}{108}$

**10.** $\frac{a_2}{a_1} = \frac{4}{1} = 4$

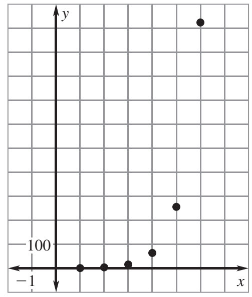
$\frac{a_3}{a_2} = \frac{16}{4} = 4$

$\frac{a_4}{a_3} = \frac{64}{16} = 4$

The ratios of consecutive terms are constant so the series is geometric where $r = 4$.

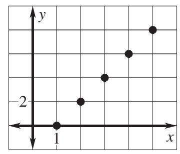
$a_n = 1 \cdot 4^{n-1} = 4^{n-1}$

| Position, $x$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Term, $y$ | 1 | 4 | 16 | 64 | 256 | 1024 |

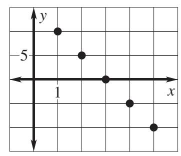



### *Second Extension for the lesson "Write and Graph Exponential Decay Functions"*

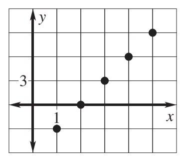
**1.** 0, 2, 4, 6, 8



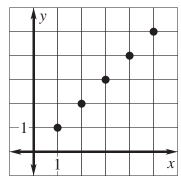

**2.** 10, 5, 0, −5, −10



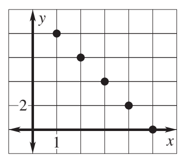

**3.** −3, 0, 3, 6, 9



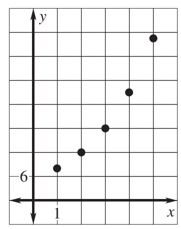

**4.** 1, 2, 3, 4, 5



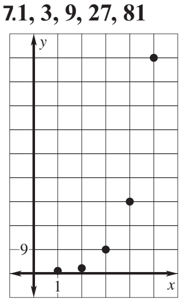

**5.** 8, 6, 4, 2, 0



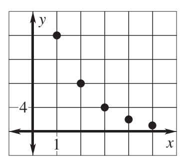

**6.** 8, 12, 18, 27, 40.5



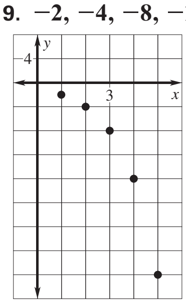

**7.**1, 3, 9, 27, 81




**8.** 16, 8, 4, 2, 1




**9.** −2, −4, −8, −16, −32



**10.** $a_1 = 8,$
$a_n = a_{n-1} + 20$

**11.** $a_1 = 256,$
$a_n = 0.25a_{n-1}$

**12.** $a_1 = 0,$
$a_n = a_{n-1} - 4$

**13.** $a_1 = 3,$
$a_n = a_{n-1} + 4$

**14.** $a_1 = 81,$
$a_n = \frac{1}{3}a_{n-1}$

**15.** $a_1 = -5,$
$a_n = a_{n-1} + 2$

**16.** $a_1 = 16, a_n = 1.5a_{n-1}$

**17.** $a_1 = -2, a_n = -2a_{n-1}$

**18.** $a_1 = 0.5, a_n = a_{n-1} + 1.25$

**19.** $a_1 = 1, a_2 = 3, a_n = a_{n-2} + a_{n-1}$; 18, 29

**20.** $a_1 = 1, a_2 = 4, a_n = (a_{n-2})(a_{n-1})$; 1024, 65,536

**21.** $a_1 = 1, a_2 = 1, a_3 = 1, a_n = a_{n-3} + a_{n-2} + a_{n-1}$; 17, 31

**22.** $a_1 = 10, a_2 = 9, a_n = a_{n-2} - a_{n-1}$; $-22$, 37

**23.** $a_1 = 64, a_2 = 16, an = \frac{a_{n-2}}{a_{n-1}}$; 4, 0.25

**24.** $a_1 = 2, a_2 = 4, an = a_{n-2} + 2a_{n-1}$; 140, 338

**25.** 3, 12, 48, 192, 768, 3072, 12,288, 49,152, 196,608, 786,432

**26.** $a_1 = 200, a_n = 2a_{n-1}$; 6400 bacteria

**27.** This is the explicit rule with $n - 1$ substituted for $n$.

### *Mixed Review of Problem Solving for the lessons "Write and Graph Exponential Growth Functions" and "Write and Graph Exponential Decay Functions"*

**1. a.** Let $y$ represent the amount of medication in patient's bloodstream (in milligrams) and let $t$ represent the time since the medication was taken. The amount of medication is halved every 8 hours, so

$y = 500(0.5)^{t/8}$.

**b.** $y = 500(0.5)^{24/8} = 500(0.5)^3 = 62.5$

After 24 hours, the patient's bloodstream will have 62.5 mg of the medication.

**2. a.** The $y$-intercept is 25,000, so $a = 25,000$.

$y = ab^x$

$23,750 = 25,000b^1$

$0.95 = b$

A function rule is $y = 25,000(0.95)^x$.

**b.** The decay factor, $1 - r$, for the truck is 0.95. So,

$1 - r = 0.95$

$0.05 = r$

The decay rate is 0.05, or 5%.

**3.** Decay factor $= 1 - r$

$0.82 = 1 - r$

$r = 0.18$

**4.** *Sample answer:* Let $y$ represent the value of the house and $t$ represent the number of quarters since 2001.

$y = a(1 + 0.04)^t = a(1.04)^t$

When $t = 4$,

$275,000 = a(1.04)^4$

$275,000 = 1.1699a$

$235,071.15 = a$

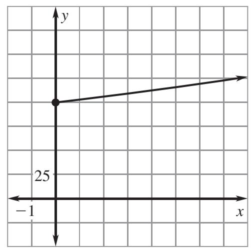
The value of the house can be modeled by $y = 235,000(1.04)^t$. A house with a value of \$235,000 at the end of 2001 would have a value of about \$275,000 at the end of 2002.

**5. a.** Let $y$ represent the amount of money in the account and $t$ represent the number of years since the \$100 was deposited,

$y = a(1 + r)^t = 100(1 + 0.03)t = 100(1.03)t$

**b.**

| *x* | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| *y* | 100 | 103 | 106.09 | 109.27 | 112.55 |




**c.** No, after 3 years the musician will only have \$109.27.

**6. a.** The graph rises from left to right, so it represents exponential growth.

**b.** The $y$-intercept is 15,000, so $a = 15,000$.

$y = ab^x$

$19,500 = 15,000b^1$

$1.3 = b$

A function rule is $y = 15,000(1.3)^x$.

**c.** $y = 15,000(1.3)^x = 15,000(1.3)^4 = 42,841.50$

The business is worth \$42,841.50 after 4 years.

### *Chapter Review for the chapter "Exponents and Exponential Functions"*

**1.** The function $y = 1200(0.3)^t$ is an exponential *decay* function, and the base 0.3 is called the *decay factor*.

**2.** *Sample answer:* A table represents a linear function if the output values change by the addition of the same number. A table represents an exponential function if the output values change by the multiplication of the same number.

**3.** The function $y = 3(0.85)^x$ represents exponential decay because the base, 0.85, is less than 1.

**4.** The function $y = \frac{1}{2}(1.01)^x$ represents exponential growth because the base, 1.01, is greater than 1.

**5.** The function $y = 2(2.1)^x$ represents exponential growth because the base, 2.1, is greater than 1.

**6.** $4^4 \cdot 4^3 = 4^{4+3} = 4^7$

**7.** $(-3)^7(-3) = (-3)^{7+1} = (-3)^8$

**8.** $z^3 \cdot z^5 \cdot z^5 = z^{3+5+5} = z^{13}$

**9.** $(y^4)^5 = y^{4 \cdot 5} = y^{20}$

**10.** $[(-7)^4]^4 = (-7)^{4 \cdot 4} = (-7)^{16}$

**11.** $[(b+2)^8]^3 = (b+2)^{8 \cdot 3} = (b+2)^{24}$

**12.** $(6^4 \cdot 31)^5 = 6^{4 \cdot 5} \cdot 31^5 = 6^{20} \cdot 31^5$

**13.** $-(8xy)^2 = -8^2x^2y^2 = -64x^2y^2$

**14.** $(2x^2)^4 \cdot x^5 = 2^4x^{2 \cdot 4}x^5 = 16x^8x^5 = 16x^{8+5} = 16x^{13}$

**15.** $10^{18} \cdot 10^3 = 10^{18+3} = 10^{21}$ kilograms

**16.** $\frac{(-3)^7}{(-3)^3} = (-3)^{7-3} = (-3)^4$

**17.** $\frac{5^2 \cdot 5^4}{5^3} = \frac{5^{2+4}}{5^3} = \frac{5^6}{5^3} = 5^{6-3} = 5^3$

**18.** $\left(\frac{m}{n}\right)^3 = \frac{m^3}{n^3}$

**19.** $\frac{17^{12}}{17^8} = 17^{12-8} = 17^4$

**20.** $\left(-\frac{1}{x}\right)^4 = \frac{(-1)^4}{x^4}$

$= \frac{1}{x^4}$

**21.** $\left(\frac{7x^5}{y^2}\right)^2 = \frac{7^2x^{5 \cdot 2}}{y^{2 \cdot 2}}$

$= \frac{49x^{10}}{y^4}$

**22.** $\frac{1}{p^2} \cdot p^6 = p^{6-2} = p^4$

**23.** $\frac{6}{7r^{10}} \cdot \left(\frac{r^5}{s}\right)^5 = \frac{6}{7r^{10}} \cdot \frac{r^{5 \cdot 5}}{s^5} = \frac{6r^{25}}{7r^{10}s^5} = \frac{6r^{25-10}}{7s^5} = \frac{6r^{15}}{7s^5}$

**24.** $\frac{10^{10}}{10^6} = 10^{10-6} = 10^4$

The order of magnitude of the mean personal income in Montana in 2003 was $\$10^4$.

**25.** $14^0 = 1$

**26.** $3^{-4} = \frac{1}{3^4} = \frac{1}{81}$

**27.** $\left(\frac{2}{3}\right)^{-3} = \left(\frac{3}{2}\right)^3 = \frac{3^3}{2^3} = \frac{27}{8}$

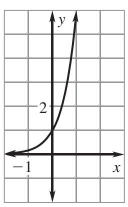
**28.** $7^{-5} \cdot 7^5 = 7^{-5+5} = 7^0 = 1$

**29.** $\frac{1 \text{ femtogram}}{10^{-18} \text{ kilogram}} \cdot \frac{10^{-12} \text{ kilogram}}{1 \text{ nanogram}} = 10^{-12-(-18)}$

$= \frac{10^6 \text{ femtogram}}{1 \text{ nanogram}}$

**30.** $y = 6^x$

| **x** | $-1$ | 0 | 1 | 2 |
|---|---|---|---|---|
| **y** | $\frac{1}{6}$ | 1 | 6 | 36 |

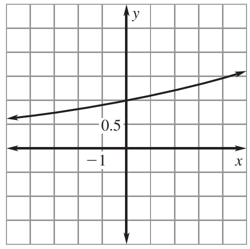



Domain: all real numbers

Range: all positive real numbers

**31.** $y = (1.1)^x$

| **x** | $-1$ | 0 | 1 | 2 |
|---|---|---|---|---|
| **y** | 0.91 | 1 | 1.1 | 1.21 |

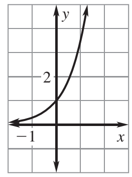



Domain: all real numbers

Range: all positive real numbers

**32.** $y = (3.5)^x$

| **x** | $-1$ | 0 | 1 | 2 |
|---|---|---|---|---|
| **y** | 0.29 | 1 | 3.5 | 12.25 |

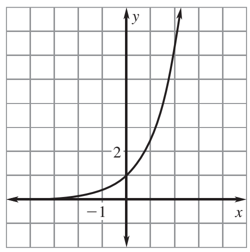



Domain: all real numbers

Range: all positive real numbers

**33.** $y = \left(\frac{5}{2}\right)^x$

| **x** | $-1$ | 0 | 1 | 2 |
|---|---|---|---|---|
| **y** | $\frac{2}{5}$ | 1 | $\frac{5}{2}$ | $\frac{25}{4}$ |

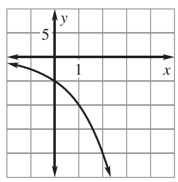



Domain: all real numbers

Range: all positive real numbers

**34.** $y = -5 \cdot 2^x$

| **x** | $-1$ | 0 | 1 | 2 |
|---|---|---|---|---|
| **y** | $-\frac{5}{2}$ | $-5$ | $-10$ | $-20$ |




The graph of $y = -5 \cdot 2^x$ is a vertical stretch with a reflection in the $x$-axis of the graph of $y = 2^x$.

**35.** The graph represents exponential growth. The $y$-intercept is 1, so $a = 1$.

$y = ab^x$

$4 = 1 \cdot b^1$

$4 = b$

A function rule is $y = 4^x$.

**36.** The graph represents exponential decay. The $y$-intercept is 3, so $a = 3$.

$y = ab^x$

$1 = 3b^1$

$\frac{1}{3} = b$

A function rule is $y = 3\left(\frac{1}{3}\right)^x$.

**37.** Let $V$ represent the value of the car (in dollars) and $t$ represent the number of years since the car was purchased.

$V = a(1 - r)^t = 13{,}000(1 - 0.15)^t = 13{,}000(0.85)^t$

Subsitute 4 for $t$.

$V = 13{,}000(0.85)^t = 13{,}000(0.85)^4 \approx 6786.08$

The approximate value of the car in 4 years is $6786.08.

### *Chapter Test for the chapter "Exponents and Exponential Functions"*

**1.** $(62 \cdot 17)^4 = 62^4 \cdot 17^4$

**2.** $(-3)(-3)^6 = (-3)^{1+6} = (-3)^7$

**3.** $\frac{8^4 \cdot 8^5}{8^3} = \frac{8^{4+5}}{8^3} = 8^{9-3} = 8^6$

**4.** $(8^4)^3 = 8^{4 \cdot 3} = 8^{12}$

**5.** $\frac{2^{15}}{2^8} = 2^{15-8} = 2^7$

**6.** $5^3 \cdot 5^0 \cdot 5^5 = 5^{3+0+5} = 5^8$

**7.** $[(-4)]^{3\,2} = (-4)^{3 \cdot 2} = (-4)^6$

**8.** $\frac{(-5)^{10}}{(-5)^3} = (-5)^{10-3} = (-5)^7$

**9.** $t^2 \cdot t^6 = t^{2+6} = t^8$

**10.** $\left(\frac{s}{t}\right)^6 = \frac{s^6}{t^6}$

**11.** $\frac{1}{9^{-2}} = 9^2 = 81$

**12.** $-(6p)^2 = -6^2p^2 = -36p^2$

**13.** $(5xy)^2 = 5^2x^2y^2 = 25x^2y^2$

**14.** $\frac{1}{z^7} \cdot z^9 = z^{9-7} = z^2$

**15.** $(x^5)^3 = x^{5 \cdot 3} = x^{15}$

**16.** $\left(\frac{-4}{c}\right)^2 = \frac{(-4)^2}{c^2} = \frac{16}{c^2}$

**17.** $\left(\frac{a^{-3}}{3b}\right)^4 = \frac{a^{-3 \cdot 4}}{(3b)^4} = \frac{a^{-12}}{3^4b^4} = \frac{1}{81a^{12}b^4}$

**18.** $\frac{3}{4d} \cdot \frac{(2d)^4}{c^3} = \frac{3}{4d} \cdot \frac{2^4d^4}{c^3} = \frac{3 \cdot 16d^4}{4dc^3} = \frac{12d^{4-1}}{c^3} = \frac{12d^3}{c^3}$

**19.** $y^0 \cdot (8x^6y^{-3})^{-2} = 1 \cdot 8^{-2}x^{6(-2)}y^{(-3)(-2)}$

$= \frac{1}{64} \cdot x^{-12}y^6$

$= \frac{y^6}{64x^{12}}$

**20.** $(5r^5)^3 \cdot r^{-2} = 5^3r^{5 \cdot 3}r^{-2}$

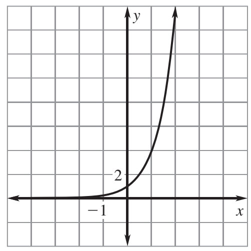
$= 125r^{15}r^{-2}$

$= 125r^{15-2}$

$= 125r^{13}$

**21.** $y = 4^x$

| x | −1 | 0 | 1 | 2 |
|---|---|---|---|---|
| y | $\frac{1}{4}$ | 1 | 4 | 16 |

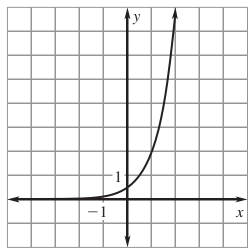



Domain: all real numbers

Range: all positive real numbers

**22.** $y = \frac{1}{2} \cdot 4^x$

| x | −1 | 0 | 1 | 2 |
|---|---|---|---|---|
| y | $\frac{1}{8}$ | $\frac{1}{2}$ | 2 | 8 |




The graph of $y = \frac{1}{2}. 4^x$ is a vertical shrink of the graph of $y = 4^x$.

**23.** $\frac{10^7 \text{ bytes}}{1 \text{ frame}} \cdot \frac{10^3 \text{ frames}}{1 \text{ min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}}$

$= 60 \times 10^7 \times 10^3$

$= 60 \times 10^{10}$

$= 6 \times 10^{11}$

There are about $6 \times 10^{11}$ bytes of data in 1 hour of an animated film.

**24. a.** Let $y$ represent the yearly salary and $t$ represent the number of years since accepting the job.

$y = 32{,}000(1 + 0.03)^t$

$y = 32{,}000(1.03)^t$

**b.** $y = 32{,}000(1.03)^t = 32{,}000(1.03)^5 = 37{,}096.77$

The employee's salary after 5 years is $37,096.77.

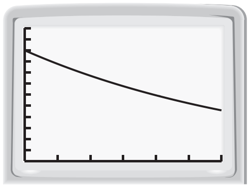
**25. a.** $P = (0.99987)^a$

Initial amount: 1

Decay factor: $1 - r = 0.99987$

Decay rate: $r = 0.00013$

**b.**



**c.** At $a = 5332$ meters, the atmospheric pressure is about 0.5 atmosphere.

## Extra Practice for the chapter "Exponents and Exponential Functions"

**1.** $5^3 \cdot 5^4 = 5^{3+4} = 5^7$

**2.** $6 \cdot 6^7 = 6^1 \cdot 6^7 = 6^{1+7} = 6^8$

**3.** $(-2)^3 \cdot (-2)^6 = (-2)^{3+6} = (-2)^9$

**4.** $(2^8)^2 = 2^{8 \cdot 2} = 2^{16}$

**5.** $[(-4)^3]^2 = (-4)^{3 \cdot 2} = (-4)^6$

**6.** $(8 \cdot 4)^5 = 8^5 \cdot 4^5 = (2^3)^5 \cdot (2^2)^5$

$= 2^{3 \cdot 5} \cdot 2^{2 \cdot 5} = 2^{15+10} = 2^{25}$

**7.** $m^5 \cdot m^2 = m^{5+2} = m^7$

**8.** $n^2 \cdot n^4 \cdot n^5 = n^{2+4+5} = n^{11}$

**9.** $(y^3)^5 = y^{3 \cdot 5} = y^{15}$

**10.** $(-2x)^3 = (-2)^3 \cdot x^3 = -8x^3$

**11.** $(3d^2)^3 \cdot 2d^2 = 3^3 \cdot (d^2)^3 \cdot 2d^2 = 27 \cdot d^6 \cdot 2d^2 = 54d^8$

**12.** $(-4s^2)^3(2s^3)^6 = (-4)^3 \cdot (s^2)^3 \cdot 2^6 \cdot (s^3)^6$

$= -64 \cdot s^6 \cdot 64 \cdot s^{18} = -4096s^{24}$

**13.** $\frac{8^7}{8^2} = 8^{7-2} = 8^5$

**14.** $\frac{4^6 \cdot 4^2}{4^3} = \frac{4^8}{4^3} = 4^{8-3} = 4^5$

**15.** $\left(-\frac{2}{3}\right)^5 = -\frac{2^5}{3^5}$

**16.** $10^{12} \cdot \frac{1}{10^7} = \frac{10^{12}}{10^7} = 10^{12-7} = 10^5$

**17.** $7^9 \cdot \left(\frac{1}{7}\right)^4 = 7^9 \cdot \frac{1^4}{7^4} = \frac{7^9}{7^4} = 7^{9-4} = 7^5$

**18.** $\frac{1}{t^9} \cdot t^{13} = \frac{t^{13}}{t^9} = t^{13-9} = t^4$

**19.** $\left(\frac{p}{8}\right)^7 = \frac{p^7}{8^7}$

**20.** $\left(\frac{6x^9}{3y^4}\right)^2 = \frac{(6x^9)^2}{(3y^4)^2} = \frac{6^2 \cdot (x^9)^2}{3^2 \cdot (y^4)^2} = \frac{36x^{18}}{9y^8} = \frac{4x^{18}}{y^8}$

**21.** $\left(\frac{4y^5}{3}\right)^3 \cdot \frac{1}{y^6} = \frac{(4y^5)^3}{3^3} \cdot \frac{1}{y^6} = \frac{4^3(y^5)^3}{27} \cdot \frac{1}{y^6}$

$= \frac{64y^{15}}{27} \cdot \frac{1}{y^6} = \frac{64y^{15}}{27y^6} = \frac{64y^9}{27}$

**22.** $\left(\frac{2}{u^2}\right)^3 \cdot \left(\frac{3u^4}{z^2}\right)^4 = \frac{2^3}{(u^2)^3} \cdot \frac{(3u^4)^4}{(z^2)^4} = \frac{8}{u^6} \cdot \frac{3^4(u^4)^4}{z^8}$

$= \frac{8}{u^6} \cdot \frac{81u^{16}}{z^8} = \frac{8.81u^{16}}{u^6z^8} = \frac{648u^{10}}{z^8}$

**23.** $\left(\frac{5x^3y^4}{2x^2y}\right)^2 = \frac{(5x^3y^4)^2}{(2x^2y)^2} = \frac{5^2(x^3)^2(y^4)^2}{2^2(x^2)^2y^2} = \frac{25x^6y^8}{4x^4y^2} = \frac{25x^2y^6}{4}$

**24.** $\frac{6a^4b^5}{ab} \cdot \left(\frac{2ab}{a^2b^2}\right)^3 = 6a^3b^4 \cdot \left(\frac{2}{ab}\right)^3 = 6a^3b^4 \cdot \frac{2^3}{(ab)^3}$

$= 6a^3b^4 \cdot \frac{8}{a^3b^3} = \frac{48a^3b^4}{a^3b^3} = 48b$

**25.** $3^{-4} = \frac{1}{3^4} = \frac{1}{81}$

**26.** $(-5)^{-3} = \frac{1}{(-5)^3} = -\frac{1}{125}$

**27.** $7^0 = 1$

**28.** $4^{-5} \cdot 4^3 = 4^{-5+3} = 4^{-2} = \frac{1}{4^2} = \frac{1}{16}$

**29.** $\left(\frac{1}{2}\right)^3 = \frac{1}{\left(\frac{1}{2}\right)^3} = \frac{1}{\frac{1}{8}} = 8$

**30.** $(3^{-2})^3 = 3^{-2 \cdot 3} = 3^{-6} = \frac{1}{3^6} = \frac{1}{729}$

**31.** $\frac{1}{2^{-5}} = 2^5 = 32$

**32.** $\frac{8^{-4}}{8^{-6}} = 8^{-4-(-6)} = 8^2 = 64$

**33.** $y^{-10} = \frac{1}{y^{10}}$

**34.** $(3c)^{-4} = \frac{1}{(3c)^4} = \frac{1}{3^4c^4} = \frac{1}{81c^4}$

**35.** $10b^{-3}c^5 = \frac{10c^5}{b^3}$

**36.** $(2d^5e^{-2})^{-3} = \frac{1}{(2d^5e^{-2})^3} = \frac{1}{2^3 \cdot (d^5)^3 \cdot (e^{-2})^3}$

$= \frac{1}{8d^{15}e^{-6}} = \frac{e^6}{8d^{15}}$

**37.** $\frac{x^{-4}}{y^{-5}} = \frac{y^5}{x^4}$

**38.** $\frac{1}{6t^{-5}u^3} = \frac{t^5}{6u^3}$

**39.** $\frac{3}{(-2z)^{-5}} = 3(-2z)^5 = 3 \cdot (-2)^5z^5$

$= 3 \cdot (-32) \cdot z^5 = -96z^5$

**40.** $\frac{(2e)^{-4}g^5}{e^5g^{-3}} = \frac{g^{5-(-3)}}{(2e)^4e^5} = \frac{g^8}{2^4 \cdot e^4 \cdot e^5} = \frac{g^8}{16e^9}$

**41.** $0.87 = 8.7 \times 10^{-1}$

**42.** $378.4 = 3.784 \times 10^2$

**43.** $0.000359 = 3.59 \times 10^{-4}$

**44.** $465{,}000{,}000 = 4.65 \times 10^8$

**45.** $5.3 \times 10^5 = 530{,}000$

**46.** $1.67 \times 10^{-4} = 0.000167$

**47.** $8\ 10^{-6} = 0.000008$

**48.** $9.0001 \times 10^2 = 900.01$

**49.** $\frac{3 \times 10^2}{8 \times 10^6} = \frac{3}{8} \times \frac{10^2}{10^6}$

$= 0.375 \times 10^{-4}$

$= (3.75 \times 10^{-1}) \times 10^{-4}$

$= 3.75 \times (10^{-1} \cdot 10^{-4})$

$= 3.75 \times 10^{-5}$

**50.** $(8.5 \times 10^{10})(3.7 \times 10^{-5}) = (8.5 \cdot 3.7) \times (10^{10} \cdot 10^{-5})$

$= 31.45 \times 10^5$

$= (3.145 \times 10^1) \times 10^5$

$= 3.145 \times (10^1 \cdot 10^5)$

$= 3.145 \times 10^6$

**51.** $\frac{2.4 \times 10^{-5}}{6 \times 10^{-8}} = \frac{2.4}{6} \times \frac{10^{-5}}{10^{-8}}$

$= 0.4 \times 10^3$

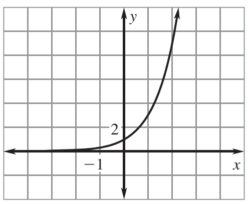
$= (4 \times 10^{-1}) \times 10^3$

$= 4 \times (10^{-1} \cdot 10^3)$

$= 4 \times 10^2$

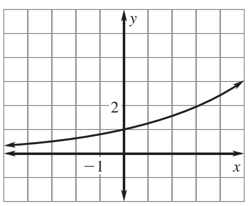
**52.** $y = 3^x$

| **x** | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| **y** | $\frac{1}{9}$ | $\frac{1}{3}$ | 1 | 3 | 9 |



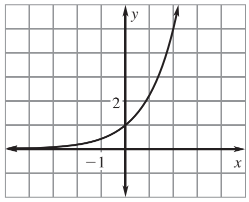

**53.** $y = 1.25^x$

| **x** | −8 | −4 | 0 | 4 | 8 |
|---|---|---|---|---|---|
| **y** | 0.17 | 0.41 | 1 | 2.44 | 5.96 |



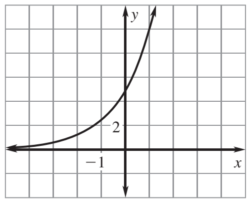

**54.** $y = \left(\frac{9}{4}\right)^x$

| **x** | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| **y** | 0.20 | 0.44 | 1 | 2.25 | 5.06 |



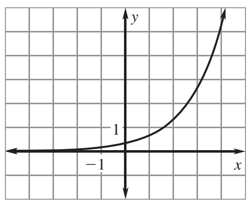

**55.** $y = 5 \cdot 2^x$

| **x** | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| **y** | 1.25 | 2.5 | 5 | 10 | 20 |



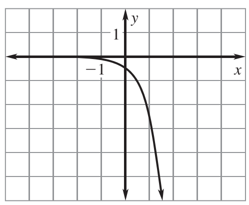

**56.** $y = \frac{1}{3} \cdot 2^x$

| **x** | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| **y** | 0.08 | 0.17 | 0.33 | 0.67 | 1.33 |



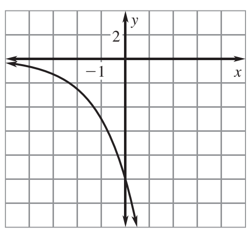

**57.** $y = -\frac{1}{2} \cdot 5^x$

| **x** | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| **y** | −0.02 | −0.1 | −0.5 | −2.5 | −12.5 |



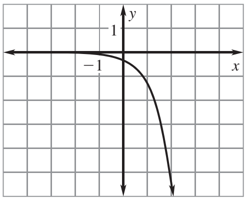

**58.** $y = -5 \cdot 2^x$

| **x** | 3 | −2 | −1 | 0 | 1 |
|---|---|---|---|---|---|
| **y** | −0.63 | −1.25 | −2.5 | −5 | −10 |



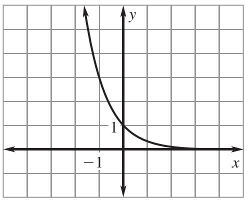
**59.** $y = -\frac{1}{3} \cdot 4^x$

| x | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| y | −0.02 | −0.08 | −0.33 | −1.33 | −5.33 |



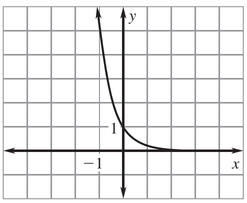

**60.** $y = \left(\frac{1}{3}\right)^x$

| x | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| y | 9 | 3 | 1 | 0.33 | 0.11 |



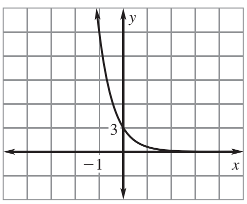

**61.** $y = (0.2)^x$

| x | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| y | 25 | 5 | 1 | 0.2 | 0.04 |



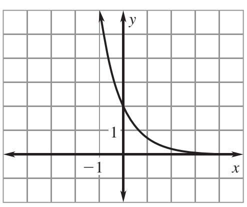

**62.** $y = 3 \cdot (0.2)^x$

| x | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| y | 75 | 15 | 3 | 0.6 | 0.12 |



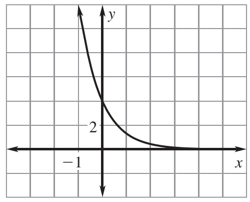

**63.** $y = 2 \cdot \left(\frac{1}{3}\right)^x$

| x | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| y | 18 | 6 | 2 | 0.67 | 0.22 |



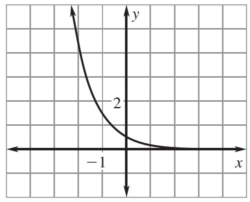

**64.** $y = 4 \cdot \left(\frac{1}{3}\right)^x$

| x | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| y | 36 | 12 | 4 | 1.33 | 0.44 |



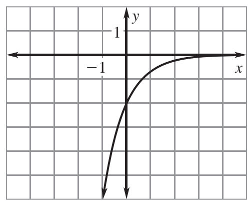

**65.** $y = \frac{1}{2} \cdot \left(\frac{1}{3}\right)^x$

| x | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| y | 4.5 | 1.5 | 0.5 | 0.17 | 0.06 |



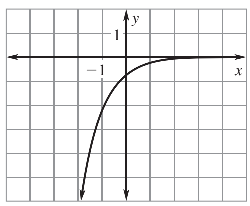

**66.** $y = -2 \cdot \left(\frac{1}{3}\right)^x$

| x | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| y | −18 | −6 | −2 | −0.66 | −0.22 |




**67.** $y = -\frac{3}{4} \cdot \left(\frac{1}{3}\right)^x$

| x | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| y | −6.75 | −2.25 | −0.75 | −0.25 | −0.08 |




**68.**

| x | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|
| y | $\frac{5}{2}$ | 5 | 10 | 20 | 40 |

×2 ×2 ×2 ×2

The $y$-value are multipled by 2 for each increase of 1 in $x$, so the table represents an exponential in the form $y = ab^x$ with $b = 2$.

The value when $x = 0$ is 5, so $a = 5$.

The table represents the exponential function

$y = 5 \cdot 2^x$.